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Influence of physical parameters on the collapse of a spherical bubble

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This paper examines the influence of physical parameters on the collapse dynamics of a spherical bubble filled with diatomic gas ($\kappa = 7/5$). The present numerical investigation shows that each physical parameter affects the bubble collapse dynamics differently. After comparing the contribution of each physical parameter, it appears that, of all the parameters, the surrounding liquid environment affects the bubble collapse dynamics the most. Meanwhile, surface tension has the weakest influence and can be ignored in the bubble collapse dynamics. However, surface tension must be retained in the initial analysis since this, as well as the pressure difference jointly control initial bubble formation. As an essential part of this study, a general Maple code is provided.

Keywords: Bubble collapse, Rayleigh's modelling, physical parameters, numerical simulation, Maple

I. INTRODUCTION

In liquid format, bubble cavitation is the formation of vapor cavities, which usually occur when a liquid is subjected to rapid changes of pressure, causing cavities to form and where the pressure is relatively low. When subjected to higher pressure, these cavities implode, and can generate intense shock waves[1–10]. Although collapsing bubbles generally do not remain spherical, nevertheless, it is often argued that the spherical analysis represents the maximum possible consequences of bubble collapse in terms of the pressure, temperature, noise, or damage potential[2]. For this understanding, the Rayleigh's modelling is used in this study, where Rayleigh's modelling assumes that the bubble remains spherical shown in Figure 1.

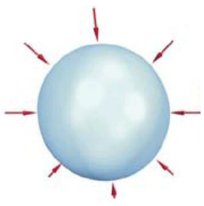

FIG. 1. Bubble collapse

Assuming that a bubble in oscillating surroundings is spherical and remains so as it collapses, the bubble radius is R_0 at time $t = 0$ and $R(t)$, subsequently. The liquid density around the bubble is ρ, the surface tension is σ, and the fluid viscosity is μ. In the surrounding liquid the oscillation amplitude and frequency are η and ω, respectively. With the assumption of polytropic behavior and with the neglect of thermal and acoustic dissipation, the Rayleigh-Plesset dynamical equation assumes the following form in an oscillating pressure field:

$$\rho R\frac{d^2R}{dt^2} + \rho\frac{3}{2}(\frac{dR}{dt})^2 = p_0(\frac{R_0}{R})^{3\kappa} - \frac{2\sigma}{R} - \frac{4\mu}{R}\frac{dR}{dt} - p_\infty(1+\eta\cos\omega t), \quad (1)$$

with the initial conditions being $R = R_0$ and $dR/dt = 0$ at $t = 0$.

If the pressure–volume relationship is taken to be adiabatic, then the polytropic exponent κ equals γ, which is the ratio of the specific heats of the gas, hence $\kappa = 5/3$ for a monatomic gas and $\kappa = 7/5$ for a diatomic gas.

Equation (1) is a nonlinear differential equation, as merely a few analytical solutions are obtained for the cases at $\kappa = 4/3$, when both the surface tensor σ and liquid viscosity μ are neglected[9]. However, if keeping both σ and μ, no analytic solutions have been obtained for bubbles filled with diatomic gas ($\kappa = 7/5$). In order to solve Eq.1, numerical method has to be applied. The references of numerical studies can be seen in the Refs[1–10].

If you check the literature[1–10] carefully, it is not difficulty to find that there are no comprehensive numerical solutions for the equation (1), and no published commutating programmes as well. Even for some cases, there are some drawings, however, the drawings are plotted in improper scaling which makes the comparison study impossible.

In order to overcome this situation, this study revisits the spherical bubble dynamics and writes a general Maple code, studies the influence of different physical parameters. A simple and general programm is highly demanded to allow the readers to compute their own problems, therefore, this paper provides a general user-friendly Maple code for bubble dynamics simulation.

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II. NUMERICAL INVESTIGATIONS OF RAYLEIGH COLLAPSE DYNAMICS OF GAS-FILLED BUBBLE

In this reserch paper, unless stated otherwise, all the simulations use the following data: $\kappa = 7/5$, $p_0 = 1$ bar $p_\infty = 1.01325$ bar $R_0 = 0.01$ cm, $\rho = 0.001$ kg/cm^3, $\mu = 0.001$ kg/cm s, $\sigma = 0.0725\ kg/s$, and $\omega_0 = \sqrt{\frac{3\kappa p_0}{\rho R_0^2}}$.

In the following sequence of figures, only one parameter is changed at a time while the rest remain unchanged.

For easy use of the results obtained from this research paper, and to simulate different modelling of bubble collapse dynamics, a general Maple code is provided to allow for an easy bubble collapse simulation.

```
with(DEtools):
with(plots):
Equ:=ρR(t)*diff(R(t),t,t)+(3/2)*ρ*diff(R(t),t)*
diff(R(t),t) = p0(R0/R(t))^(3κ) − p∞(1+η cos ωt) − 2σ/R(t) −
4μ/R(t) * diff(R(t),t):
Ins:= R(0) = R0, (D(R))(0) = 0:
Sol:=dsolve(`union`(Equ, Ins), numeric, method =
rkf45, relerr = 0.1e-12, abserr = 0.1e-12, maxfun =
0)
odeplot(Sol, [t, R(t)], t = 0 .. 0.9e-5, color = red)
display(Plot)
```

With the Maple code, we have done comprehensive numerical investigations for the bubble collapse dynamics. Fig. 2 below shows that the profile of the radius $R(t)$ is simulated numerically as $\kappa = 7/5$, $4/3$, and $3/2$. The results show that the polytropic exponent, κ, has little influence on the bubble collapse.

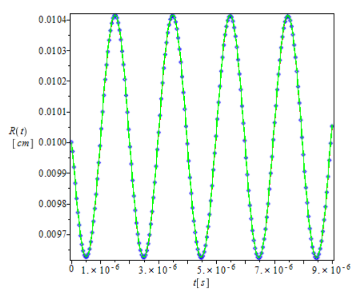

FIG. 2. Bubble radius versus time for $\kappa = 7/5$, $4/3$ and $3/2$

Fig. 3 illustrates that the profile of radius $R(t)$ is simulated numerically as $\sigma = 0$, 0.058, and 0.145. The results show that the surface tension, σ, also has little influence on the bubble collapse.

Fig. 4 below indicates that the profile of radius $R(t)$ is simulated numerically as $\mu = 0$, 0.8016, and 0.4008. The results show that the liquid viscosity, μ, has an obvious influence on the bubble collapse. In particular, the bubble collapse for $\mu = 0$ (shown by the red solid line) is

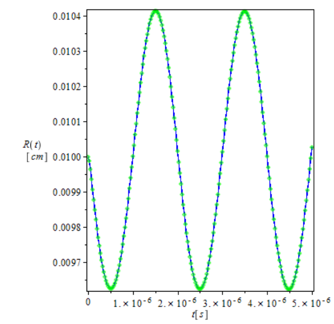

FIG. 3. Bubble radius versus time for $\sigma = 0$ (solid line), 0.058 (dashed line), and 0.145 (star line)

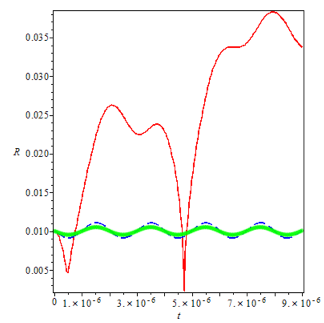

FIG. 4. Bubble radius versus time for $\mu = 0$ (solid line), 0.8016 (dashed line), and 0.4008 (star line)

dramatically different, and is much larger than the other two cases. The physics behind this phenomena is that zero viscosity offers little damping, which is unrealistic since a liquid always has a certain viscosity. This study reveals that bubble collapse dynamics must considers liquid viscosity, else the outcome cannot be trusted.

Fig. 5 below shows that the profile of radius $R(t)$ is simulated numerically as $\eta = 5$, 2.5, 1.25, and 0.625. The results show that the oscillation amplitude, η, of the surrounding liquid has a strong impact on the bubble collapse dynamics owing to the huge difference in magnitude.

Fig. 6 below illustrates that the profile of radius $R(t)$ is simulated numerically as $\omega = 1.54\omega_0[1/s]$, $1.54\omega_0/2[1/s]$, $1.54\omega_0/4[1/s]$, and $1.54\omega_0/8[1/s]$, where $\omega_0 = 2.05 \times 10^6[1/s]$. The results show that the oscillation frequency, ω, of the surrounding liquid also has a strong impact on the bubble collapse dynamics' magnitude. Together with the amplitude influence, it is concluded that the bubble collapse is sensitive to any change in the surrounding liquid environment.

Fig. 7 below projects that the profile of radius $R(t)$ is simulated numerically as $\rho = \rho_0$, $\rho_0/2$, and $\rho_0/4$, where $\rho_0 = 0.998 \times 10^{-3}$ kg/cm^3. The results show that the

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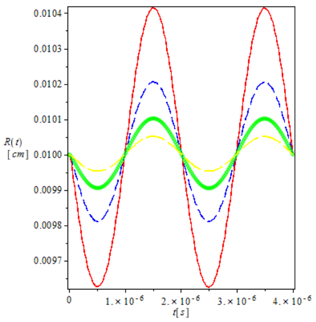

FIG. 5. Bubble radius versus time for $\eta = 5$ (solid line), 5/2 (dashed line), 5/4 (star line), and 5/8 (dot-dash line)

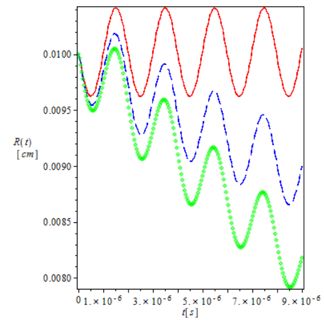

FIG. 8. Bubble radius versus time for $R_0 = 0.01$ (solid line), 0.008 (dashed line), and 0.005 cm (star line).

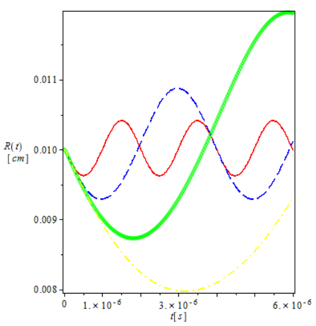

FIG. 6. Bubble radius versus time for $\omega = 1.54\omega_0[1/s]$ (solid line), $1.54\omega_0/2[1/s]$ (dashed line), $1.54\omega_0/4[1/s]$ (star line), and $1.54\omega_0/8[1/s]$ (dot-dash line), where $\omega_0 = 2.05 \times 10^6[1/s]$

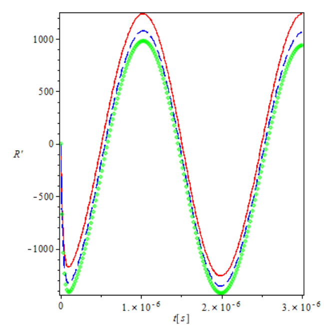

FIG. 9. Bubble velocity versus time for $R_0 = 0.01$ (solid line), 0.008 (dashed line), and 0.005 cm (star line).

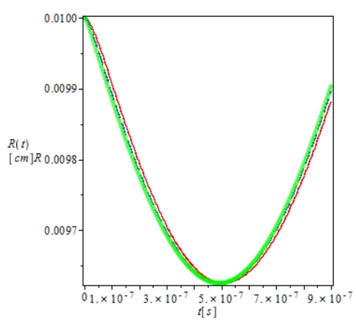

FIG. 7. Bubble radius versus time for $\rho = \rho_0$ (solid line), $\rho_0/2$ (dashed line), and $\rho_0/4$ (star line), where $\rho_0 = 0.998 \times 10^{-3}$ kg/cm^3.

liquid density, ρ, has little influence on the bubble dynamics.

Fig. 8 below demonstrates that the profile of radius $R(t)$ is simulated numerically as $R_0 = 0.01$, 0.008, and 0.005 cm. The results show that the initial radius, R_0, has a strong influence on the bubble dynamics as the smaller the R_0, the less energy in the bubble.

Fig. 9 below shows the collapse velocity profile is simulated numerically as $R_0 = 0.01$, 0.008, and 0.005 cm. The results show that the collapse velocity is affected by the initial bubble radius R_0: the greater the R_0, the faster the collapse.

Fig. 10 below indicates that the profile of radius $R(t)$ is simulated numerically for a surrounding liquid with zero surface tension σ and viscosity μ. The results clearly show that the bubble radius, without surface tension and viscosity, is much larger when these are included.

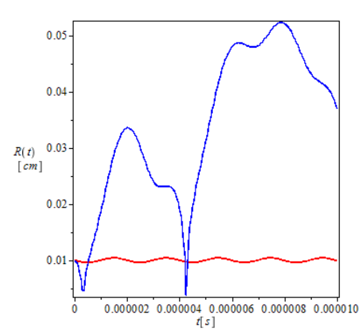

FIG. 10. Blue line is for bubble radius in liquid with no surface tension and viscosity, namely $\sigma = 0$ and $\mu = 0$, respectively.

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III. MAGNITUDE ANALYSIS OF EACH TERM IN RAYLEIGH-PLESSET EQUATION AND ROLE OF LIQUID SURFACE TENSION

The above numerical investigations show that surface tension has little influence on collapse dynamics. The reason for this different influence is that surface tension contributes much less than those terms containing p_0, p_∞, and μ. The numerical values of each term are plotted and compared, as shown in Fig. 11 below.

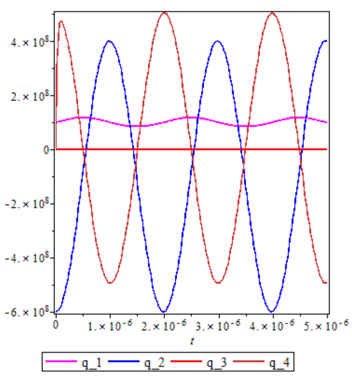

FIG. 11. $q_1 = p_0(\frac{R_0}{R})^{3\kappa}$, $q_2 = -p_\infty(1+\eta\cos\omega t)$, $q_3 = -\frac{2\sigma}{R}$, and $q_4 = -\frac{4\mu}{R}\frac{dR}{dt}$; the physical data are $\kappa = 7/5$, $p_0 = 1$ bar, $p_\infty = 0.999855$ bar, $\rho = 0.998\times10^{-3}$ kg/cm^3, $\mu = 1.0002$, $\sigma = 72.5\times10^{-3}$, $R_0 = 0.01$ cm, and $\omega_0 = \sqrt{\frac{3\kappa p_0}{\rho R_0^2}}$.

The numerical results show clearly that $q_3 \ll q_1$, q_2, and q_4, implying that liquid surface tension can be omitted when analyzing bubble collapse. Therefore, the mathematical model of bubble collapse can be simplified to the following: $\rho R\frac{d^2R}{dt^2} + \rho\frac{3}{2}(\frac{dR}{dt})^2 + \frac{4\mu}{R}\frac{dR}{dt} - p_0(\frac{R_0}{R})^{3\kappa} = -p_\infty(1+\eta\cos\omega t)$.

Although surface tension can be ignored in collapse dynamics, its contribution is essential in forming the initial bubble radius, namely $R_0 = 2\sigma/(p_0 - p_\infty)$, which implies that surface tension is an initial source of bubble formation, while the other is the pressure difference $p_0 - p_\infty$. Because we have $R(0) = R_0$ and $\dot{R}(0) = 0$ at $t = 0$, Eq. 1 gives the initial acceleration as $(\frac{d^2R}{dt^2})_{t=0} = \frac{p_0 - p_\infty(1+\eta)}{\rho R_0} = \frac{(p_0-p_\infty)[p_0-p_\infty(1+\eta)]}{2\rho\sigma}$ which reveals the joint work of the pressure difference, as well as the surface tension σ.

IV. CONCLUSIONS

Our comprehensive numerical investigations show that bubble collapse dynamics are influenced by the combined effects of surrounding liquid environment, surface tension, viscosity, as well as the initial bubble radius. However, each parameter affects the bubble collapse dynamics differently. Of all the parameters, the study shows that the surrounding liquid environment has the most influence. For a better idea of how the physical parameters influence bubble collapse, the strength of each physical parameter's influence is illustrated in the radar chart, below in Fig. 12.

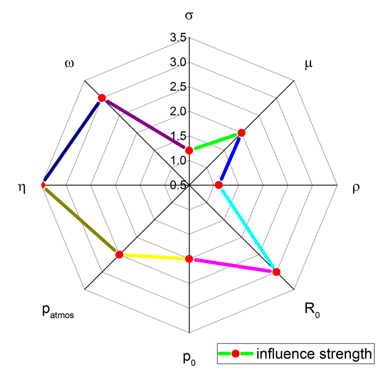

FIG. 12. Radar chart of influence strength of physical-parameter vector $(\sigma, \mu, \rho, R_0, p_0, p_\infty, \eta, \omega)$

Availability of data: There is no additional data available for this study.

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