"Dividing and conquering" and "caching" in molecular modeling

Xiaoyong Cao[†] and Pu Tian[*,†,‡]

†*School of Life Sciences, Jilin University, Changchun, China 130012*

‡*School of Artificial Intelligence, Jilin University, Changchun, China 130012*

E-mail: tianpu@jlu.edu.cn

Phone: +86 (0)431 85155287

Abstract

Molecular modeling is widely utilized in subjects including but not limited to physics, chemistry, biology, materials science and engineering. Impressive progress has been made in development of theories, algorithms and software packages. To divide and conquer, and to cache intermediate results have been long standing principles in development of algorithms. Not surprisingly, Most of important methodological advancements in more than half century of molecular modeling are various implementations of these two fundamental principles. In the mainstream classical computational molecular science, tremendous efforts have been invested on two lines of algorithm development. The first is coarse graining, which is to represent multiple basic particles in higher resolution modeling as a single larger and softer particle in lower resolution counterpart, with resulting force fields of partial transferability at the expense of some information loss. The second is enhanced sampling, which realizes "dividing and conquering" and/or "caching" in configurational space with focus either on reaction coordinates and collective variables as in metadynamics and related algorithms, or on the transition matrix

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and state discretization as in Markov state models. For this line of algorithms, spatial resolution is maintained but results are not transferable. Deep learning has been utilized to realize more efficient and accurate ways of "dividing and conquering" and "caching" along these two lines of algorithmic research. We proposed and demonstrated the local free energy landscape approach, a new framework for classical computational molecular science. This framework is based on a third class of algorithm that facilitates molecular modeling through partially transferable in resolution "caching" of distributions for local clusters of molecular degrees of freedom. Differences, connections and potential interactions among these three algorithmic directions are discussed, with the hope to stimulate development of more elegant, efficient and reliable formulations and algorithms for "dividing and conquering" and "caching" in complex molecular systems.

Introduction

Impact of molecular modeling in scientific research is clearly embodied by the number of publications. Results of a Web of Science (www.webofknowledge.com) search with various relevant key words is listed in Table 1. However, despite widespread applications, we remain far from accurately predicting and designing molecular systems in general. Further methodological development is highly desired to tap its full potential. Historically, molecular modeling has been approached from a physical or application point of view, and numerous excellent reviews are available in this regard.[1–16] From an algorithmic perspective, "dividing and conquering" (DC) and "caching" intermediate results that need to be computed repetitively are two fundamental principles in development of many important algorithms (e.g. dynamic programming[17]). As a matter of fact, the major focus of modern statistical machine learning is to learn ("caching" relevant information) and then carry out inference on top of which.[18] In this review, we provide a brief discussion of important methodological development in molecular modeling as specific applications of these two principles. The content will be organized as the following. Part II describes fundamental challenges in molecular

42 modeling; Part III summarizes application of these two fundamental algorithmic principles

43 in two lines of methodological research, coarse graining (CG)[18–27] and enhanced sampling

44 (ES);[28–31] Part IV covers how machine learning, particularly deep learning, facilitate DC

45 and "caching" in CG and ES,[30,32–36] part V introduces local free energy landscape (LFEL)

46 approach, a new framework for computational molecular science based on partially trans-

47 ferable in resolution "caching" of local sampling, and the first implementation of this new

48 framework by generalized solvation free energy (GSFE) theory[37] is briefly discussed; and part

49 VI discusses connections among these three lines of algorithmic development, their specific

50 advantages and prospective explorations. Due to the large body of literature and limited

51 space, we apologize to authors whose excellent work are not cited here.

Table 1: Number of publications from web of science search on Sep, the 8th, 2020

Key words	Number of publications
Molecular dynamics simulation	241,748
Monte Carlo simulation	189,550
QM-MM (quantum mechanical - molecular mechanical) simulation	9907
Dissipative particle dynamics simulation	3693
Langevin dynamics simulation	3893
Molecular modeling	2,072,091
All of the above	2,243,182

52 Challenges in molecular modeling

53 Accurate description of molecular interactions

Molecular interactions may be accurately described with high level molecular orbital theories
(e.g. coupled cluster theory[38,39]) or sophisticated density functionals combined with large
basis sets.[40–44] However, such quantum mechanically detailed computation is prohibitively
expensive for any realistic complex molecular systems. Molecular interactions are tradition-
ally represented by explicit functions and pairwise approximations as exemplified by typical

physics based atomistic molecular mechanical (MM) force fields (FF):[45–48]

$$
\begin{aligned}
U(\vec{R}) = &\sum_{bonds} K_b(b-b_0)^2 + \sum_{angles} K_\theta(\theta-\theta_0)^2 \\
&+ \sum_{dihedral} K_\chi(1+cos(n\chi-\delta)) \\
&+ \sum_{impropers} K_{imp}(\phi-\phi_0)^2 \\
&+ \sum_{nonbonded} \left(\epsilon_{ij} \left[\left(\frac{Rmin_{ij}}{r_{ij}}\right)^{12} - \left(\frac{Rmin_{ij}}{r_{ij}}\right)^{6} \right] \right) + \frac{q_i q_j}{\epsilon r_{ij}}
\end{aligned} \tag{1}
$$

54 or knowledge based potential functions:[49–51] These simple functions, while being amenable

55 to rapid computation and are physically sound grounded near local energy minima (e.g.

56 harmonic behavior of bonding, bending near equilibrium bond lengths and bend angles),

57 are problematic for anharmonic interactions, which are very common in many molecular

58 systems.[52] It is well understood that properly parameterized Lennard-Jones potentials are

59 accurate only near the bottom of its potential well. Frustration are ubiquitous in biomolec-

60 ular systems and are likely fundamental driving force for conformational fluctuations.[53–55]

61 One may imagine that a molecular system with all its comprising particles at their respective

62 "happy" energy minima positions would likely be a stable "dead" molecule, which may be a

63 good structural support but is likely not able to provide dynamic functional behavior.

64   Pairwise approximations are usually adopted for its computational convenience, both in

65 terms of dramatically reduced computational cost and tremendously smaller (when com-

66 pared with possible many body potentials) number of necessary parameters to be fit in

67 FF parameterization. It is widely acknowledged that construction of traditional FF (e.g.

68 equation 1) is a laborious process. Development of polarizable[9,56,57] and more complex FF

69 with larger parameter sets[58] alleviate some shortcomings of earlier counterpart. Expansion

70 based treatments were incorporated to address anharmonicity.[59] However, to tackle limi-

71 tation of explicit simple functional form and pairwise approximation for better description

72 of molecular interactions remain challenges to be met for molecular modeling community.

73 Additionally, even atomistic simulations are prohibitively expensive for large biomolecular

74 complexes at long time scales (e.g. milliseconds and beyond).[60–62]

Inherent low efficiency in sampling of configurational space

76 Complexity of molecular systems is rooted in their molecular interactions, which engenders

77 complex and non-linear correlations among molecular degrees of freedom (DOFs). Con-

78 sequently, effective number of DOFs are greatly reduced. Therefore, complex molecular

79 systems are confined to manifolds[63] of much lower dimensionality with near zero measure in

80 corresponding nominal high dimensional space (NHDS). Consequently, sufficient brute force

81 random sampling in NHDS of interested molecular systems is hopeless.

82   In stochastic trajectory generation by Monte Carlo (MC) simulations or candidate struc-

83 tural model proposal in protein structure prediction and refinement (or other similar scenar-

84 ios), new configuration proposal are carried out in NHDS, a lot of effort is inevitably wasted

85 due to sampling outside the actual manifold occupied by the target molecular system. Such

86 wasting may be avoided if we understood all correlations. However, understanding all cor-

87 relations implicates accurate description of global free energy landscape (FEL) and there

88 is no need to investigate it further! Due to preference of lower energy configurations by

89 typical importance sampling strategies (e.g. Metropolis MC), stochastic trajectories tend

90 to be trapped in local minima of FEL, this is especially true for complex molecular (e.g.

91 biomolecular) systems which have hierarchical rugged FEL with many local minima.[64,65] In

92 trajectory generation by molecular dynamics (MD) simulations, configurational space is ex-

93 plored by laws of classical mechanics and no wasting due to random moves exists. However,

94 molecular systems may well drift away from their true manifolds due to insufficient accuracy

95 of FF. Similar to stochastic trajectory generation, it takes long simulations to map FEL

96 since molecular system tend to staying at any local minimum, achieve equilibrium among

97 many local minima is just as challenging as in the case of stochastic counterpart.

DC and "caching" in traditional molecular modeling

To cope with fundamental difficulties in molecular modeling, two distinct lines of methodological development (CG and ES) based on DC and "caching" strategies have been conducted and tremendous progress has been made in understanding of molecular systems. As summarized below:

Coarse graining, a partially transferable "caching" strategy

Atomistic FF parameterization is the most well established coarse graining with a strong theoretical foundation, the Born-Oppenheimer approximation. Theoretically, MMFF are potential of mean force (PMF) obtained by averaging over many electronic DOFs for given atomic configurations. In practice, due to the fact that *ab initio* calculations are expensive and may have significant error when level of theory (and/or basis set) is not sufficient, reference data usually include results from both quantum mechanical (QM) calculations and well-established experimental data.[66–68] The DC strategy is utilized by selecting atomic clusters of various size to facilitate generation of QM reference data. The essential information learned from reference data is then permanently and approximately "cached" in FF parameters through the parameterization process. Due to the separation of time scales between electronic and atomic motion, elimination of electronic DOFs is straight forward but comes with the price of incapability in describing chemical reactions. To harvest benefits of both quantum and atomistic simulations, a well-established DC strategy is to treat a small region involved interested chemical reaction at QM detail and its surrounding with MMFF.[69–74] This series of pioneering work was awarded Nobel prize in 2013, and QM-MM treatment continues to be the mainstream methodology for computational description of chemical reactions.[75,76]

The united atom model (UAM) is the next step in coarse graining,[77] where hydrogen atoms are merged into bonded heavy atoms. This is quite intuitive since hydrogens have

123 much smaller mass on the one hand, and are difficult to see by experimental detection tech-

124 niques utilizing electron diffraction (e.g. X-ray crystallography) on the other hand. For

125 both polymeric and biomolecular systems, UAM remains to be expensive for many inter-

126 ested spatial and temporal scales. Therefore, further coarse graining in various forms have

127 been constructed. As a matter of fact, CG is usually used to denote modeling with particles

128 that representing multiple atoms in contrast to atomistic simulations, and the same con-

129 vention will be adopted in the remaining part of this review unless stated otherwise. Both

130 "Top-down" (that based on reproducing experimental data) and "bottom-up" (that based

131 on reproducing certain properties of atomistic simulations) approaches are utilized.[21,78] For

132 polymeric materials, beads are either utilized to represent monomers or defined on consider-

133 ation of persistent length,[79] and dissipative particle dynamics (DPD) were proposed to deal

134 with complexities arise from much larger particles.[80] For biomolecular systems, a wide vari-

135 ety of coarse grained models have been developed.[20,21,23,24,81,82] Another important subject of

136 CG methodology development is materials science.[83,84] Earlier definition of CG particles are

137 rather *ad hoc*.[20] More formulations with improved statistical mechanical rigor appeared later

138 on,[22] with radial distribution function based inversion,[79,85–87] entropy divergence[88] and force

139 matching algorithm[89–91] being outstanding examples of systematic development. Present

140 CG is essentially to realize the following mapping as disclosed by equation (4) in ref.[22] :

$$141 \qquad exp\left[-\beta V_{CG}(\mathbf{R}_{CG})\right] \equiv \int d\mathbf{r}\delta(M_R(\mathbf{r}) - \mathbf{R}_{CG})exp\left[-\beta V(\mathbf{r})\right] \qquad (2)$$

142 with $\mathbf{r}$ and $\mathbf{R}$ being coordinates in higher resolution and CG coordinates, $M_R(\mathbf{r})$ being the

143 map operator from $\mathbf{r}$ to $\mathbf{R}$, V and V_{CG} being potential energy of higher resolution and CG

144 representation respectively. Due to lack of time scale separation (see Fig. 1) for essentially all

145 CG mapping, strict realization of this equation/mapping is not rigorously possible. A naive

146 treatment of CG particles as basic units (with no internal degrees of freedom) would result

147 in wrong thermodynamics.[22] Due to corresponding significant loss of information, it is not

possible to develop a definition of CG and corresponding FF parameterization for compre-
hensive reproduction of atomistic description of corresponding molecular systems. Different
coarse graining have distinct advantages and disadvantages, so choosing proper CG strategy
is highly dependent upon specific goal in mind. CG particles are usually isotropic larger
and softer particles with pairwise interactions, or simple convex anisotropic object (e.g. soft
spheroids) that may be treated analytically.[24,92–94] Such simplifications provide both con-
venience of computation and certain deficiency for capturing physics of target molecular
systems. CG may be carried out iteratively to address increasingly larger spatial scales
by "caching" lower resolution CG distributions with ultra CG (UCG) FF.[22,95–99] Pairwise
approximation and explicit simple function form remain to be limitations of interaction de-
scription for traditional CG FF. When compared with atomistic FF, pairwise approximation
deteriorate further due to lack of time scale separation (Fig. 1).

Another simple and powerful type of CG model for biomolecular systems is Gō model[100,101]
and elastic network models (ENM)[102–104] or gaussian network models (GNM)[105,106] with na-
tive structure being defined as the equilibrium state, and with quadratic/harmonic interac-
tions between all residues within given cutoff. Only a few parameters (e.g. cutoff distance,
spring constant) are needed. Such models "caching" the experimental structures and are
proved to be useful in understanding major conformational transitions and slow dynamics
of many biomolecular systems.[107,108]

Enhanced sampling, a nontransferable in resolution DC and "caching" strategy

Umbrella sampling (US)[109] is probably the first combination of DC and "caching" strategy
for better sampling of molecular system along a given reaction coordinate (RC) (or order
parameter) s. DC strategy is first applied by dividing s into windows, information for each
window is then partially "cached" by corresponding bias potentials and local statistics. Later
on, adaptive US (AUS)[110,111] and weighted histogram analysis method (WHAM)[112] was de-

veloped to improve both efficiency and accuracy. MBAR[113,114] was developed to achieve error bound analysis which is lack in WHAM. Further development including adaptive bias force (ABF)[115–117] and metadynamics.[118–120] Details of these methodologies were well explained by excellent reviews.[121–124] The common trick to all of these algorithms (and their variants) is to "cache" visited configurational space with bias potentials/force and local statistics, thus dramatically accelerate sampling of interested rare events. Denote CV as $\mathbf{s}(\mathbf{r})$ ($\mathbf{r}$ being physical coordinates of atoms/particles in the target molecular system), equilibrium distribution and free energy on the CV may be expressed as:[30]

$$p_0(\mathbf{s}) = \int d\mathbf{r}\delta\left[\mathbf{s} - \mathbf{s}(\mathbf{r})\right] p_0(\mathbf{r}) = \langle\delta\left[\mathbf{s} - \mathbf{s}(\mathbf{r})\right]\rangle \tag{3}$$

$$p_0(\mathbf{r}) = \frac{e^{-\beta U(\mathbf{r})}}{\int dr e^{-\beta U(\mathbf{r})}} \tag{4}$$

$$F(\mathbf{s}) = -\frac{1}{\beta}log[p_0(\mathbf{s})] \tag{5}$$

$$F(\mathbf{s}) = -\frac{1}{\beta}log[p(\mathbf{s})] - V(\mathbf{s}) \tag{6}$$

169 with $p(\mathbf{s})$ being the sampled distribution in simulation with corresponding bias potential
170 $V(\mathbf{s})$ for "caching" of visited configurational space.

171   The starting point of these "caching" algorithms is specification of reaction coordinates
172 (RC) or collective variables (CVs), which is a very challenging task for complex molecular
173 systems in most cases. Traditionally, principle component analysis (PCA)[125] is the most
174 widely utilized and a robust way for disclosing DOFs associated with the largest variations.
175 To deal with ubiquitous nonlinear correlations, kernels are often used albeit with the difficulty
176 of choosing proper kernels.[126] Additional methodologies, include multidimensional scaling
177 (MDS),[127] isomap,[128] locally linear embedding (LLE),[129] diffusion map[130,131] and sketch
178 map[132] have been developed to map out manifold for high dimensional data. However, each
179 has it own limitations. For example, LLE[129] is sensitive to noise and therefore has difficulty
180 with molecular simulation trajectories which are quite noisy; Isomap[128] requires relatively

9

181   homogeneously sampled manifold to be accurate. Both LLE and Isomap do not provide
182   explicit mapping between molecular coordinates and CVs; Diffusion and sketch maps are
183   likely to be more suitable to analyze molecular simulation trajectories. Nonetheless, their
184   successful application for large and complex molecular systems remain to be tested. All of
185   above non-linear mapping algorithm are mainly suitable for manifold on a single scale. When
186   we are interested in finding paths for transitions among known metastable states, transition
187   path sampling (TPS)[133–135] methodology maybe utilized to establish CV.

188   Apparently, RC and/or CV based ES is a different path for facilitate simulation of com-
189   plex molecular systems on longer time scales from coarse graining. One apparent plus side
190   is that these algorithms are "in resolution" as no systematic discarding of molecular DOFs
191   occur. With specification of RC and/or CVs, computational resource is directed toward the
192   presumably most interesting dynamics of the target molecular system, and RC and/or CV
193   maybe repetitively refined to obtain mechanistic understanding of interested molecular pro-
194   cesses. However, the down side is that "cached" information on local configurational space is
195   not transferable to other similar molecular systems. While rigorous transferability may not
196   be easily established for any CG FF, practical utility of CG FF for molecular systems with
197   similar composition and thermodynamic conditions have been quite common and useful.[24]
198   Therefore, CG FF may be deemed as partially transferable.

199   An important recent development of DC strategy for enhanced sampling is Markov state
200   models (MSM),[136–139] one great advantage of which is that no RC or CV is needed. Instead,
201   it extracts long-time dynamics from independent short trajectories distributed in configu-
202   rational space. Many important biomolecular functional processes have been characterized
203   with this great technique.[140–142] The most fundamental assumption is that all states for a
204   target molecular system form an ergodic Markov chain:

$$\pi(t + \tau) = \pi(t)\mathbf{P} \tag{7}$$

206 with $\pi(t)$ and $\pi(t + \tau)$ being a vector of probabilities for all states at time t and $t + \tau$ re-
207 spectively. $\mathbf{P}$ is the transition matrix with its element P_{ij} being probability of the molecular
208 system being found in state j after an implied lag time (τ) from the previous state i. Ap-
209 parently as t goes to infinity for an equilibrium molecular system, a stationary distribution
210 π will arise as defined below:

211
$$\pi = \pi P \tag{8}$$

212 The advantage of not needing RC/CV does not come for free but with accompanying dif-
213 ficulties. Firstly, one has to distribute start point of trajectories to statistically important
214 and different part of configurational space, then select proper (usually hierarchical, with each
215 level of hierarchy corresponds to a specific lag time) partition of configurational space into
216 discrete states. This is the key step of DC strategy in MSM. No formal rule is available and
217 experience is important. In many cases some try and error is necessary. Secondly, within
218 each discrete state at a given level of hierarchy, equilibration is assumed to be achieved in-
219 stantly and this assumption causes systematic discretization error, which fortunately may
220 be controlled with proper partition and sufficiently long lag time.[143] Apparently, metastable
221 states obtained from MSM analysis is molecular system specific and thus not transferable.

222   Another important class of enhanced sampling is to facilitate sampling with non-Boltzmann
223 distributions and restore property at targeted thermodynamic condition through proper
224 reweight.[144] Most outstanding examples are Tsallis statistics,[145,146] parallel tempering,[147,148]
225 replica exchange molecular dynamics,[149,150] Landau-Wang algorithm[151] and integrated tem-
226 pering sampling (ITS).[152–154] These algorithms are not direct applications of DC and "caching"
227 strategies and are not discussed further here.

Machine learning improves "caching"

Toward *ab initio* accuracy of molecular simulation potentials

Fixed functional form and pairwise approximation of non-bonded interactions are two major factors limiting the accuracy of molecular interaction description in both atomistic and CG FF. Neural network (NN) has capability of approximate arbitrary functions and therefore has the potential to address these two issues. Not surprisingly, significant progress has been made in this regard as summarized by recent excellent reviews.[155–160] Cutoff and attention to local interactions remains the DC strategy for development of machine learning potentials. The major improvement over traditional FF is better "caching" that overcomes pairwise approximation and fixed functional form limitations. NN FF naturally tackle both issues as explicit functions are not necessary since NNs are universal approximators. The significance of many-body potentials[161] and extent of pairwise contributions were analyzed.[162,163] There are also efforts to search for proper simple functional forms, which are expected to be more accurate than present functional forms in traditional FF on the one hand, and alleviate overfitting/generalization difficulty and reduce computational cost of complex NN FF on the other hand,[164,165] especially when training dataset is small. While most machine learning FF are trained by energy data,[155,159] gradient-domain machine learning (GDML) approach[166] directly learns from forces and realizes great savings of data generation.

Just as in the case of traditional FF, transferability and accuracy is always a tradeoff. More transferability implicates less attention is paid to "cache" detailed differences among different molecular systems, hence less accuracy. Exploration in this regard, however, remains not as much as necessary.[167–169] Unlike manual fitting of traditional FF, systematic investigation of tradeoff strategy is potentially feasible for machine learning fitting,[170] and yet to be done for many interesting molecular systems. With expediency of NN training, development of a NN FF hierarchy with increasing transferability/accuracy and decreasing accuracy/transferability is likely to become a pleasing reality in the near future. Rapid fur-

254 ther development of machine learning potentials, particularly NN potentials, are expected.

255 However, significant challenges for NN potentials remain on better generalization capability,

256 description/treatment of long range interactions,[171,172] wide range of transferability,[173] faster

257 computation[174] and proper characterization of their error bounds. Should further significant

258 progress be made on these issues, it is promising we may have routine molecular simulations

259 with both classical efficiency and *ab initio* accuracy in the near future.

260 Machine learning and coarse graining

261 As in the case of constructing atomic level potentials, machine learning has been applied

262 to address two outstanding pending issues in coarse graining, which are definition of CG

263 sites/particles and parameterization of corresponding interactions between/among these

264 sites/particles. Traditional CG FF, suffers from both pairwise approximation and accu-

265 racy ceiling of simple fixed functional forms which are easy to fit. By using more complex

266 (but fixed functional form) potentials with a machine learning fitting process, Chan *et. al.*[175]

267 developed ML-BOP CG water model with great success. Deep neural network (DNN) was

268 utilized to facilitate parameterization of CG potentials when given radial distribution func-

269 tions (RDF) from atomistic simulations.[176] CGnet demonstrated great success with simple

270 model systems (alanine dipeptide).[177] DeePCG model was developed to overcome pair ap-

271 proximation and fixed functional form and demonstrated with water.[178] Using oxygen site

272 to represent water is rather intuitive. However, for more complex biomolecules such as pro-

273 teins, possibility for selection of CG site explodes. To improve over intuitive or manual

274 try and error definition of CG sites, a number of studies have been carried out[179–182] to

275 provide better and faster options for choosing CG sites. However, no consensus strategy

276 is available up to date and more investigations are desired. The fundamental difficulty is

277 that there is no sufficient time scale separation between explicit CG DOFs and discarded

278 implicit DOFs, regardless of specific selection scheme being utilized. Intuitively, one would

279 expect CG FF parameters to be dependent upon definition of CG sites/particles. In this re-

gard, auto-encoders were utilized to construct a generative framework that accomplishes CG representation and parameterization in a unified way.[34] The spirit of generative adversarial networks was utilized to facilitate CG construction and parameterization, particularly with virtual site representation.[183] It was found that description of off-target property by CG exhibit strong correlation with CG resolution, to which on-target property being much less sensitive.[184] Such observation suggests that adjust CG for specific target properties might be a better strategy than searching for a single best CG representation. Despite potentially more severe impact of pairwise approximation for CG FF than in atomistic FF, quantitative analysis in this regard remain to be done.

Machine learning in searching for RC/CVs and construction of MSM

To overcome difficulties of earlier nonlinear CV construction algorithms[128–130,132] and to reduce reliance on human experience, auto-encoders, which is well-established for trainable (non-linear) dimensionality reduction, are utilized in a few studies.[185–188] Chen and Ferguson[186] first utilized autoencoders to learn nonlinear CVs that are explicit and differentiable functions of molecular coordinates, thus enabling direct utility in molecular simulations for more effective exploration of configurational space. Further improvement[185] was achieved through circular network nodes and hierarchical network architectures to rank-order CVs. Wehmeyer and Noé[187] developed time-lagged auto-encoder to search for low dimensional embeddings that capture slow dynamics. Ribeiro *et. al.*[188] proposed the reweighted autoencoded variational Bayes to iteratively refine RC and demonstrated in computation of the binding free energy profile for a hydrophobic ligand-substrate system. Building a MSM for any specific molecular system requires tremendous experience and many steps in process are error prone. To overcome these pitfalls, VAMPnet that based on variational approach for Markov process was developed to realize the complete mapping steps from molecular trajectories to Markov states.[189]

The local free energy landscape approach

Both CG and ES methodologies facilitate molecular simulation by effectively reducing local sampling. In CG, it is realized through "caching" (integration) of distributions for faster/discarded DOFs with proper CG FF, and thus has the inevitable cost of losing resolution (information), accompanied by the desired attribute of (partial) transferability to various extent. ES reduces lingering time of molecular systems in local minima through "caching" visited local configurational space, which is usually defined by relevant DC strategies, with biasing potentials. When compared with CG, there is no resolution loss. However, "cached" manifold of configurational space is molecular process specific and thus not transferable at all. In molecular modeling community, these two lines of methodologies are developed quite independently. Nevertheless, one might want to ask why not have both advantages in one method, that is to reduce repetitive local sampling without loss of resolution and with "cached" results being partially transferable. The local free energy landscape (LFEL) approach[190] is proposed with this intention in mind. Historically, parameterization of FF by coarse graining has been the only viable framework due to two fundamental constraints. Firstly, in earlier days of molecular modeling, typical computers have memory space of megabytes or less, render it impossible to accommodate millions or more parameters needed to fit complex LFEL; Secondly, while both neural network and autodifferentiation were invented decades ago, the computational molecular science community did not master these techniques for fitting large number of parameters efficiently until recently. With these two constraints removed, possibility for alternative path arise to break monopoly of classical molecular modeling by FF parameterization via coarse graining. Specifically, one may carry out direct fitting of LFEL and all important information on local distributions of molecular DOFs obtained from expensive local sampling may be "cached". This is in strong contrast to coarse graining based parameterization, in which local distributions are substituted by averaging in relevant lower dimensional space projection (e.g. pairwise distances among CG sites). However, it is essential to assemble LFEL and construct FEL of the interested molec-

ular system, and this is the core of the LFEL approach. For a molecular system with N DOFs, this LFEL approach may be expressed as:

$$P(r_1, r_2, \cdots, r_N) = P(R_1, R_2, \cdots, R_M)(M \leq N) \tag{9}$$

$$R_i = (r_{i_1}, r_{i_2}, \cdots, r_{i_l}) \tag{10}$$

$$
\begin{aligned}
P(R_1, & R_2, \cdots, R_M) \\
&= \prod_{i=1}^{M} P(R_i) \frac{P(R_1, R_2, \cdots, R_M)}{\prod_{i=1}^{M} P(R_i)} \\
&\approx \prod_{i=1}^{M} P(R_i), \text{and sampling all } R\text{s with mediated GCF}
\end{aligned}
\tag{11} \tag{12}
$$

$$
\begin{aligned}
G &= -k_B T ln P(r_1, r_2, \cdots, r_N) \\
&= -k_B T ln P(R_1, R_2, \cdots, R_M) \\
&\approx -k_B T \sum_{i}^{M} ln P(R_i), \text{and sampling all } R\text{s with mediated GCF}
\end{aligned}
\tag{13}
$$

a N-DOF molecular system is reorganized into M overlapping regions (Equation 9), each region has some number of DOFs (Equation 10). The key step of LFEL approach is expressed in Equation 11, in which the first product term (addressed as "local term(s)" hereafter) treat M regions as if they were independent, and all correlations among different regions are incorporated by the fraction term, which is termed global correlation fraction (GCF) and is extremely difficult, if ever possible, to be calculated directly. However, GCF is a unnormalized probability distribution, when all molecular DOFs in local terms are (approximately) sampled according to GCF, then we do not need GCF explicitly anymore (Equations 12 and 13). GCF represents two types of global correlations. The first type is mediated correlations among different regions by the fact that they overlap, and relevant molecular DOFs in such

16

overlapping space shared by different regions should have exact same state for all concerning regions. The second type is direct global correlations among molecular DOFs in different regions caused by genuine long-range interactions (e.g. electrostatic interactions). Satisfying the first type with sampling is trivial, and ensuring all overlapping regions share the exact same state is sufficient (Equations 12 and 13). The second type of global correlations need more involved treatment. These equations are apparently of general utility for any multiple-variable (high-dimensional) problem. In the specific case of a complex molecular system, using one set of coordinates realizes the mediated contribution of GCF. The approximation in Equation 12 is made by ignoring the second type of global correlations. Free energy minimization of a molecular system in thermodynamic equilibrium may be treated as maximization of joint probability (eq. 13). For molecular systems (or biological systems) off equilibrium, the joint distribution remains our focus despite free energy is not well defined anymore. A schematic representation of the LFEL approach in contrast to FF framework is shown in (Fig.2). While we only demonstrated GSFE implementation of LFEL at residue level for protein structural refinement. LFEL approach may be utilized to "cache" local distributions at any spatial scales. Just as there are many methodological developments in the mainstream FF framework, there are certainly many possible ways to develop algorithms in the LFEL approach. We explored a first step toward this direction through a neural network implementation of the generalized solvation free energy (GSFE) theory.[37] In GSFE theory, each comprising unit in a complex molecular system is solvated by its neighboring units. Therefore, each unit is both a solute itself and a comprising solvent unit of its solvent units. Let $(x_i, y_i) = R_i$ denote a region i defined by a solute x_i and its solvent y_i, a molecular system of N units has N overlapping regions. Each local term may be further expanded:

$$P(R_i) = P(x_i, y_i)$$
$$= P(x_i|y_i)P(y_i) \tag{14}$$

337 Both terms may be learned from either experimental or computational datasets, as long

338 as they are sufficiently representative and reliable. The first term in Equation 14 is the

339 likelihood term when x_i is the given, it quantifies the extent of match between the solute x_i

340 and its solvent y_i. The second term is the local prior term, it quantifies the stability of the

341 solvent environment y_i. Computation of the prior term is more difficult than the likelihood

342 term, but certainly learnable when sufficient data is available. A local maximum likelihood

343 approximation of GSFE (LMLA-GSFE) is to simply ignore local prior terms.

344 A particular implementation of the LMLA-GSFE for protein structure refinement with

345 residues defined as comprising unit was conducted.[190] In this scheme, GSFE is integrated

346 with autodifferentiation and coordinate transformation to construct a computational graph

347 for free energy optimization. With fully trainable LFEL derived from backbone and C_β

348 atom coordinates of selected experimental protein structures, we achieved superb efficiency

349 and competitive accuracy when compared with state of the art atomistic protein refinement

350 refinement methodologies. With our newly developed pipeline, refinement of typical protein

351 structure decoys (within 300 amino acids) takes a few seconds on a single CPU core, in

352 contrast to a few hours by typical efficient sampling/minimization based algorithms (e.g.

353 FastRelax[191]) and thousands of hours for MD based refinement.[192] In the latest CASP14

354 refinement contest (predictioncenter.org/casp14/index.cgi), our method ranked the the first

355 for the 13 targets with start GDT-TS score larger than 60. We expect incorporation of

356 complete heavy atom information and local prior terms to further improve this method in

357 the future. GSFE theory in particular and the LFEL approach in general, are certainly

358 extendable to modeling of other soft matter molecular systems.

359 More on connections among CG, ES and LFEL approach

360 All of these algorithms have a common goal of accelerating computation of a joint distribution

361 for a given molecular system at some target resolution, albeit from distinct perspectives. The

362 fundamental underpinning is the fact that molecular correlations among its various DOFs

363 limit a molecular system to a manifold of significantly lower dimension. Both ES and CG in

364 the FF framework and the LFEL approach are distinct strategies to "cache" manifolds from

365 either configurational space (Fig. 3) or physical space perspective (Fig. 4). Commonality

366 and differences of these strategies are summarized in Table 2 and discussed below.

Table 2: Commonality and difference among three types of algorithms

Algorithm	Coarse graining	Enhanced sampling	LFEL approach
Resolution	Lower	In	In
Transferable?	Partial	No	Partial
Dividing space	Physical	Configurational	Physical
Free energy unit	Partially Specified	Specified	Arbitrary

367 Both MSM and RC/CV based ES are designed to first describe local parts of the ap-

368 proximate manifold in the configurational space formed by all molecular DOFs of the target

369 molecular system. Information for such local configurational space is partially "cached" ei-

370 ther as bias potentials or transition counts, which are further processed to map FEL and

371 dynamics of interested molecular processes. Computational process (or educated guess) for

372 establishment of RC/CVs is essentially "caching" results from sampling/guessing local parts

373 of the configurational as approximate relevant manifold(Fig. 3B). Subsequent sampling along

374 RC/CV is hoped to disclose our interested molecular processes (e.g. biomolecular confor-

375 mational transitions, substrate binding/release in catalysis). Involved molecular DOFs for

376 RC/CVs are not necessarily spatially adjacent on the one hand, and may be different for

377 different molecular processes of the same molecular system. Apparently, RC/CVs are molec-

378 ular process specific and not transferable, even among different molecular process of the same

379 molecular system. Nonetheless, the methodology for searching CVs may be applied to many

380 different molecular processes/systems.

381 In contrast, both CG in the FF framework and the LFEL approach are motivated to

382 "cache" relevant information on the complete configurational distribution for local clusters of

383 molecular DOFs. Such local clusters are building blocks for many similar molecular systems

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384 (e.g. AAs in protein molecular systems) and consequently have limited and approximate

385 transferability. In CG, strongly correlated local clusters of molecular DOFs are represented

386 as a single particle, complex many body correlations/interactions of CG particles within

387 selected cutoff distances are represented by simplified CG FF in a lower resolution and longer

388 range correlations/interactions are incorporated either through more coarser CG models or

389 by separate long-range interaction computation. In LMLA-GSFE implementation of LFEL,

390 all complex many body correlations within selected regions (i.e. each solute and its specific

391 solvent) are decomposed into two terms in Equation 14, local likelihoods and local priors

392 in the same resolution, with local priors and direct genuine long-range interactions simply

393 ignored, and LFEL being approximated by local likelihood terms. More and better ways for

394 implementing LFEL are expected in the future.

395   The first step of CG is to partition atoms/particles of high resolution representation into

396 highly correlated local clusters that will be represented by corresponding single CG parti-

397 cles, and moderately correlated regions define interaction cutoff for CG particles; The second

398 step is to select a site (usually one of the comprising high resolution particles) to represent

399 the corresponding highly correlated cluster; The third step is to select functional forms to

400 describe molecular interactions among newly defined CG particles, and parameters are op-

401 timized by selected loss functions (e.g. differences of average force in force matching[89,90])

402 based on sampling in the whole configurational space of molecular systems and hopefully

403 to be transferable to some extent. One may imagine that both best clustering and optimal

404 representation sites of clusters may vary with different functional forms used to describe CG

405 particle interactions and in different part of configurational space. Neural network based

406 CG potentials do not have limitation of fixed functional form and pairwise approximations.

407 However, the need to partition molecular systems into transferable clusters and to specify

408 representation site/particle remain. For all different forms of CG, the fundamental essence is

409 to "cache" many body potential of mean force (PMF) in simplified CG FF at a lower resolu-

410 tion. In contrast, LFEL approach is to first using a DC strategy to divide molecular systems

411 into local regions, then directly "cache" many body PMF (or LFEL) of such local regions

412 in the original resolution. The cached complex local multivariate distributions in NN are

413 subsequently utilized to construct FEL of target molecular system through dynamic puzzle

414 assembly based on sampling with GCF as expressed in Equations 12 and 13. In language

415 of statistical machine learning. Training of LFEL is the learning step, while construction of

416 global FEL is the inference step. The advantage of CG is a simpler resulting physical model,

417 but is inflexible due to fixed clustering and representation on the one hand, and lost resolu-

418 tion/information on the other hand. Properly implemented LFEL while has selected spatial

419 regions comprising many molecular DOFs, composition of such regions are fully dynamic.

420 For example, in GSFE implementation of LFEL, a region is defined by a solute unit and

421 all of its solvent units, and comprising units for the solvent is dynamically updated in each

422 iteration of free energy optimization. Additionally, no loss of resolution is involved for LFEL

423 approach. Hence all difficulties and uncertainties associated with molecular DOF partition,

424 CG site selection and time scale separation, all of which apparently limit transferability of

425 CG FF, disappear. Correspondingly, the extent of transferability of a LFEL model is in

426 principle at least no worse than CG FF. Differences of CG and specific implementation of

427 LFEL by GSFE theory is schematically illustrated in Fig. 4. The superior efficiency of LFEL

428 approach comes with a price. The assembled global FEL has arbitrary unit for two reasons.

429 Firstly, it is extremely difficult to obtain the partition function (normalization constant) for

430 local regions directly during the training/caching stage, therefore we effectively obtain the

431 LFEL up to an unknown constant. Secondly, for two different molecular systems, the number

432 of local regions are usually different and so is the corresponding normalization constant.

433 These three lines of algorithms may be combined to facilitate molecular modeling. For

434 example, one might first utilize deep learning based near quantum accuracy many body FF

435 to perform atomistic simulations for protein molecular systems, and then extracting local

436 distributions properly with some form of LFEL, which may potentially be utilized to analyze

437 protein molecular systems with near-quantum accuracy and at regular amino-acid based CG

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438 or even much faster speed! Similarly, one may extract and "cache" large body of information
439 from residue level CG simulations with proper LFEL implementation, which may be utilized
440 to achieve ultra CG (UCG) efficiency with residue resolution. Application of CV and MSM
441 based ES algorithm for CG models is straight forward. Combination of LFEL with CV or
442 MSM based ES is more subtle and yet to be investigated.

Conclusions and prospect

444 The application of "dividing and conquering" and "caching" principle in development of
445 molecular modeling algorithms is briefed. Historically, coarse graining and enhanced sam-
446 pling have been two independent lines of methodological development in the mainstream
447 FF framework. While they share the common goal of reducing local sampling, the formula-
448 tions are completely different with distinct (dis)advantages. Coarse graining obtains partial
449 transferable FF but loses resolution, enhanced sampling retains resolution but results are not
450 transferable. The LFEL approach suggests a third strategy to directly approximate global
451 joint distribution by superposition of LFEL, which may be learned from available dataset
452 of either experimental or computational origin. Through integration of coordinate transfor-
453 mation, autodifferentiation and neural network implementation of GSFE, our recent work
454 of protein structure refinement demonstrated that simultaneous realization of transferable
455 in-resolution "caching" of local sampling is not only feasible, but also highly efficient due
456 to replacement of local sampling by differentiation. It is hoped that this review stimulates
457 further development of better "dividing and conquering" strategies for complex molecular
458 systems through more elegant, efficient and accurate ways of "caching" potentially repetitive
459 computations in molecular modeling at various spatial and temporal scales. With diverse
460 molecular systems (e.g. nanomaterials, biomolecular systems), specialization of methodology
461 is essential to take advantage of distinct constraints and characteristics.

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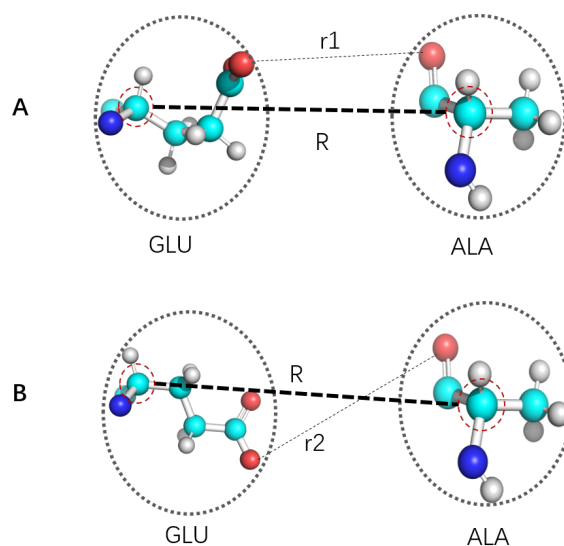
Figure 1: Schematic illustration of time scale separation issue in CG. A) and B) show two situations with C_α distances between two amino acids GLU and ALA being R, but with GLU have different conformations. If C_α atoms were defined as CG site, then these two relative conformation with distinct interactions would be treated as the same. In A) and B), CG site distance in both A) and B) are R, but many other pairs of atoms have distinct distances as exemplified by r_1 and r_2. Such treatment would only be true if for any small amount of displacement of C_α, side chains accomplished many rotations and thus may be accurately represented by averaging (i.e. with good time scale separation). This issue is apparently not limited to the specific definition of C_α being CG site, but rather general for essentially all CG development.
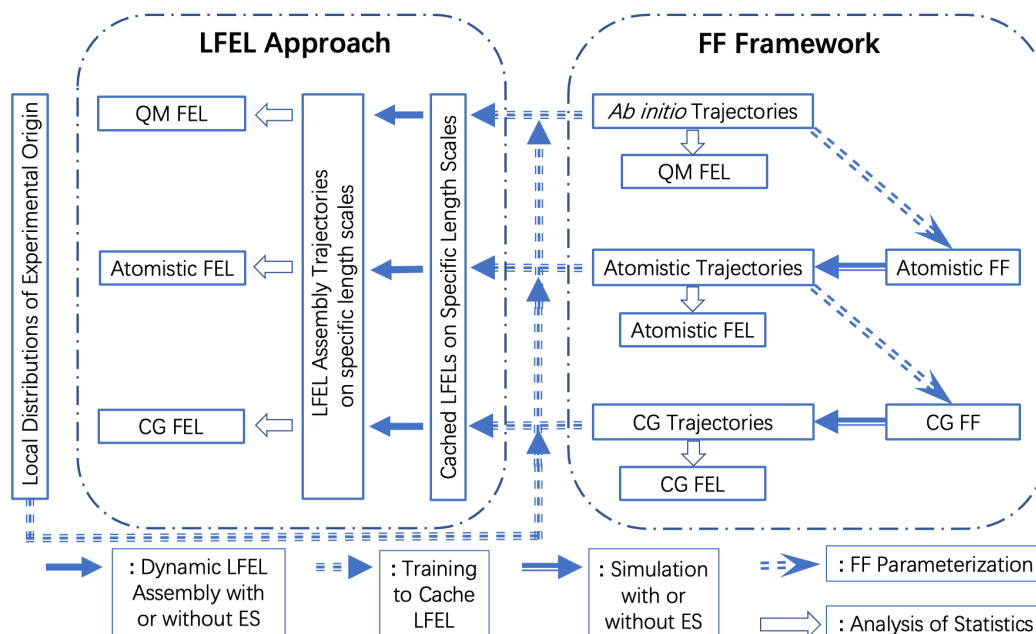
Figure 2: Schematic illustration of the LFEL approach in contrast to present mainstream FF framework. FF parameterization is the foundation for present classical computational molecular science. Training of neural network for "caching" LFEL is the foundation for LFEL approach, the source data can be either of experimental or computational origin. In FF framework, simulation (with or without ES) is driven by FF, in LFEL approach, propagation of molecular systems to minimize free energy (or maximize joint probability) is driven by compromise among many LFELs. Expensive repetitive local sampling in FF framework is substituted by differentiation w.r.t. LFELs.
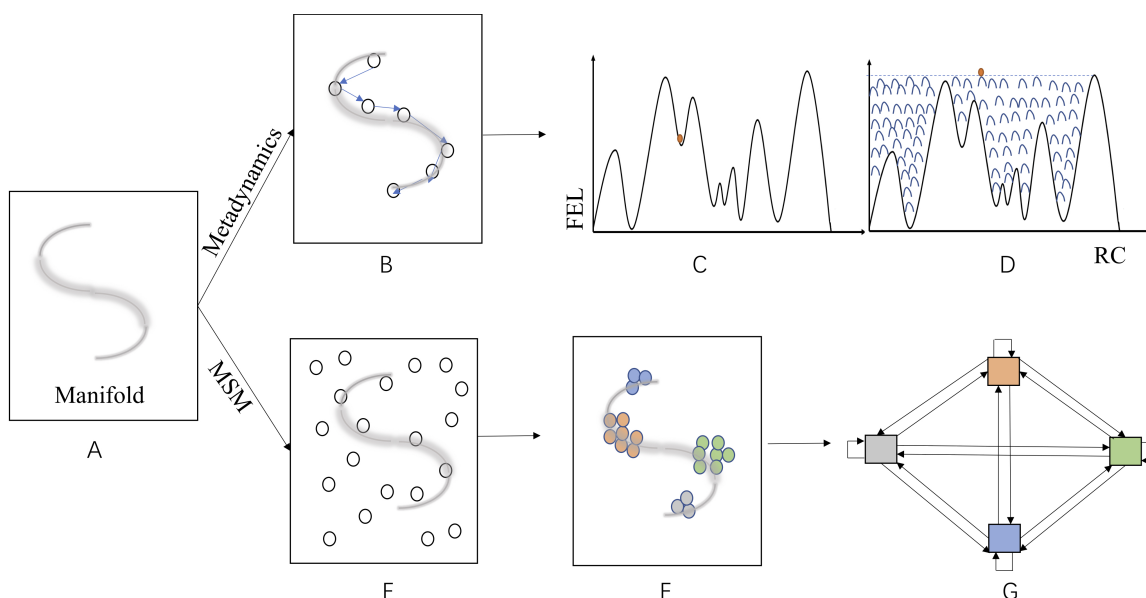
Figure 3: Schematic illustration of essential features for enhanced sampling by Metadynamics and MSM. A) The "S" shape grey line represents the unknown manifold in the configurational space (represented by the square) of a molecular system. B) Small circles connected by blue arrows represent computed (guessed) RC/CVs for the molecular system, which is utilized to conduct Metadynamics simulations. C) The FEL of the molecular system along the computed/selected RC/CV in B). D) "Caching" of the LFEL by bias potentials (gaussians represented by blue bell shaped lines) accumulated in the course of Metadynamics simulations. E) Distribution of the molecular system to the whole configurational space at the start of a MSM simulation, small circles represent initial start points for short MSM trajectories. F) Sampling results of short MSM trajectories fall mainly near the manifold, distinct "states" are represented by different colors. G) Establishment of transition matrix by transition counts between "states" obtained from short trajectories.
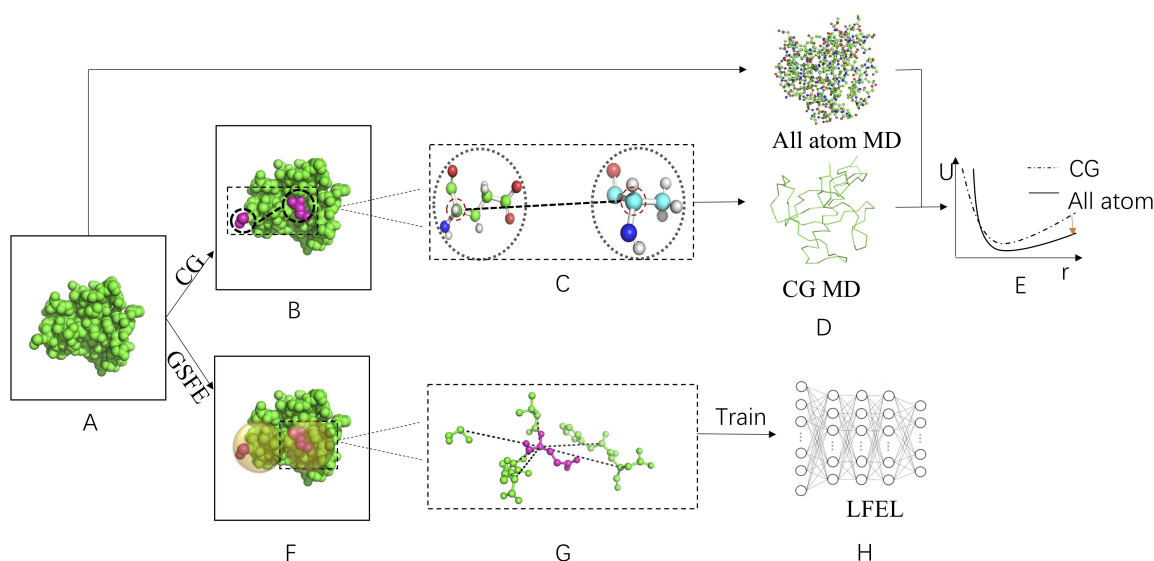
Figure 4: Schematic illustration of difference between CG and GSFE implementation of LFEL using protein as an example. A) Target molecular systems in physical space. Due to the goal of constructing partially transferable models and/or force fields, usually many different but similar molecular systems are considered. B) Selection of local atom/particle clusters to be represented as one particle in CG model. C) Selection of CG sites. D) Comparison between atomistic (or higher resolution) simulation results and CG (lower resolution) results. E) Adjust of CG FF parameter according to comparison from D). F)Definition of solvent region for each solute unit. G) Feature extraction for each solute. H) "Caching" of LFEL with neural network by training with prepared data sets.