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Cluster Flows and Multiagent Technology

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Abstract: Multiagent technologies give a new way to study and control complex systems. Local interactions between agents often lead to group synchronization also known as clusterization, which usually is a more rapid process in comparison with relatively slow changes in external environment. Usually, the goal of system control is defined by the behaviour of a system on long time intervals. When these time intervals are much longer than the time of cluster formation, clusters may be considered as new variables in a "slow" time model. We call such variables "mesoscopic" to emphasize their scale laying between the level of the whole system (macroscopic scale) and the level of individual agents (microscopic scale). Thus, it allows us to reduce significantly the dimensionality of a system by omitting considerations of each separated agent, so that we may hope to reduce the required amount of control inputs. Thus, we are often able to consider a system as a collection of "flowing" (morphing) clusters emerged form behaviour of a huge amount of individual agents. In this paper, we contrast such approach to the one where a system is considered as a network of elementary agents. We develop a mathematical framework for analysis of cluster flows in multiagent networks and use it to analyze the Kuramoto model as an attracting example of a complex networked system. In this model, a clusterization leads to sparse representation of dynamic trajectories in the whole quantized state space. With that in mind, compressive sensing allows to restore the trajectories in a high-dimensional discrete state space based on significantly lower amount of randomized integral mesoscopic observations. We propose a corresponding algorithm of quantized dynamic trajectory compression. It could allow us to efficiently transmit the state space data to a data center for further control synthesis. The theoretical results are illustrated for a simulated multiagent network with multiple clusters.

Keywords: cluster flows, mesoscopic observations, data compression

1. Introduction

It is common in a sociological (or physical) investigation to differ amid three primary societal (or system) levels: the micro-level, the meso-level, and the macro-level. In sociology, the analysis deals with a person in a social context or with a small people unit in a particular social setting. The consequences across a large population resulting in significant inner transmissions are a matter of the macro-level commonly associated with the so-named global level. In turn, a meso-level study is provided for groups lying in their size amid the micro and macro levels, and often explicitly invented to disclose relations between micro and macro levels. Following this general taxonomy, we can analogously consider the Information (Digital) Age (a famous appellation of the modern period) as the macro-level Industrial Revolution, expressed by quick transmission from the traditional industry

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to the massive application of information technology. Thus, humanity reconsiders the old-fashioned machinery aiming for new manufacturing solutions for new, regularly arising problems. Modern technologies lead to difficult supplementary tasks in the "global balancing" in humans' interactions and nature. Most people hardly comprehend their lifestyle undergo significantly modifying on the micro-level than the prior industrial standard of living. Modern discoveries in natural sciences, such as in high-energy physics or cosmology, are barely considered seriously by the majority. For instance, even though the quantum theory has proven itself essential in nanotechnology [1] [2] as well as particle accelerators in tumor treatment [3] [4], yet for many people, these fundamental achievements remain "abstract nonsense". Consecutively, the scientific community facing a severe problem cannot often deliver further strict theoretical evaluation regarding new previously unknown phenomena. It takes place due to the knowing crisis of modern scientific look at the "picture of the world." Aiming to handle this global difficulty, it seems reasonable to pay attention primarily to an appropriate theory based on a suitable mathematical model. Note that a real-world system often cannot be described sufficiently entirely with a partial model that raises a need to have one with the lowest potential bias. While discussing the model concept, it appears to be essential to mention such a notion as information. According to [5], data are inextricably presented as a reflection of a mutable object and can be described through other primaries notions such as "matter", "structure," or "system". Information can be stored and transferred with a material carrier, giving a form further perceived. It is also worth noting such the data concept is dedicated (from a system) to be supplementary exploited. Generally speaking, it is a complex of ideas and representations of patterns discovered by a person in the real world. Thus, one may visualize a typical personal information process as follows:

- Data gathering.
- Data studying.
- Data linkage to knowledge.

Thus a model is nothing else as a way of information processing to knowledge. It is important to note that the human "perception" is an uninterrupted procedure of data accumulating with an "integration" needed to attain the required awareness.

Now the question is whether we can decompose the information process further. In fact, psychology and cybernetics makes attempts to do so, however, the further it goes, the harder it is to track the relation between a complex cognitive system and its fundamental components.

1.1. A New Approach To Compression

We can uncover the common features and fundamental differences at the outset, comparing Data and Control Science with Physics. The central distinction between them is that Control Science studies systems with different complexity levels, commonly perceiving it as a scale. While physics deals in some extent with various systems in space and time, expressed in scales like "length". For example, the galaxy is immense in its extent in space, and the brain is immense in the complexity of its inner performance. Being co-existing measures, these gradations are indeed tools uniting Physics with Data and Control Science in comprehending the world around us. Following the conventional view of the history of the Universe, we can suggest that the initial chaos of elementary particles strives to arrange (clusterize) into the molecular connectivity studied by chemistry. This structure led later to the origin (clusterization?) of life as a highly organized form of matter, studied by biology. The described clustering process is a subject of Control Science, which seemly can connect the natural Sciences studying each one the scale of complexity of the organization of matter from its own standpoint.

It appears that "clustering" is closely related to such term "compression". With the aim to understand compression, it is helpful to mention that that a human usually is not capable of accumulation every detail in the personal sensory perception (registered by each cone and rod) but only essential details. Such details could be considered as clusterization of low-level data from retina into high-level features. Moreover, the general tendency is to forget even these points mainly due

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to the brain's limited capabilities. So, the data is being distorted or lost, which leads to a deterioration in the quality of information recovery. In general, as it was discussed in [6], our mind quite often compresses data using induction or deduction, in order to simplify information regarding the world.

This peculiarity of human perception is broadly exploited in computer systems. The famous Kotelnikov-Nyquist-Shannon sampling theorem [7] [8] dictates genaral rules of signal digitization, which allow to compress continuous signal flow into a discrete set of numbers without loss of information. However, this approach exhausts its limits over the years: it is exponentially harder to quantize signals when their dimensionality grows linearly, so that where a 1-D signal requires 10^3 samples to be preserved, a corresponding 2-D signal would require 10^6 samples, not to mention signals of higher dimensionality, which are being routinely transmitted today. Thus, compression algorithms are incredebly relevant nowadays, since they allow to speed up information exchange at fixed bandwith: e.g. MP3 [9] [10] allows to get rid of unnecessary high sound frequencies in music, while JPEG [11] [12] reduce picture file size at the cost of coarsening high frequency components responsible for small details. In this way, a kind of compression is believed to occur in the flow of visual information (if we follow the most popular hypothesis) among the brain's ventral and dorsal streams. However, such processes perform "on the fly" in the brain. On the other side, the computer can manipulate only preliminarily given data to be compressed.

Mathematically, a real-time compression can be understood as selecting essential parts of the surrounding big data containing the comprehensive information and then storing linear combinations of these parts (possibly with random weights). In order to implement this idea, such method as *compressive sensing* was proposed and discussed in [13] [14] [15]. In this paradigm, it is possible to compress a signal gradually, as it is read or perceived by a sensing system, according to (1). Assume that a signal $f \in \mathbb{R}^N$ is s-sparse in some domain $\Psi \in \mathbb{R}^{N\times N}$, namely $f = \Psi x$, where x has at most s ($s \ll N$) nonzero elements (thus being called an s-sparse vector). In our interpretation it means that in some basis the comprehensive information is stored in only s units of data out of N. Then compressive sensing can be described as:

$$y = \Phi\Psi x = Ax, \qquad (1)$$

where Φ is an $m \times N(m \ll N)$ sampling matrix, A is the so-called measurement matrix and $y \in \mathbb{R}^m$ is called the measurement vector or vector of compressed observations. Since $m \ll N$, the estimation of x by given y drives infinitely many solutions. However, in case x is s-sparse, it becomes possible to reconstruct it without much loss of useful data with only $m \sim s\log(N/s)$ variables required. According to [13–15], amid the solutions of (1) we prefer those maximizing the number of zero elements in x. While one would be interested in minimizing an ℓ_0 norm, as it provides the sparsest solution, this problem is NP-hard and a linear relaxation via the ℓ_1 norm provides a good compromise between sparsity and computations complexity [16] [13]. It is possible to compute ℓ_1-optimized solution in polynomial time using, for example, the interior-point method [17] [18]. Optimization with the ℓ_1 metric for the task of pattern recognition was first discussed in [19]. An optimal sparse ℓ_1-stabilizing controller for a non-minimum phase system (a solution for the task of ℓ_1-optimization in an infinite-dimensional space) was proposed in [20].

While design of Φ is in general complicated, some randomized choices of Φ are possible according to the restricted isometry property (RIP) and minimization of incoherence [5]. It was shown that only $m = 2cs(\log N)^2$ $c > 0$ measurements are sufficient to reconstruct x. In a special case when the elements a_{ij} of A are identically independently distributed with respect to the normal distribution

$$a_{ij} \sim \mathcal{N}\left(0, \frac{1}{m}\right),$$

then $m = c_1 s\log(N/s)$ measurements are sufficient. On practice, it is known that $m \approx 4s$ measurements are often enough for the initial sparse data to be effectively reconstructed.

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1.2. Multiagent Systems

Studied in Data and Control Science, the complexity levels in the matter's configuration lead to the emergence of the "complex system" notion. It is important to note that the complexity is not ascribed to the system comprehended as a static object but, to a greater extent, to the dynamic processes occurring within the system. They are the primary purpose of our research. A lack of computing power and limitations of the cognition methods encourage many severe difficulties in such research. In this regard, an approach intended to study a particular complex process resorts to various generalizations simplifying the considered task. For a long time, systems consisting of many almost identical parts with obscure chaotic interaction have been only statistically described. It appears to be impossible to describe the system's overall behavior using just macroscopic statistical characteristics of its composite elements. This approach can merely explain a minimal class of the perceived effects. While many complex systems properties may significantly disagree with relatively simple states of equilibrium or chaos, the statistical approaches deal with actually averaged or integrated subsystems. This circumstance can lead to the omission of important details if, for example, the system forms a pattern or structure of clusters.

The above-described manifestation of the whole system properties not inherent to its separate elements is called emergence [21] [22] [23]. Indeed, any non-comprehended characteristic of a structure can be considered emergent. For example, if a person cannot understand why a car moves, then moving is understood as an emergent behavior of the system-the car. However, this process is undoubtedly well apprehended from an engineering standpoint of the smooth operations in the engine, transmission, and control system. Thus, emergence may turn out to be a common subjective misunderstanding, and it is quite another matter if the system's behavior is a mystery beyond the capacity of any scientist. Say, consciousness is an emergent behavior of the brain from a naturalist's point of view. Up to date, there are no apparent means to explain the concept of consciousness itself resting upon just elements composing the brain. Moreover, even there is no way to check whether such a system is whatever cognizable with the help of somewhat arbitrarily powerful existing intelligence–or whether it is fundamentally, objectively unknowable. In the last case, the emergence is ostensive [22].

Assuming that the surrounded systems are still subjectively emergent, we can critically appraise the statistical methods applied to study complex systems. As an alternative to the accepted statistical methodology, the current article suggests an agent-based approach designed to overcome this practice's limitations. In the agent-oriented framework, a complex system (called a multiagent system) is considered as a network of simple autonomous units (called intelligent agents or simple agents), and the system of interacting agents [24] [25] [26]. The current study aims to develop a unified general theory connecting the macroscopic scale of the whole system and the individual components' microscopic scale. It aims to understand the reasons for "complexity" occurring in multiagent networks. An expected result of such an approach consists of constructing managing multiagent systems (MAS) without an in-depth investigation of the agents' profound nature.

The known belief–desire–intention model (BDI) of intelligent agents is a model of intelligent agents suggesting that each agent has a "convinced" achievable goal, including excitation of particular states, stabilization, or mutual synchronization. The latter of particular interest [27] [28] subject is frequently associated with the agents' self-organization in a network. The idea behind this phenomenon lies in the behavior of agents within a MAS. I.e., agents can regulate their internal states by exchanging messages aiming to form a system structure on a macroscopic scale. It is worth noting that a system can start from an arbitrary initial state, and not all agents are required to interact with others. These structures may include persistent global states (for example, task distribution in load balancing for a computer network [29]) or synchronized oscillations (Kuramoto networks [30]).

Since dynamic systems usually describe multiagent networks, the synchronization state signifies a convergence of dynamic trajectories to some unique synchronized one. This consequence is well studied in the case of linear systems [24] [31] [32]. However, the linear systems case corresponds to a particular instance of the nonlinear control rules, which have not yet been thoroughly studied.

A particular type of such procedures is cluster synchronization, essential in control tasks. The actual dynamics of complex, large-scale systems, such as groups of robots operating in a continuously changing environment, are often too complex to be controlled by conventional methods, including, for example, approximation by classical ODE models. However, sometimes some groups in a population synchronize in clusters. So, the same group agents are synchronized, while there is no coordination between agents from different clusters. Since these clusters can be deemed separate variables, it is possible to significantly reduce the control system inputs.

Thus, we can now introduce three levels of complex multiagent systems: microscopic (the level of individual agents), mesoscopic (the level of clusters), and macroscopic (the level of the system as a whole). The following equation can write down the described relation

$$N \gg m \gg 1, \tag{2}$$

where N is the number of agents and m is the number of clusters, which is supposed to be of the same order as the sparsity of the system.

In this paper, we formalize the framework of multiagent networks aiming to apply compressive sensing for efficient observations of dynamic trajectories. For this purpose, we propose an algorithm, on the first step of which the trajectories are quantized, then compressed.

1.3. The Kuramoto Model

A straightforward yet versatile nonlinear model of coupled oscillators is proposed by Y. Kuramoto [30] for their oscillatory dynamics. Given a network of N agents (by $\mathcal{N} = \{1, ..., N\}$ here and further we denote a set of N agents) each having one degree of freedom often called a phase of an oscillator, its dynamics is described by the following system of differential equations:

$$\dot{\theta}_i(t) = w_i + \sum_{j=1}^{N} K_{ij} \sin\left(\theta_j(t) - \theta_i(t)\right), \tag{3}$$

where $\theta_i(t)$ is a phase of an agent i, K_{ij} is a weighted adjacency matrix of the network and w_i is an own (natural) frequency of an oscillator. According to [33], [34] and [35], agents approach the state of frequency ($\dot{\theta}_i = \dot{\theta}_j \; \forall i, j \in \mathcal{N}$) or phase ($\theta_i = \theta_j \; \forall i, j \in \mathcal{N}$) synchronization under certain conditions on w_i and K_{ij}.

There are numerous extensions of this model, e.g. time-varying coupling constants (adjacency matrix $K_{ij}(t)$) and frequencies $w_i(t)$ [36]. It is also known that the Kuramoto model with phase delays in the sine function [37] [38] was studied. Besides, this model has extraordinary multiplex [39] or quantum [40] variations. In [41], cluster synchronization under specific conditions was discussed.

The model of Kuramoto appeared to be a successful tool for description of cortical activity in the human brain [39], being able to reveal three regimes of brain activity, corresponding to three states of the Kuramoto network: unsynchronized, highly synchronized and cluster synchronizatio. This model also found its application in robotics: in [42] authors discuss the task of pattern formation on a circle using rules inspired by the model of Kuramoto, while [43] proposes an idea of an artificial brain for robots made of Kuramoto oscillators. Thus, the simplicity and usefullness of this model caught our attention, so that we have chosen it to demonstrate how compressive sensing can be applied to multiagent clustering.

2. Cluster Flows

Intending to investigating the Kuramoto model, we discuss at the beginning general multiagent systems. Throughout, we consider non-isolated systems consisting of N agents, whose evolution is

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determined by their current state, their overall configuration, and the environment's external influence. The following system of differential equations characterizes the dynamics of the agents' interaction:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t), U_i(t), \eta_i(t)), \quad (4)$$

where t is unidirectional time, $x_i(t) \in \mathbb{R}^{n_i}$ is a state vector of an agent $i \in \mathcal{N}$; $u_i(t)$ is a microscopic control input describing local interactions between agents; $U_i(t)$ is another (macro- or mesoscopic) control input simultaneously affecting large groups of agents; $\eta_i(t) \in \mathbb{R}^{m_i}$ is an uncertain vector, which adds stochastic disturbances to the model. Despite $u_i(t)$ and $U_i(t)$ can be included in the state vector of a system, it is essential to separate them from the state vectors for two reasons: in the context of information models, it is necessary to distinguish both the local process of communication ($u_i(t)$) and external effects ($U_i(t)$) usually caused by a macroscopic agent with an actuator, allowing to influence the whole system; more than that, it allows to clarify the model itself. The state vector $x_i(t)$ may also have a continuous index, so that it becomes a field $x(t, a)$, where a is a continuous set of numbers. This could allow to reduce summation to integration in some cases.

As it widely accepted, the topology of an agents' network is presented by means of a directed interaction graph: $\mathcal{G}(t) = (\mathcal{N}, \mathcal{E}(t))$, where $\mathcal{N}$ is an agent vertex set, and $\mathcal{E}(t)$ is the set of directed arcs. Introduce $\mathcal{N}_i(t) \subseteq \mathcal{N}$ as a (time-dependent) neighborhood of an agent i consisting of a set of in-neighbors of this agent. We denote the in-degree of a vertex i ($i \in \mathcal{E}$) by $d(i) = \sum_{l=1}^{N} a_{il}$, where a_{ij} are the elements of an adjacency matrix A of $\mathcal{G}$ (the sum of the weights of the corresponding arcs). Likewise, the in-degree of a vertex i excluding j ($i, j \in \mathcal{E}$) is $d_j(i) = \sum_{l=1, l\neq j}^{N} a_{il}$. A strongly connected graph is said to be if there exists a "path" between all pairs of vertices.

Our next purpose is to introduce two agents outputs, one intending to their communication, and another evaluating the synchronization between agents. For this purpose, we associate with each agent i its output function g_i:

Definition 1. *A function $g_i(x_i(t), \eta_i(t))$ is called an output of an agent i if $g : \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \mapsto \mathbb{R}^l$, where l does not depend on i.*

Since the state vectors of agents may be of different dimensionality, we want to define two outputs: one for communication and one for measurement of synchronization between agents.

Let $y_j(t) = g_j(x_j(t), \eta_j(t))$ $j \in \mathcal{N}_i(t)$ be a communication output of an agent j from the neighborhood of i, used for communications. By "coupling" between agents, we assume that the dynamics of i (at time t) is affected by the outputs $y_j(t)$ of the agents j from the neighborhood. In practice, these outputs can be transmitted from j to i or shown by the agent j and then recognized by i. Mathematically, the transmission rules are usually outlined in u_i (see (4)):

$$u_i(t) = f_i\left(\{y_j(t)\}_{j \in \mathcal{N}_i(t)}\right), \quad (5)$$

where $f_i(\cdot)$ is a function of the outputs $y_j(t)$ $j \in \mathcal{N}_i(t)$. The equation (5) is also referred to as a *coupling protocol* in a sense it provides control rules for i based on all outputs j received by i. Thus, we present the following definition of a multiagent network:

Definition 2. *The triple consisting of 1) family of agents (see* (4)*); 2) interaction graph $\mathcal{G}$ and 3) coupling protocol defined as in equation* (5) *is called a multiagent network.*

Henceforth, we denote a multiagent network by the letter $\mathcal{N}$, corresponding to the set of agents of such network.

Let $z_i(t) = h_i(x_i(t), \eta_i(t))$ be an output of i, which is introduced for measurement of synchronization.

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Definition 3. *Let $\Delta_{ij}(t_*) = \|z_i(t_*) - z_j(t_*)\|$ stand for the deviation between outputs z_i and z_j at time t_*, where $\|\cdot\|$ is a corresponding norm. Then:*

1. *agents i and j are (output) synchronized, or reach (output) consensus at time t_* if $\Delta_{ij}(t_*) = 0$; similarly, agents i and j are asymptotically (output) synchronized if $\Delta_{ij}(\infty) = \overline{\lim}_{t_*\to\infty}\Delta_{ij}(t_*) = 0$;*
2. *agents i and j are (output) ε-synchronized, or reach (output) ε-consensus at time t_* if $\Delta_{ij}(t_*) \leq \varepsilon$; similarly, agents i and j are asymptotically (output) ε-synchronized if $\Delta_{ij}(\infty) = \overline{\lim}_{t_*\to\infty}\Delta_{ij}(t_*) \leq \varepsilon$;*

Summarizing the above, cluster synchronization of microscopic agents may lead to mesoscopic-scale patterns recognizable by a macroscopic sensor. However, there is no need to restrict those patterns to be static, since the most interesting cases appear when patterns evolve and change in time. Such alterations may be caused either by external impacts, or as a result of critical changes inside the system.

Definition 4. *A family of subsets $\mathcal{M}(t) = \left\{\mathcal{M}_\alpha(t) : \mathcal{M}_\alpha(t) \subseteq \mathcal{N}\ \forall t \geq 0\ \forall \alpha \in \overline{1, M(t)}\right\}_{\alpha=1}^{M(t)}$ of $\mathcal{N}$ is told to be a time-dependent partition over $\mathcal{N}$ (in this work, time-dependent partition is also called just "partition" for the sake of simplicity) at time t if the following conditions are respected:*

1. *$\nexists \mathcal{M}_\alpha(t) \in \mathcal{M}(t) : \mathcal{M}_\alpha(t) = \varnothing$;*
2. *$\bigcup_{\alpha=1}^{K(t)} \mathcal{M}_\alpha(t) = \mathcal{N}\ \forall t \geq 0$;*
3. *$\mathcal{M}_\alpha(t) \cap \mathcal{M}_\beta(t) = \varnothing\ \alpha \neq \beta$.*

With that in mind, we also propose an additional definition for a specific case of cluster synchronization.

Definition 5. *A multiagent network with a partition $\mathcal{M}(t_*)$ over $\mathcal{N}$ is (output) (ε, δ)-synchronized, or reach (output) (ε, δ)-consensus at time t_* for some $\delta \geq \varepsilon \geq 0$ if*

1. *$\Delta_{ij}(t_*) \leq \varepsilon\ i, j \in \mathcal{M}_\alpha(t_*)\ \mathcal{M}_\alpha(t_*) \in \mathcal{M}(t_*)$ and*
2. *$\Delta_{ij}(t_*) > \delta\ i \in \mathcal{M}_\alpha(t_*)\ j \in \mathcal{M}_\beta(t_*)\ \mathcal{M}_\alpha(t_*), \mathcal{M}_\beta(t_*) \in \mathcal{M}(t_*)\ \alpha \neq \beta$.*

A $(0, 0)$-synchronization is henceforth referred to as cluster synchronization. We also say that the $\mathcal{M}(t_*)$ is a clustering over $\mathcal{N}$.

Each cluster in a system with $M(t)$ clusters can also has a set $\overline{x}_\alpha$, $\alpha \in \overline{1, M(t)}$ of cluster integrals: such an approach is usually used for dimensionality reduction in physical models.

3. Compressive Sensing within the Kuramoto Model

The Kuramoto oscillator system with mesoscopic control function has been studied in paper [44] resting upon the canonical representation (4) and the proposed earlier definition of a multiagent network. As was demonstrated, such a system can undertake cluster synchronization under certain conditions (see, Theorem 2). An appropriate simulation was also provided to illustrate this circumstance. We exploit these results in the subsequent consideration of compressive sensing.

3.1. Clusterization and Mesoscopic Control of the Kuramoto model

Following the definition of a multiagent network, we introduce the connectivity graph between oscillator agent $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ and suggest for simplicity that this graph is time-independent. The corresponding adjacency matrix is denoted by Y, an element Y_{ij} of this matrix taking just values 0 and 1. The value 0 may be interpreted as "the agent j can not communicate to i ($j \notin \mathcal{N}_i$)", while the value 1 means "signals from the agent j reach i ($j \in \mathcal{N}_i$)". The communication between agents is implemented by transmitting their outputs $y_i(t)$ (see (5)) coinciding in this situation with the phase $\theta_i(t)$.

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Let $\mathcal{M}(t_1)$ be a clustering of the system of oscillators at time t and let this clustering does not change over the interval $T = [t_1, +\infty)$, and $t \in T$. We introduce the following local and mesoscopic control functions:

$$u_i(t) = w_i + \rho \sum_{j \in \mathcal{N}_i} \Upsilon_{ij} \sin\left(\theta_j(t) - \theta_i(t)\right), \qquad U_i(t) = \mu_i \mathcal{F}_\alpha(t, \overline{x}_\alpha), \tag{6}$$

were $\rho > 0$ is a constant, w_i – the own (natural) oscillator frequency $\mathcal{F}_\alpha(\cdot)$ – a mesoscopic function taking the same value for all elements in each cluster $\mathcal{M}_\alpha$, μ_i is the sensitivity of agent i to the control function $\mathcal{F}_\alpha(\cdot)$. An additional argument $\mathcal{F}_\alpha(\cdot)$ is $\overline{x}_\alpha(t)$ containing integral characteristics of a cluster α. It could be, for example, the position of the cluster centroid. Note that in systems with a large number of agents in a cluster, its integral characteristics are weakly dependent on the state of individual agents.

Looking at the classical Kuramoto model (3), we can see that its right side is similar to the expression of a protocol in (6). In fact, we consider the right-hand side of (3) entirely as a protocol corresponding to various adjacency matrix. For example, in the classical model (3), it is a weighted adjacency matrix K containing arbitrary real numerical values. We simplify the model by considering a binary adjacency matrix Υ multiplied by ρ. Thus, the Kuramoto model in the canonical multiagent representation with the addition of the meso-control function ($i \in \mathcal{M}_\alpha$) is:

$$\dot{\theta}_i(t) = \mu_i \mathcal{F}_\alpha(t, \overline{x}_\alpha(t)) + w_i + \rho \sum_{j \in \mathcal{N}_i(t)} \Upsilon_{ij} \sin\left(\theta_j(t) - \theta_i(t)\right). \tag{7}$$

Assume that the system has established a state of a cluster (ε, δ) synchronization. However, dynamics of some agents in the cluster $\mathcal{M}_\alpha$ under the control of $\mathcal{F}(\cdot)$ can become "destructive" for the whole cluster in case the values of their sensitivity highly differ from those among the rest. Besides, a significant difference in the values of w_i can also lead to undesirable effects, up to the chaotic behavior of the system. The following Theorem, which is also presented in [44], formalizes such scenarios by providing conditions for the model's parameters sufficient for the $(0, 0)$ cluster synchronization to remain.

Theorem 1. *Consider a multiagent network corresponding to (7). Let $t \in T$, output $z_i(t) = \dot{\theta}_i(t)$ and $\Delta_{ij}(t) = \left|z_i(t) - z_j(t)\right|$. Let also $\mathcal{F}_\alpha$ does not depend on θ_i $\forall i$. The following conditions are sufficient for this network to be output $(0, 0)$-synchronized.*

1. *In case $i, j \in \mathcal{M}_\alpha$,*

$$\left|w_i - w_j + (\mu_i - \mu_j)\mathcal{F}_\alpha\right| \le \rho C_{ij} \left(\sum_{l \in \mathcal{N}_i(t)} \Upsilon_{il} + \sum_{l \in \mathcal{N}_j(t)} \Upsilon_{jl} \right), \tag{8}$$

where $C_{ij} = 1$ in case $\Upsilon_{ij} = \Upsilon_{ji} = 0$; otherwise,

$$C_{ij} = \max\left\{ \sqrt{1 - (\Gamma_i(j))^2}, \sqrt{1 - (\Gamma_j(i))^2}, \frac{\sqrt{2}}{2} \right\}, \tag{9}$$

where

$$\Gamma_i(j) = \frac{-d_i(j) + \sqrt{(d_i(j))^2 + 8(\Upsilon_{ij} + \Upsilon_{ji})^2}}{4(\Upsilon_{ij} + \Upsilon_{ji})}. \tag{10}$$

2. *For $i \in \mathcal{M}_\alpha$, $j \in \mathcal{M}_\beta$, $\alpha \neq \beta$*

$$|w_i - w_j + \mu_i \mathcal{F}_\alpha(t, \overline{x}_\alpha) - \mu_j \mathcal{F}_\beta(t, \overline{x}_\beta)| > 0. \tag{11}$$

280 3. *Graph $\mathcal{G}$ is strongly connected.*

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At the next step, we consider simulations of the dynamic trajectories together with the simulations of their mesoscopic observations based on compression sensing.

3.2. Algorithm of Compressive Sensing Application for Mesoscale Observations

Aiming to model observations of the multiagent system by a data center, we similarly quantize its dynamic trajectories as it is done in the classical signal processing theory. With that being said, we heuristically bound and discretize the state space of the multiagent network of oscillators. Since the desired one-dimensional outputs are $\{\dot{\theta}_i(t)\}_{i\in\mathcal{N}}$, the corresponding dynamical trajectories lay on a coordinate plane with the horizontal axis being t and the vertical one being $\dot{\theta}$. We call the half-plane $t > 0$ the whole state space, and denote it as $\mathfrak{S}$. Next, we wish to extract the bounded region of $\mathfrak{S}$ with corresponding infimum and supremum along $\dot{\theta}$ denoted as $\dot{\Theta}_{\min}$ and $\dot{\Theta}_{\max}$; at the same time, corresponding infimum and supremum along t are $t_{\min}$ and $t_{\max}$. Such a region is denoted $\mathfrak{R}$ and is formally defined as $\mathfrak{R} = \left[\dot{\Theta}_{\min}, \dot{\Theta}_{\max}\right) \times [t_{\min}, t_{\max}) \subset \mathfrak{S}$. Thus, for each $t \in T = [t_{\min}, t_{\max})$ a point of a trajectory $\dot{\theta}_i(t)$ belongs to $\left[\dot{\Theta}_{\min}, \dot{\Theta}_{\max}\right)$. The values of the infima and the suprema are such that all the trajectories $\{\dot{\theta}_i(t)\}_{i\in\mathcal{N}}$ are contained within $\mathfrak{B}$ and are visually resolvable on a plot. We choose not to include the suprema in $\mathfrak{R}$ for the sake of simplicity of the further notion.

In our case discretization of $\mathfrak{R}$ implies its splitting into cells (sampling):

$$\begin{aligned} \left[\dot{\Theta}_{\min}, \dot{\Theta}_{\max}\right) &= \left[\dot{\Theta}_{\min}, \dot{\Theta}_1\right) \cup ... \cup \left[\dot{\Theta}_{p-1}, \dot{\Theta}_{\max}\right), \\ [t_{\min}, t_{\max}) &= [t_{\min}, t_1) \cup ... \cup [t_{q-1}, t_{\max}), \end{aligned} \tag{12}$$

where the values p and q are determined by sampling steps $\epsilon = \dot{\Theta}_i - \dot{\Theta}_{i-1}$ and $\tau = t_j - t_{j-1}$. In turn, the values of ϵ and τ depend on the form of trajectories $\{\dot{\theta}_i(t)\}_{i\in\mathcal{N}}$ and are chosen empirically. For example, the time τ should be much smaller than that during which a dynamical process of interest occurs. Such processes may include intra-cluster disturbances or inter-cluster flows of agents (cluster flows). Concerning ϵ, its value should be such that trajectories of agents from the same cluster lay in a single cell or, at least, in closely located ones. As for the agents from different clusters, their trajectories should be separated by a significant number of cells. Finally, we introduce matrix $\mathfrak{B} \in \{0,1\}^{p\times q}$ with an element $b_{i,j}$ being 1 in case at least a single trajectory lays in the cell $\left[\dot{\Theta}_{i-1}, \dot{\Theta}_i\right) \times [t_{j-1}, t_j)$, otherwise this element equals 0.

Thus, the columns of $\mathfrak{B}$ are ready to be compressed according to (1). It is clear that by choosing lower values of ϵ and τ the dimensionality of $\mathfrak{B}$ increases, leading to an increase of resolution and "readability" of the corresponding discrete trajectory portrait. What is more important, the sparsity of $\mathfrak{B}$ would also increase, which could allow to apply compression more effectively, while preserving decent amount of details.

4. Simulations

Consider model (7) and its solutions on $T = [0, 60]$. Let $N = 16$, and topology of the graph $\mathcal{G}$ be as on Figure 1. Let also $\rho = 0.5$ and natural frequencies $\{w_i\}_{i\in\mathcal{N}}$ be as follows: $w_1, ..., w_4 = \{4.1, 4.2, 4.3, 4.4\}$, $w_5, ..., w_8 = \{8.1, ..., 8.4\}$, $w_9, ..., w_{12} = \{12.1, ..., 12.4\}$, $w_{13}, ..., w_{16} = \{16.1, ..., 16.4\}$, so that agents from one "square" (see Figure 1) satisfy the condition of equation (8), however, agents from different squares does not. We obtain initial phases $\theta_i(0)$ from uniform distribution on S^1. Assuming that the synchronization output z_i coincides with $\dot{\theta}_i$, in abscence of any mesoscopic control ($U_i = 0$) such a configuration leads to a clustering $\mathcal{M}$ with four (ε, δ)-synchronized clusters for some ε and δ such that $\varepsilon \ll \delta$, as it was shown in [44]. However, in this work we focus on non-trivial "flowing" cluster patterns, which we obtain by adding a sinusoidal control function $U_i(t)$ being "turned on" at $t = 20$ (when (ε, δ)-synchronization estabilish):

$$U_i = \mu_i \mathcal{F}_\alpha(t, \overline{x}_\alpha(t)) = \mu_i \sin(2\pi f_\alpha(t - 20)),$$

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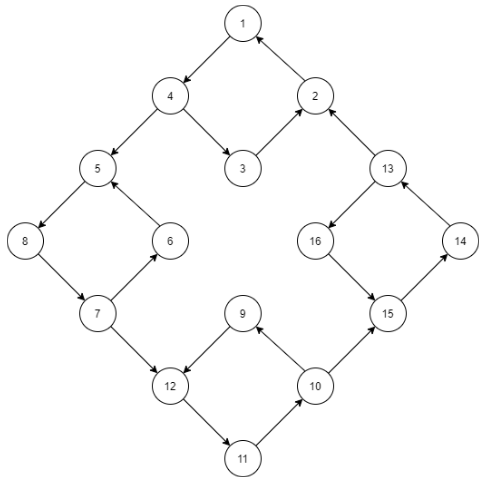

Figure 1. Topology of the graph developed for simulations

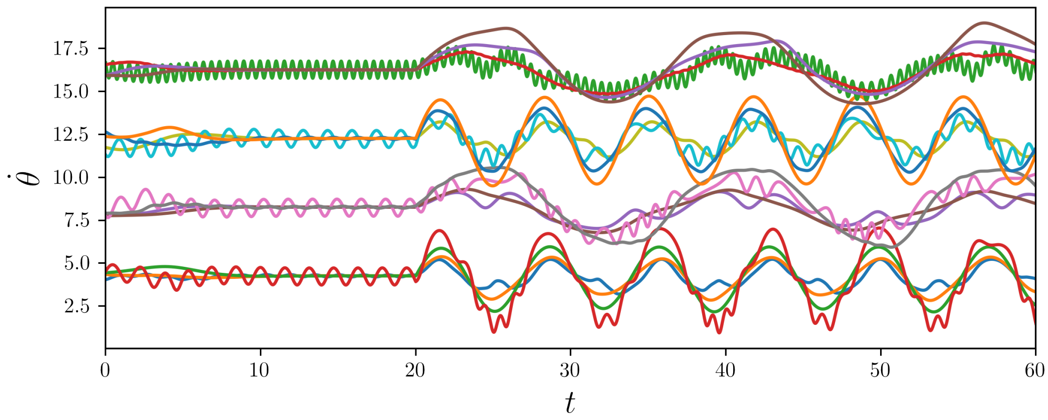

Figure 2. Trajectories of equation (7) with sinusoidal U_i and large differences between μ_i: the clusters overlapped as ε became greater than δ

where f_α are from uniform distribution on $[0, 1]$. The set of values $\{\mu_i\}_{i \in \mathcal{N}}$ is constructed as follows: $\{\mu_1, ..., \mu_4\} = \{\mu_5, ..., \mu_8\} = \{\mu_9, ..., \mu_{12}\} = \{\mu_{13}, ..., \mu_{16}\} = \{0.625, 1.25, 1.875, 2.5\}$. As it may be concluded from the equations (8) and (11), such values of agent's sensibility may break cluster invariance, and they in fact do (see Figure 2).

We denote the parameters for quantization of the state space:

- $\dot{\Theta}_{\text{min}} = 0$, $\dot{\Theta}_{\text{max}} = 20$, $\epsilon = 0.02$;
- $t_{\text{min}} = 20$, $t_{\text{max}} = 60$, $\tau = 0.1$.

The corresponding 1000×400 matrix $\mathfrak{B}$ constructed according to the algorithm described above is shown on Figure 3.

It turned out that 93% of $\mathfrak{B}$'s elements are zero, so that its columns has sparse representation in the standard basis. We choose to compress the columns of $\mathfrak{B}$, since they represent states of the considered systems observed during the time interval $[t_{i-1}, t_i)$.

The next step is to generate Φ, which is chosen to origin its elements from normal distribution $\mathcal{N}(0, \frac{1}{m})$ (according to [5]), where m is the number of measurements, which we choose as $2s$. we choose s to be the average number of non-zero elements among the columns of $\mathfrak{B}$. As it will be shown on the image with reconstructed trajectories, the multiplier 2 is sufficient, despite in [5] it was equal to 4. This empirical decision can be justified by the fact that the dynamic trajectories have very sparse representation, unlike ordinary images. Thus, we provide decent compression for each column,

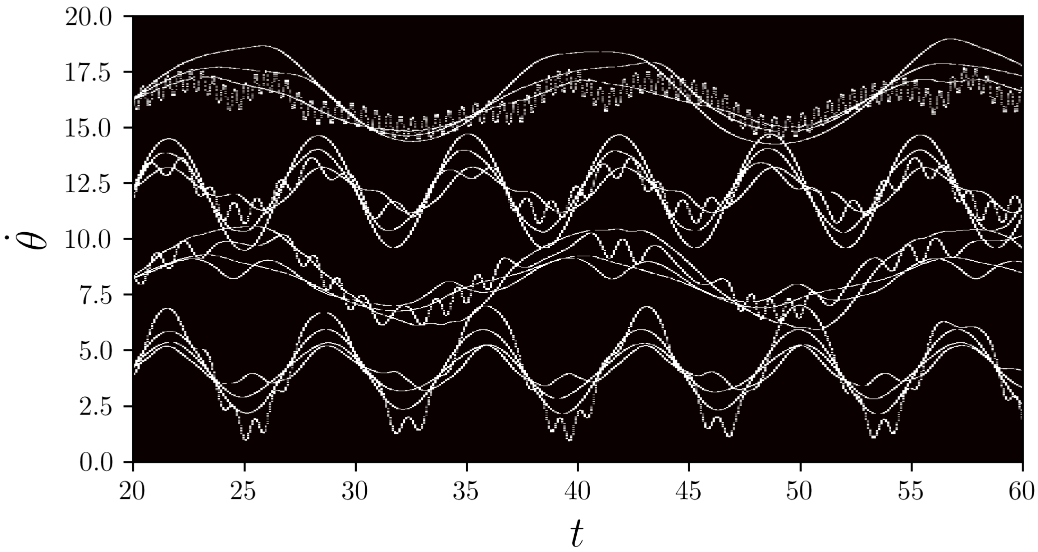

Figure 3. Matrix $\mathfrak{B}$ represented as a binary histogram (white bins does have trajectories, while black ones does not)

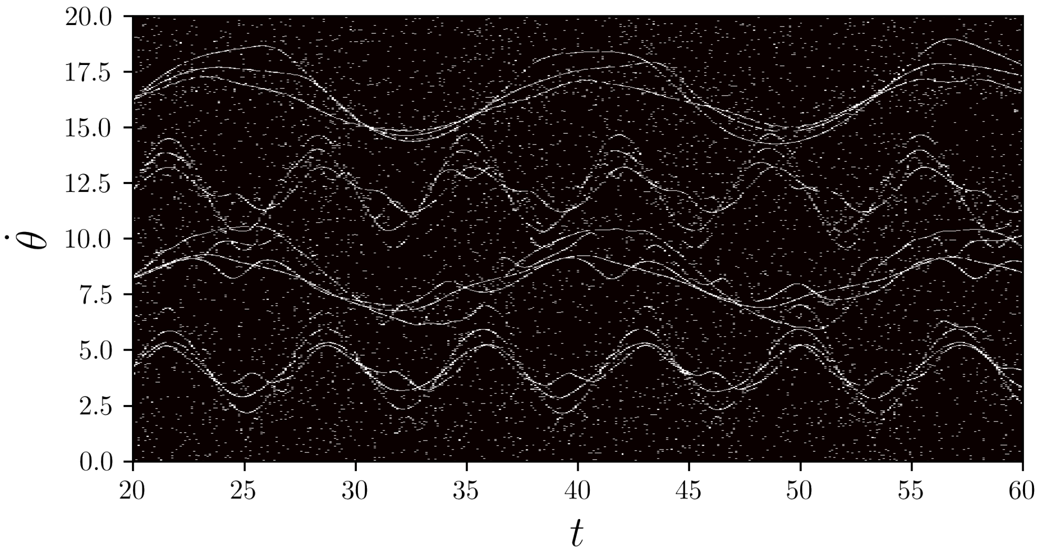

Figure 4. Decompressed matrix $\overline{\mathfrak{B}}$ represented as a binary histogram (white bins correspond to the values higher than or equal to $7 \cdot 10^{-3}$, while black ones represent values less than $7 \cdot 10^{-3}$)

 doi:10.20944/preprints202011.0707.v1

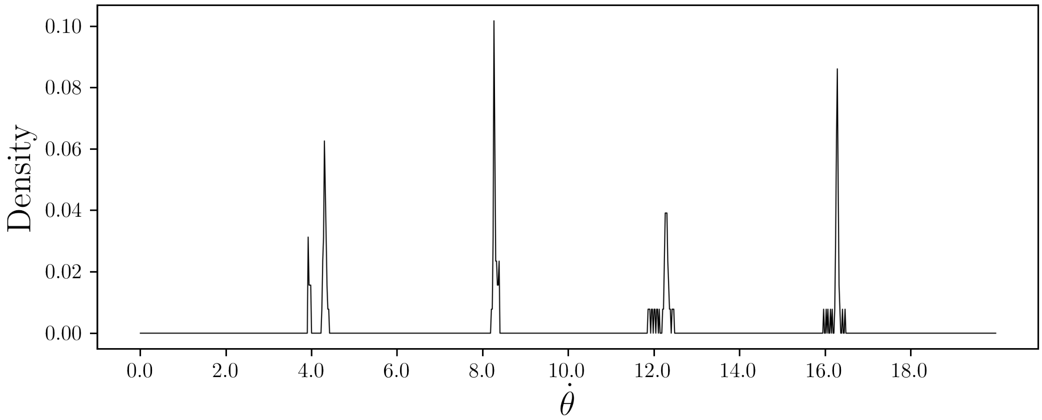

Figure 5. The values of the first column of $\mathfrak{B}$

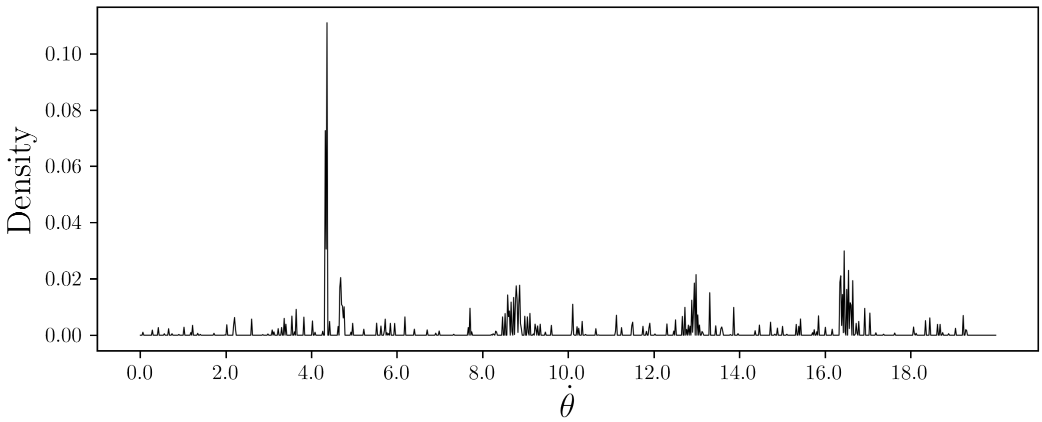

Figure 6. The values of the first column of $\overline{\mathfrak{B}}$

reducing its length from 1000 to 139. Such data reduction brings us closely from the level of individual agents to the mesoscopic scale of clusters. Decompressed matrix $\overline{\mathfrak{B}}$ is obtained as a collection of restored columns as ℓ_1-optimized vectors, numerically calculated with the interior-point method; it is shown on Figure 4. Despite the decompressed matrix appears to be noisy, the overall features of the trajectories are still perceptable, which could tell about close to lossless compression.

It is also helpful to compare the first columns of $\mathfrak{B}$ and $\overline{\mathfrak{B}}$, which correspond to low values of U_i, thus four initial clusters can be easily distinguished. Figures 5 and 6 represent those columns before and after compression. It can be seen that the four clusters are visually recognizable even after reconstruction.

5. Conclusion

We proposed a mathematical framework for analysis of complex systems as multiagent networks. Besides, we proposed an algorithm of compressive sensing application for centralized observations of the agent trajectories, exploiting their sparsity in the discretized state space caused by clustering. We also shown how this algorithm can be applied to the Kuramoto model: the achieved degree of compression could make it possible to efficiently transmit compressed state space data to a data center, where it can be reconstructed without much loss of information for further mesoscopic control decisions. Furthermore, we plan to consider more in-depth study of compression in application to multiagent networks, which may bring us to the point of simple rules that link and explain all the scales complex systems operate on. We hope that our results would push further interest in researches regarding compressed observations of agent states for mesoscopic control.

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