

Article

Causal intuition and delayed-choice experiments

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Abstract: The conventional explanation of delayed-choice experiments seems to violate our causal intuition. This apparent violation is caused by a misinterpretation of the conventional formulation of quantum mechanics. I reanalyze these experiments using advanced and time-symmetric formulations of quantum mechanics. All three formulations give the same experimental predictions, but the advanced and time-symmetric formulations violate our causal intuition that effects only happen after causes. I explore reasons why our causal intuition may be wrong at the quantum level. I also suggest how conventional causation might be recovered in the classical limit, and speculate on cosmological boundary conditions.

Keywords: delayed-choice; causality; retrocausality; advanced action; numerical simulation

1. Introduction

One of the grand challenges of modern physics is to resolve the conceptual paradoxes in the foundations of quantum mechanics [1]. Some of these paradoxes concern our causal intuition. For example, in 1926 Lewis proposed a delayed-choice gedankenexperiment which appeared to show retrocausation in the Conventional Formulation of quantum mechanics [2,3]. Retrocausation, also known as future input dependence [4], is when a model parameter associated with time t depends on model inputs associated with times greater than t . He considered a double-slit interference experiment using a single photon from a distant star. One thousand years after the photon has left the star, but just before it reaches the two slits (A and B) on Earth, we randomly choose to either keep both slits A and B open, or intervene to close slit A only, or intervene to close slit B only. We repeat this experiment for a large number of single photons to obtain an ensemble of experimental results. In the sub-ensemble where we chose to keep both slits open, we see an interference pattern, implying each photon took both routes from the star. In the sub-ensemble where we intervened to keep only one slit open, we do not see an interference pattern, implying each photon took only one route from the star. Lewis concluded that "in some manner the atom in the source S can foretell before it emits its quantum of light whether one or both of the slits A and B are going to be open." Our intervention appears to cause the photon to change its route before the intervention actually happens, in violation of our causal intuition that effects never happen before interventions. This is the delayed-choice paradox. Weizsäcker and Wheeler later rediscovered and elaborated on Lewis's gedankenexperiment [5–8]. This paradox has been confirmed in experiments with photons, neutrons, and atoms [9–20]. The most recent review of delayed-choice experiments says "It is a general feature of delayed-choice experiments that quantum effects can mimic an influence of future actions on past events" [21].

2. The Conventional Formulation of the delayed-choice Experiment

Let us consider gedankenexperiments with the neutron Mach-Zehnder interferometer (MZI) shown in Figure 1, where a single particle is emitted from either source $S1$ or source $S2$. The Conventional Formulation (CF) postulates that a single free particle with mass m is described by a wavefunction $\psi(\vec{r}, t)$ which satisfies the initial conditions and evolves forwards in time according to the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi. \quad (1)$$

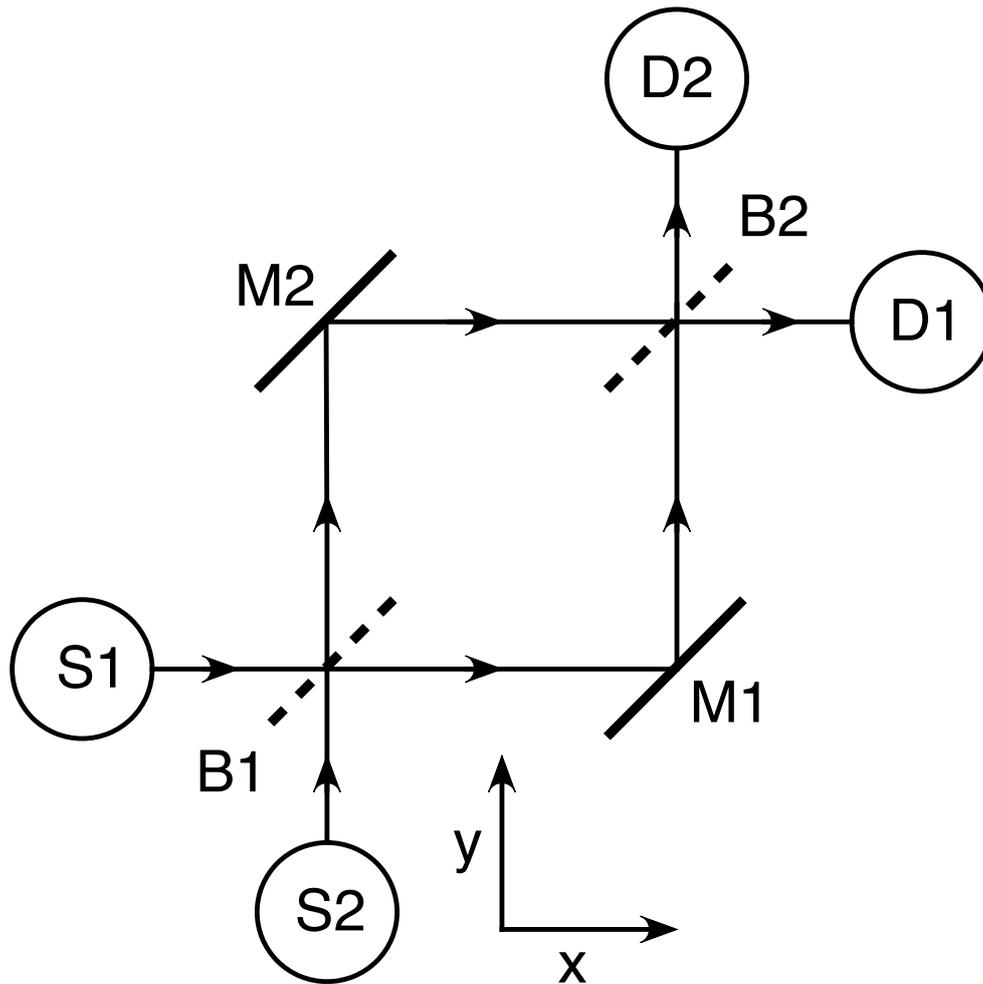


Figure 1. The neutron Mach-Zehnder interferometer (MZI). S1 and S2 are neutron sources, B1 and B2 are beam-splitters, M1 and M2 are mirrors, and D1 and D2 are detectors. The sources S1 and S2 can each emit a single neutron on command. The MZI is constructed such that neutrons emitted from S1 are always detected at D1, while neutrons emitted from S2 are always detected at D2.

The Schrödinger equation only has solutions which evolve forwards in time, so the CF is a retarded formulation of quantum mechanics. We will use units where $\hbar = 1$ and assume the retarded wavefunction $\psi(\vec{r}, t)$ is a traveling gaussian with an initial standard deviation $\sigma = 50$, momentum $k_x = 0.4$, and mass $m = 1$. The CF assumes this retarded wavefunction is ontic: it is a real, physical object.

Figure 2 shows how the particle's CF probability density $\psi^*\psi$ evolves over time in the MZI, assuming the initial condition is localization in source $S1$. At $t = 0$, $|\psi^*\psi|$ is localized inside the source $S1$. At $t = 3000$, $\psi^*\psi$ has been split in half by beam-splitter $B1$, and the halves are traveling towards mirrors $M1$ and $M2$. At $t = 5000$, the two halves have been reflected by $M1$ and $M2$ and are both traveling towards beam-splitter $B2$. At $t = 7000$, the two halves have been recombined by $B2$, with $\psi^*\psi$ interfering constructively towards detector $D1$ and destructively towards detector $D2$. At $t = 8000 - \delta t$, $\psi^*\psi$ arrives at $D1$, but is not localized inside $D1$. Upon measurement at $t = 8000$, ψ collapses to a different wavefunction ζ , with $\zeta^*\zeta$ localized inside $D1$. Similarly, if $\psi^*\psi$ had been localized inside the source $S2$ at $t = 0$, it would have taken both routes and collapsed to being localized inside $D2$ at $t = 8000$.

To analyze delayed-choice experiments, we need to define a model for causality and explain its connection with causal intuition. Let us define the system as the two sources, the MZI, the two detectors, and the particles. We can intervene on this system from outside the system. For example, we can intervene by sending a command to a source to emit a particle. Let us define an intervention as a cause. Let us also define the effects as whatever is correlated with the intervention. For example, the emission of a particle, the motion of that particle through the MZI, and the detection of that particle. Many of our causal intuitions are based on interventions, not on temporal order. These causal intuitions are correlated with but not caused by temporal order [22].

There are four possible ensembles of completed MZI experiments: (1) a particle is emitted from $S1$ and detected in $D1$; (2) a particle is emitted from $S2$ and detected in $D2$; (3) a particle is emitted from $S1$ and detected in $D2$; and (4) a particle is emitted from $S2$ and detected in $D1$. Repeated MZI experiments show that only ensembles 1 and 2 occur. Wheeler said this is "evidence that each arriving [particle] has arrived by both routes"[8].

Now consider a modified experiment where $B2$ is removed for the entire experiment. Repeated experiments show that ensembles 1, 2, 3, and 4 occur. Wheeler said either "one counter goes off, or the other. Thus the [particle] has traveled only one route"[8].

Finally, consider a CF delayed-choice experiment where $B2$ is removed before each particle is emitted at time $t = 0$. At $t = 5000$, we randomly choose to either reinsert or not reinsert $B2$. For the runs where we chose to not reinsert $B2$, we know that ensembles 1, 2, 3, and 4 occur. For the runs where we intervene to reinsert $B2$, only ensembles 1 and 2 occur. Wheeler said "Thus one decides the [particle] 'shall have come by one route, or by both routes' after it has 'already done its travel'[8]. How could a particle at $B1$ know if an intervention will or will not occur before it reaches the point of intervention at $B2$? Wheeler said "we have a strange inversion of the normal order of time. We, now, by moving the [beam-splitter] in or out have an unavoidable effect on what we have a right to say about the *already* past history of that [particle]" [8]. This is the presumed delayed-choice paradox.

However, Wheeler implicitly assumed that a measurement occurs at beam-splitter $B1$ when $B2$ is removed for the entire experiment, causing the wavefunction to either completely reflect from or completely pass through $B1$. This is not true. What actually happens in the CF delayed-choice experiment, according to a correct interpretation of the CF, is the following: At $t = 0$, we intervene on the system by sending a command to $S1$ or $S2$ to emit a particle. The particle wavefunction reaches $B1$ at $t = 2000$, where half of it is transmitted towards $M1$ and the other half is reflected towards $M2$. These two halves reflect from $M1$ and $M2$, and then travel towards $D1$ and $D2$. If we do not intervene at $t = 5000$, one half reaches $D1$ while the other half reaches $D2$ at $t = 8000$, then one half collapses to a full wavefunction while the other half collapses to no wavefunction. If we do intervene at $t = 5000$ by inserting $B2$, the two halves recombine at $B2$ and interfere constructively towards $D1$ and destructively

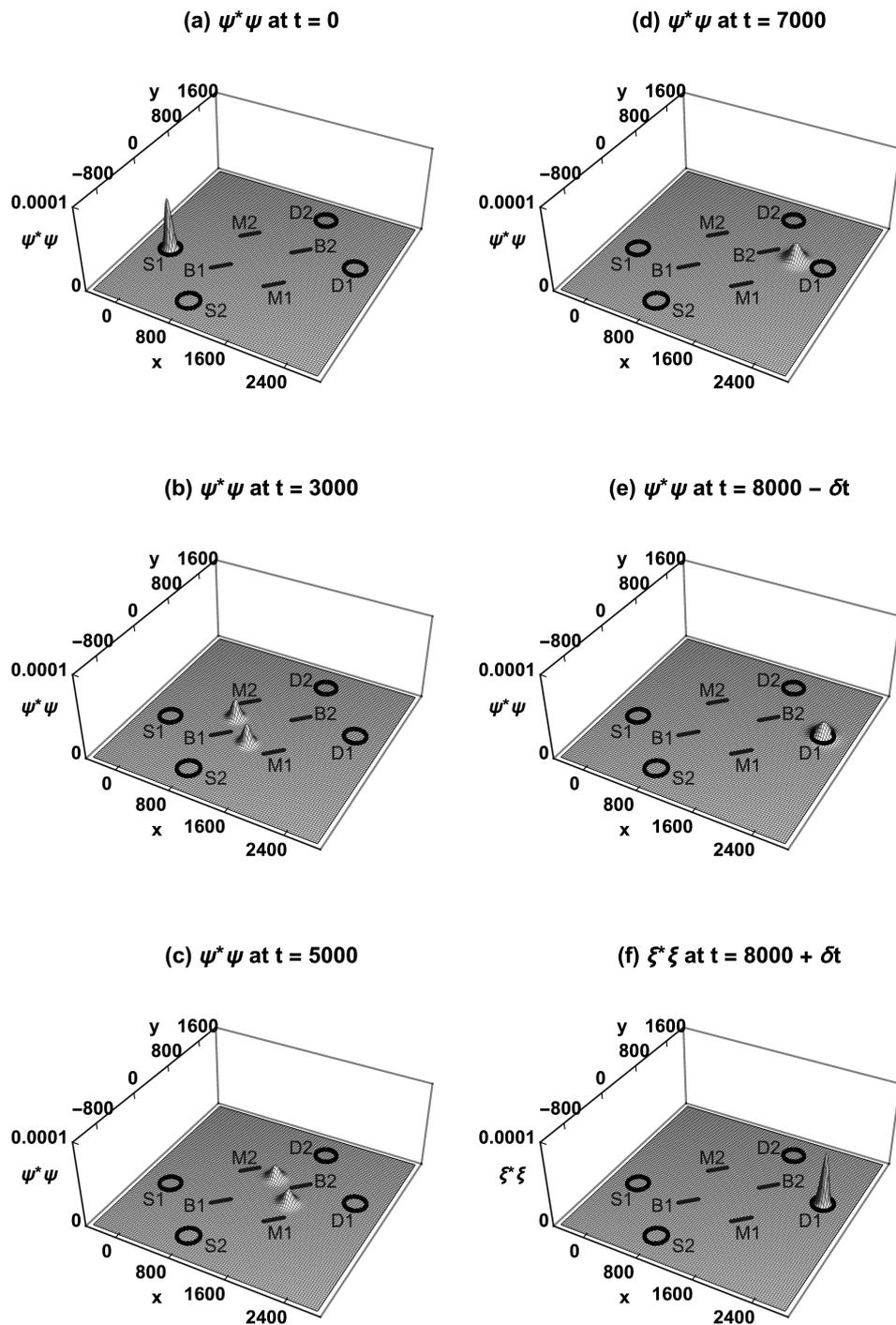


Figure 2. The Conventional Formulation (CF) of the MZI experiment, with a single particle emitted from S1. (a) The probability density $\psi^* \psi$ is localized inside S1. (b) $\psi^* \psi$ is split in half by B1. (c) The two halves are reflected by M1 and M2. (d) The recombined $\psi^* \psi$ interferes constructively towards D1 and destructively towards D2. (e) $\psi^* \psi$ arrives at D1, but is not localized inside D1. (f) Upon measurement at $t = 8000$, ψ collapses to a different wavefunction ξ , localized inside D1. Wavefunction collapse is a postulate of the CF, and is required to obtain agreement between CF predictions and experimental results.

towards $D2$ if the particle came from $S1$, or vice versa if the particle came from $S2$. Our intervention at $t = 5000$ is uncorrelated with anything the wavefunction did for $0 \leq t < 5000$. This means there is no delayed-choice paradox in the CF delayed-choice experiment. This has been explained before by Ellerman [23], but does not seem to be widely known.

3. The Advanced Formulation of the delayed-choice Experiment

Penrose pointed out that many quantum experiments can be explained equally well by an Advanced Formulation (AF) of quantum mechanics [24]. The AF postulates that a single free particle with mass m is described by an advanced wavefunction $\phi^*(\vec{r}, t)$ which satisfies the final conditions and evolves backwards in time according to the advanced Schrödinger equation:

$$-i\hbar \frac{\partial \phi^*}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \phi^*. \quad (2)$$

The advanced Schrödinger equation only has solutions which evolve backwards in time from the final conditions. We will use natural units and assume $\phi^*(\vec{r}, t)$ is a traveling gaussian with a final standard deviation $\sigma = 50$, momentum $k_x = 0.4$, and mass $m = 1$. The AF assumes this advanced wavefunction is ontic: it is a real, physical object.

Figure 3 shows how the particle's AF probability density $\phi^*\phi$ evolves over time in the MZI, assuming the final condition is localization in detector $D1$. At time $t = 8000$, $\phi^*\phi$ is localized inside the detector $D1$. At $t = 7000$, $\phi^*\phi$ is traveling towards beam-splitter $B2$. At $t = 5000$, $\phi^*\phi$ has been split in half by $B2$, and the halves are traveling towards mirrors $M1$ and $M2$. At $t = 3000$, the two halves have been reflected by $M1$ and $M2$ and are both traveling towards $B1$. At $t = 0 + \delta t$, the two halves have been recombined by $B1$, with $\phi^*\phi$ interfering constructively towards source $S1$ and destructively towards source $S2$. Upon preparation at $t = 0$, ϕ^* collapses to the different wavefunction ζ^* , with $\zeta^*\zeta$ localized inside $S1$. Similarly, if $\phi^*\phi$ had been localized inside the detector $D2$ at $t = 8000$, it would have taken both routes and collapsed to being localized inside $S2$ at $t = 0$. Wheeler could correctly say this is "evidence that each arriving [particle] has arrived by both routes" [8].

Now consider a modified experiment where $B2$ is removed for the entire experiment. At time $t = 8000$, let us assume the particle's AF probability density $\phi^*\phi$ is localized inside the detector $D1$. At $t = 7000$, $\phi^*\phi$ will travel towards mirror $M2$ via the upper route. At $t = 5000$, $\phi^*\phi$ will continue traveling towards $M2$ via the upper route. At $t = 3000$, $\phi^*\phi$ has been reflected from $M2$ and is traveling towards $B1$ via the upper route. At $t = 2000$, $\phi^*\phi$ will reach $B1$, where it will be split in half, one half passing through $B1$ and traveling towards $S2$, while the other half reflecting from $B1$ and traveling towards $S1$. At $t = 0 + \delta t$, the two halves will reach $S1$ and $S2$. At $t = 0$, one half will collapse to a full wavefunction in either $S1$ or $S2$, while the other half will collapse to nothing. If the particle's AF probability density $\phi^*\phi$ had instead been localized inside the detector $D2$, it would have taken the lower route only. For any combination of source and detector, the advanced wavefunction always takes either the upper route or the lower route, never both routes. For the AF, Wheeler's analysis is now true: either "one counter goes off, or the other. Thus the [particle] has traveled only one route"[8].

Finally, consider an AF delayed-choice experiment. When $B2$ is not present at times $0 \leq t < 5000$ and not reinserted at $t = 5000$, we know each particle always takes either the upper route or the lower route. When $B2$ is not present at times $0 \leq t < 5000$, but we intervene to reinsert $B2$ for $5000 \leq t \leq 8000$, then each particle always takes both routes. Our intervention to reinsert $B2$ at $t = 5000$ causes the wavefunction to change from taking either route to taking both routes for $2000 \leq t < 6000$: some of the effects occur *before* the intervention occurs, violating our causal intuition that effects never happen before interventions. There is a true delayed-choice paradox in the AF delayed-choice experiment. Wheeler could correctly say "we have a strange inversion of the normal order of time. We, now, by moving the [beam-splitter] in or out have an unavoidable effect on what we have a right to say about the *already* past history of that [particle]" [8].

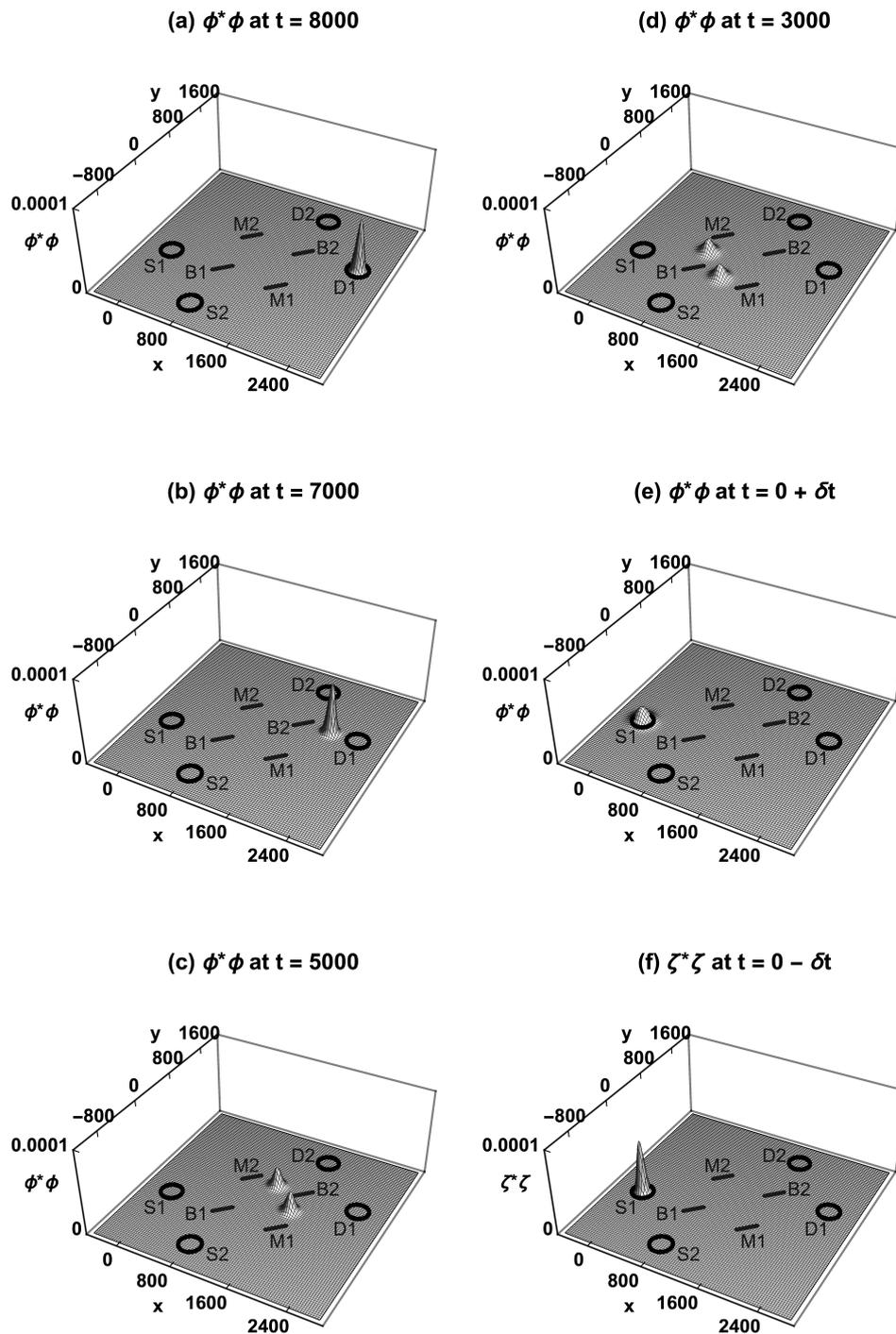


Figure 3. The Advanced Formulation (AF) of the MZI experiment, with a single particle detected at $D1$. (a) The probability density $\phi^* \phi$ is localized inside $D1$. (b) $\phi^* \phi$ is split in half by $B2$. (c) The two halves are reflected by $M1$ and $M2$. (d) The recombined $\phi^* \phi$ interferes constructively towards $S1$ and destructively towards $S2$. (e) $\phi^* \phi$ arrives at $S1$, but is not localized inside $S1$. (f) Upon preparation at $t = 0$, ϕ^* collapses to a different wavefunction ζ^* , localized inside $S1$. Wavefunction collapse is a postulate of the AF, and is required to obtain agreement between AF predictions and experimental results.

4. The Time-symmetric Formulation of the delayed-choice Experiment

It is also possible to explain delayed-choice experiments using the Time-symmetric Formulation (TF) of quantum mechanics described in [25]. This TF is a type IIB model, in the classification system of Wharton and Argaman [4]. The CF and AF implicitly assume that quantum mechanics is a theory about particles, while the TF explicitly assumes that quantum mechanics is a theory about transitions of particles (transition amplitude densities). The CF and AF postulate that a particle is described by one boundary condition and one wavefunction, while the TF postulates that the transition of a particle is described by two boundary conditions and the algebraic product of two wavefunctions: a retarded wavefunction $\psi(\vec{r}, t)$ that obeys the retarded Schrödinger equation and satisfies only the initial boundary condition; and an advanced wavefunction $\phi^*(\vec{r}, t)$ that obeys the advanced Schrödinger equation and satisfies only the final boundary condition. The TF assumes this product wavefunction (aka transition amplitude density) is ontic: it is a real, physical object. The CF and AF postulate that the wavefunction collapses instantaneously, indeterministically, and irreversibly into a different wavefunction at one of the boundary conditions, while the TF postulates that wavefunctions never collapse. Consequently, the CF and AF have intrinsic arrows of time, while the TF has no intrinsic arrow of time.

Figure 4 shows how the absolute value of the particle's TF product wavefunction $|\phi^*\psi|$ evolves over time in the MZI experiment, assuming the initial condition is localization in source $S1$ and the final condition is localization in detector $D1$. At time $t = 0$, $|\phi^*\psi|$ is localized inside the source $S1$. At $t = 3000$, $|\phi^*\psi|$ has been split in half by beam-splitter $B1$, and the halves are traveling towards mirrors $M1$ and $M2$. At $t = 5000$, the two halves have been reflected by $M1$ and $M2$ and are both traveling towards $B2$. At $t = 7000$, the two halves have been recombined by $B2$, and the whole wavefunction travels towards detector $D1$. At $t = 8000 - \delta t$, $|\phi^*\psi|$ arrives at $D1$ and is localized inside $D1$. A second measurement at $t = 8000 + \delta t$ gives the same $|\phi^*\psi|$: there is no wavefunction collapse. Similarly, if $S2$ emits a particle, it will always go to $D2$ via both routes. Wheeler could correctly say this is "evidence that each arriving [particle] has arrived by both routes" [8].

Now consider a modified TF experiment where $B2$ is removed for the entire experiment. Figure 5 shows how the particle's TF product wavefunction $|\phi^*\psi|$ evolves over time in this modified experiment, assuming the initial condition is localization in $S1$ and the final condition is localization in $D1$. At $t = 0$, $|\phi^*\psi|$ is localized inside source $S1$. At $t = 3000$, $|\phi^*\psi|$ has been completely reflected by $B1$ and is traveling towards $M2$. At $t = 5000$, $|\phi^*\psi|$ has been reflected by $M2$ and is traveling towards $D1$. At $t = 7000$, $|\phi^*\psi|$ is still traveling towards $D1$. At $t = 8000$, $|\phi^*\psi|$ arrives at $D1$ and is localized inside $D1$. A second measurement at $t = 8000 + \delta t$ would give the same $|\phi^*\psi|$: there is no wavefunction collapse upon measurement. The TF product wavefunction $\phi^*\psi$ takes only the upper route. If the final state is changed to $D2$, the product wavefunction would take only the lower route. For any combination of source and detector, the product wavefunction always takes either the upper route or the lower route, never both routes. For this formulation, Wheeler's analysis is now true: either "one counter goes off, or the other. Thus the [particle] has traveled only one route" [8].

Finally, consider a TF delayed-choice experiment. When $B2$ is not present at times $0 \leq t < 5000$ and not reinserted at $t = 5000$, we know each particle always takes either the upper route or the lower route. When $B2$ is not present at times $0 \leq t < 5000$, but we intervene to reinsert $B2$ for $5000 \leq t \leq 8000$, then each particle always takes both routes. Our intervention to reinsert $B2$ at $t = 5000$ causes the wavefunction to change from taking either route to taking both routes for $2000 \leq t < 6000$: some of the effects occur *before* the intervention occurs, violating our causal intuition that effects never happen before interventions. There is a true delayed-choice paradox in the TF delayed-choice experiment. Wheeler could correctly say "we have a strange inversion of the normal order of time. We, now, by moving the [beam-splitter] in or out have an unavoidable effect on what we have a right to say about the *already* past history of that [particle]" [8].

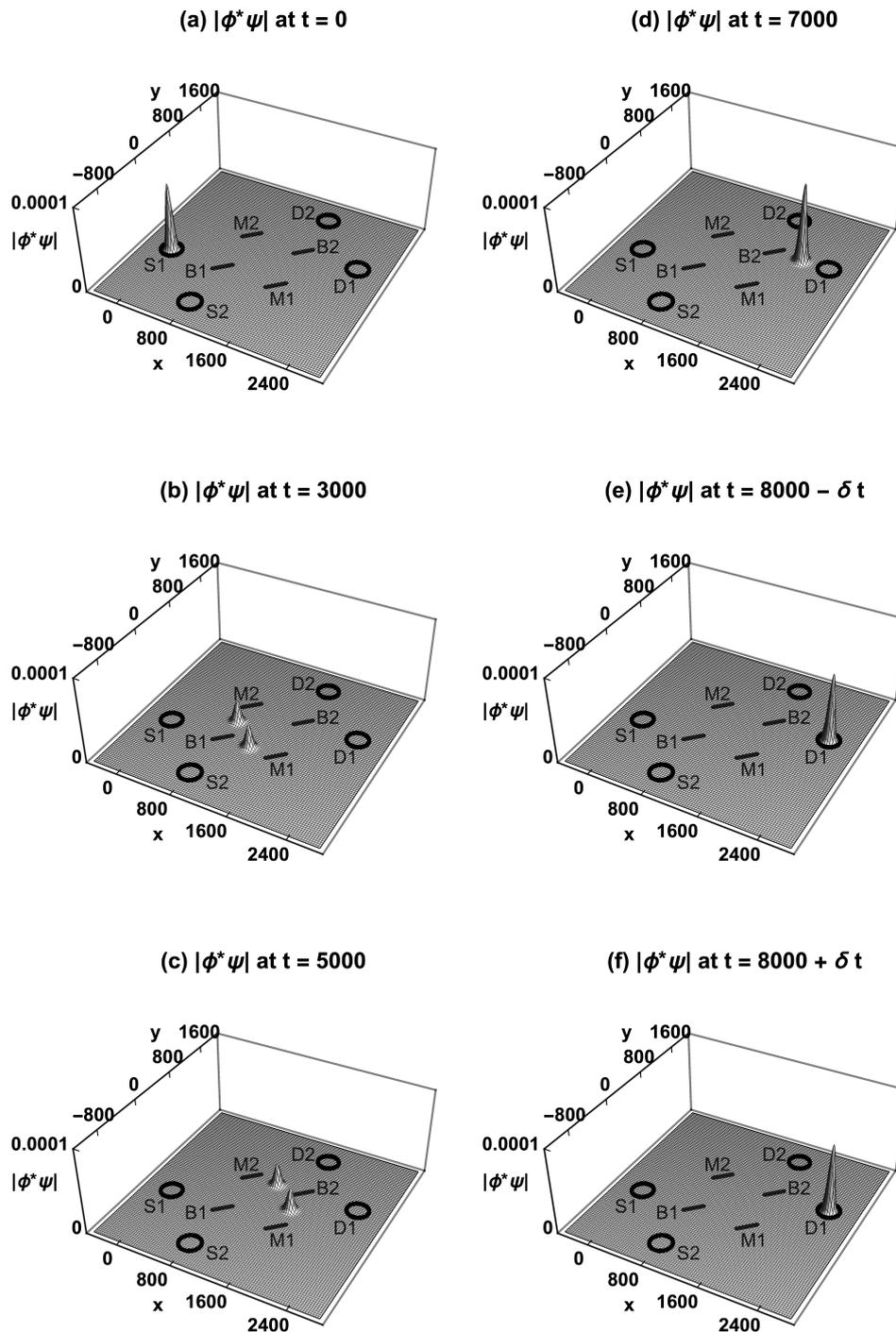


Figure 4. The Time-symmetric Formulation of the MZI experiment. (a) The absolute value of the product wavefunction $|\phi^* \psi|$ is localized inside $S1$. (b) $|\phi^* \psi|$ is split in half by $B1$. (c) The two halves are reflected by $M1$ and $M2$. (d) The recombined $|\phi^* \psi|$ interferes constructively towards $D1$ and destructively towards $D2$. (e) $|\phi^* \psi|$ arrives at $D1$ and is localized inside $D1$. (f) A second measurement immediately afterward gives the same $|\phi^* \psi|$: there is no wavefunction collapse at any time. The probability for the transition is normalized to one.

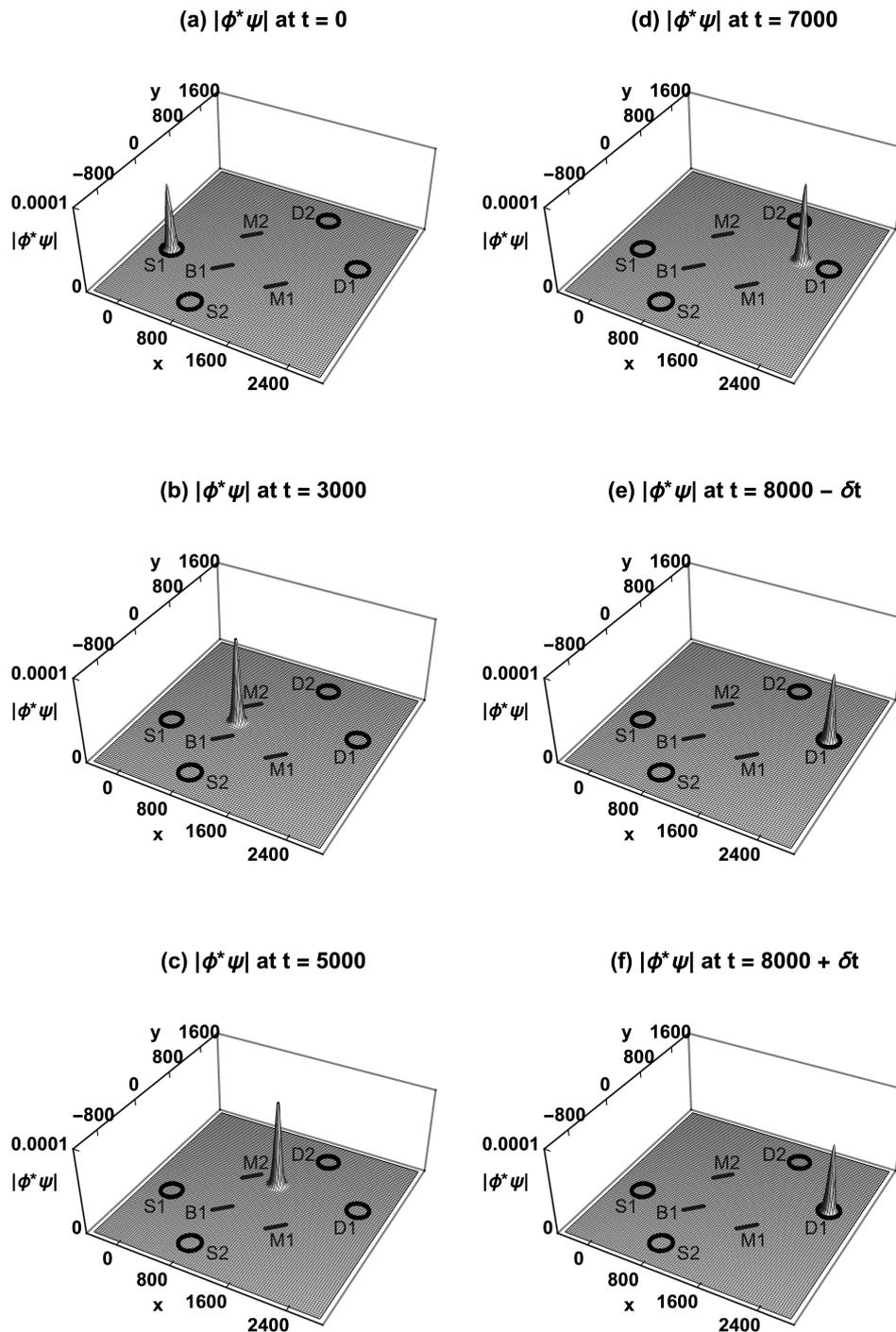


Figure 5. The Time-symmetric Formulation of a modified experiment with $B2$ absent. The particle is emitted by $S1$ and later detected by $D1$. (a) $|\phi^* \psi|$ is localized inside $S1$. (b) $|\phi^* \psi|$ is completely reflected by $B1$. (c) $|\phi^* \psi|$ is completely reflected by $M2$. (d) $|\phi^* \psi|$ travels towards $D1$. (e) $|\phi^* \psi|$ arrives at $D1$, and is localized inside $D1$. (f) A second measurement immediately afterward would give the same $|\phi^* \psi|$: there is no wavefunction collapse upon measurement. If the particle had been emitted by $S1$ and later detected by $D2$, $|\phi^* \psi|$ would have taken only the lower route.

5. Discussion

The original Conventional Formulation (CF) explanations of the delayed-choice experiment by Lewis, Weizsäcker, and Wheeler appeared to violate our causal intuition: the effects of an intervention seemed to occur before the intervention. But this was due to a mistaken interpretation of the CF. The correct CF explanation does not violate our causal intuition: the effects of an intervention always occur after the intervention. The Advanced Formulation (AF) and Time-symmetric Formulation (TF) explanations of the same experiment say the effects of an intervention can occur before the intervention, violating our causal intuition. Some may see this as reason to discard the AF and TF, but other aspects of quantum mechanics also violate our causal intuition (see papers in this issue). Perhaps there is something wrong with our causal intuition. What might be wrong?

The analyses in this paper suggest two overlapping areas. First, our causal intuition that there is an arrow of time in the quantum world may be at fault. Humans live and develop intuitions in a macroscopic world with an omnipresent arrow of time set by the second law of thermodynamics and the low entropy past. It is natural that we would implicitly assume this arrow of time extends to the quantum world. But the few-quantum world need not necessarily obey the second law, and indeed seems to allow hitherto unknown types of causal structure which do not have the cause-effect relationship ingrained in our classical intuition. Second, it is also ingrained in human intuition that nature is composed of objects which live in 3-dimensional space and evolve in time, because the velocities we experience are insignificant compared to the speed of light. The CF extends this intuition into the quantum world by assuming that a wavefunction which lives in configuration space and evolves in time gives the most complete description of a quantum that is in principle possible. But the main lesson of the special theory of relativity is that nature is fundamentally (3+1)-dimensional. Extending this lesson to the quantum world suggests the fundamental quantum beable lives in a (3N+1)-dimensional configuration spacetime, where N is the number of degrees of freedom. This is the block universe viewpoint, where nothing evolves in time and so there is no arrow of time. The TF product wavefunction in (3N+1)-dimensional configuration spacetime could be the quantum equivalent of the world tube of a classical particle in (3+1)-dimensional spacetime. This fits in with the fact that the low velocity approximation of any relativistic wave equation gives both retarded and advanced wave equations.

The Conventional and Advanced Formulation both require wavefunction collapse, while the Time-symmetric Formulation does not. If we ignore the time asymmetry of wavefunction collapse, the Conventional and Advanced Formulations are still not time-symmetric, due to the superposition states produced by the beam-splitters. The Time-symmetric Formulation is time-symmetric and gives the same experimental predictions for the delayed-choice experiment as the Conventional and Advanced Formulations. Heisenberg said "Since the symmetry properties always constitute the most essential features of a theory, it is difficult to see what would be gained by omitting them in the corresponding language [26]."

How might conventional causation be recovered in the classical limit? First, as the number of particles in a quantum system increases, the second law of thermodynamics comes into play, creating an effective arrow of time. Second, as the number of particles increases, the mean free distance between initial and final states decreases. This decreases the coherence length of the particle's product wavefunctions, so quantum phenomena which depend on delocalization are suppressed at macroscopic length scales. Third, as the quantum system interacts with the environment, decoherence effects will occur that make the quantum system behave in a more classical way [27].

Finally, the wavefunction of the universe is believed to depend on the initial conditions at the Big Bang. If the TF is correct, the final conditions of the universe should play an equally important role in its evolution.

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