Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 May 2021 doi:10.20944/preprints202011.0602.v4

Towards room-temperature superconductivity

Jacob Szeftel[1] , Nicolas Sandeau[2] , Michel Abou Ghantous[3] and Muhammad El-Saba[4]

[1] *ENS Paris-Saclay/LuMIn, 4 avenue des Sciences, 91190 Gif-sur-Yvette, France*
[2] *Aix Marseille Univ, CNRS, Centrale Marseille, Institut Fresnel, F-13013 Marseille, France*
[3] *American University of Technology, AUT Halat, Highway, Lebanon*
[4] *Ain-Shams University, Cairo, Egypt*

PACS `74.25.Bt` – Thermodynamic properties
PACS `74.25.Jb` – Electronic structure
PACS `74.62.Bf` – Effects of material synthesis, crystal structure, and chemical composition

Abstract –By taking advantage of a stability criterion established recently, the critical temperature T_c is reckoned with help of the microscopic parameters, characterising the normal and superconducting electrons, namely the independent-electron band structure and a *repulsive* two-electron force. The emphasis is laid on the sharp T_c dependence upon electron concentration and inter-electron coupling, which might offer a practical route toward higher T_c values and help to understand why high-T_c compounds exhibit such remarkable properties.

Introduction. – The BCS theory [1], despite its impressive success, does not enable one to predict [2] superconductivity occurring in any metallic compound. As shown elsewhere [3], such a drawback ensues from an *attractive* interaction, assumed to couple electrons together, which is not only at loggerheads with the sign of the Coulomb repulsion, but in addition leads to inconsistencies to be recalled below. Therefore this work is intended at investigating the T_c dependence upon the parameters, characterising the motion of electrons correlated together through a *repulsive* force, in order to devise, within the framework of a two-fluid picture developed elsewhere [3–6], a *single* model, valid for *all* superconductors, including low and high T_c materials, as well. This is in marked contrast with the mainstream approach, tending to seek a particular model, dedicated to each superconducting compound [7–9].

In order to reach the hereabove mentioned goal, a historical survey, aimed at identifying the cause of the BCS failure and its far-reaching consequences, is needed. Moreover, it will enable the reader to understand why our previous work [3–6] is not only unrelated to the mainstream view, but also runs afoul at it. As a matter of fact, the original sin was to purport with *no scientific proof* whatsoever that the many bound electron state, coming out of the BCS variational calculation [1], conveyed the signature of superconductivity, namely it did *sustain persistent currents*. Actually, the *only* well-established property of the BCS state [1] is that its energy is *lower* than that of the Fermi gas [10] of same electron concentration at $T = 0$K, which is of course *irrelevant* to the issue of persistent currents. Accordingly, since the BCS conclusion requires by all means to assume an *attractive* two-electron coupling, all authors [7–9], in the wake of BCS, lacking anyhow a reliable criterion of superconductivity, took for granted that any contrived two-electron interaction would ensure superconductivity, provided it is *attractive*. Illustrative examples of this surmise can be found, in particular, in several extensive studies [11–17] dealing with the electron-phonon coupling that has been believed to mediate the *attractive* force between electrons since Froehlich's work [18]. Unfortunately, such wishful thinking proved a fatal mistake, whenever it was shown [3] that superconductivity *cannot* arise, if the electrons are coupled together through an *attractive* force. As an appalling consequence, the whole research work in superconductivity decayed eventually into a muddled, religious war, vividly documented by Zaanen [8] and Lederer [9].

This barren bickering might have lasted till doomsday, were it not for the recent discovery [4] of a long-awaited criterion of superconductivity, saying that persistent currents are observed, *not* because the resistivity vanishes (it has been shown [19–22] actually to be finite, albeit much smaller than in the corresponding normal metallic phase), but rather because their being destroyed by the Joule effect would *violate* the second law of thermodynamics. Likewise, a simple experiment was proposed [4] to validate this conclusion and to bring evidence for the anomalous properties of the Joule effect, taking place in a superconductor. Furthermore it proved gratifying to realize that a prerequisite for a thermodynamically stable superconducting phase, discovered subsequently [6], turned out to be consistent with the above mentioned criterion of superconductivity [4]. The discussion below will take full advantage of this achievement but it could not be overemphasized that our thermodynamical criterion [4] acts as a watershed, distinguishing our work from that of all other authors who, unlike us, never question the BCS assumption of an *attractive* inter-electron force and in addition disregard the issue of *persistent currents*.

The outline is as follows : the conditions, warranting thermal stability in a superconductor, are recalled in section 1, while the parameters, needed to calculate T_c, are derived in section 2; the T_c dependences on electron concentration and inter-electron coupling are discussed in section 3, 4, respectively; the main results are summarised in the conclusion.

1-Thermal stability. – The conduction electrons are taken to comprise [3, 4, 6] bound and independent electrons, in respective temperature dependent concentration $c_s(T), c_n(T)$, such that

$$c_0 = c_s(T) + c_n(T) \quad ,$$

with c_0 being the total concentration of conduction electrons. They are organized, respectively, as a many bound electron [3] (MBE) state, characterised by its chemical potential $\mu(c_s)$, and a Fermi gas [10] of Fermi energy $E_F(T, c_n)$. The Helmholz free energy of independent electrons per unit volume F_n and E_F on the one hand, and the eigenenergy per unit volume $\mathcal{E}_s(c_s)$ of bound electrons and μ on the other hand, are related [10, 23], respectively, by $E_F = \frac{\partial F_n}{\partial c_n}$ and $\mu = \frac{\partial \mathcal{E}_s}{\partial c_s}$. Then a stable equilibrium is conditioned [4] by Gibbs and Duhem's law

$$E_F(T, c_n(T)) = \mu(c_s(T)) \quad , \qquad (1)$$

which expresses [23] that the total free energy $F_n + \mathcal{E}_s$ is minimum at T kept fixed, provided $\frac{\partial E_F}{\partial c_n} + \frac{\partial \mu}{\partial c_s} > 0$. Noteworthy is that $\frac{\partial \mu}{\partial c_s} < 0$ has been shown to be a prerequisite for persistent currents [4], thermal equilibrium [3], the Josephson effect [5] and a stable [6] superconducting phase. Likewise, Eq.(1) reads [3, 4, 6] for $T = T_c$

$$E_F(T_c, c_0) = \mu(c_s = 0) = \varepsilon_B/2 \quad , \qquad (2)$$

with ε_B being the energy of a *bound* electron pair [3]. Note that Eqs.(1,2) are consistent with the superconducting transition being of second order [23], whereas it has been shown [3] to be of first order at $T < T_c$ ($\Rightarrow E_F(T, c_0 - c_s) > \mu(c_s)$), if the sample is flown through by a finite current ($\Rightarrow c_s < c_s(T)$).

The binding energy of the superconducting state $E_B(T < T_c)$ has been worked out [3, 24] as

$$E_B(T) = \int_T^{T_c} \left(C_s(u) - C_n(u)\right) du \quad ,$$

with $C_s(T), C_n(T)$ being the electronic specific heat of a superconductor, flown through by a vanishing current [3] and that of a degenerate Fermi gas [10]. A stable phase ($\Rightarrow E_B > 0$) requires $C_s(T_c) > C_n(T_c)$, which can be secured *only* by fulfilling the following conditions [6]

$$\frac{\partial E_F}{\partial c_n}(T_c, c_0) = -\frac{\partial \mu}{\partial c_s}(0), \quad \rho'(E_F(T_c, c_0)) > 0 \quad , \qquad (3)$$

with $\rho(\epsilon), \epsilon$ being the *one-electron* density of states and energy [10], respectively, and $\rho' = \frac{d\rho}{d\epsilon}$.

2-Microscopic parameters. – Since the remaining analysis relies heavily on Eqs.(2,3), explicit expressions are needed for $E_F(T_c, c_0), \frac{\partial E_F}{\partial c_n}(T_c, c_0), \varepsilon_B, \frac{\partial \mu}{\partial c_s}(0)$. Because the independent electrons make up a degenerate Fermi gas ($\Rightarrow T << E_F/k_B$ with k_B being Boltzmann's constant), applying the Sommerfeld expansion [10] up to T^2 yields

$$\begin{array}{c} E_F(T_c, c_0) = E_F(0, c_0) - \frac{\rho'}{\rho}\frac{(\pi k_B T_c)^2}{6} \\ \frac{\partial E_F}{\partial c_n}(T_c, c_0) = \left(\rho + \rho'' \frac{(\pi k_B T_c)^2}{6}\right)^{-1} \end{array} \quad , \qquad (4)$$

with $\rho = \rho(E_F(0, c_0)), \rho' = \frac{d\rho}{dE_F}(E_F(0, c_0)), \rho'' = \frac{d^2\rho}{dE_F^2}(E_F(0, c_0))$. As for $\varepsilon_B, \frac{\partial \mu}{\partial c_s}(0)$, a truncated Hubbard Hamiltonian H_K, introduced previously [25–27], will be used. The main features of the calculation [3] are summarised below for self-containedness.

The independent electron motion is described by the Hamiltonian H_d

$$H_d = \sum_{k,\sigma} \epsilon(k) c^+_{k,\sigma} c_{k,\sigma} \quad .$$

$\epsilon(k), k$ are the *one-electron* energy ($\epsilon(k) = \epsilon(-k)$) and a vector of the Brillouin zone, respectively, $\sigma = \pm$ is the electron spin and the sum over k is to be carried out over the whole Brillouin zone. Then $c^+_{k,\sigma}, c_{k,\sigma}$ are creation and annihilation operators on the Bloch state $|k, \sigma\rangle$

$$|k, \sigma\rangle = c^+_{k,\sigma} |0\rangle \quad , \quad |0\rangle = c_{k,\sigma} |k, \sigma\rangle \quad ,$$

with $|0\rangle$ being the no electron state. The Hamiltonian H_K reads then

$$H_K = H_d + \frac{U}{N} \sum_{k,k'} c^+_{k,+} c^+_{K-k,-} c_{K-k',-} c_{k',+} \quad ,$$

with $N >> 1, U > 0$ being the number of atomic sites, making up the three-dimensional crystal, and the Hubbard constant, respectively. Note that the Hamiltonian used by Cooper [28] is identical to $H_{K=0}$, but with $U < 0$.

H_K sustains [3] a single *bound* pair eigenstate, the energy $\varepsilon_B(K)$ of which is obtained by solving

$$\frac{1}{U}=\frac{1}{N}\sum_k \frac{1}{\varepsilon_B(K)-\varepsilon(K,k)}=\int_{-t_K}^{t_K}\frac{\rho_K(\varepsilon)}{\varepsilon_B(K)-\varepsilon}d\varepsilon. \quad (5)$$

$\pm t_K$ are the upper and lower bounds of the *two-electron* band, i.e. the maximum and minimum of the *two-electron* energy $\varepsilon(K,k)=\epsilon(k)+\epsilon(K-k)$ over k with K kept fixed, whereas $\rho_K(\varepsilon)$ is the corresponding *two-electron* density of states, taken equal to

$$\rho_K(\varepsilon)=\frac{2}{\pi t_K}\sqrt{1-\left(\frac{\varepsilon}{t_K}\right)^2} \quad , \quad -t_K\leq\varepsilon\leq t_K \quad .$$

The dispersion curves $\varepsilon_B(K)$ are plotted in Fig.1. Though Eq.(5) is identical to the equation yielding the Cooper pair energy [28], their respective properties are quite different :

- the data in Fig.1 have been calculated with $U>0$, rather than $U<0$ favoured by Cooper [28] and BCS [1], because, due to the inequality [3] $U\frac{\partial\mu}{\partial c_s}<0$, choosing $U<0$ entails $\frac{\partial\mu}{\partial c_s}>0$, which has been shown *not* to be consistent with persistent currents [4], thermal equilibrium [3], the Josephson effect [5] and occurence [6] of superconductivity. As a further consequence of $U>0$, $\varepsilon_B(K)$ shows up in the upper gap of the *two-electron* band structure ($\Rightarrow\varepsilon_B(K)>t_K$) rather than in the lower gap ($\Rightarrow\varepsilon_B(K=0)<-t_K$) in case of the Cooper pair [28]. Nevertheless the bound pair is thermodynamically *stable*, because every *one-electron* state of energy $\epsilon(k)\leq E_F(T_c,c_0)$ is *occupied* for a degenerate Fermi gas [10], so that, due to Pauli's principle, a bound electron *pair* of energy $\varepsilon_B(K)=2E_F(T_c,c_0)$, according to Eq.(2), *cannot* decay into two *one-electron* states $\epsilon(k)\leq E_F,\epsilon(K-k)\leq E_F$ such that
$$\epsilon(k)+\epsilon(K-k)\leq\varepsilon_B(K) \quad ;$$
- a remarkable feature in Fig.1 is that $\varepsilon_B(K)\rightarrow t_K$ for $U\rightarrow t_K/2$, so that there is *no* bound pair for $U<t_K/2$ (accordingly, the dashed curve is no longer defined in Fig.1 for $\frac{Ka}{\pi}<.13$), in marked contrast with the opposite conclusion drawn by Cooper [28], that there is a Cooper pair, even for $U\rightarrow 0$. This discrepancy results from the *three-dimensional* Van Hove singularities, showing up at both *two-electron* band edges $\rho_K(\varepsilon\rightarrow\pm t_K)\propto\sqrt{t_K-|\varepsilon|}$, unlike the *two-electron* density of states, used by Cooper [28] which is constant and thence displays no such singularity. Likewise the width of Cooper's *two-electron* band is equal to a Debye phonon energy $2t_{K=0}=\omega_D\approx 30\text{meV}<<E_F\approx 3\text{eV}$. Hence the resulting small concentration of superconducting electrons, $\frac{c_s(T=0)}{c_0}\approx\frac{\omega_D}{E_F}\approx .01$, entails that London's length should be at least 10 times *larger* than observed values [19–22];
- at last Cooper's assumption $U<0$ implies $\varepsilon_B/2<E_F(T_c)$, which is typical of a *first order* transition but runs afoul at all measurements, proving conversely the superconducting transition to be of *second order* ($\Rightarrow\varepsilon_B/2=E_F(T_c)$ in accordance with Eq.(2)).

The bound pair of energy $\varepsilon_B(K)$ turns [3], at finite concentration c_s, into a MBE state, characterised by $\mu(c_s)$. Its properties have been calculated thanks to a variational procedure, displaying several merits with respect to that used by BCS [1] :

- it shows that $\mu(0)=\varepsilon_B/2$;
- the energy of the MBE state has been shown to be exact for $|U|\rightarrow\infty$;
- an analytical expression has been worked out for $\frac{\partial\mu}{\partial c_s}(K,c_s=0)$ as :
$$\frac{\partial\mu}{\partial c_s}(K,c_s=0)=-\frac{\int_{-t_K}^{t_K}\frac{\rho_K(\varepsilon)}{(\varepsilon_B(K)-\varepsilon)^3}d\varepsilon}{2\left(\int_{-t_K}^{t_K}\frac{\rho_K(\varepsilon)}{(\varepsilon_B(K)-\varepsilon)^2}d\varepsilon\right)^2} \quad . \quad (6)$$

Note that $\varepsilon_B(K)\rightarrow t_K\Rightarrow\left|\frac{\partial\mu}{\partial c_s}(K,c_s=0)\right|\rightarrow\infty$.

At last, focussing on H_K instead of the Hubbard Hamiltonian H, as done above, deserves a special comment. H can be written as

$$H=H_d+\sum_K(H_K-H_d) \quad .$$

The bound electron pair of eigenenergy $\varepsilon_B(K)$ is indeed an eigenstate of both H_K *and* H, whereas the BCS scheme [1], introduced to approximate the *many bound electron* eigenstate ψ_K of H_K, *cannot* be applied to H. However H_K is not a *realistic* Hamiltonian, because it involves four-electron forces, whenever it is Fourier transformed back into real space. Thus ψ_K is seen to be of little significance, unless it happens to be an eigenstate of H too. Such a claim, which has been demonstrated [25] but within a Hilbert space, containing the usual [10] one S, as a subspace, will be proved in S too, in a forthcoming publication.

3-T_c versus electron concentration. – The T_c dependence on c_0 will be discussed by assigning to the *one-electron* density of states the expression valid for free electrons [10]

$$\rho(\epsilon)=\eta\sqrt{\epsilon-\epsilon_b}\Rightarrow c_0=\frac{2}{3}\eta\left(E_F\left(0,c_0\right)-\epsilon_b\right)^{\frac{3}{2}} \quad , \quad (7)$$

with $\eta=\frac{\sqrt{2}m^{\frac{3}{2}}V}{\pi^2\hbar^3}$, whereas $\epsilon_b,m,V=17\text{Å}^3$ stand for the bottom of the conduction band, electron mass and volume of the unit-cell, respectively. With help of Eq.(4), Eqs.(2,3) can be recast into a system of two equations

$$\begin{array}{l} E_F(0,c_0)-\frac{\rho'}{\rho}\frac{(\pi k_BT_c)^2}{6}-\frac{\varepsilon_B(K)}{2}=0 \\ \left(\rho+\rho''\frac{(\pi k_BT_c)^2}{6}\right)^{-1}+\frac{\partial\mu}{\partial c_s}(K,c_s=0)=0 \end{array} \quad , \quad (8)$$

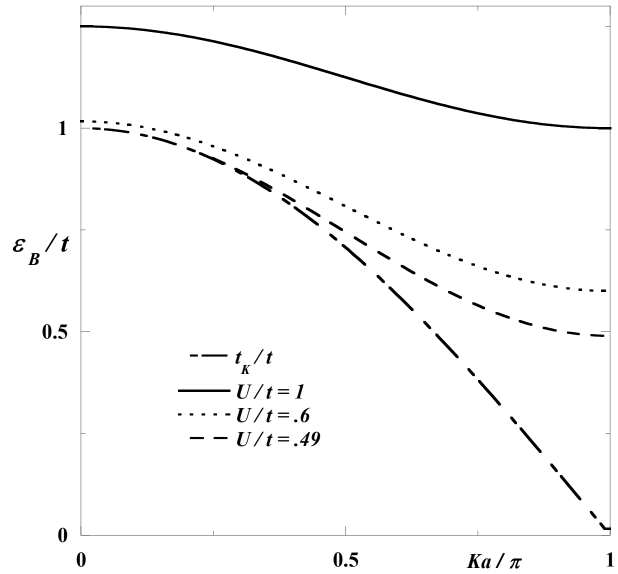

Fig. 1: Dispersion curves of t_K as a dashed-dotted line, and of $\varepsilon_B(K)$ as solid, dotted and dashed lines, associated with various U values, respectively; those data have been obtained with $t_K = t\cos(Ka/2)$, where t, a are the *one-electron* bandwidth and the lattice parameter, respectively.

to be solved for the two unknowns $c_0(T_c), t_K(T_c)$ with T_c being dealt with as a disposable parameter.

To that end, starting values are assigned to U, t_K, which gives access to $\varepsilon_B(K)$, $\frac{\partial\mu}{\partial c_s}(K, c_s = 0))$ and thence to $E_F(0, c_0)$, ϵ_b and finally to c_0, owing to Eqs.(2,3,7). Those values of c_0, t_K are then fed into Eqs.(8) to launch a Newton procedure, yielding the solutions $c_0(T_c), t_K(T_c)$. The results are presented in table 1. Since we intend to apply this analysis to high-T_c compounds [7], we have focused upon low concentrations $c_0 < 0.2$, which entails, in view of Eqs.(4,7), that $\left|\frac{\partial\mu}{\partial c_s}\right|$ takes a high value. This requires in turn $\varepsilon_B(K) \to t_K$ (see Eq.(6)) and thence [3] $U \to \frac{t_K}{2}$, in agreement with $\frac{t_K}{U} \approx 2$ in table 1.

A remarkable property of the data in table 1 is that c_0, t_K are barely sensitive to large variations of T_c, i.e. $|\delta c_0| < 10^{-3}, |\delta t_K| < 10^{-5}$ for $\delta T_c \approx 400$K. This can be understood as follows : taking advantage of Eqs.(2,4,7) results into

$$\frac{2E_F(0,c_0)}{\varepsilon_B(K)} - 1 = \frac{\pi^2}{12}\left(\frac{k_B T_c}{\Delta(T_c)}\right)^2 \quad ,$$

which, due to $\frac{dt_K}{dT_c} \approx 0, \Delta(T_c) \approx 1$eV, $T_c = 400$K, yields indeed $\delta c_0 = c_0(400\text{K}) - c_0(1\text{K}) \approx 10^{-3}$, in agreement with the data in table 1. Such a result is significant in two respects, regarding high-T_c compounds, for which c_0 can be varied over a wide range :

- because of $\frac{dc_0}{dT_c} \approx 0$, the *one-electron* band structure can be regarded safely as c_0 independent, which enhances the usefulness of the above analysis;

Table 1: Solutions $c_0(T_c), t_K(T_c), \Delta(T_c)$ $(\Delta(T_c) = E_F(0, c_0(T_c)) - \epsilon_b)$ of Eqs.(8); t_K, Δ, U are expressed in eV, whereas the unit for c_0 is the number of conduction electrons per atomic site.

$T_c(K)$	c_0	t_K	Δ
1	0.10215	6	1.1976
400	0.10225	5.9999	1.1984

$U = 3.39$

$T_c(K)$	c_0	t_K	Δ
1	0.14897	2	1.5402
400	0.14906	1.9999	1.5407

$U = 1.04$

$T_c(K)$	c_0	t_K	Δ
1	0.19158	4	1.8214
400	0.19167	3.9999	1.8219

$U = 2.2$

- the large doping rate up to ≈ 0.2 is likely to give rise to local fluctuations of c_0. Hence, in view of the utmost sensitivity of T_c with respect to c_0, this will result into a heterogeneous sample, consisting in domains, displaying T_c varying from 0 up to a few hundreds of K. Thus the observed T_c turns out to be the upper bound of a broad distribution of T_c values, associated with superconducting regions, the set of which makes up a percolation path throughout the sample. However, if the daunting challenge of making samples, wherein local c_0 fluctuations would be kept well below 10^{-4}, could be overcome, this might pave the way to superconductivity at *room* temperature.

4-T_c versus inter-electron coupling. – The T_c dependence upon U will be analysed with the *one-electron* density of states

$$\rho(\epsilon) = \frac{4}{\pi t}\sqrt{1 - \left(1 - \frac{\epsilon}{t}\right)^2} \quad ,$$

where $2t$ stands for the *one-electron* bandwidth $(\Rightarrow 0 \leq \epsilon \leq 2t)$. Our purpose is to determine the unknowns $t_K(E_F, T_c), U(E_F, T_c)$ with

$$E_F = E_F(T = 0, c_0) \quad , \quad c_0 = \int_0^{E_F} \rho(\epsilon) d\epsilon \quad .$$

To that end, Eq.(3) will first be solved for t_K by replacing $\frac{\partial E_F}{\partial c_n}(T_c, c_0)$, $\frac{\partial\mu}{\partial c_s}(0)$ by their expressions given by Eqs.(4,6), while taking advantage of Eq.(2). Then the obtained t_K value is fed into Eq.(5) to determine U. The results are presented in Fig.2.

It can be noticed that there is no solution for $c_0 > .75$, because $\frac{\partial E_F}{\partial c_n}(T_c, c_0) \approx \frac{1}{\rho}(E_F(0, c_0))$ and $\frac{\partial\mu}{\partial c_s}(0) > \frac{U}{2}$ decrease and increase, respectively, with increasing c_0, so that Eq.(3) can no longer be fulfilled eventually. This property is indeed *observed* in high T_c compounds, for which T_c decreases down to 0 for $c_0 > .25$.

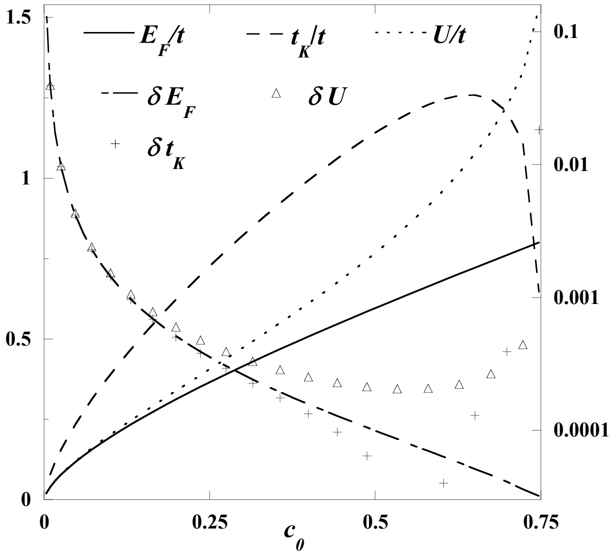

Fig. 2: Plots of $E_F(T_c, c_0), t_K(T_c, c_0), U(T_c, c_0)$ calculated for $T_c = 1$K and $t = 3$eV; the unit for c_0 is the number of conduction electrons per atomic site; δf with $f = E_F, t_K, U$ is defined as $\delta f = \left|1 - \frac{f(300K, c_0)}{f(1K, c_0)}\right|$; the scale is linear for $\frac{E_F}{t}, \frac{t_K}{t}, \frac{U}{t}$ on the left hand side, but logarithmic for dimensionless $\delta E_F, \delta t_K, \delta U$ on the right hand side.

But the most significant feature is that δU is almost insensitive to large T_c variation, except for $E_F \rightarrow 0$, i.e. for E_F close to the Van Hove singularity, located at the bottom of the *one-electron* band, which has two consequences:

- c_0 cannot be varied in most superconducting materials, apart from high-T_c compounds, so that U is unlikely to be equal to $U(c_0)$, indicated in Fig.2. Conversely, since high-T_c compounds allow for wide c_0 variation, c_0 can be tuned so that $U = U(c_0)$;
- the only possibility for a non high-T_c material to turn superconducting is then offered at the bottom of the band, because δU becomes large due to $\frac{\rho'}{\rho}(E_F \rightarrow 0) \propto \frac{1}{E_F}$ in Eq.(4). Such a conclusion, that superconductivity was likely to occur in the vicinity of a Van Hove singularity in low-T_c materials, had already been drawn independently, based on magnetostriction data [3].

It will be shown now that the *one* and *two*-electron densities of states $\rho(\epsilon), \rho_K(\varepsilon)$ cannot stem from the same *one-electron* band. The proof is by contradiction. As a matter of fact the *one-electron* density of states should read in that case

$$\rho(\epsilon) = \frac{4}{\pi t}\sqrt{1 - \left(\frac{\epsilon}{t}\right)^2} \quad .$$

Hence $U > 0$ entails, in view of Fig.1 and Eq.(2), that there is $\frac{\varepsilon_B}{2} = E_F > 0$, which implies $\rho'(E_F) < 0$ in contradiction with Eq.(3). Accordingly, since the two different *one-electron* bands, giving rise to $\rho(\epsilon), \rho_K(\varepsilon)$, respectively, display a sizeable overlap, they should in addition belong to *different* symmetry classes of the crystal point group, so that superconductivity *cannot* arise if there are only s-like electrons at E_F (no alcali or noble metal is indeed superconducting) or if the point group reduces to identity. At last superconductivity is inferred *not* to occur in case of an almost full conduction band, because it is tantamount to E_F being located near the upper band-edge and thence implies $\rho'(E_F(T_c, c_0)) < 0$, in contradiction with Eq.(3). Noteworthy is that all of those hereabove conclusions had already been drawn *empirically* [2].

Conclusion. – This work, combined with our previous publications [3–6, 19–22], is aimed at presenting the reader with a comprehensive, albeit elementary solution, based mainly on thermodynamics, of the long-standing riddle of superconductivity. The critical temperature T_c has been calculated for conduction electrons, coupled via a *repulsive* force, within a model based on two conditions, expressed in Eqs.(2,3) and characterising a second order transition happening at T_c. It should be noted that Eq.(3), unlike Eq.(2), is *difficult* to fulfil. Likewise, the unfortunate consequences of an *attractive* inter-electron force, assumed by BCS [1], have been analysed.

Superconductivity occurring in conventional materials has been shown to require $E_F(T_c)$ being located near a Van Hove singularity of the *one-electron* band structure, whereas a practical route towards still higher T_c values has been delineated in high-T_c compounds, provided the *local* electron concentration can be controlled accurately. At last, the thermodynamical criterions in Eqs.(2,3) unveil the close interplay between independent and bound electrons in giving rise to superconductivity.

Since magnetic (ferromagnetic, antiferromagnetic, ferrimagnetic, static spin-wave...) transitions are also of second order and involve only conduction electrons [29], it might be tempting to apply the thermodynamical conditions in Eqs.(2,3) to the study of this case. However this cannot be done, because the electrons, responsible for the magnetic long-range order, are localised or Wannier-like [10], unlike the normal and bound electrons which are itinerant and Bloch-like [10]. Hence tackling the magnetic issue will require a dedicated approach.

REFERENCES

[1] SCHRIEFFER J.R., *Theory of Superconductivity* (Addison-Wesley) 1993

[2] MATTHIAS B. T., GEBALLE T. H. and COMPTON V. B., *Rev.Mod.Phys.*, **35** (1963) 1

[3] SZEFTEL J., SANDEAU N. and ABOU GHANTOUS M., *J.Supercond.Nov.Magn.*, **33** (2020) 1307

[4] SZEFTEL J., SANDEAU N. and ABOU GHANTOUS M., *Eur.Phys.J.B*, **92** (2019) 67

[5] SZEFTEL J., SANDEAU N. and ABOU GHANTOUS M., *arXiv* : 2003.07196

[6] Szeftel J., Sandeau N., Abou Ghantous M. and El-Saba M., *J.Supercond.Nov.Magn.*, **34** (2021) 37
[7] Armitage N.P., Fournier P. and Greene R.L., *Rev.Mod.Phys.*, **82** (2010) 2421
[8] Zaanen J., *100 years of superconductivity*, edited by Rochalla H. and Kes P.H. (Chapman and Hall) 2011 (also available in *arXiv* : 1012.5461)
[9] Lederer P., *arXiv* : 1510.08808
[10] Ashcroft N.W. and Mermin N. D., *Solid State Physics* (Saunders College) 1976
[11] Carbotte J.P., *Rev.Mod.Phys.*, **62** (1990) 1027
[12] Pietronero L., Straessler S. and Grimaldi C., *Phys.Rev.B*, **52** (1995) 10516
[13] Grimaldi C., Pietronero L. and Straessler S., *Phys.Rev.B*, **52** (1995) 10530
[14] Ruiz H.S., Giraldo J.J. and Baquero R., *J.Supercond.Nov.Magn.*, **21** (2008) 21
[15] Szczesniak D. and Szczesniak R., *Phys.Rev.B*, **99** (2019) 224512
[16] Schrodi F., Oppeneer P.M. and Aperis A., *Phys.Rev.B*, **102** (2020) 024503
[17] Sadovskii M.V., *J.Supercond.Nov.Magn.*, **33** (2020) 19
[18] Froehlich H., *Phys.Rev.*, **79** (1950) 845
[19] Szeftel J., Sandeau N. and Khater A., *Phys.Lett.A*, **381** (2017) 1525
[20] Szeftel J., Sandeau N. and Khater A., *Prog.In.Electro.Res.M*, **69** (2018) 69
[21] Szeftel J., Abou Ghantous M. and Sandeau N., *Prog.In.Electro.Res.L*, **81** (2019) 1
[22] Szeftel J., Sandeau N., Abou Ghantous M. and Khater A., *EuroPhys.Lett.*, **131** (2020) 17003
[23] Landau L.D. and Lifshitz E.M., *Statistical Physics* (Pergamon Press) 1959
[24] Gorter C.J. and Casimir H., *Physica*, **1** (1934) 306
[25] Szeftel J. and Khater A., *Phys.Rev.B*, **54** (1996) 13581
[26] Szeftel J., *Electron Correlations and Material Properties 2*, edited by Gonis A. and Kioussis N. and Ciftan M. (Kluwer Academic, New York) 2003
[27] Szeftel J. and Caffarel M., *J.Phys.A*, **37** (2004) 623
[28] Cooper L.N., *Phys.Rev.*, **104** (1956) 1189
[29] Lévy L.P., *Magnetism and Superconductivity* (Springer) 2000