

IDENTIFICATION OF COMPOSITE COMBINATIONS: KEY TO VALIDATE GOLDBACH CONJECTURE

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Abstract:

This paper discusses a possible approach to validate the Goldbach conjecture which states that all even numbers can be expressed as a summation of two prime numbers. For this purpose the paper begins with the concept of successive-addition-of-digits-of-an-integer-number (SADN) and its properties in terms of basic algebraic functions like addition, multiplication and subtraction. This concept of SADN forms the basis for classifying all odd numbers into 3 series- the S1, S3 and S5 series- which comprise of odd numbers of SADN(7,4,1), SADN(3,9,6) and SADN(5,2,8) respectively and follow a cyclical order. The S1 and S5 series are of interest in the analysis since they include both prime and composite numbers while the S3 series exclusively consists of composite numbers. Furthermore, the multiplicative property of SADN shows why composites on the S1 series are derived as products of intra-series elements of the S1 and S5 series while composites on the S5 series are derived as products of inter-series elements of the S1 and S5 series. The role of SADN is also important in determining the relevant series for identifying the combination of primes for a given even number since it shows why such combinations for even numbers of SADN(1,4,7) will be found on the S5 series while those for even numbers of SADN(2,5,8) will lie on the S1 series and both the series have a role to play in identifying the prime number combinations for even numbers with SADN(3,6,9). Thereafter, the analysis moves to calculating the total number of acceptable combinations for a given even number that would include combinations in the nature of two composites (c_1+c_2), one prime and one composite ($p+c$) and two primes (p_1+p_2). A cyclical pattern followed by even numbers is also discussed in this context. Identifying the c_1+c_2 and $p+c$ combinations and thereafter subtracting them from the total number of combinations will yield the number of p_1+p_2 combinations. For this purpose the paper discusses a general method to calculate the number of composites on the S1 and S5 series for a given number and provides a detailed method for deriving the number of c_1+c_2 combinations. The paper presents this analysis as a proof to validate the Goldbach conjecture.

Since even numbers can be of SADN 1 to 9 and the relation between nTc (i.e. total number of acceptable combinations) and nc (i.e. number of composites) for all even numbers can either be of $nTc > nc$ or $nTc \leq nc$, the paper shows that the Goldbach conjecture is true for both these categories of even numbers. In this manner this analysis is totally inclusive of all even numbers in general terms and since the analysis of every even number is common in methodology but unique in compilation, apart from being totally inclusive, it is also mutually exclusive in nature.

This proves that the Goldbach conjecture which states that all even numbers can be expressed as atleast one combination of two prime numbers holds true for all even numbers, across all categories possible. Additionally this approach proves that the identification of p_1+p_2 combinations which would validate the Goldbach conjecture lies in the identification of c_1+c_2 combinations.

Keywords: Goldbach conjecture, Goldbach problem, Primes, Distribution of Primes, Primes and integers, Additive questions involving primes

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1

Introduction**Statement of Goldbach conjecture & methodology discussed to address it**

On 7th of June, 1742, a Prussian amateur mathematician and historian Christian Goldbach, wrote a letter to Leonard Euler, the content of which is later understood to have lead to saying ‘at least it seems that every even number that is greater than 2 is the sum of two primes’[1,2]. In 1938 Nils Pipping showed that the Goldbach conjecture is true for even numbers up to and including 100,000 [3]. It has been established, using a computer search, that it is true for even numbers up to and including 4,000,000,000,000,000 [4]. The unproven Goldbach conjecture enjoyed the dual importance of being one of such theorems which were easy enough to be guessed by any fool and yet not proven [5]. It is generally because of these paradoxical properties attached with Goldbach conjecture that British Mathematicians G.H.Hardy and J.E.Littlewood attempted to prove it using partition functions and resulted in bringing this problem into contact with the then recognised methods of the Analytic Theory of Numbers [6,7] and the problem was found worthy of being included as selected problems in number theory [8]. Interestingly ternary Goldbach conjecture, also an offshoot of the abovementioned exchange of letters between Euler and Goldbach, has been proved by Helfgott to be true for odd numbers greater than 5 [9]. In this context, although the attempts by Pogorzelski have not received any objections, they have not been accepted either, till date [10, 11].

Watanabe observes that the number of prime & prime combinations for even numbers would be less in case if half of even number in consideration is a prime as compared to number of prime & prime combinations in case if half of even number in consideration is composite [12]. In present work a logical and conclusive derivation is presented for this observation as well.

The present paper’s approach is to validate the Goldbach conjecture which states that all even numbers can be expressed as a summation of two prime numbers. For this purpose the paper begins with a brief discussion of the concept of successive-addition-of-digits-of-an-integer-number (SADN) and its properties in terms of basic algebraic functions like addition, multiplication and subtraction. This concept of SADN forms the basis for classifying all odd numbers into 3 series-the S1, S3 and S5 series- which comprise of odd numbers of SADN (7,4,1), (3,9,6) and (5,2,8) respectively and follow a cyclical order. The S1 and S5 series are of interest in the analysis since they include both prime and composite numbers while the S3 series exclusively consists of composite numbers except the number ‘3’. Furthermore, the multiplicative property of SADN shows why composites on the S1 series are derived as products of intra-series elements of the S1 and S5 series while composites on the S5 series are derived as products of inter-series elements of the S1 and S5 series. The role of SADN is also important in determining the relevant series for identifying the combination of primes for

a given even number since it shows why such combinations for even numbers of SADN 1,4 and 7 will be found on the S5 series while those for even numbers of SADN 2,5 and 8 will lie on the S1 series and both the series have a role to play in identifying the prime number combinations for even numbers with SADN 3,6 and 9. Thereafter, the analysis moves to calculating the total number of acceptable combinations for a given even number that would include combinations in the nature of two composites (c_1+c_2), one prime and one composite ($p+c$) and two primes (p_1+p_2). A cyclical pattern followed by even numbers is also discussed in this context. Identifying the c_1+c_2 and $p+c$ combinations and thereafter subtracting them from the total number of combinations will yield the number of p_1+p_2 combinations. For this purpose the paper discusses a general method to calculate the number of composites on the S1 and S5 series for any given even number and provides a detailed method for deriving the number of combinations of type c_1+c_2 . The paper thereafter introduces the concept of minimum required number of combinations of type c_1+c_2 to identify a combination of type p_1+p_2 ; for any given even number. The relation between minimum required number of c_1+c_2 combinations and actual number of c_1+c_2 combinations forms the basis for identifying the possibility of existence of combination of type p_1+p_2 . The paper presents this analysis as a proof to validate the Goldbach conjecture.

2

Successive Addition of Digits of a Number (SADN)

Definition and Properties:

- What is SADN?
- What are the properties of SADN?
- Proof of the properties of SADN

The function of SADN stands for Successive Addition of Digits of (integer) Number. SADN function exhibits following properties:-

- i. Idempotence
- ii. Range of SADN function
- iii. Distribution over addition
- iv. Distribution over multiplication
- v. Additive Identity for SADN function
- vi. Interchangeability of non-positive SADN and positive SADN
- vii. Distribution over subtraction
- viii. Multiplicative Identity for SADN function

i. Property of Idempotence:-

SADN stands for successive-addition-of-digits-of-number. Addition is an operation which is operated upon multiple operands and not defined in case of a single operand alone. Here the term 'successive' implies that the digits of the number are to be added as long as the operation of addition is defined. It implies that to determine SADN of any given number, its digits are to be successively added until a single digit is obtained. This single digit is termed as SADN of the given number.

Example: Suppose the given number is 546289. Addition of its digits= $5+4+6+2+8+9=34$

Successive addition of digits= $3+4=7$

In our example, SADN of (546289)=7

In general terms: SADN of (x)= SADN of (SADN of (SADN of (...SADN of (x)))) which says that SADN function is an idempotent function.

ii. Range of SADN function:-

The property of idempotence implies that the value of SADN function for any non-zero integer number would be a single digit integer only ranging from 1 to 9.

$1 \leq \text{SADN of } (x) \leq 9$ implies that $\text{SADN of } (x) = \{1,2,3,4,5,6,7,8,9\}$

iii. Property of distribution over addition:-

Present paper identifies the phenomenon of SADN as analogous to '*valency of an atom*'. In case of an atom, its inner shell gets filled by as much number of electrons as suggested by the octet rule and the outermost shell is known as valence shell. Electrons occupying the valence-shell are called as valence-electrons. Just as valence-electrons are responsible for the properties exhibited by the corresponding atom, the current-row(as discussed below) plays a central role in portraying the properties of SADN of an integer.

Consider the natural numbers written in matrix form where number of rows is nine [as 9 is the maximum possible value of SADN of any integer] and number of columns goes on increasing. Upon writing in this form, we get a sample matrix M1 for natural numbers from one(1) to fifty(50) as following:-

Row(1) having elements of SADN(1):	1	10	19	28	37	46
Row(2) having elements of SADN(2):	2	11	20	29	38	47
Row(3) having elements of SADN(3):	3	12	21	30	39	48
Row(4) having elements of SADN(4):	4	13	22	31	40	49
Row(5) having elements of SADN(5):	5	14	23	32	41	50
Row(6) having elements of SADN(6):	6	15	24	33	42	
Row(7) having elements of SADN(7):	7	16	25	34	43	
Row(8) having elements of SADN(8):	8	17	26	35	44	
Row(9) having elements of SADN(9):	9	18	27	36	45	

Matrix M1

Through this matrix M1, we discuss about SADN as follows:

In order to determine the SADN of an integer 'n'; we write numbers, starting from one(1), till the integer 'n'. The column in which the integer 'n' exists, is called as current column. All columns filled before the current column are known as complete columns. For determining SADN of 'n'; we don't bother about the number of complete columns and count the number of rows of the current column. Number of row in which integer(n) exists; is termed as SADN of the integer(n).

In this way, the current column in case of SADN is similar to what valence-shell is in case of an atom.

Distributive over addition:

SADN of $(x+y)$ = SADN of (x) + SADN of (y)

SADN function is distributive over addition.

Proof:-

SADN of integers x and y denotes the number of rows of current columns corresponding to x and y as mentioned in matrix $M1$. In case of addition: Suppose SADN of x is given as x' and SADN of y is given as y' . Here x' and y' would be natural numbers such that the value of x' and y' cannot exceed the number 9; as there are only 9 rows in matrix $M1$. Either $x'+y'$ would be ≤ 9 (i.e. first case) or it would be > 9 (i.e. second case) but can never be greater than 18, which is the case if both x' and y' attain their maximum possible values; i.e. 9.

In first case when $x'+y'$ would be ≤ 9 :-

If we write numbers upto x as per matrix $M1$ and call this arrangement as matrix $M1x$, we may get some number of complete columns alongwith x' rows in current column and upon writing numbers upto y and call it matrix $M1y$, we may get some number of complete columns alongwith y' rows in current column. Now if we write numbers upto $x+y$ as matrices $M1x$ and $M1y$ standing side by side, the number of complete columns remains as summation of number of complete columns of $M1x$ and $M1y$. In present case the summation of number of filled rows (i.e. x') in current column of $M1x$ and that of in $M1y$ gives us $x'+y'$. As $x'+y' \leq 9$, it implies that no extra complete column is generated upto the integer $x+y$.

Hence SADN of $(x+y)$ = SADN of (x) + SADN of (y) in first case.

In second case when $x'+y'$ would be > 9 :-

If we write numbers upto x as per matrix $M1$ and call this arrangement as matrix $M1x$, we may get some number of complete columns alongwith x' rows in current column and upon writing numbers upto y and call it matrix $M1y$, we may get some number of complete columns alongwith y' rows in current column. Now if we write numbers upto $x+y$ as matrices $M1x$ and $M1y$ standing side by side, the number of complete columns remains, (till now) as addition of number of complete columns of $M1x$ and $M1y$. In present case the summation of number of filled rows (i.e. x') in current column of $M1x$ and that of in $M1y$ gives us $x'+y'$. As $x'+y' > 9$, it implies that one extra complete column is generated if we write numbers upto the integer $x+y$ as arranged in matrix $M1$. As number of rows in this extra, newly generated, complete column would be 9 only; number of filled rows of current column would become $x'+y'-9$. As in present case, $9 < x'+y' \leq 18$; there will be total of 9 subcases, corresponding to $x'+y'$ equals to either 10

or 11 or 12 or 13 or 14 or 15 or 16 or 17 or 18. In these subcases the corresponding row number of current column would be 1 or 2 or 3 or 4 or 5 or 6 or 7 or 8 or 9 respectively.

As 1 to 9 are SADN of 10 to SADN of 18 respectively i.e. 10 is the 1st number after removing initial 9 numbers and similarly 18 is the 9th number after removing initial 9 numbers. Hence in this case as well, if numbers upto $x+y$ are arranged in terms of matrix M1 and this arrangement is denoted as $M1(x+y)$, then SADN of $(x+y)$ is summation of individual SADNs of the integers x and y .

Hence SADN of $(x+y) = \text{SADN of } (x) + \text{SADN of } (y)$ in second case as well.

Abovementioned analysis allows us to say that in any of the possible cases:

SADN of $(x+y) = \text{SADN of } (x) + \text{SADN of } (y)$

i.e. SADN function is distributive over addition.

Example: SADN of (28) + SADN of (541) = SADN of (28+541) = SADN of (569)

Under property of idempotence SADN of (569) = SADN of (20) = 2

iv. Property of distribution over multiplication:-

SADN of $(x.y) = \text{SADN of } (x). \text{SADN of } (y)$

SADN function is distributive over multiplication.

Proof:

Suppose SADN of x is given as x' and SADN of y is given as y' .

Since SADN of $(x.y) = \text{SADN of } (x+x+x+ \dots y \text{ times}) = \text{SADN of } x + \text{SADN of } x + \dots y \text{ times}$
 $= y. [\text{SADN of } (x)] = y. x' \quad [\text{by property of distribution over addition, as discussed above}]$

Implies that SADN of $(x.y) = y.x' \dots \dots \dots [\text{equation 1}]$

Property of idempotence of SADN function says that:-

SADN of $(x.y) = \text{SADN of } [\text{SADN of } (x.y)] = \text{SADN of } (y.x') = x'.y' \text{ i.e. } [\text{SADN of } (x)].$
 $[\text{SADN of } (y)] \dots \dots \dots [\text{applying equation 1}]$

Implies that SADN of $(x.y) = [\text{SADN of } (x)]. [\text{SADN of } (y)]$

Example: As shown below: SADN of (12) . SADN of (15)= SADN of (12x15)=SADN of (180)

SADN of 12 = 3; SADN of 15 = 6;

[SADN of 12].[SADN of 15]= 3x6 = 18 = 9[property of idempotence]

SADN of (12x15) = SADN of (180) = 9[property of idempotence]

v. Additive Identity for SADN function:-

As SADN is primarily a type of addition operator, additive identity zero(0) acts as an additive identity for SADN as well.

Apart from zero, the number nine(9) also acts as an additive identity for SADN function.

Proof:

The reason of 9 being an additive identity for SADN function is as follows:-

In context of arranging the integers as per the matrix M1, SADN of any integer is identified by the number of row in which that integer lies in current column. Placing the digit 9 anywhere in an integer changes the integer in such a way that the new integer has more number of complete columns of 9 numbers but the number of filled rows in current column remains unchanged. As the number of filled rows of current column denotes the SADN of integer, hence introduction of digit 9 within the integer doesn't affect the SADN of that integer. This drives us to conclude that the number 9 acts as additive identity for SADN function.

Example: SADN of (52)= SADN of (529)=SADN of (5092990)=7

vi. Interchangeability of non-positive SADN and positive SADN:-

Properties of distribution over addition and identity of SADN function leads to following equivalence between nonpositive and positive values of SADN

Positive digit	SADN	1	2	3	4	5	6	7	8	9
Equivalent non-positive digit	SADN	-8	-7	-6	-5	-4	-3	-2	-1	0

Table 2.1: Equivalence between nonpositive and positive values of SADN

Both numbers of same columns are considered to be identical and replaceable substitutes of one another if need arises to consider non-positive digit for SADN.

This above-mentioned table-2.1 of interchangeability relates negative and positive SADN as follows:

$$\text{SADN of } (-a) = - \text{ SADN of } (a)$$

vii. Distribution over subtraction:-

Under application of Table-2.1 and property of distribution over addition:-

$$\text{SADN of } (x) - \text{SADN of } (y) = \text{SADN of } (x) + \text{SADN of } (-y) = \text{SADN of } (x-y)$$

$$\text{SADN of } (x) - \text{SADN of } (y) - \text{SADN of } (z) = \text{SADN of } (x-y-z) = \text{SADN of } (x) - \text{SADN of } (y+z)$$

Example1: $\text{SADN of } (724-452) = \text{SADN of } (724) - \text{SADN of } (452) = \text{SADN of } (4) - \text{SADN of } (2) = \text{SADN of } (2) = 2$

OR $\text{SADN of } (724-452) = \text{SADN of } (724) - \text{SADN of } (452) = \text{SADN of } (4) - \text{SADN of } (2) = \text{SADN of } (4) + \text{SADN of } (-2) = \text{SADN of } (4) + \text{SADN of } (7) = \text{SADN of } (11) = 2$ (refer table 1)

and $\text{SADN of } (724 - 452) = \text{SADN of } (272) = \text{SADN of } (2) = 2$

Example2: $\text{SADN of } (121-24) = \text{SADN of } (121) - \text{SADN of } (24) = \text{SADN of } (4) - \text{SADN of } (6) = -2 = 7$ (refer table 2.1)

OR $\text{SADN of } (121-24) = \text{SADN of } (121) - \text{SADN of } (24) = \text{SADN of } (4) - \text{SADN of } (6) = \text{SADN of } (4) + \text{SADN of } (-6) = \text{SADN of } (4) + \text{SADN of } (3) = \text{SADN of } (7) = 7$ (refer table 2.1)

And $\text{SADN of } (121-24) = \text{SADN of } (97) = 7$

viii. Multiplicative Identity for SADN function:-

Apart from SADN(1) as multiplicative identity for every SADN function; following are multiplicative identities as special cases:-

SADN(4,7,1) acts as multiplicative identities for SADN(3,6)

SADN(1,2,3,4,5,6,7,8,9) or SADN(n) act as multiplicative identity for SADN(9)

3

SADN of prime numbers

SADN of prime numbers

Prime numbers are a particular type of subset of natural numbers. Any prime number would be divisible by itself and by the number 1 (one) only. Primes cannot be divided by any other natural number except themselves and 1. In terms of SADN function, the divisibility test of 3 says that any natural number would be divisible by 3 only if its SADN is 3, 6 or 9. It leads to conclude that any natural number whose SADN is 3, 6 or 9 would be a composite as it would be divisible by the number 3, hence SADN of primes can never be 3, 6 or 9 (only exception to this would be the number '3' itself). This discussion in conjunction with the properties of range of SADN says that SADN of primes may be 1, 2, 4, 5, 7 or 8.

If 'p' represents a prime number, then $\text{SADN of } (p) = \{1, 2, 4, 5, 7, 8\}$

4(A).

Classifying odd numbers into 3 series based on their SADN

Classifying odd numbers into 3 series based on their SADN

Set of natural numbers $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots\}$

$$N = E + D$$

Where E is set of even numbers, $E = \{2, 4, 6, 8, 10, \dots\}$

And D = set of odd numbers = $\{1, 3, 5, 7, 9, 11, \dots\}$

All elements of the set of even numbers 'E' are composites (with exception of the number 2) whereas elements of the set of odd numbers 'D' may be prime or composite.

Now consider the following three series of odd numbers:-

$S_1 = a_1 + 6n$ where a_1 is 1 and n is a natural number implying

$$S_1 = 1 + 6n; n \in \{N\}$$

$$S_1 = \{7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, \dots\}$$

SADN of (element of S_1) = $\{7, 4, 1\}$ in cyclic order

SADN of $(a_1 + 6n) \in S_1$ implies that SADN of $(1 + 6n) \in S_1$

$S_3 = a_3 + 6n$ where a_3 is 3 and n is a natural number including zero implying

$$S_3 = \{3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, \dots\}$$

SADN of (element of S_3) = $\{3, 9, 6\}$ in cyclic order

SADN($a_3 + 6n$) $\in S_3$ implies that SADN($3 + 6n$) $\in S_3$

$S_5 = a_5 + 6n$ where a_5 is 5 and n is a natural number including zero implying

$$S_5 = 5 + 6n; n \in \{N\}$$

$$S5 = \{5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, \dots\}$$

SADN of (element of $S5$) = $\{5, 2, 8\}$ in cyclic order

SADN of $(a5+6n) \in S5$ implies that SADN of $(5+6n) \in S5$ implies that SADN of $(6n-1) \in S5$

SADN of (n) :-	n \in :-
7,4,1	S1
3,9,6	S3
5,2,8	S5

Table 4A.1: SADN of odd numbers determines its Series out of S1, S3, S5

Set of natural numbers $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots\}$

Now we say that $N = E + D = E + \{1\} + S1 + S3 + S5$

Where E is set of even numbers, $E = \{2, 4, 6, 8, 10, \dots\}$

And $D = \text{set of odd numbers} = \{1\} + S1 + S3 + S5 = \{1, 3, 5, 7, 9, 11, \dots\}$

An additional explanation for classifying odd numbers into the above-mentioned three series is discussed in Appendix 1.

4(B).

Why S3 is of only composites whereas S1 and S5 comprises of primes and composites

Why S3 is of only composites whereas S1 and S5 comprises of primes and composites

Such segregation of odd numbers in term of three series S1, S3 and S5 leads to segregation of primes. As series S3 consists of elements of $6n+3$ type or $3x(2n+1)$ type, so all elements of S3 will be multiples of the number 3. This leads to conclude that series S3 consists of composites and no prime (number '3' being the only exception). A similar logic is that as all elements of S3 are having SADN as 3,6 or 9 leading to S3 elements beings multiples of 3 and hence being composites.

Hence prime numbers may belong to only S1 and S5. Additionally as mentioned above, series S1 has elements of type $6n+1$ and series S5 has elements of type $6n-1$. So this segregation of S1, S3 and S5 series also segregates the primes of types $6n+1$ and $6n-1$.

All $6n+1$ type primes (and composites) will be only on series S1.

All $6n-1$ type primes (and composites) will be only on series S5.

5(A).

Method for calculating total number of composites on the S1 and S5 series

Method for calculating total number of composites on the S1 and S5 series

SADN function is distributive over multiplication. An application of this property is the following table which leads to the condition for the elements of series S1 and S5.

Series	SADN of elements of series	Possible combinations of SADN of divisors to yield the composite number of particular SADN of series S1 or S5			
S1	SADN1	SADN1xSADN1	SADN2xSADN5	SADN4xSADN7	SADN8xSADN8
S1	SADN7	SADN1xSADN7	SADN2xSADN8	SADN4xSADN4	SADN5xSADN5
S1	SADN4	SADN1xSADN4	SADN2xSADN2	SADN5xSADN8	SADN7xSADN7
S5	SADN5	SADN1xSADN5	SADN2xSADN7	SADN4xSADN8	XXXXXX
S5	SADN2	SADN1xSADN2	SADN4xSADN5	SADN7xSADN8	XXXXXX
S5	SADN8	SADN1xSADN8	SADN2xSADN4	SADN5xSADN7	XXXXXX

Table 5A.1: Series and SADN of series-elements alongwith possible combinations of divisors

Comparison of Table 4A.1 and Table 5A.1 concludes that:-

- (a) Composites of SADN1 can be obtained by product of SADN(1x1) or SADN(2x5) or SADN(4x7) or SADN(8x8). In this case the only four possible combinations of both the divisor elements (1 and 1 OR 2 and 5 OR 4 and 7 OR 8 and 8) are always elements of a **single series** (S1 or S5)

- (b) Composites of SADN7 can be obtained by product of SADN(1x7) or SADN(2x8) or SADN(4x4) or SADN(5x5). In this case the only four possible combinations of both the divisor elements (1 and 7 OR 2 and 8 OR 4 and 4 OR 5 and 5) are always elements of a **single series** (S1 or S5)
- (c) Composites of SADN4 can be obtained by product of SADN(1x4) or SADN(2x2) or SADN(5x8) or SADN(7x7). In this case the only four possible combinations of both the divisor elements (1 and 4 OR 2 and 2 OR 5 and 8 OR 7 and 7) are always elements of a **single series** (S1 or S5)
- (d) Composites of SADN5 can be obtained by product of SADN(1x5) or SADN(2x7) or SADN(4x8). In this case the only three possible combinations of both the divisor elements (1 and 5 OR 2 and 7 OR 4 and 8) are always elements of **two different series** (S1 and S5)
- (e) Composites of SADN2 can be obtained by product of SADN(1x2) or SADN(4x5) or SADN(7x8). In this case the only three possible combinations of both the divisor elements (1 and 2 OR 4 and 5 OR 7 and 8) are always elements of **two different series** (S1 and S5)
- (f) Composites of SADN8 can be obtained by product of SADN(1x8) or SADN(2x4) or SADN(5x7). In this case the only three possible combinations of both the divisor elements (1 and 8 OR 2 and 4 OR 5 and 7) are always elements of **two different series** (S1 and S5)

Regarding series S1: The first three (a, b and c) of above conclusions suggest that composite numbers of series S1 are either **intra-series products** of elements of series S1 or **intra-series products** of elements of series S5.

Regarding series S5: The next three (d, e and f) of above conclusions suggest that composite numbers of series S5 are always **inter-series products** of elements of S1 and S5.

Pattern of formation of composite numbers on the S1 and S5 series:

In context of Composites on S1 series:-

As composites contained in S1 are products of intra-series elements of S1 or S5, say products of intra-series elements of S1 are denoted by C1 and that of S5 are denoted by C5:-

$C1 = [(6n+1).(6n+1) + (6n+1).6n']$; for each and every $n \in \{N\}$, there exists $n' \in \{0, N\}$

Or $C1 = (6n+1).[6(n+n')+1]$; for each and every $n \in \{N\}$, there exists $n' \in \{0, N\}$

Implies that $C1 = \{49, 91, 133, 175, \dots\} + \{91, 169, 247, 325, 403, \dots\} + \{133, 247, 361, 475, \dots\} + \{175, 325, 475, 625, \dots\} + \dots + \{\} + \dots$ infinite sets of infinite elements in each set

And $C5 = [(6n-1).(6n-1) + (6n-1).6n']$; for each and every $n \in \{N\}$, there exists $n' \in \{0, N\}$

Or $C5 = (6n-1).[6(n+n')-1]$; for each and every $n \in \{N\}$, there exists $n' \in \{0, N\}$

Implies that $C5 = \{25, 55, 85, 115, \dots\} + \{55, 121, 187, \dots\} + \{85, 187, 289, \dots\} + \{115, 253, 391, \dots\} + \dots + \{\} + \dots$ infinite sets of infinite elements in each set

From above discussion:

Primes on series $S1 = S1$ - composites of $S1 = S1 - C1 - C3$

In context of Composites on $S5$ series:-

As composites contained in $S5$ are products of interseries elements of $S1$ and $S5$, say interseries products of $S1$ and $S5$ are denoted by $C15'$

$C15' = [5(6n+1) + (6n+1).6n']$; for each and every $n \in \{N\}$, there exists $n' \in \{0, N\}$

$C15' = (6n+1).(5+6n')$; for each and every $n \in \{N\}$, there exists $n' \in \{0, N\}$

$C15' = \{35, 77, 119, 161, 203, \dots\} + \{65, 143, 221, 299, \dots\} + \dots + \{\} + \dots$ infinite sets of infinite elements in each set

Primes on series $S5 = S5$ - composites of $S5 = S5 - C15'$

The pattern of formation of composite numbers on the $S1$ and $S5$ series as mentioned above has been confirmed both graphically (for limited number of integers) and also through a computer program (again, for limited but rather large number of integers).

```
#Program for finding primes lying on S1 series and smaller than the given
even number
```

```
import csv
U_LIMIT = 20000

#initialize set with first values
s = {7}
t1 = {25}
t2 = {91}

# open file for writing results
f = open("result.txt", "w")

# generating set 's'
for x in range(2, U_LIMIT):
    s.add(1+6*x)

# generating set 't1'
for x in range(1, U_LIMIT):
    for y in range(0, U_LIMIT):
        t1.add((6*x-1)*(6*(x+y)-1))

# generating set 't2'
for x in range(1, U_LIMIT):
    for y in range(0, U_LIMIT):
        t2.add((6*x+1)*(6*(x+y)+1))

# removing t1 and t2 from s
s-=t1
s-=t2

# converting set to list for sorting
# as set are stored unordered
l=list(s)

# sorted outcome stored in list named 'result'
result=sorted(l)

# write the result from list to file

wr = csv.writer(f)
wr.writerow(result)
#for x in result:
#    f.write("%s\n" % x)

# close file
f.close()
```

```

#Program for finding primes lying on S1 series and smaller than the given
even number

import csv
U_LIMIT = 20000

#initialize set with first values
s = {5}
t1 = {25}

# open file for writing results
f = open("result.txt", "w")

# generating set 's'
for x in range(2, U_LIMIT):
    s.add(6*x-1)

# generating set 't1'
for x in range(1, U_LIMIT):
    for y in range(0, U_LIMIT):
        t1.add((6*x+1)*(5+6*y))

# removing t1 and t2 from s
s-=t1

# converting set to list for sorting
# as set are stored unordered
l=list(s)

print len(l)

# sorted outcome stored in list named 'result'
result=sorted(l)

# write the result from list to file

wr = csv.writer(f)
wr.writerow(result)
#for x in result:
#    f.write("%s\n" % x)

# close file
f.close()

```

5(B).

Number of composites for a given even number $2k$

Number of composites for a given even number $2k$

For S1 series:-

For deriving number of unique composites on S7 series:-

Composites on S7 series are derived as intra-series products of elements of the S5 and S7 series.

In order to calculate the total number of composites formed on S7 series, following steps may be followed:-

1 Deriving composites on S7 series that are formed as intra-series products of S5 series

Unique composites of S7 series formed by elements of S5 series

Step 1:-Here we first find out the total number of composites formed by a particular prime element of the S5 series. This can be done by the following formula:-

$$\text{Floor function of } [1/6\{\text{floor function of } (2k/(6n-1))\}] = I$$

$\text{ff}(2k/N)$ will tell us that how many numbers on a natural number line are such that their product with 'N' would be less than or equal to $2k$. Since elements of S5 series are denoted as $6n-1$, hence if we wish to arrive at number of such numbers, we divide $2k$ by $6n-1$. Hence we get $\text{ff}\{2k/(6n-1)\}$. Now as there is a gap of 6 numbers between any two consecutive elements of S5 series, we further divide $\text{ff}\{2k/(6n-1)\}$ by 6 and consider its ff to ensure that we end up at an element of S5 series only. This is denoted as 'I' here.

Hence $I = \text{Floor function of } [1/6\{\text{floor function of } (2k/(6n-1))\}]$

Consider the floor function 'I'. This integer 'I' indicates the total number of composites of which $6n-1$ is a factor and values of all these composites are less than $2k$. As the biggest number also needs to be on S5 series, so biggest number that is smaller than $6I$ is given as $6I-1$. The biggest such number of which $6n-1$ is a factor and whose value is less than $2k$ can be derived as

$$(6n-1) \times (6I-1)$$

For the first element of the S5 series, 'I₁' will be the total number of unique composites due to first element. When we move to the second element of the S5 series, 'I₂' will again indicate the total number of composites due to second element but not necessarily the total number of unique composites since there may be some composites that are common to both these first and second elements. The number of such composites needs to be identified and subtracted from 'I₂' to avoid double

addition and thereby over-stating of the number of total composites. The conditions to be satisfied by such composites are as follows:-

- Such a composite should be divisible by both prime numbers; $6n-1$ under consideration and $6n'-1$ which is a previous prime element.
- Such a composite should lie on S7 series.

For example; while identifying the common composites formed by 5 and 11; the first such composite would be 11×5 since it has already been derived as 5×11 while considering composites formed by 5.

Thereafter the product of 11 with every 5th (or 30th) element of S5 series would satisfy the above condition. This is because for any particular element $6n-1$ of S5 series, the periodicity of obtaining common composites would be $6 \times (6n-1)$ because of the difference of 6 between any two consecutive elements of S5 series.

In general terms, these common composites can be identified as follows:-

$(6n-1) \times [(6n'-1) + 6m(6n'-1)] \leq (6n-1) \cdot (6I_2-1)$; for any n ; $1 \leq n' < n$; m belongs to set of natural numbers including zero.

where $6n-1$ is the current element, $6n'-1$ is the previous prime element.

Number of possible values of m may be arrived as:-

$$\text{Number of such composites} = \text{Number of possible values of 'm'} = q = \left[\frac{(6I_2-1) - (6n'-1)}{6(6n'-1)} \right] + 1$$

Unique composites for the 2nd element of the S5 series can be derived as:-

$$N(\text{u.c.2S5}) = I_2 - q$$

Number of Unique Composites (UC) for n th element of S5 can be derived as:

$$N(\text{u.c. n S5}) = I_n - \sum_{n'=1 \text{ to } n-1} \left[\left\{ \frac{(6I_2-1) - (6n'-1)}{6(6n'-1)} \right\} + 1 \right]; \text{ such that } (6n'-1) \text{ is prime}$$

From the 3rd element (i.e. 17) onwards an additional step needs to be followed. This is because while identifying and subtracting composites already derived by earlier numbers, there may be some composites that are common to more than one earlier element number of S5. Just as it is important to avoid multiple/double counting of composites to avoid over-stating the total number of unique composites, it is equally important to identify those common composites and add the number of such composites to avoid multiple/double subtraction and thereby under-stating the total number of unique composites. This may be done as follows:-

Such composites need to satisfy the following two conditions:

If we denote such composites as 'A' then:-

- 'A' should be divisible by p_1 and p_2 and $6n-1$. Here p_1 and p_2 are any two prime elements on S5 series lying prior to $6n-1$ under consideration.
- 'A' divided by $6n-1$ under consideration, would be a composite number on the S5 series of which p_1 and p_2 are factors.

Considering $p_1 < p_2$; the product of p_1^2 (which would lie on the S7 series) and p_2 (which lies on the S5 series) would give the first composite that lies on the S5 series which is divisible by both p_1 and p_2 . The product of this composite with $6n-1$ under consideration would lie on S7 series and would be a composite that had been identified while calculating q for both p_1 and p_2 . Since q would be subtracted from I_3 ,

to arrive at the number of unique composites for $6n-1$ under consideration, this composite would be subtracted twice if not identified and adjusted for. Thereafter every $(6p_1p_2)$ th number would also satisfy these conditions as long as their value remains less than corresponding $6I-1$. In general terms, the composites common to p_1 and p_2 can be identified as follows:

$$p_1p_2(p_1+6m') \leq 6I_3-1; \text{ where } m' = \{0, N\}$$

$$\text{Number of such composites} = \text{Number of possible values of } m' = q' = \sum [\{(6I_n-1)-p_1^2p_2\}/6p_1p_2]+1$$

where p_1 and p_2 are prime elements of the S5 series prior to the number $6n-1$ under consideration and $p_1 < p_2$. Also, all possible pairs of p_1 and p_2 that lie on the S5 series prior to $6n-1$ under consideration need to be considered.

For example if $6n-1 = 17$, p_1 and p_2 would be 5 and 11 respectively

If $6n-1 = 23$, then p_1 and p_2 pairs would be 5&11, 5&17 and 11&17 subject to the condition that $p_1^2p_2 \leq 6I-1$

Unique composites for the 3rd (and onwards) elements of S5 series would be derived as follows:-

$$N(\text{u.c.3S5}) = I_3 - \sum (q-q')$$

In general terms, unique composites due to elements of S5 series would be derived as follows:-

$$\text{Number of U.C. due to } (6m-1) = I_m - \sum_{n=1 \text{ to } m-1} \{\text{floor function of } ((6Im - 1)/6(6n-1)) + 1\} + \pi_{p_1p_2(p_1+6n) \leq (6Im-1)} p_1p_2(p_1+6n)$$

2 Deriving composites on S7 series that are formed as intra-series products of S7 series

Unique composites of S7 series formed by elements of S7 series

Here again we first calculate the total number of composites of a particular element number $6n+1$ (which belongs to S7 series) as follows:-

$$\text{Floor function of } [1/6\{\text{floor function of } (2k/(6n+1))\}] = I'$$

Consider the floor function I' which will indicate the total number of composites of which the $6n+1$ number under consideration is a factor and whose value is less than $2k$. The biggest number would be $6I'+1$ i.e. $(6n+1) \times (6I'+1)$ would be the biggest composite of which this number $6n+1$ is a factor and whose value is less than $2k$

While deriving unique composites of S7 series formed by elements of S7 series, two types of repetitions are possible. First those composites those have been already derived by elements of S5 series and second those composites that have been derived by prior elements of S7 series. These can be identified as follows:-

(a) Identifying composites which have been already derived by elements of S5 series:-

The composite number whose product with $6n+1$ under consideration has already been derived while calculating composites formed by the elements of S5 series need to satisfy following conditions:-

The composite no should lie on the S7 series

It should be divisible by $6n-1$ number for which composites already derived are being identified. The first such composite number on the S7 series formed by the elements of S5 series that have not been derived earlier by prior elements of S5 series would be the square of the $6n-1$ whose composites are being identified.

e.g., 11^2 would be the first composite on S7 series of which 11 is a factor and has not been derived by earlier element of S5 series i.e. '5'. ' $6n+1$ ' multiplied by 'this composite number' would be the first composite on S7 series that is common to both $6n+1$ and $6n-1$ under consideration. Thereafter every $6x(6n-1)$ would satisfy the above condition. All $6n-1$ numbers under consideration should be prime because if its composite then they would have been already counted while considering composites formed by its prime factors.

$$r = \sum_{n': 1 \text{ to } (6n'-1), (6n'-1) < 6l+1} [\{ (6l'+1) - (6n'-1)^2 \} / 6(6n'-1)] + 1$$

This will give the total number of such composites that have already been formed by elements of S5 series.

However, here again there may be some composites that are common to more than one elements of the S5 series. To account for such composites, following conditions may be identified:-

- The composite should be on the S7 series
- It should be divisible by any two prime elements of S5 series and the first element of S7 series, i.e. '7'.

The first such composite that satisfies the above conditions would be $7p_1p_2$ where p_1 and p_2 are prime elements of S5 series. Thereafter every $6p_1p_2$ th element would satisfy the above condition. In general terms such composites may be identified as :-

$$r' = (6n+1)p_1p_2(7+6n) \leq (6n+1)(6l'+1)$$

or number of possible values of r' ; $n(r') = [\{ (6l'+1) - 7p_1p_2 \} / 6p_1p_2] + 1$;

where p_1 is the number $6n-1$ for which r is being calculated, p_2 is prior element of the S5 series that lie before p_1 , and 7 is the first element of the S7 series. This can also be written as :-

$$n(r') = [\{ (6l'+1) - 7p_1p_2 \} / 6p_1p_2] + 1$$

Note: while compiling r' only those numbers are to be considered that are $> (6n-1)^2$ (i.e. p_1^2) and $< 6l'+1$. This is because numbers prior to $(6n-1)^2$ have already been subtracted while calculating r

Once r and r' have been computed, $r-r'$ will give the unique composites that have been already derived by elements of the S5 series and are now being repeated while calculating unique composites for the S7 series elements.

Now we turn to identify the composites already formed by prior elements of the S7 series that need to be identified and subtracted to avoid multiple counting of the same composite number.

For the first element of the S7 series (i.e. 7),

$$I' - \sum (r-r')$$

will give the number of unique composites for the number 7. From the second number 13 (i.e. 7+6) onwards, an additional step needs to be followed wherein composites already formed by 7 have to be identified and subtracted to avoid multiple counting of the same composite.

For this we follow the following steps:-

The conditions to be satisfied by such composites are as follows:-

- Such a composite should be divisible by both $6n+1$ under consideration and $6n'+1$ which is a previous prime element of S7 series.
- Such a composite should lie on S7 series.

The first such composite would be 13×7 since it has already been derived as 7×13 . Thereafter every 7x6th (or 42nd) element of S7 series would satisfy the above condition. This is because for any particular element $6n+1$ of S7 series, the periodicity of obtaining common/non-degenerate composites would be $6 \times (6n+1)$ because of the difference of 6 between any two consecutive elements of S7 series.

In general terms, these common composites can be identified as follows:-

$(6n+1) \times [(6n'+1) + 6m'(6n'+1)] \leq (6n+1) \cdot (6I_2' + 1)$; for any given n ; $1 \leq n' < n$; m' belongs to set of natural numbers including zero.

where $6n+1$ is the current element, $6n'+1$ is the previous prime element

Number of possible values of m' may be arrived as:-

Number of such composites = Number of possible values of $m' = s = [\{ (6I_2' + 1) - (6n'+1) \} / 6(6n'+1)] + 1$

Number of values of $s = \sum_{n'=1 \text{ to } n-1; \text{ such that } 6n'+1 \text{ is prime}} [\{ (6I_2' + 1) - (6n'+1) \} / 6(6n'+1)] + 1$

This will give the total number of composites formed by previous prime elements of the S7 series, lying prior to the number $6n+1$ under consideration.

However this 's' may comprise of such composites that may have been derived by earlier elements of the S5 series or S7 series. These need to be identified and accounted for since, as mentioned earlier, just as it is important to avoid multiple counting, it is equally important to avoid multiple removal.

For this we introduce the following terms:-

Conditions for deriving composites calculated by s that have already been identified and duly subtracted while counting the number of unique composites formed by elements of S5 series:-

- Such a composite should lie on S7 series

- It should be divisible by the number $6n+1$ (for which s has been calculated) and by prime elements of the $S5$ series.

The first such composite number would be the product of ' $6n+1$ ' (for which s has been calculated) denoted as $p2$ and the square of the prime elements of the $S5$ series (since the square of any element amounts to product of intra-series elements and thus leading to an $S7$ series element) denoted as $p1$. Thereafter every $6p1p2$ th number would satisfy the above conditions. In general terms this can be written as :-

$$s' = (6n+1)p1p2(p1+6n) \leq (6n+1)(6I'+1)$$

$$\text{or number of possible values of } s'; n(s') = \sum [\{ (6I'+1) - p1^2p2 \} / 6p1p2] + 1;$$

where $p1$ is a prime element of the $S5$ series and $p2$ is a prime element of the $S7$ series for which ' s ' has been calculated. This term will give the number of composites counted in ' s ' that have already been counted and subtracted while computing composites of the $S5$ series.

To identify composites already derived by previous elements of $S7$ series:-

From the 3rd element (i.e. 19) onwards an additional step needs to be followed. This is because while identifying and subtracting composites already derived by earlier numbers, there may be some composites that are common to more than one element number of $S7$. Just as it is important to avoid multiple/double counting of composites to avoid over-stating the total number of composites, it is equally important to identify those common composites and add them to avoid multiple/double subtraction and thereby under-stating the total number of composites. This may be done as follows:-

Such composites need to satisfy the following conditions:

- Such a composite should be divisible by two previous prime elements of $S7$ series
- It should lie on the $S7$ series so that its product with 19 would be a composite on $S7$ series

First such composite that would satisfy the above conditions would be $p1p2$ where $p1$ and $p2$ are two prime elements prior to the $6n+1$ under consideration. Thereafter every $6p1p2$ th number would satisfy the above condition. In general terms this can be identified as:

$$s'' = (6n+1)p1p2(1+6m'') \leq (6n+1)(6I'+1)$$

$$\text{or number of possible values of } s''; n(m'') = [\{ (6I'+1) - p1p2 \} / 6p1p2] + 1;$$

This will give the number of composites formed by prior elements of the $S7$ series. Here $p1$ and $p2$ are prime elements of the $S7$ series prior to $6n+1$ under consideration.. All previous prime pairs ($p1 \& p2$) that satisfy the condition $6p1p2 \leq 6I'+1$; are to be considered.

If $6n+1 = 19$; $p1$ and $p2$ would be 7 and 13. If $6n+1 = 31$; $p1$ $p2$ pairs would be 7&13, 7&19 and 13&19 subject to the condition that $p1p2$ should be less than $6I+1$

Composites formed by earlier elements of S7 series that need to be subtracted from I' to arrive at unique composites for a given number $6n+1$ can be derived as:-

$$n(\text{unique composites formed by } 6n+1) = I' - \sum (r-r') - \sum (s-s'-s'')$$

Deriving number of composites on S7 series for a given $2k$: An illustration

Consider the even number 16658. It is of SADN 8//8 which means the relevant series is the S1 series. Unique composites on the S7 series that are less than 16658 are shown in the following tables where the number of composites have been derived by following the steps as mentioned above:-

Element number	I	6I-1	q	q'	q-q'	Number of unique composites $\sum I - \sum (q-q')$
5	555	3329	555
11	252	1511	51	...	51	201
17	163	977	48	3	45	118
23	120	719	42	3	39	81
29	95	569	38	2	36	59
41	67	401	30	1	29	38
47	59	353	29	1	28	31
53	52	311	28	1	27	25
59	47	281	25	1	24	23
71	39	233	23	0	23	16
83	33	197	20	0	20	13
89	31	185	21	0	21	10
101	27	161	21	0	21	6
107	25	149	20	0	20	5
113	24	143	21	0	21	3
TOTAL						1184

Table 5B.1: Unique Composites formed on S7 series upto 16658 by elements of S5 series

Element number	I	6I+1	r	r'	r-r'	s	s'	s''	s-s'-s''	Number of unique composites $\sum I - \sum (r-r') - \sum (s-s'-s'')$
7	396	2377	162	19	143	253
13	213	1279	79	7	72	31	7	24	117
19	145	871	51	4	47	33	7	2	24	74
31	89	535	29	1	28	25	4	4	17	44
37	74	445	22	1	21	24	3	5	16	37
43	64	385	19	1	18	21	3	4	14	32
61	45	271	12	0	12	19	1	5	13	20
67	41	247	10	0	10	17	1	3	13	18
73	37	223	9	0	9	16	1	2	13	15
79	24	205	9	0	9	15	1	1	13	12

97	28	169	6	0	6	15	0	1	14	8
103	26	157	6	0	6	15	0	1	14	6
109	25	151	6	0	6	15	0	0	15	4
127	21	127	5	0	5	16	0	1	15	1
Total										641

Table 5B.2: Unique Composites formed on S7 series upto 16658 by elements of S7 series

Total number of composites on S7 series, less than 16658 = 1184+641=1825

This implies that there would be 1825 composite numbers on S7 series whose value is less than 16658

For S5 series:

For deriving number of unique composites on S5 series:-

Based on similar reasoning as discussed above;

Composites on the S5 series are derived as inter-series products of elements of the S5 and S7 series. In order to calculate the total number of composites formed on S5 series upto a particular even number $2k$, following steps may be followed:-

We first find out the total number of composites formed by a particular prime element of the S5 series. This can be done by the following formula:-

$$\text{Floor function of } [1/6 \text{ floor function of } [2k/(6n-1)]] = I_n''$$

Consider the floor function I_n'' . This integer I_n'' gives the total number of composites on the S5 series of which $6n-1$ is a factor and values of all of these composites are less than $2k$. These composites will be of the nature of:-

$$(6n-1) \times (6n'+1) < 2k$$

The biggest $6n+1$ number on the S7 series whose product with $6n-1$ will be $< 2k$ can be derived as:-

$$6 I_n'' + 1$$

For the first element of the S5 series, i.e. 5, I_1'' will be the total number of unique composites, on the S5 series upto $2k$, of which 5 would be a factor.

When we move to the second ($n=2$) number of the S5 series, i.e. 11, I_2'' will indicate the total number of composites on the S5 series upto $2k$ of which 11 is a factor, but not necessarily the total number of unique composites since there may be some composites that are common to both elements, 5 and 11. These composites need to be identified and number of such composites needs to be subtracted from I_2'' to avoid double counting/addition and thereby over-stating of the number of total composites.

The conditions to be followed/satisfied are as follows:

- Such a composite should lie on S7 series, so that its product with $6n-1$ number under consideration will lie on S5 series.
- It should be divisible by both ' $6n-1$ ' under consideration and the 'previous prime element' of the S5 series; denoted as p_1 .

The first such composite number would be the square of the previous prime element since it would lie on the S7 series, whose product with the '6n-1' under consideration would lie on the S5 series.

Thereafter every 6p1th number would satisfy the above conditions. In general terms these conditions will be expressed as:

$$(6n-1)(p1^2+6np1) \leq (6n-1)(6I''+1)$$

The number of such composites would be :

$$t = [(6I''+1)-p1^2]/6p1 + 1$$

These common composites can be identified as:-

$$t_n = \sum_{n'=1 \text{ to } (6n'-1), (6n'-1) < 6I_n''+1} [\{(6I_n+1) - (6n'-1)^2\}/6(6n'-1)] + 1$$

Unique composites of the S5 series for which the 2nd (n=2) element, i.e. 11, is a factor can be derived as:-

$$I_2'' - t_n$$

From the 3rd (n=3) element onwards of the S5 series an additional step needs to be followed. This is because while identifying and subtracting composites already derived by earlier numbers, there may be some composites that are common to more than one element number of S5. Just as it is important to avoid multiple/double counting of composites to avoid over-stating the total number of composites, it is equally important to identify those common composites, if any, and add them to avoid multiple/double subtraction and thereby under-stating the total number of composites. Such composites denoted as 'B'; need to satisfy the following conditions:-

- 'B' should be divisible by a pair of previous prime-elements p1 and p2 and also by 6n-1 under consideration.
- 'B' divided by '6n-1' should be a composite number on S7 series. This composite number should be a multiple of both p1 and p2.

The first such number would be 7p1p2 subject to the condition that 7p1p2 < p2p2 or in other words; 7p1 < p2

Thereafter every 6p1p2th number would satisfy the general conditions satisfying the above conditions are as follows:-

$$t'_n = \sum_{n=0 \text{ to } n < \{(6I_n+1)-7p1p2\}/6p1p2} p1p2(7+6m) \leq 6I_n''+1;$$

or number of possible values of t'; $m(t') = [\{(6I_n''+1)-7p1p2\}/6p1p2]+1$;

where p1 and p2 are prime element of the S5 series prior to the number 6n-1 under consideration. All possible pairs of p1 and p2 that lie on the S5 series prior to 6n-1 under consideration need to be considered here.

Number of unique composites for the 3rd (and onwards) elements of S5 series would be derived as follows:-

$$\sum I_n'' - \sum (t_n - t'_n)$$

An illustration:-

Consider the even number 19978 which is SADN 7//8. The relevant series would therefore be the S5 series. Unique composites have been derived by following the above mentioned steps and are summarised in the following table:-

Element number N	I_n''	$6I_n''+1$	t_n	t_n'	t_n-t_n'	Unique composites $\sum I_n'' - \sum(t_n - t_n')$
5	665	3991	665
11	302	1813	60	...	60	242
17	195	1171	55	3	52	143
23	144	865	47	3	44	100
29	114	685	38	2	36	78
41	81	487	24	1	23	58
47	70	421	21	1	20	50
53	62	373	17	0	17	45
59	56	337	16	0	16	40
71	46	277	12	0	12	34
83	39	235	10	0	10	29
89	37	223	9	0	9	28
101	32	193	8	0	8	24
107	30	181	7	0	7	23
113	29	175	7	0	7	22
131	25	151	6	0	6	19
137	24	145	6	0	6	18
149	22	143	5	0	5	17
167	19	115	4	0	4	15
173	19	115	4	0	4	15
179	18	109	3	0	3	15
191	17	103	3	0	3	14
197	16	97	3	0	3	13
227	14	85	3	0	3	11
233	14	85	3	0	3	11
239	13	79	2	0	2	11
251	13	79	2	0	2	11
257-269 (3)	12	73	2	0	2	10x3=30
281-293 (2)	11	67	2	0	2	9x2=18
311-317 (2)	10	61	2	0	2	8x2=16
347-359 (3)	9	55	2	0	2	7x3=21
383-401 (3)	8	49	1	0	1	7x3=21
419-461 (5)	7	43	1	0	1	6x5=30
467-539 (6)	6	37	1	0	1	5x6=30

557-641 (8)	5	31	1	0	1	$4 \times 8 = 32$
647-797 (11)	4	25	1	0	1	$11 \times 3 = 33$
809-1055 (20)	3	19	0	0	0	$20 \times 3 = 60$
1061-1535 (34)	2	13	0	0	0	$34 \times 2 = 68$
1553-2855 (86)	1	7	0	0	0	$86 \times 1 = 86$
Total						2196

Table 5B.3: Derivng unique composites for a given 2k: An illustration

From the above table, it can be implied that there are 2196 composites on the S5 series whose value is less than the considered even number 19978.

6

Possible combinations of p_1+p_2 for even number of particular SADN and a particular last-digit

Possible combinations of p_1+p_2 for even number of particular SADN and a particular last-digit:

Even numbers can be of SADN 1 to 9 and can end in any of the digits 2, 4, 6, 8 and 0. While an even number of a particular SADN will recur after 18 integers, an even number of a particular SADN ending in a specific digit will recur after 90 integers. For example 12 is an even number of SADN 3 which ends in the digit 2. The next even number with SADN 3 would be $12+18=30$ while the next even number with SADN 3 that ends in the digit 2 will be $12+90=102$. Therefore, if we denote an even number $2k$ as a/b , where 'a' is the SADN of the even number and 'b' is the digit in which it ends, then $2k+18$ will be in the form of $a/b-2$ (or $a/b+8$ if $b-2$ is negative); and $2k+90$ will be in the form of a/b . Here $a=1$ to 9 while $b=2, 4, 6, 8, 0$.

As mentioned earlier, prime numbers will essentially be odd numbers (with the only exception of the number 2), and can be of SADN 1, 2, 4, 5, 7 or 8 (with the only exception of the number 3) and can end in the digits 1, 3, 7 or 9 (with the only exception of the number 5 which is of SADN 5 and ends in 5). Therefore, prime numbers can be denoted as a_p/b_p where ' a_p ' denotes the SADN of the prime number while ' b_p ' denotes the last digit of the prime. Here a_p can be of value 1, 2, 4, 5, 7, 8 while b_p can be 1, 3, 7, 9.

If we leave out the exceptions, the combinations of prime numbers in which even numbers of a given SADN can be summed up can be generalized in the following table:-

SADN of the even number $2k$	SADN of combinations of prime numbers that can add up to the given $2k$
1	2+8, 5+5
2	1+1, 4+7
3	2+1, 4+8, 5+7
4	2+2, 5+8
5	1+4, 7+7
6	1+5, 2+4, 7+8
7	2+5, 8+8
8	1+7, 4+4
9	1+8, 2+7, 4+5

Table 6.1: SADN of Possible combinations of prime numbers possible for an even number of a given SADN

It may be noted here that even numbers with SADN 1,2,4,5,7 and 8 can be added up in the form of 3+odd number of a particular SADN. For instance if SADN of $2k$ is 1 then one possible combination of $p+p$ that can add up to $2k$ will be 3+prime number of SADN 7. Similarly, if $2k=\text{SADN } 2$ then a possible combination can be of 3+prime number of SADN 8. However, since odd numbers of SADN 3 are ‘generally’ composite in nature, in order to consider this combination a limiting condition needs to be placed that it will be applicable only in cases where the corresponding odd number will be a prime number so that when combined with the digit 3, it will qualify to be a $p+p$ combination i.e. such a combination can be considered for numbers where $2k-3$ will be a prime number. Since a general solution is not conceivable with such specific limiting conditions, the current line of analysis will treat these combinations as exceptions and leave them out.

Furthermore, depending on the last digit of $2k$, last digit of the odd numbers that can add up in $2k$ will also have a role to play. For instance; consider $2k=20$, it can be denoted as an even number of the form $2//0$ i.e. SADN 2 ending in 0. In this case combinations of odd numbers that can add up to $2k$ will be of $ap//1 + ap'//9$ or $ap//3 + ap'//7$. If $2k$ ends in 2 then prime numbers ending in 7 cannot be considered in a general solution since in this case $2k$ -‘the odd number’ will end in 5 and all odd numbers ending in 5 (with the exception of the number 5) will be composite numbers divisible by 5. The following table shows the possible combinations of prime numbers ending in specific digits that can be considered in a general solution:-

Last digit of $2k$	Last digit of combinations of prime numbers that are possible	Last digit that is not possible
2	$ap//1+ap'//1$, $ap//3+ap'//9$	$ap//7$
4	$ap//1+ap'//3$, $ap//7+ap'//7$	$ap//9$
6	$ap//3+ap'//3$, $ap//7+ap'//9$	$ap//1$
8	$ap//1+ap'//7$, $ap//9+ap'//9$	$ap//3$
0	$ap//1+ap'//9$, $ap//3+ap'//7$	-

Table 6.2: Possible combinations of prime numbers ending in specific digits those which can be considered, and those which are prohibited; in a general solution

A general picture of what combinations of prime numbers can add up to even numbers of a particular SADN and last digit, in terms of SADN and the last digit of the prime combination, can be summarized in the form of 45 matrices as mentioned in Appendix 2.

7

Why p_1+p_2 combinations of SADN(2,5,8) will lie on S7 series and that of SADN(7,4,1) will lie on S5 series?

Why p_1+p_2 of SADN(2,5,8) will lie on S7 series and that of SADN(7,4,1) will lie on S5 series?

The matrices mentioned in Appendix 1 show that even numbers with SADN(2,5,8) can be summed up in prime numbers with SADN(1,4,7). Even numbers with SADN(1,4,7) can be added up in terms of prime numbers with SADN(2,5,8). Even numbers with SADN(3,6,9) can be added up in prime numbers of SADN(1,2,4,5,7,8). If we consider this in perspective of the 3 series of odd numbers discussed in Section 4A, this can be re-stated to suggest that prime combinations (p_1+p_2) for even numbers with SADN(2,5,8) will be found on the S7 series of odd numbers while prime combinations for even numbers of SADN(1,4,7) will be found on the S5 series of odd numbers. Prime combinations for even numbers with SADN(3,6,9) will be in the form of p_1+p_2 where p_1 and p_2 will be found on the S5 and S7 series respectively.

8

Cyclic-Series-Element (CSE) of even numbers (defined as $2k$)**Cyclical series of even numbers:**

There is a cyclic & closed series of six numbers viz. 12,2,4,6,4,2,12; where first and last numbers are identical.

Consider a series of consecutive even numbers, e.g. 38,40,42,44,46,48,50,52,54,..... It may be written as 2×19 , 4×10 , 6×7 , 4×11 , 2×23 , 12×4 , 2×25 , 4×13 , 6×9

For any length of series of consecutive even numbers, the universal series of factors will be {2,4,6,4,2,12} acting as a unit series in cyclic order. We call this series of factors as factor-series and their elements as factor-elements (fe). Hence factor-series is given as {2,4,6,4,2,12} in this specific order and 2,4,6,12 are factor-elements (fe). This cyclic order of the factor-series may start from any element of the factor-series. First element of factor-series would depend on the first number of corresponding consecutive-even number-series selected for study.

Corresponding factor-elements are said to be 'fe'

So $fe = \{2,4,6,4,2,12\}$ with order preserved

Consider any even number ' $2k$ '

$$2k/2=k$$

Number of possible combinations resulting in ' $2k$ ' would be ' k '

Suppose factor-element of ' $2k$ ' is 'fe'

$$12/fe = c$$

$$2k = (k+nc) + (k-nc); n=1,2,3,... \text{such that } (k+nc) < 2k \text{ OR } (k-nc) > 0$$

$$(k+nc)^2 - (k-nc)^2 = 24n(2k/fe)$$

$$\text{LHS} = (k+nc)^2 - (k-nc)^2$$

$$= [(k+nc) + (k-nc)] \cdot [(k+nc) - (k-nc)]$$

$$= 2k [2nc]$$

$$= 2k [2n(12/fe)]$$

$$= 24n(2k/fe)$$

$$= \text{RHS}$$

The cyclic-relation between factor-element (fe) and SADN of even numbers can be understood with the help of following quadrant diagram:-

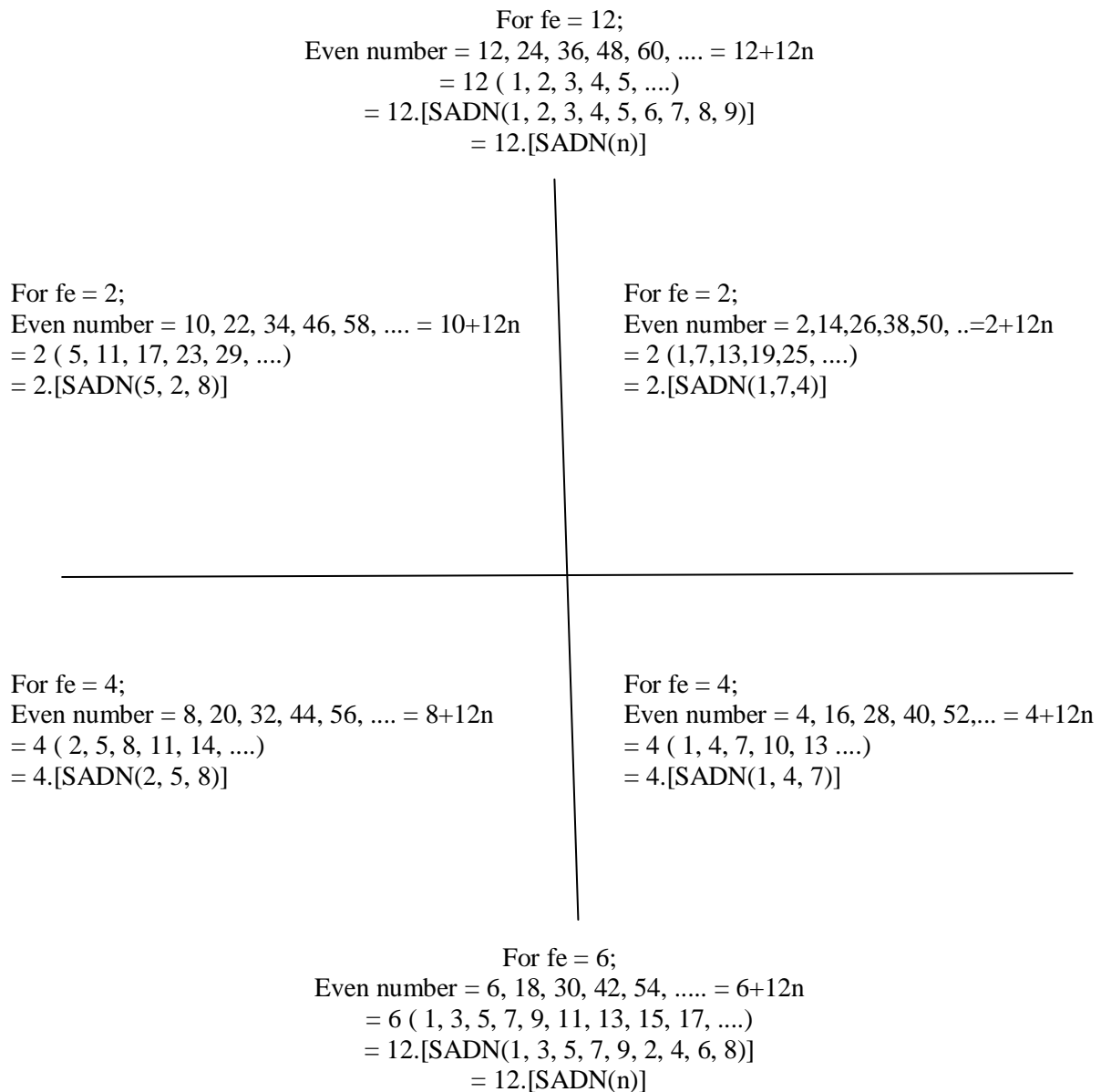


Diagram 8.1: Quadrant diagram representing cyclic series elements vis-a-vis SADN of even numbers

Identifying nTC (i.e. total number of acceptable combinations) for a given even number (2k) depending on SADN and CSE

Deriving possible combinations of primes for even numbers

Case I- Even numbers (2k) of SADN(2,5,8) that are of CSE 2 type

For even numbers of SADN(2,5,8); the relevant series of odd numbers will be the S7 series as mentioned earlier in Section 7

Here $\{(2k-2)/6\} - 1$ will give the total number of elements, worth consideration, of the S7 series up to 2k that would include both prime and composite element numbers. As evident from Figure 1, $(2k-2)/6$ gives the total number of elements of S7 series that exist up to 2k, including an element given as $2k-1$. As the number '1' is not considered to be an element in S7 series, its corresponding element (i.e. $2k-1$) of S7 series is also to be not considered. Hence total number of elements, worth consideration, of the S7 series up to 2k that would include both prime and composite element numbers would be $\{(2k-2)/6\} - 1$. For instance, if $2k=32$ (SADN 5), then $32-2/6=5$. Here 5 is the number of elements of the S7 series whose value is less than 32 and the actual numbers will be 7,13,19,25 and 31.

If 2k is a SADN(2,5,8) number of CSE 2 type, then k will be an odd number (for proof, see Appendix 3). $(k-1)/6$ will give the number of combinations of different elements of the S7 series that will add up to 2k (refer diagram 9.1). It is important to note here that all odd numbers of SADN(1,4,7) that lie on the S7 series will find a place in the combinations thus derived irrespective of whether they are prime or composite.

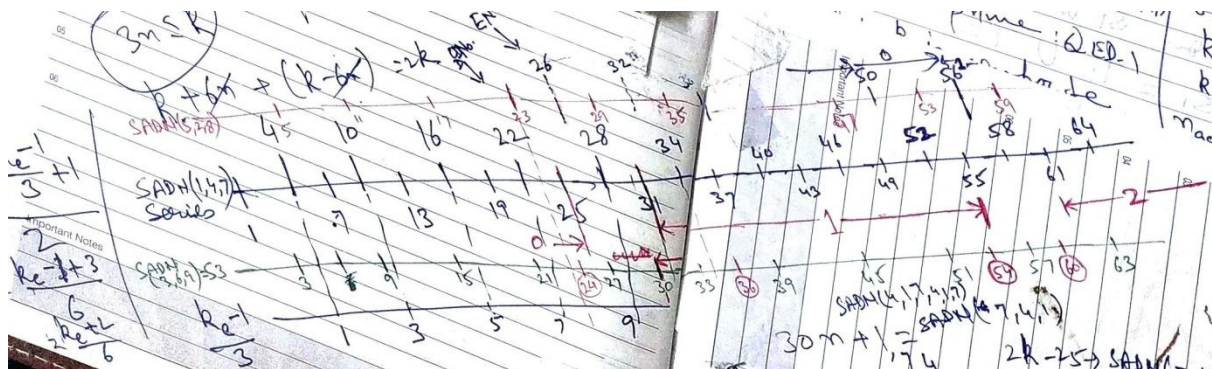


Diagram 9.1: The three series; S1, S3 and S5 alongwith number of unique composites on the relevant series

It is important to note here that since there is no consensus on whether the number 1 is a prime or composite number, this additional combination of $2k=1+(2k-1)$ is only of

academic importance and this combination does not have a role to play, in identifying $p1+p2$ combinations for the given $2k$, in present paper.

Case II – Even numbers ($2k$) of SADN(2,5,8) that are of CSE 4 type

In case $2k$ is of SADN(2,5,8) of CSE 4 type, k will be an even number (for proof, see Appendix 3). Here, $(k-4)/6$ will give the number of total combinations (nTC) of elements of the S7 series that will add up to $2k$ (refer diagram 9.1).

Therefore, for $2k$ of SADN(2,5,8); the value of nTC will be: –

For $2k$ of CSE 2 type- $nTC=(k-1)/6$

For $2k$ of CSE 4type- $nTC=(k-4)/6$

Case III- Even numbers ($2k$) of SADN(1,4,7) that are of CSE 2 type

For even numbers of SADN(1,4,7) the relevant series of odd numbers will be the S5 series as mentioned earlier in Section 7. Here the total number of odd numbers lying on the S5 series up to $2k$ which includes both prime and composite element numbers will be given by $(2k-4)/6$ (for proof, see Appendix 3 and refer diagram 9.1). For instance, consider $2k$ to be 34 (i.e. SADN 7). This implies that there are $(34-4)/6$ i.e. 5 elements of the S5 series whose value is less than 34 and the actual numbers will be 5,11,17,23 and 29.

In case $2k$ is a SADN 1,4,7 number of CSE 2 type, then k will be an odd number and $(k-1)/6$ will give the value of nTC.

Case IV- Even numbers ($2k$) of SADN(1,4,7) that are of CSE 4 type

In this case, k will be an even number and $(k-2)/6$ will give the value of nTC as is proven in Appendix 3.

Case V- Even numbers ($2k$) of SADN(3,6,9)

In case of even numbers of SADN 3, 6 and 9, the numbers would be of either CSE 6 or CSE 12 type and the relevant series will be both the S5 and S7 series since the possible prime combinations will be such that one term will lie on the S5 series while the corresponding term will lie on the S7 series, as discussed in Section 7. In this case $2k/3$ will give the number of elements of the S5 and S7 series of odd numbers whose value is less than $2k$ and $2k/6$ will give the value of nTC as is proven in Appendix 3.

It is important to note here that since nTC for all even numbers irrespective of their SADN includes all elements on the relevant series irrespective of whether they are prime or composite, these combinations are of following three types-

Combination1: $p+c$ where one element is prime and the other is composite

Combination2: $c1+c2$ where both elements being summed up are composites

Combination3: $p1+p2$ where both elements being summed up are primes

The next step would be to identify the $p1+p2$ combinations, as discussed in next section.

10

Identifying combinations of type p_1+p_2 for even number $2k$

Once we arrive at the number of unique composites on the relevant series we can now proceed to identify the p_1+p_2 combinations for any given even number ($2k$).

If the number of composites is less than the total number of acceptable combinations derived earlier in Section 9 by at least 1 then it directly follows that even if all composites are prime-eaters i.e. are paired with a prime number, there will still be at least one p_1+p_2 combination. For instance consider the even number 100. This is a SADN 1//0 type number which implies that S5 is the relevant series on which the p_1+p_2 may be identified. For the number 100 there would be $2k-4/6$ i.e. $100-4/6=16$ element numbers on the S5 series, $k-2/6$ i.e. $50-2/6=8$ acceptable combinations. Since the number of composites on the 5 series $<100=4$, if we consider all these 4 composites to be prime-eaters, they will absorb 4 out of the 8 acceptable combinations. This means that 4 combinations will still be in the nature of p_1+p_2 combinations. Rather, it would be more appropriate to state that at least 4 of the 8 combinations would be in the nature of p_1+p_2 combinations. This is because here we have considered all the composites to be prime-eaters and have not explored the possibility that some of these composites could be in the form of c_1+c_2 combinations. The number of P_1+P_2 combinations could increase if there are such C_1+C_2 combinations. In this example, two of the 4 composites (viz. 35 and 65) come together to form a C_1+C_2 combination. Therefore, in this example, the total combinations can be classified as: 1 out of 8 combinations are of type C_1+C_2 ; 2 out of 8 combinations are of type P_1+C_1 ; and 5 out of 8 combinations are of type P_1+P_2 .

While finding out the number of C_1+C_2 combinations for numbers where $TC > \text{number of composites}$ is an exercise for academic purposes, it becomes mandatory to find them out for numbers where $nTC < \text{number of composites}$.

In general terms; the above discussion can be summarised as follows:

For any even number (EN), SADN of $EN = \{7,4,1\}$ or $\{5,2,8\}$ or $\{6,3,9\}$

Case(1):

SADN of $EN = \{7,4,1\}$

$EN = 2k$

Case(1A): $EN/2 = k$ is a prime number

Case(1B): $EN/2 = k$ is a composite number

If 'k' is a composite number:

Number of acceptable combinations of elements is given as n_{acc}

Case(1BP): If number of composites is less than number of primes (i.e. $n_c < n_p$ implies that even if all composites are prime-eaters; there exists atleast one p_1+p_2 pair

Case(1BC): If number of composites is greater than or equal to number of primes (i.e. $n_c \geq n_p$) implies that we need to find total number of unique C_1+C_2 pairs

Case(2):

SADN of EN = {5,2,8}

EN = $2k$

Case(2A): EN/2 = k is a prime number

Case(2B): EN/2 = k is a composite number

If ' k ' is a composite number:

Number of acceptable combinations of elements is given as n_{acc}

Case(2BP): If number of composites is less than number of primes (i.e. $n_c < n_p$ implies that even if all composites are prime-eaters; there exists atleast one p_1+p_2 pair

Case(2BC): If number of composites is greater than or equal to number of primes (i.e. $n_c \geq n_p$) implies that we need to find total number of unique C_1+C_2 pairs

Case(3):

SADN of EN = {6,3,9}

EN = $2k$

Case(3A): EN/2 = k is a prime number, which is never possible as midpoint (k) of even numbers ($2k$) of SADN (6,3,9) would themselves be of SADN(3,6,9) respectively i.e. a composite number lying on S3 series.

Case(3B): EN/2 = k is a composite number

If ' k ' is a composite number:

Number of acceptable combinations of elements is given as n_{acc}

Case(3BP): If number of composites is less than number of primes (i.e. $n_c < n_p$ implies that even if all composites are prime-eaters; there exists atleast one p_1+p_2 pair

Case(3BC): If number of composites is greater than or equal to number of primes (i.e. $n_c \geq n_p$) implies that we need to find total number of unique c_1+c_2 pairs

11

Identifying number of unique combinations of type c_1+c_2 for a given even number

Identification of unique combinations of type c_1+c_2 (i.e. such combinations where both components are composites) for any given even number ($2k$) comprises of following three steps:-

Step1: c_1+c_2 of type 1 derived from k , i.e. mid-point of the even number $2k$

Step2: c_1+c_2 of type 2 derived from last digit of $2k$

Step3: c_1+c_2 of type 3 derived from $6p_1p_2$ where p_1 and p_2 are primes such that $6p_1p_2 \leq 2k$

11A.

Step1:

c_1+c_2 derived from k , i.e. mid-point of the even number $2k$

Step 1: This step is applicable for even numbers where midpoint k is a composite number. Suppose 'a' and 'b' are factors of the mid-point k i.e. $k=ab$

Here the following C_1+C_2 combinations can be derived:

$$a(b+6n)+a(b-6n); \text{ where } n \text{ is an element of the set of Natural numbers}$$

These combinations will be derived as long as $b+6n$ is less than $2k$ OR in other words, as long as $b-6n$ does not lead us to the first element number of the concerned series on which b is existing.

Number of such combinations is given by number of possible values of n where:

$$b+6n < 2k \text{ or } n < \text{ff of } [(2k-b)/6] \text{ (ff stands for floor function)}$$

Similarly, another set of C_1+C_2 combinations will be in the form of $b(a+6n')+b(a-6n')$. Here again the combinations can be derived till $a-6n'$ does not lead us to the first element of the series OR as long as $a+6n'$ is less than $2k$.

Number of such combinations is given by: $a+6n' < 2k$ or $n' < \text{ff of } [(2k-a)/6]$

Hence total number of c_1+c_2 combinations of type-I are given as: $n+n'$ or $\text{ff of } [(2k-b)/6] + \text{ff of } [(2k-a)/6]$

Alternately, this implies that:

C_1+C_2 of Type-1:

$k = a.b$ implies that

$$ab + ab = 2k$$

and

$$a(b+6n) + a(b-6n) = 2k; \text{ where } n \text{ is an integer}$$

and

$$b(a+6n') + b(a-6n') = 2k; \text{ where } n' \text{ is an integer}$$

As $b-6n > 0$, implies that $n < \text{floor function } [b/6]$

As $a-6n' > 0$, implies that $n' < \text{floor function } [a/6]$

Hence number of C1+C2 combinations of type-I $\leq \text{floor function } [a/6] + \text{floor function } [b/6] + 1$

Example 1:

For instance consider $2k$ to be 598 which is SADN 4//8 type number. Since $2k$ is of SADN (7,4,1) type, relevant series is the S5 series on which composites are derived as interseries elements of the S5 and S7 series. Here mid-point is 299 which is a product of 13×23 . In this case the following C1+C2 combinations can be derived

1. $13 \times 5 + 13 \times 41$
2. $13 \times 11 + 13 \times 35$
3. $13 \times 17 + 13 \times 29$
4. $23 \times 7 + 23 \times 19$

This will give us 4 C1+C2 combinations.

For a given $2k$ if k is a composite number, then $c1+c2$ combinations can be identified in the following manner:-

1. Case I: For even number ($2k$) of CSE 2 type where midpoint k is a composite odd number

For example Consider $2k = 598$ i.e. SADN 4//8. Here midpoint is a composite odd number ($k = 299$) that can be expressed as $299 = 13 \times 23$ (for $2k = 598$), the $c+c$ combinations identified from the mid-point are in the form of $13 \times (23+6n) + 13(23-6n)$ till we reach a value of n that leads $(23-6n)$ to the first element of the S5 series (since 23 is an element of the S5 series), where n is a natural number.

The actual combinations are: $13 \times 17 + 13 \times 29$

$$13 \times 11 + 13 \times 35$$

$$13 \times 5 + 13 \times 41$$

In each of these cases, the addition of the said numbers will add upto 598. Similarly another combination would be in the form of $23 \times (13+6n) + 23(13-6n)$ till we reach a value of n that leads $(13-6n)$ to the first element of the S7 series (since 13 is an element of the S7 series), where n is a natural number.

The actual combinations are: $23 \times 7 + 23 \times 19 = 598$

In addition to the 4 c_1+c_2 combinations identified as above, 13×23 itself would be a c_1+c_2 combination. The total number of c_1+c_2 combinations thus identified will be 5.

Example 2:

Consider $2k = 902$ which is of SADN $2//2$ type. Since $2k$ is a SADN(5,2,8) type number, relevant series would be the S7 series on which composites are derived as intra series elements of either S5 or S7 series. Here mid-point = 451 which is a composite odd number that can be expressed in terms of its factors as 11×41 . C_1+c_2 combinations for $2k=902$ can be derived as:

$11 \times (41+6n) + 11(41-6n)$ till we reach a value of n that leads $(41-6n)$ to the first element of the S5 series (since 41 is an element of the S5 series), where n is a natural number. The actual combinations would be:

$$11 \times 35 + 11 \times 47$$

$$11 \times 29 + 11 \times 53$$

$$11 \times 23 + 11 \times 59$$

$$11 \times 17 + 11 \times 65$$

$$11 \times 11 + 11 \times 71$$

$$11 \times 5 + 11 \times 77$$

In all these combinations the addition of the said numbers gives 902.

Similarly c_1+c_2 combinations would also be identified in the nature of $41 \times (11+6n) + 41(11-6n)$ till we reach a value of n that leads $(11-6n)$ to the first element of the S5 series (since 11 is an element of the S5 series), where n is a natural number. Such a combination will be $41 \times 5 + 41 \times 17$ leading to 902.

In addition to the above c_1+c_2 combinations identified, 11×41 itself will also be a combination of c_1+c_2 type. The total number of c_1+c_2 type combinations thus identified will be 8.

Example 3:

Consider $2k = 602$ which is of type SADN $8//2$. This implies that relevant series would be the S7 series whose composites are derived as intra series products of elements of either S5 or S7 series. Mid-point of 602 is given as 301 which can be expressed in terms of its factors as $301 = 7 \times 43$. C_1+c_2 combinations can be identified as –

$7(43+6n) + 7(43-6n)$ till we reach a value of n that leads $(43-6n)$ to the first element of the S7 series (since 43 is an element of the S7 series), where n is a natural number. The actual combinations would be:

$$7 \times 37 + 7 \times 49$$

$$7 \times 31 + 7 \times 55$$

$$7 \times 25 + 7 \times 61$$

$$7 \times 19 + 7 \times 67$$

$$7 \times 13 + 7 \times 73$$

$$7 \times 7 + 7 \times 79$$

In addition to the above, 7×43 itself will be a c_1+c_2 combination and the total number of c_1+c_2 type combinations = 7

Example 4:

In case if for a given $2k$; k is a square of a certain number, then c_1+c_2 combinations would be identified as follows:

Consider $2k = 1058$ where midpoint $529 = 23 \times 23$. C_1+c_2 type combinations for this $2k$ are:

$$23 \times 23 + 23 \times 23$$

$$23 \times 17 + 23 \times 29$$

$$23 \times 11 + 23 \times 35$$

$$23 \times 5 + 23 \times 41$$

Total number of c_1+c_2 in this case will be 4.

Example 5:

If midpoint of $2k$ is a composite odd number having more than 2 factors:

Consider $2k = 2002$ which is SADN $4//2$ whose relevant series is the S_5 series. Mid-point k is 1001 which can be expressed in terms of its factors as $7 \times 11 \times 13$.

C_1+c_2 for $2k = 2002$ would be identified as follows-

Step(i)- Consider (k) $1001 = 7 \times 143$

C_1+c_2 would be identified as $7 \times (143+6n) + 7 \times (143-6n)$ till we reach a value of n that leads $(143-6n)$ to the first element of the S_5 series (since 143 is an element of the S_5 series), where n is a natural number. C_1+c_2 combinations thus identified would be-

$$7 \times 143 + 7 \times 143$$

$$7 \times 137 + 7 \times 149$$

$$7 \times 131 + 7 \times 155$$

$$7 \times 125 + 7 \times 161$$

$$7 \times 119 + 7 \times 167$$

⋮

$$7 \times 5 + 7 \times 281$$

Total number of c_1+c_2 thus derived will be 23.

Now consider $k=1001$ as 11×91 . C_1+c_2 will be identified as $11(91+6n) + 11(91-6n)$ till we reach a value of n that leads $(91-6n)$ to the first element of the S_7 series (since 91 is an element of the S_7 series), where n is a natural number. C_1+c_2 combinations thus derived would be:

$$11 \times 85 + 11 \times 97$$

$$11 \times 79 + 11 \times 103$$

$$11 \times 73 + 11 \times 109$$

⋮

$$11 \times 7 + 11 \times 175$$

However it needs to be noted here that some c_1+c_2 combinations identified by 11 may have been already identified while calculating c_1+c_2 for 7. Those need to be identified and subtracted to avoid double counting of the same c_1+c_2 . For this purpose we need to identify a c_1+c_2 that involves a composite number divisible by both 7 and 11. The first such number would be 7×11 itself, since it is being derived while identifying c_1+c_2 corresponding to 11 and have already been identified while calculating c_1+c_2 corresponding to 7. Thereafter every $[11 \times 7] \times [1 + 6n]$ th number satisfies this condition; as long as $[11 \times 7] \cdot [1+6n] \leq k$ where n is a natural number.

In general terms the conditions for identifying composites already derived can be expressed as:

$p_2 p_1 [1 + 6n] \leq k$; where p_2 is the prime element, which is a factor of particular k . Here p_1 is the previous prime element which is a factor of k and whose c_1+c_2 have already been identified. In this equation $(11 \times 7) + (11 \times 175)$ and $(11 \times 49) + (11 \times 133)$ and $(11 \times 91) + (11 \times 91)$ would be 3 such c_1+c_2 combinations that are already identified by 7.

Now consider 1001 as 13×77 :

C1+c2 combinations would be identified as $13(77+6n) + 13(77-6n)$ till we reach a value of n that leads $(77-6n)$ to the first element of the S5 series (since 77 is an element of the S5 series), where n is a natural number. The actual combinations thus derived would be:-

13x71 + 13x83
13x65 + 13x89
13x59 + 13x95
⋮
13x5 + 13x149

Here again some combinations could be those which have been already identified by 7 and 11. Those need to be identified. Applying the logical conditions given above, we get the following combinations already derived:-

13x11 + 13x143- already derived while calculating c1+c2 for 11.
13x77 + 13x77- already derived while calculating c1+c2 for 11.
13x35 + 13x119- already identified while calculating c1+c2 for 7.

After identifying those common c1+c2 combinations we need to now calculate the number of total c1+c2 identified by the method.

The total number of c1+c2 combinations derived from the midpoint may be summarised in the following table:-

Factor of midpoint	ncc (number of c1+c2 combinations) derived, from factor of midpoint (k), from midpoint (k)	ncc already derived from previous factors of midpoint	nucc (number of unique combinations of type c1+c2)
7	24	24
11	15	3	12
13	13	3	10
Number of total combinations of type c1+c2 derived due to midpoint(k) of the even number (2k)=			24+12+10 = 46

Table 11A.1: Number of unique c1+c2 combinations of type 1 derived from composite mid-point(k) of the even number(2k)

2. Case-II: For even number (2k) of CSE 4 type where midpoint k is an even number

Example 6:

Consider $2k = 196$ i.e. SADN7//6

Midpoint $k = 98$ which is a composite even number whose factors are $7 \times 2 \times 7$.

Here we identify all the factors of k. Of these, we consider the prime factors which lie on the relevant series for derivation of c1+c2 combination due to mid-point (k).

In this equation corresponding factor as $7 \times 2 = 14$ i.e. we express k as 7×14 . Here c1+c2 combination will be in nature of:

$7 \times (14-3) + 7 \times (14+3)$
i.e. $7 \times 11 + 7 \times 17$

Here addition (and subtraction) of 3 gives us 17 (and 11) which are elements of S5 series.

Thereafter other c1+c2 combinations will be derived as $7 \times (11-6n) + 7 \times (17+6n)$ for different values of natural number n, till we reach a value of $(11-6n)$ which leads us to the first element of the S5 series.

Similarly consider the number $2k = 748$ i.e. SADN1//8. Midpoint $k = 374$ which can be expressed in terms of its factors as $11 \times 2 \times 17$ or 11×34 .

$C1+c2$ combinations that can be identified here are:-

$$11 \times (34-3) + 11(34+3)$$

$$\text{i.e. } 11 \times 31 + 11 \times 37$$

Thereafter other combinations would be:

$11 \times (31-6n) + 11(37+6n)$ for different values of natural number n , till we reach a value of $(31-6n)$ that leads us to the first element of the $S7$ series.

Similarly since k can also be expressed as 17×22 , other $c1+c2$ combinations identified are:-

$$17(22-3) + 17(22+3)$$

$$\text{i.e. } 17 \times 19 + 17 \times 25$$

Therefore other $c1+c2$ identified would be:-

$$17(19-6n) + 17(25+6n)$$

Till we reach a value of $(19-6n)$ for different values of natural number n , that leads us to the first element of the $S7$ series.

3. Case-III: For even number ($2k$) of CSE 12 type and CSE 6 type

Example7:

Deriving $C1+C2$ of type 1 for even numbers of SADN (3,6,9) of CSE 12 type

In case of even numbers of SADN (3,6,9), $c1+c2$ combinations derived from the mid-point would be identified in a different manner. Here, if even number ($2k$) is having SADN(3), its midpoint (k) would be of SADN(6); if even number ($2k$) is having SADN(6), its midpoint (k) would be of SADN(3); and if even number ($2k$) is having SADN(9), its midpoint (k) would be of SADN(9). Therefore, $k+/-6$ would also be of SADN (3,6,9). The method that would be employed here would be to find the factors of $2k$ and consider them in the following manner. Consider $2k=300$. The factors of this number would be 2,3,4,5,6,10,12,15,18,20,25,30,50,60,100,150. Of these, the factor 5 is such that it lies on the S-5 series. The only factor which lies on $S1$ or $S5$ series is 5. We find the value of $2k/5=300/5=60$. Thereafter, we identify all possible combinations of two numbers in which 60 can be expressed as a summation of two numbers in the form of $(6n+1)+[60-(6n+1)]$ where the value of n ranges from 1 to such a value where $(6n+1)$ remains < 60 . For instance the combinations would be $7+53$; $13+47$; $19+41$, ..., $55+5$. Now, by multiplying both the terms that are being summed up to 60 by 5, we can derive $c1+c2$ combinations of type 1. For instance, the combinations would be:-

$$5 \times 7(\text{i.e.} 35) + 5 \times 53(\text{i.e.} 265)$$

$$5 \times 13(\text{i.e.} 65) + 5 \times 47(\text{i.e.} 235)$$

$$5 \times 19(\text{i.e.} 95) + 5 \times 41(\text{i.e.} 205)$$

⋮

$$5 \times 55(\text{i.e.} 275) + 5 \times 5(\text{i.e.} 25)$$

In this way, the number of $c1+c2$ combinations of type 1 is equal to the number of possible values of n . So we identify total of 10 $c1+c2$ combinations of type 1.

Example 8:

Deriving C1+C2 of type 1 for even numbers of SADN (3,6,9) of CSE 6 type

Consider the even number 462. It is of SADN 3 and CSE 6 type. C1+C2 combinations derived from the mid-point can be identified as follows. Firstly identify the factors for the even number. Here, 7 and 11 are factors of $2k$. In the next step find the value of $462/7$ which would be 66. Now identify such combinations of numbers that add up to 66 and are in the nature of $(6n+1) + [66-(6n+1)]$ where the value of n would range from 1 to such an integer where value of $(6n+1)$ would be < 66 . Such combinations would be 7+59; 13+53; 19+47, ..., 61+5. Now multiplying both the terms in the summation function by 7 will yield C1+C2 combinations. These would be as follows:

$$7 \times 7(\text{i.e. } 49) + 7 \times 59(\text{i.e. } 413)$$

$$7 \times 13(\text{i.e. } 91) + 7 \times 53(\text{i.e. } 371)$$

$$7 \times 19(\text{i.e. } 133) + 7 \times 47(\text{i.e. } 329)$$

⋮

$$7 \times 61(\text{i.e. } 427) + 7 \times 5(\text{i.e. } 35)$$

In the above considered case, total number of combinations would be 10.

Further, find the value of $462/11=42$. Now find the combinations of two numbers which would add up to 42 and are in the nature of $(6n+1) + [42-(6n+1)]$ where the value of n ranges from 1 to such an integer where value of $6n+1$ would be < 42 . Such combinations would be 7+35; 13+29; 19+23, ..., 37+5. Thereafter multiplying both the terms in the summation function by 11 we can derive C1+C2 combinations. These would be identified as:

$$11 \times 7(\text{i.e. } 77) + 11 \times 35(\text{i.e. } 385)$$

$$11 \times 13(\text{i.e. } 143) + 11 \times 29(\text{i.e. } 319)$$

$$11 \times 19(\text{i.e. } 209) + 11 \times 23(\text{i.e. } 253)$$

⋮

$$11 \times 37(\text{i.e. } 403) + 11 \times 5(\text{i.e. } 55)$$

From these combinations we need to subtract those which have been derived earlier. For instance, the combination $11 \times 7(\text{i.e. } 77) + 11 \times 35(\text{i.e. } 385)$ has already been derived earlier and will therefore not be counted here. After accounting for the repetitions, the number of unique combinations would be $6-1=5$. Therefore, the total number of c1+c2 combinations for $2k$ as 462 of type 1 would be $10+5=15$.

11B.**Step2:****c1+c2 of type 2 derived from last digit of even number (2k)****Step2: c1+c2 derived from last digit of 2k**

Step 2: This step depends on the last digit of the even number which will lead us to find out c1+c2 combinations in which one of the terms is a multiple of 5. As mentioned earlier (in section 6), for any even number, depending on the last digit, some combinations of odd numbers cannot be considered in the identification of p1+p2 combinations, since the corresponding number would be a multiple of 5. For instance; for 2k ending in 2, prime numbers ending in 7 cannot be considered since:

$$(2k/2) - (ap/7) = (a/5) \text{ i.e. an odd number ending in 5}$$

Therefore, if we can identify composite numbers on the relevant series that end in 7, these would constitute c1+c2 alongwith their corresponding number that is a multiple of 5. Following table shows the last digit of composite odd numbers that need to be identified for 2k ending in a particular digit:

Last digit of 2k	Last digit of composite odd number
2	7
4	9
6	1
8	3
0	5

Table 11B.1: Last digit of composite odd number which combines with another odd number ending in digit 5, to form c1+c2 combination corresponding to last digit of given even number

Last digit of composite odd number	Last digits of corresponding factors of composite odd numbers				
1	1x1	3x7	9x9
3	1x3	7x9
5	1x5	3x5	5x5	7x5	9x5
7	1x7	3x9
9	1x9	3x3	7x7

Table 11B.2: Last digits of factors corresponding to last digit of yielding composite odd number

Table 11B.1 implies that if $2k$ ends in 2, then composite odd numbers on the relevant series that end in 7 will form part of $C1+C2$ combinations. Composite odd numbers ending in 7 can be derived by multiplying odd numbers that end in 1 with odd numbers that end in 7 (refer table 11B.2). Similarly, composite odd numbers ending in 7 can also be derived as a product of odd numbers ending in 1&7 or 3&9. For instance, consider $2k$ as 412 (i.e. SADN 7//2 which means the relevant series is the S5 series). As composites on S5 series are derived as products of inter-series elements (refer section 5A), this can be derived as $7*11$, $7*41$, $13*29$, $19*23$ and so on. Now consider $2k$ to be 422 (i.e. SADN 8//2) which means the relevant series is the S7 series. Composite odd numbers on the S7 series are formed as products of intra-series elements. These can be derived as $7*31$, $11*187$, $13*17$ and so on. In all these cases the composite odd numbers will form part of $C1+C2$ combination since the corresponding number in the combination will essentially be a multiple of 5. The only exception to this pattern will be:

$$'2k - 5'$$

since in such a case:

$2k - \text{'composite odd number ending in 7'} = 5$ and 5 itself is a prime number thus leading us to a $p+c$ type of combination.

It is important to note here, as supplement to above discussion, that if $2k-5$ happens to be prime then it gives us a $p1+p2$ combination AND if $2k-5$ happens to be composite then it gives us a $p+c$ combination.

If $2k$ is an even number ending in 4 then composite odd numbers on the relevant series ending in 9 will form part of $c1+c2$ combinations. These can be derived as products of odd numbers ending in 1&9 or 3&3 or 7&7 (refer table 11B.2).

If $2k$ is an even number ending in 6 then composite odd numbers on the relevant series ending in 1 will form part of $C1+C2$ combinations. These can be derived as products of elements ending in 1 with other element ending in 1 or as products of odd numbers ending in 3 with odd numbers ending in 7 or as products of odd numbers ending in 9 with other odd numbers ending in 9 (refer table 11B.2).

If $2k$ is an even number ending in 8 then composite odd numbers ending in 3 on the relevant series will form part of $C1+C2$ combinations which in turn can be derived as products of odd numbers ending in 3&1 or 7&9 (refer table 11B.2).

If $2k$ is an even number ending in 0, then composite odd numbers on the relevant series ending in 5 will form part of $C1+C2$ combinations. Products of all elements ending in 5 with any other odd number will end in 5 (refer table 11B.2).

Number of $C1+C2$ combinations of this type (i.e. type 2) for a given $2k$ can be generalized as follows:

Case(1):

SADN of EN = {7,4,1}

EN = 2k

Case(1A): EN/2 = k is a prime number

Case(1B): EN/2 = k is a composite number

If 'k' is a composite number:

Number of acceptable combinations of elements is given as n_{acc}

Case(1BP): If number of composites is less than number of primes (i.e. $n_c < n_p$ implies that even if all composites are prime-eaters; there exists atleast one p_1+p_2 pair

Case(1BC): If number of composites is greater than or equal to number of primes (i.e. $n_c \geq n_p$ implies that we need to find total number of unique C1+C2 pairs

C1+C2 of Type-1:

It has already been discussed in section 11A.

C1+C2 of Type-2 for even number having SADN (7,4,1):

EN= SADN(7,4,1//0) OR SADN(7,4,1//2) OR SADN(7,4,1//4) OR SADN(7,4,1//6) OR SADN(7,4,1//8)

Here EN= SADN(7,4,1//0) indicates even numbers having SADN(7,4,1) and last digit as 0

Case(1BC-iA):

EN = SADN(7,4,1//0//2) indicates even numbers of SADN(7,4,1//0) and cyclical series element as '2'

EN = SADN(7,4,1//0//2)

C1 of SADN(5,2,8//5) + C2 of SADN(5,2,8//5) = 2k

$(35+30n_a'') + [2k - (35+30n_a'')] = 2k$

$35+30n_a'' < 2k$

Number of Composites having last digit as 5 is given as $n_a'' < (2k-35)/30$

No. of type-2 C1+C2 combinations = floor function $[(1/2)*(2k-35)/30] + 1$

Case(1BC-iB):

EN = SADN(7,4,1//0//4) indicates even numbers of SADN(7,4,1//0) and cyclical series element as '4'

EN = SADN(7,4,1//0//4)

$$C1 \text{ of } \text{SADN}(5,2,8//5) + C2 \text{ of } \text{SADN}(5,2,8//5) = 2k$$

$$(35+30n_b'') + [2k - (35+30n_b'')] = 2k$$

$$(35+30n_b'') < 2k$$

$$\text{Number of } C//5 \text{ is given as } n_b'' < (2k-35)/30$$

$$\text{No. of type-2 } C1+C2 \text{ combinations} = \text{floor function } [(1/2)*(2k-35)/30]$$

Case(1BC-ii):

$$EN = \text{SADN}(7,4,1//2)$$

Here $EN = \text{SADN}(7,4,1//2)$ indicates even numbers having $\text{SADN}(7,4,1)$ and last digit as 2

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(5,2,8)//7] = 2k$$

Case(1BC-iiA) For $C2[\text{SADN}(5,2,8)//7]$ implies that $//1x//7$
 $[\text{SADN}(5,2,8)//1] \times [\text{SADN}(7,4,1)//7]$

Case(1BC-iiA-i)

For $\text{SADN}(5,2,8)//1 \times \text{SADN}(7,4,1)//7:-$

$$(11+30n''') \times (7+30n^{iv}) \leq (2k-35)$$

$$\text{For } n''' = 0; n_0^{iv} = \text{floor function of } [\{(2k-35)/(11*30)\} - (7/30)]$$

$$\text{For } n''' = 1; n_1^{iv} = \text{floor function of } [\{(2k-35)/(11+30)*30\} - (7/30)]$$

$$\text{For } n''' = 2; n_2^{iv} = \text{floor function of } [\{(2k-35)/(11+60)*30\} - (7/30)]$$

⋮

⋮

For $n''' = n'''_{\max}$ i.e. floor function of $[\{(2k-35)/(7*30)\} - (11/30)]$; $n_{\text{for max value of } n'''}^{iv} = \text{floor function of } [(2k-35)/\{(11+30n'''_{\max}) * 30\} - (7/30)]$

Hence total number of $n^{iv} = n_0^{iv} + n_1^{iv} + n_2^{iv} + \dots + n_{\text{for max value of } n'''}^{iv}$

Or total number of possible values of $n^{iv} = \sum_{n'''=0 \text{ to floor function of } [\{(2k-35)/(7*30)\} - (11/30)]} \text{floor function of } [(2k-35)/\{(11+30n''') * 30\} - (7/30)]$

Number of type-2 $c1+c2$ combinations = $\sum_{n'''=0 \text{ to floor function of } [\{(2k-35)/(7*30)\} - (11/30)]} \text{floor function of } [(2k-35)/\{(11+30n''') * 30\} - (7/30)]$

Case(1BC-iiA-ii)

For SADN(7,4,1)//1 x SADN(5,2,8)//7:-

$$(31+30n^v) \times (17+30n^{vi}) \leq (2k-35)$$

For $n^v = 0$; $n^{vi}_0 = \text{floor function of } [(2k-35)/(31*30) - (17/30)]$

For $n^v = 1$; $n^{vi}_1 = \text{floor function of } [(2k-35)/\{(31+30)*30\} - (17/30)]$

For $n^v = 2$; $n^{vi}_2 = \text{floor function of } [(2k-35)/\{(31+60)*30\} - (17/30)]$

For $n^v = n^v_{\max}$ i.e. floor function of $[(2k-35)/(17*30) - (31/30)]$; $n^{vi} = \text{floor function of } [(2k-35)/\{(31+30 n^v_{\max}) * 30\} - (17/30)]$

Hence total number of $n^{vi} = n^{vi}_0 + n^{vi}_1 + n^{vi}_2 + \dots + n^{vi}_{\text{for max value of } n^v}$

Or total number of possible values of $n^{vi} = \sum_{n^v=0 \text{ to floor function of } [(2k-35)/(17*30) - (31/30)]} \text{floor function of } [(2k-35)/\{(31+30 n^v)*30\} - (17/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^v=0 \text{ to floor function of } [(2k-35)/(17*30) - (31/30)]} \text{floor function of } [(2k-35)/\{(31+30 n^v)*30\} - (17/30)]$

Case(1BC-iiA-iii)

For SADN(7,4,1)//3 x SADN(5,2,8)//9:-

$$(13+30n^{vii}) \times (29+30n^{viii}) \leq (2k-35)$$

For $n^{vii} = 0$; $n^{viii}_0 = \text{floor function of } [(2k-35)/(13*30) - (29/30)]$

For $n^{vii} = 1$; $n^{viii}_1 = \text{floor function of } [(2k-35)/\{(13+30)*30\} - (29/30)]$

For $n^{vii} = 2$; $n^{viii}_2 = \text{floor function of } [(2k-35)/\{(13+60)*30\} - (29/30)]$

For $n^{vii} = n^{vii}_{\max}$ i.e. floor function of $[(2k-35)/(29*30) - (13/30)]$; $n^{viii} = \text{floor function of } [(2k-35)/\{(13+30 n^{vii}_{\max}) * 30\} - (29/30)]$

Hence total number of $n^{viii} = n^{viii}_0 + n^{viii}_1 + n^{viii}_2 + \dots + n^{viii}_{\text{for max value of } n^{vii}}$

Or total number of possible values of $n^{viii} = \sum_{n^{vii}=0 \text{ to floor function of } [(2k-35)/(29*30) - (13/30)]} \text{floor function of } [(2k-35)/\{(13+30 n^{vii}) * 30\} - (29/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^{vii}=0 \text{ to floor function of } [(2k-35)/(29*30) - (13/30)]} \text{floor function of } [(2k-35)/\{(13+30 n^{vii}) * 30\} - (29/30)]$

Case(1BC-iiA-iv)

For $\text{SADN}(5,2,8)//3 \times \text{SADN}(7,4,1)//9:-$

$$(23+30n^{ix}) \times (19+30n^x) \leq (2k-35)$$

For $n^{ix} = 0$; $n^x_0 = \text{floor function of } [(2k-35)/(23*30) - (19/30)]$

For $n^{ix} = 1$; $n^x_1 = \text{floor function of } [(2k-35)/\{(23+30)*30\} - (19/30)]$

For $n^{ix} = 2$; $n^x_2 = \text{floor function of } [(2k-35)/\{(23+60)*30\} - (19/30)]$

For $n^{ix} = n^{ix}_{\max}$ i.e. floor function of $[\{(2k-35)/(19*30)\} - (23/30)]$; $n^{viii} = \text{floor function of } [(2k-35)/\{(23+30n^{'''}_{\max}) * 30\} - (19/30)]$

Hence total number of $n^x = n^x_0 + n^x_1 + n^x_2 + \dots + n^x_{\text{for max value of } n^{ix}}$

Or total number of possible values of $n^x = \sum_{n^{ix}=0 \text{ to floor function of } [\{(2k-35)/(19*30)\} - (23/30)]} \text{floor function of } [(2k-35)/\{(23+30 n^{ix}) * 30\} - (19/30)]$

Number of type-2 $c1+c2$ combinations = $\sum_{n^{ix}=0 \text{ to floor function of } [\{(2k-35)/(19*30)\} - (23/30)]} \text{floor function of } [(2k-35)/\{(23+30 n^{ix}) * 30\} - (19/30)]$

Case(1BC-iii):

$EN = \text{SADN}(7,4,1//4)$

Here $EN = \text{SADN}(7,4,1//4)$ includes even numbers having $\text{SADN}(7,4,1)$ and last digit as 4

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(5,2,8)//9] = 2k$$

Case(1BC-iiiA)

For $C2[\text{SADN}(5,2,8)//9]$

Case(1BC-iiiA-i)

For $\text{SADN}(7,4,1)//1 \times \text{SADN}(5,2,8)//9$

$$(31+30n) \times (29+30n') \leq (2k-35)$$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-35)/(31*30) - (29/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-35)/\{(31+30)*30\} - (29/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-35)/\{(31+60)*30\} - (29/30)]$

For $n = n'_{\max}$ i.e. floor function of $[\{(2k-35)/(29*30)\} - (31/30)]$; $n^{'''} = \text{floor function of } [(2k-35)/\{(31+30n^{'''}_{\max}) * 30\} - (29/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0}^{\text{floor function of } [(2k-35)/(29*30)] - (31/30)} \text{floor function of } [(2k-35)/\{(31+30n)*30\} - (29/30)]$

Number of type-2 c_1+c_2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-35)/(29*30)] - (31/30)} \text{floor function of } [(2k-35)/\{(31+30n)*30\} - (29/30)]$

Case(1BC-iiiA-ii)

For $\text{SADN}(5,2,8)//1 \times \text{SADN}(7,4,1)//9:-$

$$(11+30n'') \times (19+30n''') \leq (2k-35)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-35)/(11*30)] - (19/30)$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-35)/\{(11+30)*30\} - (19/30)]$

For $n'' = 2$; $n'''_2 = \text{floor function of } [(2k-35)/\{(11+60)*30\} - (9/30)]$

For $n'' = n''_{\text{max}}$ i.e. floor function of $[(2k-35)/(19*30)] - (11/30)$; $n''' = \text{floor function of } [(2k-35)/\{(11+30n''_{\text{max}})*30\} - (19/30)]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0}^{\text{floor function of } [(2k-35)/(19*30)] - (11/30)} \text{floor function of } [(2k-35)/\{(11+30n'')*30\} - (19/30)]$

Number of type-2 c_1+c_2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-35)/(19*30)] - (11/30)} \text{floor function of } [(2k-35)/\{(11+30n'')*30\} - (19/30)]$

Case(1BC-iiiA-iii)

For $\text{SADN}(7,4,1)//7 \times \text{SADN}(5,2,8)//7:-$

$$(7+30n^{iv}) \times (17+30n^v) \leq (2k-35)$$

For $n^{iv} = 0$; $n^v_0 = \text{floor function of } [(2k-35)/(7*30)] - (17/30)$

For $n^{iv} = 1$; $n^v_1 = \text{floor function of } [(2k-35)/\{(7+30)*30\} - (17/30)]$

For $n^{iv} = 2$; $n^v_2 = \text{floor function of } [(2k-35)/\{(7+60)*30\} - (17/30)]$

For $n^{iv} = n^{iv}_{\text{max}}$ i.e. floor function of $[(2k-35)/(17*30)] - (7/30)$; $n^v = \text{floor function of } [(2k-35)/\{(7+30n^{iv}_{\text{max}})*30\} - (17/30)]$

Hence total number of $n^v = n^v_0 + n^v_1 + n^v_2 + \dots + n^v_{\text{for max value of } n^{iv}}$

Or total number of possible values of $n^v = \sum_{n^v=0 \text{ to floor function of } [(2k-35)/(17*30)]-(7/30)} \text{floor function of } [(2k-35)/\{(7+30 n^{iv}) * 30\} -(17/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^v=0 \text{ to floor function of } [(2k-35)/(19*30)]-(11/30)} \text{floor function of } [(2k-35)/\{(7+30 n^{iv}) * 30\} -(17/30)]$

Case(1BC-iiiA-iv)

For SADN(7,4,1)//3 x SADN(5,2,8)//3:-

$$(13+30n^{vi}) \times (23+30n^{vii}) \leq (2k-35)$$

For $n^{vi} = 0$; $n^{vii}_0 = \text{floor function of } [(2k-35)/(13*30) -(23/30)]$

For $n^{vi} = 1$; $n^{vii}_1 = \text{floor function of } [(2k-35)/\{(13+30)*30\} -(23/30)]$

For $n^{vi} = 2$; $n^{vii}_2 = \text{floor function of } [(2k-35)/\{(13+60)*30\} -(23/30)]$

For $n^{vi} = n^{vi}_{\max}$ i.e. floor function of $[(2k-35)/(23*30) -(13/30)]$; $n^{vii} = \text{floor function of } [(2k-35)/\{(13+30n^{vi}_{\max}) * 30\} -(23/30)]$

Hence total number of $n^{vii} = n^{vii}_0 + n^{vii}_1 + n^{vii}_2 + \dots + n^{vii}_{\text{for max value of nvi}}$

Or total number of possible values of $n^{vii} = \sum_{n^{vi}=0 \text{ to floor function of } [(2k-35)/(23*30)]-(13/30)} \text{floor function of } [(2k-35)/\{(13+30 n^{vi}) * 30\} -(23/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^{vi}=0 \text{ to floor function of } [(2k-35)/(23*30)]-(13/30)} \text{floor function of } [(2k-35)/\{(13+30 n^{iv}) * 30\} -(23/30)]$

Case(1BC-iv):

EN= SADN(7,4,1//6)

Here EN= SADN(7,4,1//6) indicates even numbers having SADN(7,4,1) and last digit as 6

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(5,2,8)//1] = 2k$$

Case(1BC-ivA)

For $C2[\text{SADN}(5,2,8)//1]$

Case(1BC-ivA-i)

For SADN(7,4,1)//3 x SADN(5,2,8)//7:-

$$(13+30n) \times (17+30n') \leq (2k-35)$$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-35)/(13*30) - (17/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-35)/\{(13+30)*30\} - (17/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-35)/\{(13+60)*30\} - (17/30)]$

For $n = n'_{\max}$ i.e. floor function of $[(2k-35)/(17*30)] - (13/30)$; $n''' = \text{floor function of } [(2k-35)/\{(13+30n'_{\max}) * 30\} - (17/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0 \text{ to floor function of } [(2k-35)/(17*30)] - (13/30)} \text{floor function of } [(2k-35)/\{(13+30n)*30\} - (17/30)]$

Number of type-2 c_1+c_2 combinations = $\sum_{n=0 \text{ to floor function of } [(2k-35)/(17*30)] - (13/30)} \text{floor function of } [(2k-35)/\{(13+30n)*30\} - (17/30)]$

Case(1BC-ivA-ii)

For $\text{SADN}(5,2,8)/3 \times \text{SADN}(7,4,1)/7:-$

$$(23+30n'') \times (7+30n''') \leq (2k-35)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-35)/(23*30) - (7/30)]$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-35)/\{(23+30)*30\} - (7/30)]$

For $n'' = 2$; $n'''_{iv_2} = \text{floor function of } [(2k-35)/\{(23+60)*30\} - (7/30)]$

For $n'' = n''_{\max}$ i.e. floor function of $[(2k-35)/(7*30)] - (23/30)$; $n''' = \text{floor function of } [(2k-35)/\{(23+30n''_{\max}) * 30\} - (7/30)]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0 \text{ to floor function of } [(2k-35)/(7*30)] - (23/30)} \text{floor function of } [(2k-35)/\{(23+30n'') * 30\} - (7/30)]$

Number of type-2 c_1+c_2 combinations = $\sum_{n''=0 \text{ to floor function of } [(2k-35)/(7*30)] - (23/30)} \text{floor function of } [(2k-35)/\{(23+30n'') * 30\} - (7/30)]$

Case(1BC-ivA-iii)

For SADN(7,4,1)//1 x SADN(5,2,8)//1:-

$$(31+30n^{iv}) \times (11+30n^v) \leq (2k-35)$$

For $n^{iv} = 0$; $n^v_0 = \text{floor function of } [(2k-35)/(31*30) - (11/30)]$

For $n^{iv} = 1$; $n^v_1 = \text{floor function of } [(2k-35)/\{(31+30)*30\} - (11/30)]$

For $n^{iv} = 2$; $n^v_2 = \text{floor function of } [(2k-35)/\{(31+60)*30\} - (11/30)]$

For $n^{iv} = n^{iv}_{\max}$ i.e. floor function of $[\{(2k-35)/(11*30)\} - (31/30)]$; $n^v = \text{floor function of } [(2k-35)/\{(31+30 n^{iv}_{\max}) * 30\} - (11/30)]$

Hence total number of $n^v = n^v_0 + n^v_1 + n^v_2 + \dots + n^v_{\text{for max value of niv}}$

Or total number of possible values of $n^v = \sum_{niv=0 \text{ to floor function of } [\{(2k-35)/(11*30)\} - (31/30)]} \text{floor function of } [(2k-35)/\{(31+30 n^{iv}) * 30\} - (11/30)]$

Number of type-2 c1+c2 combinations = $\sum_{niv=0 \text{ to floor function of } [\{(2k-35)/(11*30)\} - (31/30)]} \text{floor function of } [(2k-35)/\{(31+30 n^{iv}) * 30\} - (11/30)]$

Case(1BC-ivA-iv)

For SADN(7,4,1)//9 x SADN(5,2,8)//9:-

$$(19+30n^{vi}) \times (29+30n^{vii}) \leq (2k-35)$$

For $n^{vi} = 0$; $n^{vii}_0 = \text{floor function of } [(2k-35)/(19*30) - (29/30)]$

For $n^{vi} = 1$; $n^{vii}_1 = \text{floor function of } [(2k-35)/\{(19+30)*30\} - (29/30)]$

For $n^{vi} = 2$; $n^{vii}_2 = \text{floor function of } [(2k-35)/\{(19+60)*30\} - (29/30)]$

For $n^{vi} = n^{vi}_{\max}$ i.e. floor function of $[\{(2k-35)/(29*30)\} - (19/30)]$; $n^{vii} = \text{floor function of } [(2k-35)/\{(19+30 n^{vi}_{\max}) * 30\} - (29/30)]$

Hence total number of $n^{vii} = n^{vii}_0 + n^{vii}_1 + n^{vii}_2 + \dots + n^{vii}_{\text{for max value of nvi}}$

Or total number of possible values of $n^{vii} = \sum_{nvi=0 \text{ to floor function of } [\{(2k-35)/(29*30)\} - (19/30)]} \text{floor function of } [(2k-35)/\{(19+30 n^{vi}) * 30\} - (29/30)]$

Number of type-2 c1+c2 combinations = $\sum_{niv=0 \text{ to floor function of } [\{(2k-35)/(29*30)\} - (19/30)]} \text{floor function of } [(2k-35)/\{(19+30 n^{iv}) * 30\} - (29/30)]$

Case(1BC-v):

$$EN = \text{SADN}(7,4,1//8)$$

Here $EN = \text{SADN}(7,4,1//6)$ includes even numbers having $\text{SADN}(7,4,1)$ and last digit as 8

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(5,2,8)//3] = 2k$$

Case(1BC-vA)

$$\text{For } C2[\text{SADN}(5,2,8)//3]$$

Case(1BC-vA-i)

For $\text{SADN}(7,4,1)//1 \times \text{SADN}(5,2,8)//3:-$

$$(31+30n) \times (23+30n') \leq (2k-35)$$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-35)/(31*30) - (23/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-35)/\{(31+30)*30\} - (23/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-35)/\{(31+60)*30\} - (23/30)]$

For $n = n'_{\max}$ i.e. floor function of $[(2k-35)/(23*30) - (31/30)]$; $n'' = \text{floor function of } [(2k-35)/\{(31+30n'_{\max})*30\} - (23/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0 \text{ to floor function of } [(2k-35)/(23*30) - (31/30)]} \text{floor function of } [(2k-35)/\{(31+30n)*30\} - (23/30)]$

Number of type-2 $c1+c2$ combinations = $\sum_{n=0 \text{ to floor function of } [(2k-35)/(23*30) - (31/30)]} \text{floor function of } [(2k-35)/\{(31+30n)*30\} - (23/30)]$

Case(1BC-vA-ii)

For $\text{SADN}(5,2,8)//1 \times \text{SADN}(7,4,1)//3:-$

$$(11+30n'') \times (13+30n''') \leq (2k-35)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-35)/(11*30) - (13/30)]$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-35)/\{(11+30)*30\} - (13/30)]$

For $n'' = 2$; $n'''_2 = \text{floor function of } [(2k-35)/\{(11+60)*30\} - (13/30)]$

For $n'' = n''_{\max}$ i.e. floor function of $\{[(2k-35)/(13*30)] - (11/30)\}$; $n''' =$ floor function of $[(2k-35)/\{(11+30n''_{\max}) * 30\} - (13/30)]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0 \text{ to floor function of } \{[(2k-35)/(13*30)] - (11/30)\}}$
 floor function of $[(2k-35)/\{(11+30n'') * 30\} - (13/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n''=0 \text{ to floor function of } \{[(2k-35)/(13*30)] - (11/30)\}}$ floor function of $[(2k-35)/\{(11+30n'') * 30\} - (13/30)]$

Case(1BC-vA-iii)

For $\text{SADN}(7,4,1)/7 \times \text{SADN}(5,2,8)/9:-$

$$(7+30n^{iv}) \times (29+30n^v) \leq (2k-35)$$

For $n^{iv} = 0$; $n^v_0 =$ floor function of $[(2k-35)/(7*30) - (29/30)]$

For $n^{iv} = 1$; $n^v_1 =$ floor function of $[(2k-35)/\{(7+30)*30\} - (29/30)]$

For $n^{iv} = 2$; $n^v_2 =$ floor function of $[(2k-35)/\{(7+60)*30\} - (29/30)]$

For $n^{iv} = n^{iv}_{\max}$ i.e. floor function of $\{[(2k-35)/(29*30)] - (7/30)\}$; $n^v =$ floor function of $[(2k-35)/\{(7+30n^{iv}_{\max}) * 30\} - (29/30)]$

Hence total number of $n^v = n^v_0 + n^v_1 + n^v_2 + \dots + n^v_{\text{for max value of } n^{iv}}$

Or total number of possible values of $n^v = \sum_{n^{iv}=0 \text{ to floor function of } \{[(2k-35)/(11*30)] - (31/30)\}}$ floor function of $[(2k-35)/\{(7+30 n^{iv}) * 30\} - (29/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^{iv}=0 \text{ to floor function of } \{[(2k-35)/(29*30)] - (7/30)\}}$ floor function of $[(2k-35)/\{(7+30 n^{iv}) * 30\} - (29/30)]$

Case(1BC-vA-iv)

For $\text{SADN}(5,2,8)/7 \times \text{SADN}(7,4,1)/9:-$

$$(17+30n^{vi}) \times (19+30n^{vii}) \leq (2k-35)$$

For $n^{vi} = 0$; $n^{vii}_0 =$ floor function of $[(2k-35)/(17*30) - (19/30)]$

For $n^{vi} = 1$; $n^{vii}_1 =$ floor function of $[(2k-35)/\{(17+30)*30\} - (19/30)]$

For $n^{vi} = 2$; $n^{vii}_2 = \text{floor function of } [(2k-35)/\{(17+60)*30\} - (19/30)]$

For $n^{vi} = n^{vi}_{\max}$ i.e. floor function of $[\{(2k-35)/(19*30)\} - (17/30)]$; $n^{vii} = \text{floor function of } [(2k-35)/\{(17+30n^{vii}_{\max})*30\} - (19/30)]$

Hence total number of $n^{vii} = n^{vii}_0 + n^{vii}_1 + n^{vii}_2 + \dots + n^{vii}_{\text{for max value of nvi}}$

Or total number of possible values of $n^{vii} = \sum_{nvi=0 \text{ to floor function of } [\{(2k-35)/(29*30)\} - (17/30)]} \text{floor function of } [(2k-35)/\{(17+30n^{vi})*30\} - (19/30)]$

Number of type-2 $c1+c2$ combinations = $\sum_{niv=0 \text{ to floor function of } [\{(2k-35)/(19*30)\} - (17/30)]} \text{floor function of } [(2k-35)/\{(17+30n^{iv})*30\} - (19/30)]$

Case(2):

SADN of EN = {5,2,8}

EN = 2k

Case(2A): EN/2 = k is a prime number

Case(2B): EN/2 = k is a composite number

If 'k' is a composite number:

Number of acceptable combinations of elements is given as n_{acc}

Case(2BP): If number of composites is less than number of primes (i.e. $n_c < n_p$ implies that even if all composites are prime-eaters; there exists atleast one $p1+p2$ pair

Case(2BC): If number of composites is greater than or equal to number of primes (i.e. $n_c \geq n_p$ implies that we need to find total number of unique $C1+C2$ pairs

C1+C2 of Type-1:

It has already been discussed in section 11A.

C1+C2 of Type-2:

EN = SADN(5,2,8//0) OR SADN(5,2,8//2) OR SADN(5,2,8//4) OR SADN(5,2,8//6) OR SADN(5,2,8//8)

Here EN = SADN(5,2,8//0) includes even numbers having SADN(5,2,8) and last digit as 0

Case(2BC-iA):

EN = SADN(5,2,8//0//2) including even numbers of SADN(5,2,8//0) and cyclical series element as '2'

$$EN = \text{SADN}(5,2,8 // 0 // 2)$$

$$C1 \text{ of } \text{SADN}(7,4,1 // 5) + C2 \text{ of } \text{SADN}(7,4,1 // 5) = 2k$$

$$(25+30n_a'') + [2k - (25+30n_a'')] = 2k$$

$$25+30n_a'' < 2k$$

Number of Composites of $\text{SADN}(7,4,1 // 5)$ is given as $n_a'' < (2k-25)/30$

or $n_a'' = \text{floor function of } [(2k-25)/30]$

No. of type-2 $C1+C2$ combinations = floor function $[(1/2)*(2k-25)/30] + 1$

Or No. of type-2 $C1+C2$ combinations = floor function $[k/30] + 1$

Case(2BC-iB):

$EN = \text{SADN}(5,2,8 // 0 // 4)$ including even numbers of $\text{SADN}(5,2,8 // 0)$ and cyclical series element as '4'

$$EN = \text{SADN}(5,2,8 // 0 // 4)$$

$$C1 \text{ of } \text{SADN}(7,4,1 // 5) + C2 \text{ of } \text{SADN}(7,4,1 // 5) = 2k$$

$$(25+30n_b'') + [2k - (25+30n_b'')] = 2k$$

$$(25+30n_b'') < 2k$$

Number of C of $\text{SADN}(7,4,1 // 5)$ is given as $n_b'' < (2k-25)/30$

or $n_b'' = \text{floor function of } [(2k-25)/30]$

No. of type-2 $C1+C2$ combinations = floor function $[(1/2)*(2k-25)/30] + 1$

Or No. of type-2 $C1+C2$ combinations = floor function $[k/30]$

Case(2BC-ii):

$$EN = \text{SADN}(5,2,8 // 2)$$

Here $EN = \text{SADN}(5,2,8 // 2)$ includes even numbers having $\text{SADN}(5,2,8)$ and last digit as 2

$$C1[\text{SADN}(7,4,1 // 5)] + C2[\text{SADN}(7,4,1 // 7)] = 2k$$

Case(2BC-iiB) For $C2[\text{SADN}(7,4,1 // 7)]$ implies that

CASE(2BC-iiB-i) i.e. $// 1x // 7$ $[\text{SADN}(7,4,1 // 1)]x[\text{SADN}(7,4,1 // 7)]$ OR

CASE(2BC-iiB-ii) i.e. $//1x//7$ [SADN(5,2,8)//1] x [SADN(5,2,8)//7]

CASE(2BC-iiB-iii) i.e. $//3x//9$ [SADN(7,4,1)//3]x[SADN(7,4,1)//9] OR

CASE(2BC-iiB-iv) i.e. $//3x//9$ [SADN(5,2,8)//3] x [SADN(5,2,8)//9]

Case(2BC-iiB-i)

For SADN(7,4,1)//1 x SADN(7,4,1)//7 :-

$$(31+30n^{'''}) \times (7+30n^{iv}) \leq (2k-25)$$

For $n^{'''} = 0$; $n^{iv}_0 = \text{floor function of } [(2k-25)/(31*30) - (7/30)]$

For $n^{'''} = 1$; $n^{iv}_1 = \text{floor function of } [(2k-25)/\{(31+30)*30\} - (7/30)]$

For $n^{'''} = 2$; $n^{iv}_2 = \text{floor function of } [(2k-25)/\{(31+60)*30\} - (7/30)]$

For $n^{'''} = n^{'''_{\max}}$ i.e. floor function of $[(2k-25)/(7*30) - (31/30)]$; $n^{iv} = \text{floor function of } [(2k-25)/\{(31+30n^{'''_{\max}})*30\} - (7/30)]$

Hence total number of $n^{iv} = n^{iv}_0 + n^{iv}_1 + n^{iv}_2 + \dots + n^{iv}_{\text{for max value of } n^{'''}}$

Or total number of possible values of $n^{iv} = \sum_{n^{'''}=0 \text{ to floor function of } [(2k-25)/(7*30) - (31/30)]} \text{floor function of } [(2k-25)/\{(31+30n^{'''})*30\} - (7/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^{'''}=0 \text{ to floor function of } [(2k-25)/(7*30) - (31/30)]} \text{floor function of } [(2k-25)/\{(31+30n^{'''})*30\} - (7/30)]$

Case(2BC-iiB-ii)

For [SADN(5,2,8)//1] x [SADN(5,2,8)//7]:-

$$(11+30n^v) \times (17+30n^{vi}) \leq (2k-25)$$

For $n^v = 0$; $n^{vi}_0 = \text{floor function of } [(2k-25)/(11*30) - (17/30)]$

For $n^v = 1$; $n^{vi}_1 = \text{floor function of } [(2k-25)/\{(11+30)*30\} - (17/30)]$

For $n^v = 2$; $n^{vi}_2 = \text{floor function of } [(2k-25)/\{(11+60)*30\} - (17/30)]$

For $n^v = n^{v_{\max}}$ i.e. floor function of $[(2k-25)/(17*30) - (11/30)]$; $n^{vi} = \text{floor function of } [(2k-25)/\{(11+30 n^{v_{\max}})*30\} - (17/30)]$

Hence total number of $n^{vi} = n^{vi}_0 + n^{vi}_1 + n^{vi}_2 + \dots + n^{vi}_{\text{for max value of } n^v}$

Or total number of possible values of $n^{vi} = \sum_{n^v=0 \text{ to floor function of } [(2k-25)/(17*30) - (11/30)]} \text{floor function of } [(2k-25)/\{(11+30 n^v)*30\} - (17/30)]$

Number of type-2 c1+c2 combinations = $\sum_{n^v=0}^{\text{floor function of } [(2k-25)/(17*30)]-(11/30)}$ floor function of $[(2k-25)/\{(11+30 n^v)*30\}-(17/30)]$

Case(2BC-iiB-iii)

For [SADN(7,4,1)//3]x[SADN(7,4,1)//9]:-

$$(13+30n^{vii}) \times (19+30n^{viii}) \leq (2k-25)$$

For $n^{vii} = 0$; $n^{viii}_0 = \text{floor function of } [(2k-25)/(13*30)]-(19/30)$

For $n^{vii} = 1$; $n^{viii}_1 = \text{floor function of } [(2k-25)/\{(13+30)*30\}-(19/30)]$

For $n^{vii} = 2$; $n^{viii}_2 = \text{floor function of } [(2k-25)/\{(13+60)*30\}-(19/30)]$

For $n^{vii} = n^{vii}_{\max}$ i.e. floor function of $[(2k-25)/(19*30)]-(13/30)$; $n^{viii} = \text{floor function of } [(2k-25)/\{(13+30n^{vii}_{\max})*30\}-(19/30)]$

Hence total number of $n^{viii} = n^{viii}_0 + n^{viii}_1 + n^{viii}_2 + \dots + n^{viii}_{\text{for max value of } n^{vii}}$

Or total number of possible values of $n^{viii} = \sum_{n^{vii}=0}^{\text{floor function of } [(2k-25)/(19*30)]-(13/30)}$ floor function of $[(2k-25)/\{(13+30 n^{vii})*30\}-(19/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{vii}=0}^{\text{floor function of } [(2k-25)/(19*30)]-(13/30)}$ floor function of $[(2k-25)/\{(13+30 n^{vii})*30\}-(19/30)]$

Case(2BC-iiB-iv)

For [SADN(5,2,8)//3] x [SADN(5,2,8)//9]:-

$$(23+30n^{ix}) \times (29+30n^x) \leq (2k-25)$$

For $n^{ix} = 0$; $n^x_0 = \text{floor function of } [(2k-25)/(23*30)]-(29/30)$

For $n^{ix} = 1$; $n^x_1 = \text{floor function of } [(2k-25)/\{(23+30)*30\}-(29/30)]$

For $n^{ix} = 2$; $n^x_2 = \text{floor function of } [(2k-25)/\{(23+60)*30\}-(29/30)]$

For $n^{ix} = n^{ix}_{\max}$ i.e. floor function of $[(2k-25)/(29*30)]-(23/30)$; $n^x = \text{floor function of } [(2k-25)/\{(23+30n^{ix}_{\max})*30\}-(29/30)]$

Hence total number of $n^x = n^x_0 + n^x_1 + n^x_2 + \dots + n^x_{\text{for max value of } n^{ix}}$

Or total number of possible values of $n^x = \sum_{n^{ix}=0}^{\text{floor function of } [(2k-25)/(29*30)]-(23/30)}$ floor function of $[(2k-25)/\{(23+30 n^{ix})*30\}-(29/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{ix}=0}^{\text{to floor function of } [(2k-25)/(29*30)] - (23/30)}$ floor function of $[(2k-25)/\{(23+30 n^{ix}) * 30\} - (29/30)]$

Case(2BC-iii):

EN= SADN(5,2,8//4)

Here EN= SADN(5,2,8//4) includes even numbers having SADN(5,2,8) and last digit as 4

$C1[\text{SADN}(7,4,1)//5] + C2[\text{SADN}(7,4,1)//9] = 2k$

Case(2BC-iiiB) For $C2[\text{SADN}(7,4,1)//9]$ implies that

CASE(2BC-iiiB-i) i.e. $//1x//9 [\text{SADN}(7,4,1)//1]x[\text{SADN}(7,4,1)//9]$

CASE(2BC-iiiB-ii) i.e. $//1x//9 [\text{SADN}(5,2,8)//1] x [\text{SADN}(5,2,8)//9]$

CASE(2BC-iiiB-iii) i.e. $//3x//3 [\text{SADN}(7,4,1)//3]x[\text{SADN}(7,4,1)//3]$

CASE(2BC-iiiB-iv) i.e. $//3x//3 [\text{SADN}(5,2,8)//3]x[\text{SADN}(5,2,8)//3]$

CASE(2BC-iiiB-v) i.e. $//7x//7 [\text{SADN}(7,4,1)//7] x [\text{SADN}(7,4,1)//7]$

CASE(2BC-iiiB-vi) i.e. $//7x//7 [\text{SADN}(5,2,8)//7] x [\text{SADN}(5,2,8)//7]$

Case(2BC-iiiB-i)

For $\text{SADN}(7,4,1)//1 x \text{SADN}(7,4,1)//9$

$(31+30n) x (19+30n') \leq (2k-25)$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-25)/(31*30) - (19/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-25)/\{(31+30)*30\} - (19/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-25)/\{(31+60)*30\} - (19/30)]$

For $n = n'_{\max}$ i.e. floor function of $[(2k-25)/(19*30)] - (31/30)$; $n'''' = \text{floor function of } [(2k-25)/\{(31+30n'_{\max}) * 30\} - (19/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0}^{\text{to floor function of } [(2k-25)/(19*30)] - (31/30)}$ floor function of $[(2k-25)/\{(31+30n)*30\} - (19/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-25)/(19*30)] - (31/30)}$ floor function of $[(2k-25)/\{(31+30n)*30\} - (19/30)]$

Case(2BC-iiiB-ii)

For [SADN(5,2,8)//1] x [SADN(5,2,8)//9]:-

$$(11+30n'') \times (29+30n''') \leq (2k-25)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-25)/(11*30) - (29/30)]$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-25)/\{(11+30)*30\} - (29/30)]$

For $n'' = 2$; $n'''_2 = \text{floor function of } [(2k-25)/\{(11+60)*30\} - (29/30)]$

For $n'' = n''_{\max}$ i.e. floor function of $[(2k-25)/(29*30)] - (11/30)$; $n''' = \text{floor function of } [(2k-25)/\{(11+30n''_{\max})*30\} - (29/30)]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0}^{\text{floor function of } [(2k-25)/(29*30)] - (11/30)}$ floor function of $[(2k-25)/\{(11+30n'')*30\} - (29/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(29*30)] - (11/30)}$ floor function of $[(2k-25)/\{(11+30n'')*30\} - (29/30)]$

Case(2BC-iiiB-iii)

For [SADN(7,4,1)//3]x[SADN(7,4,1)//3]:-

$$(13+30n^{iv}) \times (13+30n^v) \leq (2k-25)$$

For $n^{iv} = 0$; $n^v_0 = \text{floor function of } [(2k-25)/(13*30) - (13/30)]$

For $n^{iv} = 1$; $n^v_1 = \text{floor function of } [(2k-25)/\{(13+30)*30\} - (13/30)]$

For $n^{iv} = 2$; $n^v_2 = \text{floor function of } [(2k-25)/\{(13+60)*30\} - (13/30)]$

For $n^{iv} = n^{iv}_{\max}$ i.e. floor function of $[(2k-25)/(13*30)] - (13/30)$; $n^v = \text{floor function of } [(2k-25)/\{(13+30n^{iv}_{\max})*30\} - (13/30)]$

Hence total number of $n^v = n^v_0 + n^v_1 + n^v_2 + \dots + n^v_{\text{for max value of } n^{iv}}$

Or total number of possible values of $n^v = \sum_{n^{iv}=0}^{\text{floor function of } [(2k-25)/(13*30)] - (13/30)}$ floor function of $[(2k-25)/\{(13+30n^{iv})*30\} - (13/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0}^{\text{floor function of } [(2k-25)/(13*30)] - (13/30)}$ floor function of $[(2k-25)/\{(13+30n^{iv})*30\} - (13/30)]$

Case(2BC-iiiB-iv)

For [SADN(5,2,8)//3]x[SADN(5,2,8)//3]:-

$$(23+30n^{vi}) \times (23+30n^{vii}) \leq (2k-25)$$

For $n^{vi} = 0$; $n^{vii}_0 = \text{floor function of } [(2k-25)/(23*30) - (23/30)]$

For $n^{vi} = 1$; $n^{vii}_1 = \text{floor function of } [(2k-25)/\{(23+30)*30\} - (23/30)]$

For $n^{vi} = 2$; $n^{vii}_2 = \text{floor function of } [(2k-25)/\{(23+60)*30\} - (23/30)]$

For $n^{vi} = n^{vi}_{\max}$ i.e. floor function of $[(2k-25)/(23*30)] - (23/30)$; $n^{vii} = \text{floor function of } [(2k-25)/\{(23+30n^{vi}_{\max}) * 30\} - (23/30)]$

Hence total number of $n^{vii} = n^{vii}_0 + n^{vii}_1 + n^{vii}_2 + \dots + n^{vii}_{\text{for max value of nvi}}$

Or total number of possible values of $n^{vii} = \sum_{nvi=0 \text{ to floor function of } [(2k-25)/(23*30)] - (13/30)} \text{floor function of } [(2k-25)/\{(23+30 n^{vi}) * 30\} - (23/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{niv=0 \text{ to floor function of } [(2k-25)/(23*30)] - (13/30)} \text{floor function of } [(2k-25)/\{(23+30 n^{iv}) * 30\} - (23/30)]$

Case(2BC-iiiB-v)

For [SADN(7,4,1)//7] x [SADN(7,4,1)//7]:-

$$(7+30n) \times (7+30n') \leq (2k-25)$$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-25)/(7*30) - (7/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-25)/\{(7+30)*30\} - (7/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-25)/\{(7+60)*30\} - (7/30)]$

For $n = n'_{\max}$ i.e. floor function of $[(2k-25)/(7*30)] - (7/30)$; $n'' = \text{floor function of } [(2k-25)/\{(7+30n'_{\max}) * 30\} - (7/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of n}}$

Or total number of possible values of $n' = \sum_{n=0 \text{ to floor function of } [(2k-25)/(7*30)] - (7/30)} \text{floor function of } [(2k-25)/\{(7+30n) * 30\} - (7/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-25)/(7*30)] - (7/30)}$ floor function of $[(2k-25)/\{(7+30n)*30\} - (7/30)]$

Case(2BC-iiiB-vi)

For $[SADN(5,2,8)//7] \times [SADN(5,2,8)//7]$:-

$$(17+30n'') \times (17+30n''') \leq (2k-25)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-25)/(17*30) - (17/30)]$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-25)/\{(17+30)*30\} - (17/30)]$

For $n'' = 2$; $n'''^{iv}_2 = \text{floor function of } [(2k-25)/\{(17+60)*30\} - (17/30)]$

For $n'' = n''_{\max}$ i.e. floor function of $[(2k-25)/(17*30)] - (17/30)$; $n''' = \text{floor function of } [(2k-25)/\{(17+30n''_{\max}) * 30\} - (17/30)]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0}^{\text{floor function of } [(2k-25)/(17*30)] - (17/30)}$ floor function of $[(2k-25)/\{(17+30n'') * 30\} - (17/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(17*30)] - (17/30)}$ floor function of $[(2k-25)/\{(17+30n'') * 30\} - (17/30)]$

Case(2BC-iv):

$$EN = SADN(5,2,8//6)$$

Here $EN = SADN(5,2,8//6)$ includes even numbers having $SADN(5,2,8)$ and last digit as 6

$$C1[SADN(7,4,1)//5] + C2[SADN(7,4,1)//1] = 2k$$

Case(2BC-ivB) For $C2[SADN(7,4,1)//1]$ implies that

CASE(2BC-ivB-i) i.e. $//1x//1 [SADN(7,4,1)//1] \times [SADN(7,4,1)//1]$

CASE(2BC-ivB-ii) i.e. $//1x//1 [SADN(5,2,8)//1] \times [SADN(5,2,8)//1]$

CASE(2BC-ivB-iii) i.e. $//3x//7 [SADN(7,4,1)//3] \times [SADN(7,4,1)//7]$

CASE(2BC-ivB-iv) i.e. $//3x//7 [SADN(5,2,8)//3] \times [SADN(5,2,8)//7]$

CASE(2BC-ivB-v) i.e. $//9x//9 [SADN(7,4,1)//9] \times [SADN(7,4,1)//9]$

CASE(2BC-ivB-vi) i.e. $//9x//9 [SADN(5,2,8)//9] \times [SADN(5,2,8)//9]$

Case(2BC-ivB-i)

For $[SADN(7,4,1)//1] \times [SADN(7,4,1)//1]$

$$(31+30n) \times (31+30n') \leq (2k-25)$$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-25)/(31*30) - (31/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-25)/\{(31+30)*30\} - (31/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-25)/\{(31+60)*30\} - (31/30)]$

For $n = n'_{\max}$ i.e. floor function of $[(2k-25)/(31*30)] - (31/30)$; $n''' = \text{floor function of } [(2k-25)/\{(31+30n'_{\max}) * 30\} - (31/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0 \text{ to floor function of } [(2k-25)/(31*30)] - (31/30)} \text{floor function of } [(2k-25)/\{(31+30n)*30\} - (31/30)]$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n=0 \text{ to floor function of } [(2k-25)/(31*30)] - (31/30)} \text{floor function of } [(2k-25)/\{(31+30n)*30\} - (31/30)]$

Case(2BC-ivB-ii)

For $SADN(5,2,8)//1 \times SADN(5,2,8)//1:-$

$$(11+30n'') \times (11+30n''') \leq (2k-25)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-25)/(11*30) - (11/30)]$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-25)/\{(11+30)*30\} - (11/30)]$

For $n'' = 2$; $n'''_{iv_2} = \text{floor function of } [(2k-25)/\{(11+60)*30\} - (11/30)]$

For $n'' = n''_{\max}$ i.e. floor function of $[(2k-25)/(11*30)] - (11/30)$; $n''' = \text{floor function of } [(2k-25)/\{(11+30n''_{\max}) * 30\} - (11/30)]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0 \text{ to floor function of } [(2k-25)/(11*30)] - (11/30)} \text{floor function of } [(2k-25)/\{(11+30n'')*30\} - (11/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(11*30)] - (11/30)}$ floor function of $[(2k-25)/\{(11+30n'')*30\} - (11/30)]$

Case(2BC-ivB-iii)

For SADN(7,4,1)//3 x SADN(7,4,1)//7:-

$$(13+30n^{iv}) \times (7+30n^v) \leq (2k-25)$$

For $n^{iv} = 0$; $n^v_0 = \text{floor function of } [(2k-25)/(13*30) - (7/30)]$

For $n^{iv} = 1$; $n^v_1 = \text{floor function of } [(2k-25)/\{(13+30)*30\} - (7/30)]$

For $n^{iv} = 2$; $n^v_2 = \text{floor function of } [(2k-25)/\{(13+60)*30\} - (7/30)]$

For $n^{iv} = n^{iv}_{\max}$ i.e. floor function of $[(2k-25)/(7*30)] - (13/30)$; $n^v = \text{floor function of } [(2k-25)/\{(13+30 n^{iv}_{\max}) * 30\} - (7/30)]$

Hence total number of $n^v = n^v_0 + n^v_1 + n^v_2 + \dots + n^v_{\text{for max value of niv}}$

Or total number of possible values of $n^v = \sum_{niv=0}^{\text{floor function of } [(2k-25)/(7*30)] - (13/30)}$ floor function of $[(2k-25)/\{(13+30 n^{iv}) * 30\} - (7/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{niv=0}^{\text{floor function of } [(2k-25)/(7*30)] - (13/30)}$ floor function of $[(2k-25)/\{(13+30 n^{iv}) * 30\} - (7/30)]$

Case(2BC-ivB-iv)

For SADN(5,2,8)//3 x SADN(5,2,8)//7:-

$$(23+30n^{vi}) \times (17+30n^{vii}) \leq (2k-25)$$

For $n^{vi} = 0$; $n^{vii}_0 = \text{floor function of } [(2k-25)/(23*30) - (17/30)]$

For $n^{vi} = 1$; $n^{vii}_1 = \text{floor function of } [(2k-25)/\{(23+30)*30\} - (17/30)]$

For $n^{vi} = 2$; $n^{vii}_2 = \text{floor function of } [(2k-25)/\{(23+60)*30\} - (17/30)]$

For $n^{vi} = n^{vi}_{\max}$ i.e. floor function of $[(2k-25)/(17*30)] - (23/30)$; $n^{vii} = \text{floor function of } [(2k-25)/\{(23+30 n^{vi}_{\max}) * 30\} - (17/30)]$

Hence total number of $n^{vii} = n^{vii}_0 + n^{vii}_1 + n^{vii}_2 + \dots + n^{vii}_{\text{for max value of nvi}}$

Or total number of possible values of $n^{vii} = \sum_{n^{vi}=0 \text{ to floor function of } [\{(2k-25)/(17*30)\}-(23/30)]}$
 floor function of $[(2k-25)/\{(23+30 n^{vi}) * 30\} - (17/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0 \text{ to floor function of } [\{(2k-25)/(17*30)\}-(23/30)]}$
 floor function of $[(2k-25)/\{(23+30 n^{iv}) * 30\} - (17/30)]$

Case(2BC-ivB-v)

For [SADN(7,4,1)//9] x [SADN(7,4,1)//9]:-

$$(19+30n) \times (19+30n') \leq (2k-25)$$

For $n = 0$; $n'_0 = \text{floor function of } [(2k-25)/(19*30) - (19/30)]$

For $n = 1$; $n'_1 = \text{floor function of } [(2k-25)/\{(19+30)*30\} - (19/30)]$

For $n = 2$; $n'_2 = \text{floor function of } [(2k-25)/\{(19+60)*30\} - (19/30)]$

For $n = n'_{\max}$ i.e. floor function of $[\{(2k-25)/(19*30)\} - (19/30)]$; $n''' = \text{floor function of } [(2k-25)/\{(19+30n'_{\max}) * 30\} - (19/30)]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0 \text{ to floor function of } [\{(2k-25)/(19*30)\}-(19/30)]}$ floor
 function of $[(2k-25)/\{(19+30n)*30\} - (19/30)]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0 \text{ to floor function of } [\{(2k-25)/(19*30)\}-(19/30)]}$
 floor function of $[(2k-25)/\{(19+30n)*30\} - (19/30)]$

Case(2BC-ivB-vi)

For [SADN(5,2,8)//9] x [SADN(5,2,8)//9]:-

$$(29+30n'') \times (29+30n''') \leq (2k-25)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-25)/(29*30) - (29/30)]$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-25)/\{(29+30)*30\} - (29/30)]$

For $n'' = 2$; $n'''_2 = \text{floor function of } [(2k-25)/\{(29+60)*30\} - (29/30)]$

For $n'' = n''_{\max}$ i.e. floor function of $\left[\frac{(2k-25)}{(29*30)}\right] - (29/30)$; $n''' =$ floor function of $\left[\frac{(2k-25)}{\{(29+30n''_{\max}) * 30\}} - (29/30)\right]$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0 \text{ to floor function of } \left[\frac{(2k-25)}{(29*30)}\right] - (29/30)}$
 floor function of $\left[\frac{(2k-25)}{\{(29+30n'') * 30\}} - (29/30)\right]$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0 \text{ to floor function of } \left[\frac{(2k-25)}{(29*30)}\right] - (29/30)}$ floor function of $\left[\frac{(2k-25)}{\{(29+30n'') * 30\}} - (29/30)\right]$

Case(2BC-v):

EN= SADN(5,2,8//8)

Here EN= SADN(5,2,8//6) includes even numbers having SADN(5,2,8) and last digit as 8

$C1[\text{SADN}(7,4,1)//5] + C2[\text{SADN}(7,4,1)//3] = 2k$

Case(2BC-vB)

For $C2[\text{SADN}(7,4,1)//3]$ implies that

CASE(2BC-vB-i) i.e. $//1x//3 [\text{SADN}(7,4,1)//1]x[\text{SADN}(7,4,1)//3]$

CASE(2BC-vB-ii) i.e. $//1x//3 [\text{SADN}(5,2,8)//1] x [\text{SADN}(5,2,8)//3]$

CASE(2BC-vB-iii) i.e. $//7x//9 [\text{SADN}(7,4,1)//7]x[\text{SADN}(7,4,1)//9]$

CASE(2BC-vB-iv) i.e. $//7x//9 [\text{SADN}(5,2,8)//7]x[\text{SADN}(5,2,8)//9]$

Case(2BC-vB-i)

For $\text{SADN}(7,4,1)//1 x \text{SADN}(7,4,1)//3:-$

$(31+30n) x (13+30n') \leq (2k-25)$

For $n = 0$; $n'_0 =$ floor function of $\left[\frac{(2k-25)}{(31*30)} - (13/30)\right]$

For $n = 1$; $n'_1 =$ floor function of $\left[\frac{(2k-25)}{\{(31+30)*30\}} - (13/30)\right]$

For $n = 2$; $n'_2 =$ floor function of $\left[\frac{(2k-25)}{\{(31+60)*30\}} - (13/30)\right]$

For $n = n'_{\max}$ i.e. floor function of $\left[\frac{(2k-25)}{(13*30)} - (31/30)\right]$; $n''' =$ floor function of $\left[\frac{(2k-25)}{\{(31+30n'_{\max}) * 30\}} - (13/30)\right]$

Hence total number of $n' = n'_0 + n'_1 + n'_2 + \dots + n'_{\text{for max value of } n}$

Or total number of possible values of $n' = \sum_{n=0}^{\text{floor function of } [(2k-25)/(13*30)] - (31/30)} \text{floor function of } [(2k-25)/\{(31+30n)*30\} - (13/30)]$

Number of combinations of type-2 $c1+c2$ combinations = $\sum_{n=0}^{\text{floor function of } [(2k-25)/(13*30)] - (31/30)} \text{floor function of } [(2k-25)/\{(31+30n)*30\} - (13/30)]$

Case(2BC-vB-ii)

For $\text{SADN}(5,2,8)//1 \times \text{SADN}(5,2,8)//3:-$

$$(11+30n'') \times (23+30n''') \leq (2k-25)$$

For $n'' = 0$; $n'''_0 = \text{floor function of } [(2k-25)/(11*30)] - (23/30)$

For $n'' = 1$; $n'''_1 = \text{floor function of } [(2k-25)/\{(11+30)*30\}] - (23/30)$

For $n'' = 2$; $n'''_{iv}_2 = \text{floor function of } [(2k-25)/\{(11+60)*30\}] - (23/30)$

For $n'' = n''_{\text{max}}$ i.e. floor function of $[(2k-25)/(23*30)] - (11/30)$; $n''' = \text{floor function of } [(2k-25)/\{(11+30n''_{\text{max}})*30\}] - (23/30)$

Hence total number of $n''' = n'''_0 + n'''_1 + n'''_2 + \dots + n'''_{\text{for max value of } n''}$

Or total number of possible values of $n''' = \sum_{n''=0}^{\text{floor function of } [(2k-25)/(23*30)] - (11/30)} \text{floor function of } [(2k-25)/\{(11+30n'')*30\}] - (23/30)$

Number of combinations of type-2 $c1+c2$ combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(23*30)] - (11/30)} \text{floor function of } [(2k-25)/\{(11+30n'')*30\}] - (23/30)$

Case(2BC-vB-iii)

For $\text{SADN}(7,4,1)//7 \times \text{SADN}(7,4,1)//9:-$

$$(7+30n^{iv}) \times (19+30n^v) \leq (2k-25)$$

For $n^{iv} = 0$; $n^v_0 = \text{floor function of } [(2k-25)/(7*30)] - (19/30)$

For $n^{iv} = 1$; $n^v_1 = \text{floor function of } [(2k-25)/\{(7+30)*30\}] - (19/30)$

For $n^{iv} = 2$; $n^v_2 = \text{floor function of } [(2k-25)/\{(7+60)*30\}] - (19/30)$

For $n^{iv} = n_{\max}^{iv}$ i.e. floor function of $\left[\frac{(2k-25)}{(19*30)}\right] - (7/30)$; $n^v = \text{floor function of } \left[\frac{(2k-25)}{(7+30n_{\max}^{iv})*30}\right] - (19/30)$

Hence total number of $n^v = n_0^v + n_1^v + n_2^v + \dots + n_{\text{for max value of niv}}^v$

Or total number of possible values of $n^v = \sum_{niv=0 \text{ to floor function of } \left[\frac{(2k-25)}{(11*30)}\right] - (31/30)} \text{floor function of } \left[\frac{(2k-25)}{(7+30n^{iv})*30}\right] - (19/30)$

Number of combinations of type-2 c1+c2 combinations = $\sum_{niv=0 \text{ to floor function of } \left[\frac{(2k-25)}{(19*30)}\right] - (7/30)} \text{floor function of } \left[\frac{(2k-25)}{(7+30n^{iv})*30}\right] - (19/30)$

Case(2BC-vB-iv)

For $\text{SADN}(5,2,8)/7 \times \text{SADN}(5,2,8)/9:-$

$$(17+30n^{vi}) \times (29+30n^{vii}) \leq (2k-25)$$

For $n^{vi} = 0$; $n_0^{vii} = \text{floor function of } \left[\frac{(2k-25)}{(17*30)}\right] - (29/30)$

For $n^{vi} = 1$; $n_1^{vii} = \text{floor function of } \left[\frac{(2k-25)}{(17+30)*30}\right] - (29/30)$

For $n^{vi} = 2$; $n_2^{vii} = \text{floor function of } \left[\frac{(2k-25)}{(17+60)*30}\right] - (29/30)$

For $n^{vi} = n_{\max}^{vi}$ i.e. floor function of $\left[\frac{(2k-25)}{(29*30)}\right] - (17/30)$; $n^{vii} = \text{floor function of } \left[\frac{(2k-25)}{(17+30n_{\max}^{vii})*30}\right] - (29/30)$

Hence total number of $n^{vii} = n_0^{vii} + n_1^{vii} + n_2^{vii} + \dots + n_{\text{for max value of nvi}}^{vii}$

Or total number of possible values of $n^{vii} = \sum_{nvi=0 \text{ to floor function of } \left[\frac{(2k-25)}{(29*30)}\right] - (17/30)} \text{floor function of } \left[\frac{(2k-25)}{(17+30n^{vi})*30}\right] - (29/30)$

Number of combinations of type-2 c1+c2 combinations = $\sum_{niv=0 \text{ to floor function of } \left[\frac{(2k-25)}{(29*30)}\right] - (17/30)} \text{floor function of } \left[\frac{(2k-25)}{(17+30n^{iv})*30}\right] - (29/30)$

Case(3):

SADN of $EN = \{6,3,9\}$ and EN not equal to the number '6'

$$EN = 2k$$

$EN/2 = k$ can never be a prime and will always be a composite number

'k' is a composite number

Number of acceptable combinations of elements is given as n_{acc}

Case(3A): If number of composites is less than number of primes (i.e. $n_c < n_p$ implies that even if all composites are prime-eaters; there exists atleast one p_1+p_2 pair

Case(3B): If number of composites is greater than or equal to number of primes (i.e. $n_c \geq n_p$ implies that we need to find total number of unique C_1+C_2 pairs

C1+C2 of Type-1:

It has already been discussed in section 11A.

C1+C2 of Type-2:

EN= SADN(6,3,9//0) OR SADN(6,3,9//2) OR SADN(6,3,9//4) OR SADN(6,3,9//6) OR SADN(6,3,9//8)

Here EN= SADN(6,3,9//0) indicates even numbers having SADN(6,3,9) and last digit as 0

Case(3B-iA):

EN = SADN(6,3,9//0//6) including even numbers of SADN(6,3,9//0) and cyclical series element as '6'

EN = SADN(6,3,9//0//6)

C_1 of SADN(7,4,1//5) + C_2 of SADN(7,4,1//5) = $2k$

$(25+30n_a'') + [2k - (25+30n_a'')] = 2k$

$25+30n_a'' < 2k$

Number of Composites of SADN(7,4,1//5) is given as $n_a'' < (2k-25)/30$

or $n_a'' = \text{floor function of } [(2k-25)/30]$

No. of type-2 C_1+C_2 combinations = floor function $[(1/2)*(2k-25)/30] + 1$

Or No. of type-2 C_1+C_2 combinations = floor function $[k/30] + 1$

Case(3B-iB):

EN = SADN(6,3,9//0//12) including even numbers of SADN(6,3,9//0) and cyclical series element as '12'

EN = SADN(6,3,9//0//12)

C_1 of SADN(7,4,1//5) + C_2 of SADN(7,4,1//5) = $2k$

$(25+30n_b'') + [2k - (25+30n_b'')] = 2k$

$$(25+30n_b'') < 2k$$

Number of C of SADN(7,4,1//5) is given as $n_b'' < (2k-25)/30$

or $n_b'' = \text{floor function of } [(2k-25)/30]$

Number of type-2 C1+C2 combinations = floor function $[(1/2)*(2k-25)/30] + 1$

Or Number of type-2 C1+C2 combinations = floor function $[k/30]$

Case(3B-ii):

$$EN = \text{SADN}(6,3,9//2)$$

Here EN= SADN(6,3,9//2) includes even numbers having SADN(5,2,8) and last digit as 2

$$\text{Case(3B-ii-A)} \quad C1[\text{SADN}(7,4,1)//5] + C2[\text{SADN}(5,2,8)//7] = 2k$$

$$\text{Case(3B-ii-B)} \quad C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(7,4,1)//7] = 2k$$

Case(3B-ii-A)

$$C1[\text{SADN}(7,4,1)//5] + C2[\text{SADN}(5,2,8)//7] = 2k$$

$$25 + C2[\text{SADN}(5,2,8)//7] = 2k$$

$$\text{Or} \quad C2[\text{SADN}(5,2,8)//7] = 2k - 25$$

For $C2[\text{SADN}(5,2,8)//7]$ implies that $//1x//7$ OR $//3x//9$

With Reference from Cases(1BC-iiA-i) upto (1BC-iiA-iv):-

Case(3B-ii-A-i)

For SADN(5,2,8)//1 x SADN(7,4,1)//7:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(7*30)] - (11/30)}$ floor function of $[(2k-25)/\{(11+30n'')*30\} - (7/30)]$

Case(3B-ii-A-ii)

For SADN(7,4,1)//1 x SADN(5,2,8)//7:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^v=0}^{\text{floor function of } [(2k-25)/(17*30)] - (31/30)}$ floor function of $[(2k-25)/\{(31+30n^v)*30\} - (17/30)]$

Case(3B-ii-A-iii)

For SADN(7,4,1)//3 x SADN(5,2,8)//9:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{vii}=0}^{\text{floor function of } [(2k-25)/(29*30)] - (13/30)}$ floor function of $[(2k-25)/\{(13+30 n^{vii}) * 30\} - (29/30)]$

Case(3B-ii-A-iv)

For SADN(5,2,8)//3 x SADN(7,4,1)//9:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{ix}=0}^{\text{floor function of } [(2k-25)/(19*30)] - (23/30)}$ floor function of $[(2k-25)/\{(23+30 n^{ix}) * 30\} - (19/30)]$

Case(3B-ii-B)

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(7,4,1)//7] = 2k$$

$$35 + C2[\text{SADN}(7,4,1)//7] = 2k$$

$$\text{Or } C2[\text{SADN}(7,4,1)//7] = 2k - 35$$

For $C2[\text{SADN}(7,4,1)//7]$ implies that $//1x//7$ OR $//3x//9$

With Reference from Cases(2BC-iiB-i) upto (2BC-iiB-iv):-

Case(3B-iiB-i)

For SADN(7,4,1)//1 x SADN(7,4,1)//7 :-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{''}=0}^{\text{floor function of } [(2k-35)/(7*30)] - (31/30)}$ floor function of $[(2k-35)/\{(31+30 n^{''}) * 30\} - (7/30)]$

Case(3B-iiB-ii)

For $[\text{SADN}(5,2,8)//1] \times [\text{SADN}(5,2,8)//7]$:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^v=0}^{\text{floor function of } [(2k-35)/(17*30)] - (11/30)}$ floor function of $[(2k-35)/\{(11+30 n^v) * 30\} - (17/30)]$

Case(3B-iiB-iii)

For $[\text{SADN}(7,4,1)//3] \times [\text{SADN}(7,4,1)//9]$:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{vii}=0}^{\text{floor function of } [(2k-35)/(19*30)] - (13/30)}$ floor function of $[(2k-35)/\{(13+30 n^{vii}) * 30\} - (19/30)]$

Case(3B-iiB-iv)

For $[\text{SADN}(5,2,8)//3] \times [\text{SADN}(5,2,8)//9]$:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{ix}=0 \text{ to floor function of } [(2k-35)/(29*30)]}^{(23/30)}$ floor function of $[(2k-35)/\{(23+30 n^{ix}) * 30\} - (29/30)]$

Case(3B-iii):

$$EN = \text{SADN}(6,3,9//4)$$

Here EN= SADN(6,3,9//4) includes even numbers having SADN(6,3,9) and last digit as 4

$$\text{Case(3B-iii-A)} \quad C1[\text{SADN}(7,4,1)//5] + C2[\text{SADN}(5,2,8)//9] = 2k$$

$$\text{Case(3B-iii-B)} \quad C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(7,4,1)//9] = 2k$$

Case(3B-iii-A)

$$C1[\text{SADN}(7,4,1)//5] + C2[\text{SADN}(5,2,8)//9] = 2k$$

$$25 + C2[\text{SADN}(5,2,8)//9] = 2k$$

$$\text{Or} \quad C2[\text{SADN}(5,2,8)//9] = 2k - 25$$

For $C2[\text{SADN}(5,2,8)//9]$ implies that $//1x/9$ OR $//3x/3$

With Reference from Cases(1BC-iiiA-i) upto (1BC-iiiA-iv):-

Case(3B-iii-A-i)

$$\text{For } C2[\text{SADN}(5,2,8)//9]$$

Case(3B-iiiA-i)

$$\text{For } \text{SADN}(7,4,1)//1 \times \text{SADN}(5,2,8)//9$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0 \text{ to floor function of } [(2k-25)/(29*30)]}^{(31/30)}$ floor function of $[(2k-25)/\{(31+30n) * 30\} - (29/30)]$

Case(3B-iiiA-ii)

$$\text{For } \text{SADN}(5,2,8)//1 \times \text{SADN}(7,4,1)//9:-$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0 \text{ to floor function of } [(2k-25)/(19*30)]}^{(11/30)}$ floor function of $[(2k-25)/\{(11+30n'') * 30\} - (19/30)]$

Case(3B-iiiA-iii)

$$\text{For } \text{SADN}(7,4,1)//7 \times \text{SADN}(5,2,8)//7:-$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0 \text{ to floor function of } \{ \{(2k-25)/(19*30) \} - (11/30) \}} \text{floor function of } [(2k-25)/\{(7+30 n^{iv}) * 30\} - (17/30)]$

Case(3B-iiiA-iv)

For SADN(7,4,1)//3 x SADN(5,2,8)//3:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0 \text{ to floor function of } \{ \{(2k-25)/(23*30) \} - (13/30) \}} \text{floor function of } [(2k-25)/\{(13+30 n^{iv}) * 30\} - (23/30)]$

Case(3B-iii-B)

$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(7,4,1)//9] = 2k$

$35 + C2[\text{SADN}(7,4,1)//9] = 2k$

Or $C2[\text{SADN}(7,4,1)//9] = 2k - 35$

For $C2[\text{SADN}(7,4,1)//9]$ implies that $//1x//9$ OR $//3x//3$ OR $//7x//7$

With Reference from Cases(2BC-iiiB-i) upto (2BC-iiiB-vi):-

Case(3B-iiiB-i)

For SADN(7,4,1)//1 x SADN(7,4,1)//9

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0 \text{ to floor function of } \{ \{(2k-35)/(19*30) \} - (31/30) \}} \text{floor function of } [(2k-35)/\{(31+30n) * 30\} - (19/30)]$

Case(3B-iiiB-ii)

For $[\text{SADN}(5,2,8)//1] \times [\text{SADN}(5,2,8)//9]$:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0 \text{ to floor function of } \{ \{(2k-35)/(29*30) \} - (11/30) \}} \text{floor function of } [(2k-35)/\{(11+30n'') * 30\} - (29/30)]$

Case(3B-iiiB-iii)

For $[\text{SADN}(7,4,1)//3] \times [\text{SADN}(7,4,1)//3]$:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0 \text{ to floor function of } \{ \{(2k-35)/(13*30) \} - (13/30) \}} \text{floor function of } [(2k-35)/\{(13+30 n^{iv}) * 30\} - (13/30)]$

Case(3B-iiiB-iv)

For $[\text{SADN}(5,2,8)//3] \times [\text{SADN}(5,2,8)//3]$:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0 \text{ to floor function of } [(2k-35)/(23*30)] - (13/30)}$ floor function of $[(2k-35)/\{(23+30n^{iv}) * 30\} - (23/30)]$

Case(3B-iiiB-v)

For [SADN(7,4,1)//7] x [SADN(7,4,1)//7]:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0 \text{ to floor function of } [(2k-35)/(7*30)] - (7/30)}$ floor function of $[(2k-35)/\{(7+30n) * 30\} - (7/30)]$

Case(3B-iiiB-vi)

For [SADN(5,2,8)//7] x [SADN(5,2,8)//7]:-

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0 \text{ to floor function of } [(2k-35)/(17*30)] - (17/30)}$ floor function of $[(2k-35)/\{(17+30n'') * 30\} - (17/30)]$

Case(3B-iv):

EN= SADN(6,3,9//6)

Here EN= SADN(6,3,9//6) includes even numbers having SADN(6,3,9) and last digit as 6

Case(3B-iv-A) $C1[SADN(7,4,1)//5] + C2[SADN(5,2,8)//1] = 2k$

Case(3B-iv-B) $C1[SADN(5,2,8)//5] + C2[SADN(7,4,1)//1] = 2k$

Case(3B-iv-A)

$C1[SADN(7,4,1)//5] + C2[SADN(5,2,8)//1] = 2k$

$25 + C2[SADN(5,2,8)//1] = 2k$

Or $C2[SADN(5,2,8)//1] = 2k - 25$

For $C2[SADN(5,2,8)//1]$ implies that //3x//7 OR //1x//1 OR //9x//9

With Reference from Cases(1BC-ivA-i) upto (1BC-ivA-iv):-

For $C2[SADN(5,2,8)//1]$

Case(3B-ivA-i)

For SADN(7,4,1)//3 x SADN(5,2,8)//7:-

$(13+30n) \times (17+30n') \leq (2k-25)$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-25)/(17*30)] - (13/30)}$ floor function of $[(2k-25)/\{(13+30n)*30\} - (17/30)]$

Case(3B-ivA-ii)

For SADN(5,2,8)//3 x SADN(7,4,1)//7:-

$$(23+30n'') \times (7+30n''') \leq (2k-25)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(7*30)] - (23/30)}$ floor function of $[(2k-25)/\{(23+30n'')*30\} - (7/30)]$

Case(3B-ivA-iii)

For SADN(7,4,1)//1 x SADN(5,2,8)//1:-

$$(31+30n^{iv}) \times (11+30n^v) \leq (2k-25)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0}^{\text{floor function of } [(2k-25)/(11*30)] - (31/30)}$ floor function of $[(2k-25)/\{(31+30n^{iv})*30\} - (11/30)]$

Case(3B-ivA-iv)

For SADN(7,4,1)//9 x SADN(5,2,8)//9:-

$$(19+30n^{vi}) \times (29+30n^{vii}) \leq (2k-25)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0}^{\text{floor function of } [(2k-25)/(29*30)] - (19/30)}$ floor function of $[(2k-25)/\{(19+30n^{iv})*30\} - (29/30)]$

Case(3B-iv-B)

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(7,4,1)//1] = 2k$$

$$35 + C2[\text{SADN}(7,4,1)//1] = 2k$$

$$\text{Or } C2[\text{SADN}(7,4,1)//1] = 2k - 35$$

For $C2[\text{SADN}(7,4,1)//1]$ implies that //1x//1 OR //3x//7 OR //9x//9

With Reference from Cases(2BC-ivB-i) upto (2BC-ivB-vi):-

Case(3B-ivB-i)

For $[SADN(7,4,1)//1] \times [SADN(7,4,1)//1]$

$$(31+30n) \times (31+30n') \leq (2k-35)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-35)/(31*30)] - (31/30)}$ floor function of $[(2k-35)/\{(31+30n)*30\} - (31/30)]$

Case(3B-ivB-ii)

For $SADN(5,2,8)//1 \times SADN(5,2,8)//1:-$

$$(11+30n'') \times (11+30n''') \leq (2k-35)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-35)/(11*30)] - (11/30)}$ floor function of $[(2k-35)/\{(11+30n'')*30\} - (11/30)]$

Case(3B-ivB-iii)

For $SADN(7,4,1)//3 \times SADN(7,4,1)//7:-$

$$(13+30n^{iv}) \times (7+30n^v) \leq (2k-35)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n^{iv}=0}^{\text{floor function of } [(2k-35)/(7*30)] - (13/30)}$ floor function of $[(2k-35)/\{(13+30n^{iv})*30\} - (7/30)]$

Case(3B-ivB-iv)

For $SADN(5,2,8)//3 \times SADN(5,2,8)//7:-$

$$(23+30n^{vi}) \times (17+30n^{vii}) \leq (2k-35)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n^{iv}=0}^{\text{floor function of } [(2k-35)/(17*30)] - (23/30)}$ floor function of $[(2k-35)/\{(23+30n^{iv})*30\} - (17/30)]$

Case(3B-ivB-v)

For $[SADN(7,4,1)//9] \times [SADN(7,4,1)//9):-$

$$(19+30n) \times (19+30n') \leq (2k-35)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-35)/(19*30)] - (19/30)}$ floor function of $[(2k-35)/\{(19+30n)*30\} - (19/30)]$

Case(3B-ivB-vi)

For $[SADN(5,2,8)//9] \times [SADN(5,2,8)//9]$:-

$$(29+30n'') \times (29+30n''') \leq (2k-35)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-35)/(29*30)]} [(2k-35)/\{(29+30n'')*30\} - (29/30)]$

Case(3B-v):

$$EN = SADN(6,3,9//8)$$

Here $EN = SADN(6,3,9//6)$ includes even numbers having $SADN(6,3,9)$ and last digit as 8

$$\text{Case(3B-v-A)} \quad C1[SADN(7,4,1)//5] + C2[SADN(5,2,8)//3] = 2k$$

$$\text{Case(3B-v-B)} \quad C1[SADN(5,2,8)//5] + C2[SADN(7,4,1)//3] = 2k$$

Case(3B-v-A)

$$C1[SADN(7,4,1)//5] + C2[SADN(5,2,8)//3] = 2k$$

$$25 + C2[SADN(5,2,8)//3] = 2k$$

$$\text{Or} \quad C2[SADN(5,2,8)//3] = 2k - 25$$

For $C2[SADN(5,2,8)//3]$ implies that $//1x//3$ OR $//7x//9$

With Reference from Cases(1BC-vA-i) upto (1BC-vA-iv):-

Case(3B-vA-i)

For $SADN(7,4,1)//1 \times SADN(5,2,8)//3$:-

$$(31+30n) \times (23+30n') \leq (2k-25)$$

Number of combinations of type-2 c_1+c_2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-25)/(23*30)]} [(2k-25)/\{(31+30n)*30\} - (23/30)]$

Case(3B-vA-ii)

For $SADN(5,2,8)//1 \times SADN(7,4,1)//3$:-

$$(11+30n'') \times (13+30n''') \leq (2k-25)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-25)/(13*30)] - (11/30)}$ floor function of $[(2k-25)/\{(11+30n'')*30\} - (13/30)]$

Case(3B-vA-iii)

For SADN(7,4,1)//7 x SADN(5,2,8)//9:-

$$(7+30n^{iv}) \times (29+30n^v) \leq (2k-25)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{niv=0}^{\text{floor function of } [(2k-25)/(29*30)] - (7/30)}$ floor function of $[(2k-25)/\{(7+30n^{iv})*30\} - (29/30)]$

Case(3B-vA-iv)

For SADN(5,2,8)//7 x SADN(7,4,1)//9:-

$$(17+30n^{vi}) \times (19+30n^{vii}) \leq (2k-25)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{niv=0}^{\text{floor function of } [(2k-25)/(19*30)] - (17/30)}$ floor function of $[(2k-25)/\{(17+30n^{iv})*30\} - (19/30)]$

Case(3B-v-B)

$$C1[\text{SADN}(5,2,8)//5] + C2[\text{SADN}(7,4,1)//3] = 2k$$

$$35 + C2[\text{SADN}(7,4,1)//3] = 2k$$

$$\text{Or } C2[\text{SADN}(7,4,1)//3] = 2k - 35$$

For $C2[\text{SADN}(7,4,1)//3]$ implies that //1x/3 OR //7x/9

With Reference from Cases(2BC-vB-i) upto (2BC-vB-iv):-

Case(3B-vB-i)

For SADN(7,4,1)//1 x SADN(7,4,1)//3:-

$$(31+30n) \times (13+30n') \leq (2k-35)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n=0}^{\text{floor function of } [(2k-35)/(13*30)] - (31/30)}$ floor function of $[(2k-35)/\{(31+30n)*30\} - (13/30)]$

Case(3B-vB-ii)

For SADN(5,2,8)//1 x SADN(5,2,8)//3:-

$$(11+30n'') \times (23+30n''') \leq (2k-35)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n''=0}^{\text{floor function of } [(2k-35)/(23*30)]}$ floor function of $[(2k-35)/\{(11+30n'')*30\} - (23/30)]$

Case(3B-vB-iii)

For SADN(7,4,1)//7 x SADN(7,4,1)//9:-

$$(7+30n^{iv}) \times (19+30n^v) \leq (2k-35)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{iv}=0}^{\text{floor function of } [(2k-35)/(19*30)]}$ floor function of $[(2k-35)/\{(7+30n^{iv})*30\} - (19/30)]$

Case(3B-vB-iv)

For SADN(5,2,8)//7 x SADN(5,2,8)//9:-

$$(17+30n^{vi}) \times (29+30n^{vii}) \leq (2k-35)$$

Number of combinations of type-2 c1+c2 combinations = $\sum_{n^{vi}=0}^{\text{floor function of } [(2k-35)/(29*30)]}$ floor function of $[(2k-35)/\{(17+30n^{vi})*30\} - (29/30)]$

To find unique C1+C2 combination for a given even number:

Consider the even number 16658, i.e. SADN8//8 and its relevant series is S7 series.

In this case, composites would be derived as intraseries products of elements of S5 and S7 series. c1+c2 of type2 would be in the nature of 5-p where 5 and some other prime number (p) would be factors on either side of the combination.

Since 2k ends in 8, composite odd numbers on the S7 series that end in '3' will find such C+C combinations:

For S7 series: $7x(19+30n) \leq 7x(6I+1)$

ncc7; i.e. number of c1+c2 combinations identified by 7 = $[(6I+1) - 19]/30 + 1$

Similarly, ncc11; i.e. number of c1+c2 combinations identified by 11 = $[(6I-1) - 23]/30 + 1$

There would be some c1+c2 combinations derived by 11 that may have been already derived while calculating c1+c2 by 7. The first such combination would have a composite number of

which 7 is a factor, ends in 3 and lies on the S5 series. Thus the 4 conditions for such a common $c1+c2$ combination are that the composite component:-

- i. Should be divisible by 7
- ii. Should be a composite that ends in 3
- iii. Should lie on the S5 series
- iv. Should be greater than 11 in value

Such a number can be identified as follows:-

The number 7 ends in the digit 7 and will form a composite ending in 3 when multiplied with an odd number ending in 9. The first odd number on the S5 series that ends in 9 is 29. So $7 \times 29 = 203$ will be a number on the S5 series that will have to be removed from the $c+c$ combinations formed by 11 to avoid double/multiple counting. Thereafter every $7 \times (29+30n)$ th number will be a number divisible by 7, ending in 3 lying on the S5 series and greater than 11 and will have to be removed as long as its value is $< 6I-1$

Therefore numbers that need to be removed are obtained by the expression $= 7(29+30n) \leq 6I-1$

$$\text{i.e. } n'(c1+c2) = \text{floor function of } [\{(6I-1) - 203\}/30] + 1$$

General formula for identifying $c1+c2$ which have been already derived by previous prime elements:

$$N'(c1+c2) (p_n=741) = [\{(6I+1) - (p_e \times p_a)\}/30p_e] + 1$$

$$N'(c1+c2) (p_n=528) = [\{(6I-1) - (p_e \times p_a)\}/30p_e] + 1$$

$$p_e < p_n < p_a$$

- where $n'(c1+c2)$ will give the number of $c1+c2$ combinations derived for a particular prime p_n which have already been derived by earlier prime elements p_e ;
- p_n is the current prime element for which $c1+c2$ is being derived;
- p_e is the previous prime element for which such $c1+c2$ combination has been identified which are being repeated in case of p_n ;
- p_a is the factor whose product with p_e will give the first such composite component of $c1+c2$ which is common to $c1+c2$ derived by both p_n and p_e .

While calculating unique $c1+c2$ formed by the number 13, we first calculate $6I+1$ and remove those $c1+c2$ that have been already derived by 7 and 11.

Total $c1+c2$ formed by 13:

$$(31+30n) \leq 6I+1$$

$$n \leq [(6I+1)-31/30]+1$$

Composites already formed by 7:

There would be some $c1+c2$ combinations derived by 13 that may have been already derived while calculating $c1+c2$ by 7. The first such combination would have a composite number of which 7 is a factor, ends in digit 1 and lies on the S7 series. Thus the 4 conditions for such a common $c1+c2$ combination are that the composite component:-

- i. Should be divisible by 7
- ii. Should be a composite that ends in 1
- iii. Should lie on the S7 series since 13 is on S7 series
- iv. Should be greater than 13 in value
- v. Should be smaller than $6I+1$

Such a number can be identified as follows:-

The number 7 ends in the digit 7 and will form a composite ending in 1 when multiplied with an odd number ending in 3. The first odd number on the S7 series that ends in 3 is 13. So $7 \times 13 = 91$ will be a number on the S7 series that will have to be removed from the $c1+c2$ combinations formed by 13 to avoid double/multiple counting. Thereafter every $7 \times (13+30n)$ th number will be a number divisible by 7, ending in 1 lying on the S7 series and greater than 13 and will have to be removed as long as its value is $< 6I+1$

Therefore numbers that need to be removed are obtained by the expression $= 7(13+30n) \leq 6I+1$

$$\text{i.e. } n'(c1+c2) = \text{floor function of } [\{(6I+1) - 91\}/210] + 1$$

Composites already formed by 11:

Applying the similar 5 conditions as mentioned above;

$$11(11+30n) \leq 6I+1$$

$$n \leq [\{(6I+1)-121\}/330] + 1$$

There would be some combination that may be common to both 7 and 11 and will be removed twice if not identified and adjusted accordingly. Just as it is important to avoid double-counting, it is equally important to avoid double removal.

Conditions for identifying a combination that has been counted twice:-

- Should be divisible by both 7 and 11. So this number will by default be a multiple of 77.
- Should end in a digit whose multiplication with 13 will end in 3. Therefore should end in 1.

- Should be a number on the S7 series. Since 13 is an S7 element number and its multiplication with other S7 series numbers is being considered. Therefore such a number would be 77x S5 series number that ends in 3

$$7 \times 11 \times 23 = 1771$$

Hence 1771 will be the first such number on the S7 series which will be common to both 7 and 11 and needs to be added for arriving at unique $c1+c2$ for the number 13. Thereafter every $(7 \times 11 \times 30)$ th number will satisfy the condition. The general condition therefore is:

$$77(23+30n) \leq 6I+1$$

$$\text{Or } n \leq \{[(6I+1)-1771]/2310\}+1$$

This number needs to be added to avoid double subtraction.

Suppose we denote the number of $c1+c2$ combinations that are common to two previous prime elements and are therefore been removed twice, as $n''(c1+c2)$. The number of such $c1+c2$ combinations would be:

$$n'(c1+c2) = [\{(6I-1) - p_{e1}p_{e2}p_x\}/30p_{e1}p_{e2}] + 1$$

where p_{e1} and p_{e2} are two previous prime elements for which common $c1+c2$ combinations are being identified.

p_x is the factor whose product with $p_{e1}p_{e2}$ will give first such common composite component.

In general terms:

In order to avoid double counting of the same $c1+c2$ the following conditions have been identified:-

p_n is prime factor of the composite component of $c1+c2$ combination

p_e is earlier prime element of the relevant series for which $c1+c2$ combination identified for p_n have already been derived i.e. while deriving $c1+c2$ for p_n , some $c1+c2$ will be such that they have already been identified for previous elements p_e where value of e may range from 1 to $n-1$. These needs to be identified on the basis of following:-

- SADN and last digit of $2k$
- SADN and last digit of p_n and p_e
- Series (S1 or S5) on which p_n lies
- Relevant series for $2k$

We will first consider the case of $2k = \text{SADN } (5,2,8)$ which implies that relevant series on which $c1+c2$ are to be identified is the S7 series.

For p_n on S5 series:

$$N'(c1+c2) = [\{(6I-1) - (p_e \times p_a)\}/30p_e]+1$$

For p_n on S7 series:

$$N'(c1+c2) = [\{(6I+1) - (p_e \times p_a)\} / 30p_e] + 1$$

SADN(5,2,8)//2:

$p_n//1$ for which $n(c1+c2)$ is being calculated

p_e = earlier prime elements where $p_e < p_n$. $C1+c2$ identified for p_n which have already been derived by p_e

p_a = element whose product with p_e will give the first composite of $c1+c2$ that has already been identified for p_e and is being repeated in p_n .

In order to find unique $c1+c2$ combinations; we introduce the following matrix:

First row i.e.	R1:	1	7	3	9
Second row i.e.	R2:	7	9	1	3
Third row i.e.	R3:	3	1	9	7
Fourth row i.e.	R4:	9	3	7	1

We further introduce a term 'A'; which would follow a sequence of 1,3,7 and 9

There would be a prefix (5 or 7) before R and A. This prefix indicates the series of S5 or S7; implies that:

5//R1 means 5//1; 5//7; 5//3 and 5//9

Similarly 7//A means 7//1; 7//3; 7//7 and 7//9

The correspondence between R and A would be as shown in the following table:

5//A	5//R1
5//1	5//1
5//3	5//7
5//7	5//3
5//9	5//9

Table 11B.3: Correspondence between R and A considering specific example of R1

In order to identify and remove $c1+c2$ already derived by previous elements:

p_n = the prime factor of the composite component of $c1+c2$ combination

p_e = earlier prime elements where $p_e < p_n$. $C1+c2$ identified for p_n which have already been derived by p_e

p_a = such a factor whose product with p_e will give the first such composite component which is a part of $c1+c2$ combination and the same $c1+c2$ combination is being identified (or repeated) for deriving $c1+c2$ wherein p_n is a factor of component composites **OR** element whose product with p_e will give the first composite of $c1+c2$ that has already been identified for p_e and is being repeated in p_n .

We would now express the number of $c1+c2$ combinations in terms of the above matrix. We begin by derivation of total number of $c1+c2$ as below.

General conditions for identifying number of $c1+c2$ combinations of type 2 in which a prime element denoted as p_n is a factor of composite components may be summarised as :-

$$N(c+c) \text{ for } p_n = [\{(6I \pm 1) - d\} / 30] + 1$$

Here d is the factor of the first such composite component such that $p_n \times d$ forms part of first $c1+c2$ combination for p_n .

In order to find value of 'd', we draw the following table:

Series and last digit of P_n	Finding value of 'd' for 2k of:-	
	SADN(5,2,8)//2	SADN(7,4,1)//2
5//A	5//R2	7//R2
7//A	7//R2	5//R2

Series and last digit of P_n	Finding value of 'd' for 2k of:-	
	SADN(5,2,8)//4	SADN(7,4,1)//4
5//A	5//R4	7//R4
7//A	7//R4	5//R4

Series and last digit of P_n	Finding value of 'd' for 2k of:-	
	SADN(5,2,8)//6	SADN(7,4,1)//6
5//A	5//R1	7//R1
7//A	7//R1	5//R1

Series and last digit of P_n	Finding value of 'd' for 2k of:-	
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	SADN(5,2,8)//8	SADN(7,4,1)//8
5//A	5//R3	7//R3
7//A	7//R3	5//R3

Note:

- I. $d > p_n$ in all cases.
- II. Whether we need to consider the term $6I+1$ or $6I-1$ would depend on the SADN of the even number and thereby its relevant series. This is summarized in the following table:

SADN of $2k$	SADN of p_n	Whether $6I+1$ or $6I-1$
2,5,8	2,5,8	$6I-1$
7,4,1	2,5,8	$6I+1$
2,5,8	7,4,1	$6I+1$
7,4,1	7,4,1	$6I-1$

In order to identify and remove $c1+c2$ already derived by previous elements:

Series and last digit of P_e	Finding value of p_a for $2k=\text{SADN}(5,2,8)//2$							
	$P_n=5//1$	$P_n=7//1$	$P_n=5//3$	$P_n=7//3$	$P_n=5//7$	$P_n=7//7$	$P_n=5//9$	$P_n=7//9$
7//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
5//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

Series and last digit of P_e	Finding value of p_a for $2k=\text{SADN}(5,2,8)//4$							
	$P_n=5//1$	$P_n=7//1$	$P_n=5//3$	$P_n=7//3$	$P_n=5//7$	$P_n=7//7$	$P_n=5//9$	$P_n=7//9$
7//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1
5//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1

Series and last digit of P_e	Finding value of p_a for $2k=\text{SADN}(5,2,8)//6$							
	$P_n=5//1$	$P_n=7//1$	$P_n=5//3$	$P_n=7//3$	$P_n=5//7$	$P_n=7//7$	$P_n=5//9$	$P_n=7//9$

	Pn=5//1	Pn=7//1	Pn=5//3	Pn=7//3	Pn=5//7	Pn=7//7	Pn=5//9	Pn=7//9
7//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4
5//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4

Series and last digit of Pe	Finding value of pa for 2k=SADN(5,2,8)//8							
	Pn=5//1	Pn=7//1	Pn=5//3	Pn=7//3	Pn=5//7	Pn=7//7	Pn=5//9	Pn=7//9
7//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
5//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series and last digit of Pe	Finding value of pa for 2k=SADN(7,4,1)//2							
	Pn=5//1	Pn=7//1	Pn=5//3	Pn=7//3	Pn=5//7	Pn=7//7	Pn=5//9	Pn=7//9
7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3

Series and last digit of Pe	Finding value of pa for 2k=SADN(7,4,1)//4							
	Pn=5//1	Pn=7//1	Pn=5//3	Pn=7//3	Pn=5//7	Pn=7//7	Pn=5//9	Pn=7//9
7//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1
5//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1

Series and last digit of Pe	Finding value of pa for 2k=SADN(7,4,1)//6							
	Pn=5//1	Pn=7//1	Pn=5//3	Pn=7//3	Pn=5//7	Pn=7//7	Pn=5//9	Pn=7//9
7//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4
5//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4

Series and last digit of Pe	Finding value of pa for 2k=SADN(7,4,1)//8							
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	$P_n=5//1$	$P_n=7//1$	$P_n=5//3$	$P_n=7//3$	$P_n=5//7$	$P_n=7//7$	$P_n=5//9$	$P_n=7//9$
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2

Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=7$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=11$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4
7//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4

Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=13$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3

7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3
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Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=17$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2
7//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2

Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=19$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1
7//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1

Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=23$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

7//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
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Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=29$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1
7//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1

Series and last digit of P_n	Finding value of p_x for $p_e'=7$ and $p_e''=31$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4
7//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4

Series and last digit of P_n	Finding value of p_x for $p_e'=11$ and $p_e''=11$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

Series	Finding value of p_x for $p_e'=11$ and $p_e''=13$							
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and last digit of P_n								
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1
7//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1

Series and last digit of P_n	Finding value of p_x for $p_e'=11$ and $p_e''=17$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4
7//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4

Series and last digit of P_n	Finding value of p_x for $p_e'=11$ and $p_e''=19$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2
7//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2

Series	Finding value of p_x for $p_e'=11$ and $p_e''=23$
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and last digit of Pn								
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1
7//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1

Series and last digit of Pn	Finding value of px for $pe'=11$ and $pe''=29$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series and last digit of Pn	Finding value of px for $pe'=11$ and $pe''=31$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3
7//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3

Series and last digit	Finding value of px for $pe'=13$ and $pe''=13$							
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of Pn								
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series and last digit of Pn	Finding value of px for pe'=13 and pe''=17							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3
7//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
Series and last digit of Pn	Finding value of px for pe'=13 and pe''=19							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4
7//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4

Series and last digit of Pn	Finding value of px for pe'=13 and pe''=23							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8

5//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2
7//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2

Series and last digit of Pn	Finding value of px for $pe'=13$ and $pe''=29$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4
7//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4

Series and last digit of Pn	Finding value of px for $pe'=13$ and $pe''=31$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1
7//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1

Series and last digit of Pn	Finding value of px for $pe'=17$ and $pe''=17$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series	Finding value of px for $pe'=17$ and $pe''=19$							
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and last digit of Pn								
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1
7//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1

Series and last digit of Pn	Finding value of px for pe'=17 and pe''=23							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

Series and last digit of Pn	Finding value of px for pe'=17 and pe''=29							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1
7//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1

Series and last digit of Pn	Finding value of px for pe'=17 and pe''=31							
	2k=	2k=	2k=	2k=	2k=	2k=	2k=	2k=

	SADN (5,2,8) //2	SADN (7,4,1) //2	SADN (5,2,8) //4	SADN (7,4,1) //4	SADN (5,2,8) //6	SADN (7,4,1) //6	SADN (5,2,8) //8	SADN (7,4,1) //8
5//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4
7//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4

Series and last digit of Pn	Finding value of px for pe'=19 and pe''=19							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

Series and last digit of Pn	Finding value of px for pe'=19 and pe''=23							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4
7//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4

Series and last digit of Pn	Finding value of px for pe'=19 and pe''=29							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3
7//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3

Series and last digit of Pn	Finding value of px for $pe'=19$ and $pe''=31$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series and last digit of Pn	Finding value of px for $pe'=23$ and $pe''=23$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2
7//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2

Series and last digit of Pn	Finding value of px for $pe'=23$ and $pe''=29$							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R1	7//R1	5//R2	7//R2	5//R3	7//R3	5//R4	7//R4
7//A	7//R1	5//R1	7//R2	5//R2	7//R3	5//R3	7//R4	5//R4

Series and last	Finding value of px for $pe'=23$ and $pe''=31$							
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digit of Pn								
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R4	5//R4	7//R3	5//R3	7//R2	5//R2	7//R1	5//R1
7//A	5//R4	7//R4	5//R3	7//R3	5//R2	7//R2	5//R1	7//R1

Series and last digit of Pn	Finding value of px for pe'=29 and pe''=29							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

Series and last digit of Pn	Finding value of px for pe'=29 and pe''=31							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1) //4	2k= SADN (5,2,8) //6	2k= SADN (7,4,1) //6	2k= SADN (5,2,8) //8	2k= SADN (7,4,1) //8
5//A	7//R3	5//R3	7//R1	5//R1	7//R4	5//R4	7//R2	5//R2
7//A	5//R3	7//R3	5//R1	7//R1	5//R4	7//R4	5//R2	7//R2

Series and last digit of Pn	Finding value of px for pe'=31 and pe''=31							
	2k= SADN (5,2,8) //2	2k= SADN (7,4,1) //2	2k= SADN (5,2,8) //4	2k= SADN (7,4,1)	2k= SADN (5,2,8)	2k= SADN (7,4,1)	2k= SADN (5,2,8)	2k= SADN (7,4,1)

				//4	//6	//6	//8	//8
5//A	5//R2	7//R2	5//R4	7//R4	5//R1	7//R1	5//R3	7//R3
7//A	7//R2	5//R2	7//R4	5//R4	7//R1	5//R1	7//R3	5//R3

Only those values of 'n' are to be considered, whose value is greater than $p_n.d$ and less than $6I+1$

Only those values of p_e' , p_e'' and p_x are to be considered, whose value is greater than $p_e.d$ and less than $6I\pm 1$

Number of unique c_1+c_2 combinations of type2 = $n(c+c) - [n'(c+c) - n''(c+c)]$

11C.

Step3:

c1+c2 of type 2 derived from $6p_1p_2$ where p_1 and p_2 are primes

Step3: c1+c2 derived from $6p_1p_2$:

Suppose an even number is denoted as $2k$

Suppose p_1 and p_2 are two primes which are factors of the two composites c_1 and c_2 respectively, which form the combination as $c_1+c_2 = 2k$

Now consider the following:-

Floor function of $(1/6 (\text{floor function of } ((2k/p_1)))) = I_1$

$$2k - p_1(6I_1+1) = a$$

$[a+6np_1]/p_2 = \text{an Integer}$; for particular value of n viz. n_1 ($n_1=4$ in case of $p_1=13$ & $p_2=7$)

$$(6I_1+1) - 6n_1 = \text{integer}_1$$

And

Floor function of $(2k/6p_2) = I_2$

$$2k - p_2(6I_2+1) = b$$

$[b+6p_2n']/p_1 = \text{Integer}$ for particular value of n viz. n_2 ($n_2=11$ in case of $p_1=13$ & $p_2=7$)

$$(6I_2+1) - 6n_2 = \text{integer}_2$$

Now first combination of type c_1+c_2 would be :-

$$(7 \text{ integer}_2) + (13 \text{ integer}_X) = 2k$$

And last combination would be:-

$$(7 \text{ integer}_X) + 13\text{integer}_1 = 2k$$

.....
....

Example 1:

Suppose there's an even number as 16658

Say a prime number p_1 is 7 and another prime p_2 is 11

Now floor function (ff) of $[(1/6)[\text{ff of } [2k/p_1]]] = \text{ff of } [(1/6) [\text{ff of } (16658/7)]] = 396 = I$

$$2k - [p_1(6I+1)] = 16658 - [7 \times 2377] = 19 = d_1$$

$$(2k - d_1)/p_1 = (16658 - 19)/7 = \text{Integer} = 6I+1$$

$$[\text{As } 2k - d_1 = 2k - [2k - p_1(6I+1)] = p_1(6I+1) \text{ or } (2k - d_1)/p_1 = 6I+1]$$

$$(16658 - 19)/7 = \text{Integer} = 6I+1$$

Now:-

$$[d_1 + 6p_1n]/p_2 = \text{an integer for a value of } n \text{ such that } 0 \leq n < p_2$$

$$[19 + 42n]/11 = \text{an integer for a value of } n \text{ such that } 0 \leq n < 11$$

Here for $n=0$; $19/11$ is not an integer

$$n=1; (19+42)/11 = 61/11 \text{ not an integer}$$

$$n=2; (19+84)/11 = 103/11 \text{ not an integer}$$

$$n=3; (19+126)/11 = 145/11 \text{ not an integer}$$

$$n=4; (19+168)/11 = 187/11 = 17 = \text{an integer}$$

It implies that:-

$$16658 - 187 = 7 \times \text{integer}'$$

$$16658 - (11 \times 17) = 7 \times \text{integer}'$$

$$16658 = (7 \times \text{integer}') + (11 \times 17)$$

$$\text{which is an example of } 2k = c_1 + c_2 = (p_1 \times \text{integer}') + (p_2 \times 17)$$

This leads us to a third type of $c_1 + c_2$ combinations where any two primes p_1 and p_2 would be factors of two composites which form $c_1 + c_2$ combinations to give us $2k$.

Any such combination will be formed for primes p_1 and p_2 if the condition of $2k/6p_1p_2 \geq 1$ is fulfilled.

In the above example when $7 \times 11 \times 6n$ (i.e. 462 times n) is added to 187, it will give us further $C_1 + C_2$ combinations in which 7 and 11 are factors of C_1 and C_2 respectively.

Number of such combinations would be given by $n+1$, where 'n' would be floor function of $[(2k-187)/462]$

Generalised as:

Number of type-3 $c_1 + c_2$ combinations formed by any two prime numbers p and p' would be floor function of $[2k/6pp']$

Suppose an even number $(EN) = 2k$

$2k/6pp' = q.r$ where q is an integer and r is fractional part i.e. $0 \leq r < 1$

Then there exists ' q ' number of combinations such that $pa + p'b = 2k$

Let these combinations be given as:

$$pa_1 + p'b_1 = 2k$$

$$pa_2 + p'b_2 = 2k$$

$$pa_3 + p'b_3 = 2k$$

\vdots

$$pa_q + p'b_q = 2k$$

Since least-common-multiple (lcm) of pp' and $6 = 6pp'$

Hence here:

$$\begin{array}{ll} pa_2 = pa_1 + 6pp' & \text{OR} \quad a_2 = a_1 + 6p' \\ pa_3 = pa_2 + 6pp' & a_3 = a_2 + 6p' \\ pa_4 = pa_3 + 6pp' & a_4 = a_3 + 6p' \\ \vdots & \vdots \\ pa_q = pa_{q-1} + 6pp' & a_q = a_{q-1} + 6p' \\ & \text{OR} \quad a_q = a_1 + 6(q-1)p' \end{array}$$

On similar logic:-

$$\begin{array}{ll} p'b_2 = p'b_1 - 6pp' & \text{OR} \quad b_2 = b_1 - 6p \\ p'b_3 = p'b_2 - 6pp' & b_3 = b_2 - 6p \\ p'b_4 = p'b_3 - 6pp' & b_4 = b_3 - 6p \\ \vdots & \vdots \\ p'b_q = p'b_{q-1} - 6pp' & b_q = b_{q-1} - 6p \\ & \text{OR} \quad b_q = b_1 - 6(q-1)p \end{array}$$

In addition to above :

$$b_{q+1} = b_1 - 6qp$$

and

$$b_{q+n} = b_1 - 6(q+n-1)p$$

If $r = pa_1 + p'b_q - 6pp'$; then total number of combinations of c_1+c_2 type-3 becomes q

If $r = pa_1 + p'b_{q+1}$; then total number of combinations of c_1+c_2 type-3 becomes $q+1$

If

$$b_{q+n(\max)} = b_1 - 6(q+n_{\max} - 1)p > 0$$

then number of c_1+c_2 combinations of type-3 = $q+n_{\max}$

where n_{\max} is given by:

$$b_1 - 6(q+n_{\max} - 1)p > 0$$

$$\text{OR } n_{\max} \leq \text{floor function of } [(b_1/6p) - q + 1]$$

To find unique c_1+c_2 combinations of type-3:

As mentioned earlier c_1+c_2 combinations of type3 are formed by any 2 prime elements irrespective of their SADN or SADN of $2k$ or its last digit. In these combinations p_1 and p_2 are factors of the components on either side of the summation sign used in the combination. The general method for calculating $n(c_1+c_2)$ of third type has been discussed above and can be generally derived as $2k/6p_1p_2=q.r$ where q is the number of c_1+c_2 combinations that would be formed by the pair of primes p_1p_2 for a given $2k$ and r is the fractional part. Also if the fractional part of the quotient i.e. r can be split into two numbers that are both composite in nature and are such that p_1 and p_2 are factors of either composite, then $n(c_1+c_2) = q+1$

Once we arrive at $n(c_1+c_2)$ for a given pair of p_1p_2 , we need to identify the unique c_1+c_2 combinations. Two types of repetitions are possible while deriving such c_1+c_2 combinations. First, those that have been already derived while calculating $n(c_1+c_2)$ for combinations of type-2 and second, those that have been derived by earlier pairs of p_1p_2 while calculating c_1+c_2 combinations of type-3. These combinations need to be identified and removed to avoid double counting.

Since c_1+c_2 of type-2 depends on the last digit of composite odd numbers and their series vis-à-vis the last digit of $2k$ and its relevant series, those 2 factors would play an important role in identifying c_1+c_2 already derived.

If $2k = S5//2$ (i.e. even number lies on $S5$ series and has its last digit as 2), then all such c_1+c_2 combinations which include a composite odd number ending in 7, would form a c_1+c_2 combination of type2.

Consider $2k = 16658$

Number of c_1+c_2 for $p_1=7, p_2=11$:

$$2k/6p_1p_2 = 36.05$$

Total number of c_1+c_2 formed by 7&11 upto $2k$ is given as 36. Since $2k=S5//8$, then composite odd numbers ending in 3 will form c_1+c_2 combinations of type-2.

To find such c_1+c_2 combinations the following steps may be followed:

Step A: Identify the first c_1+c_2 using the method discussed above.

The first such combination = 187 (i.e. 11×17) + 16471 (i.e. 7×2353)

When we add 462(i.e. $7 \times 11 \times 6$) to 187 and subtract 462 from 16471; we get the next c_1+c_2 combination:-

$$649(\text{i.e. } 11 \times 59) + 16009(\text{i.e. } 7 \times 2287)$$

By adding $462n$ to 187, and subtracting $462n$ from 16471; we get c_1+c_2 combinations of which 7 and 11 are factors.

Since the first number of the combinations is 187 i.e. ends in 7, adding 462, a number ending in 2 will give an odd number in the nature of $7+2=9$ and the corresponding number would be $16471(\text{number ending } 1) - 462(\text{number ending } 2)$ yielding an odd number ending in 9.

To this odd number ending in 9 when we add 462, we get an odd number ending in 1 and when we subtract 462 from the corresponding number ending 9, we get an odd number ending 7. Since none of those numbers identified so far ends in 3, they can not be derived as c_1+c_2 combination of type-2.

When we add 462 to the former term of the third c_1+c_2 ending in 1, we get an odd number ending in 3. By subtracting 462 from the corresponding number ending in 7 we get an odd number ending in 5.

This is a c_1+c_2 combination already identified.

Further when we add 462 to the former term of the c_1+c_2 ending in 3 (i.e. having last digit as '3'), we get an odd number ending in 5 and by subtracting 462 from the corresponding number ending in 5 will end in 3. This again would be a c_1+c_2 combination already derived. The combinations thus derived would be as follows:-

Adding $462n$

- i. $187+462 = 649$
- ii. $649+462 = 1111$
- iii. $1111+462 = 1573$
- iv. $1573+462 = 2035$
- v. $2035+462 = 2497$

Subtracting $462n$

- i. $16471-462 = 16009$
- ii. $16009-462 = 15547$
- iii. $15547-462 = 15085$
- iv. $15085-462 = 14623$
- v. $14623-462 = 14161$

There is a similarity between the 1st and 6th combination in that the composite numbers in the combination end in the same digit. This is because odd numbers end in any one of the following 5 digits- 1,3,5,7,9 and this sequence will occur in a cyclic manner. If we start adding the same even digit to any odd number, a cyclical series of odd numbers ending in different digits will be formed.

Consider the following sequences for odd number ending in digit '1':

- (1) If even number ends in digit 2 then last digits will appear as:

$$1+2 = 3$$

$$3+2 = 5$$

$$5+2 = 7$$

$$7+2 = 9$$

$$9+2 = 1$$

$$1+2 = 3$$

⋮

(2) If even number ends in digit 4 then last digits will appear as:

$$1+4 = 5$$

$$5+4 = 9$$

$$9+4 = 3$$

$$3+4 = 7$$

$$7+4 = 1$$

$$1+4 = 5$$

⋮

(3) If even number ends in digit 6 then last digits will appear as:

$$1+6 = 7$$

$$7+6 = 3$$

$$3+6 = 9$$

$$9+6 = 5$$

$$5+6 = 1$$

⋮

(4) If even number ends in digit 8 then last digits will appear as:

$$1+8 = 9$$

$$9+8 = 7$$

$$7+8 = 5$$

$$5+8 = 3$$

$$3+8 = 1$$

⋮

Therefore when we add the even number ending in the same digit to any odd number we get odd numbers ending in odd digits in a specific sequence which repeats itself after every 5 numbers (*this is because every even number would be a multiple of 2, and adding the even number 5 times is equivalent to adding an even number having last digit as '0', which implies that the last digit remains unchanged as the last digit of added number is a '0' which is an additive identity*). That is, if we add any even number ending in 2,4,6,8 to any odd number ending in a particular digit, last digit of the 1st, $1+5=6^{\text{th}}$, $6+5=11^{\text{th}}$, $11+5=16^{\text{th}}$, $16+5=21^{\text{st}}$ number will end in the same digit. Similarly when we add the same even number to an odd number the 2nd, $2+5=7^{\text{th}}$, $7+5=12^{\text{th}}$, $12+5=17^{\text{th}}$, $17+5=22^{\text{nd}}$ number will end in the same digit. This cyclical series will be infinite in nature and the sequence will remain the same no matter which number we begin from.

Further when we add the same even number to an odd number and study the pattern of resultant number, it follows that one number out of every 5 numbers will end in the digit 5 and thus the resulting odd numbers will end in 1,3,5,7,9 in same order depending on the value of the even number that is being added.

Therefore it can be concluded that 1 out of every 5 resulting numbers will end in 1,3,5,7 and 9. We now return to the rationale behind $c1+c2$ combinations of type-2 involving composites ending in 5 and same odd composite number ending in a particular digit depending on the SADN//last digit of $2k$ i.e. if $2k=S5$ or $S7//2$, then $c1+c2$ combinations of type-2 include composites ending in 7 and corresponding composites ending in 5.

Putting the above patterns together it is possible to conclude that 1 out of every 5 consecutive odd numbers derived by adding the same even number to an odd number, will end in 5 and also 1 out of the same set of 5 odd numbers will end in a composite odd number.

This brings us to the pattern that since $c1+c2$ combinations of type-3 are derived by adding $6p_1p_2$ to the former term of the first $c1+c2$ combination, 2 out of every 5 resulting combinations would be such that those have been already derived as $c1+c2$ combination of type-2.

Therefore if $n(c1+c2)$ of type-3 is 'a', i.e. number of $c1+c2$ combinations of type-3 is 'a'; then $a/5=b.r$ where 'r' is the fractional part. It will give the number of blocks of 5 consecutive odd numbers that would end in 5 different odd digits.

Since 2 out of 5 such consecutive odd numbers have already been derived as $c1+c2$ combination of type-2, it follows that $2b$ will be the number of $c1+c2$ combinations already derived by the method for calculating $c1+c2$ of type-2.

In the above example,

$$16658/(7 \times 11 \times 6) = 36$$

$$36/5 = 7.2$$

There are 7 blocks of 5 consecutive odd numbers ending in 5 different odd digits. Therefore $2b = 2 \times 7 = 14$

14 out of 36 $c1+c2$ combinations formed by 7&11 are those that have already been derived while calculating $c1+c2$ combinations of type-2.

It is important to note that the exact number of c_1+c_2 already derived while calculating c_1+c_2 combinations of type-2 would depend on the last digit of the first c_1+c_2 derived while finding c_1+c_2 combinations of type-3. However it can be generalized that the number of c_1+c_2 already derived as c_1+c_2 of type-2 would be $2b$ or $2b+1$ or $2b+2$ maximum.

The second type of possible repetitions is those c_1+c_2 combinations that have been already derived by earlier p_1p_2 combinations.

Identifying c_1+c_2 combinations already derived by earlier $p_1&p_2$:

In the above example, 7&11 is the first p_1p_2 pair whose c_1+c_2 of type-3 is being identified. Therefore c_1+c_2 combination already derived earlier would be those identified while calculating c_1+c_2 combinations of type-2.

Therefore unique c_1+c_2 pair derived by prime pair of 7&11 would be:

$$\text{Total } c_1+c_2 = 2k/462 = 36$$

$$N(c_1+c_2) \text{ of type-2} = 14$$

$$36-14=22$$

The next pair of primes would be 7&13. While deriving c_1+c_2 of type-3 for this pair, 2 varieties of repetitions are possible. First, those already derived as c_1+c_2 of type-2. Second, those already derived as c_1+c_2 of type-3 for the previous pair of primes 7&11. This may be done as follows:-

Step A: Calculate total c_1+c_2 of type-3 for the pair of primes 7&13

$$16658/(7 \times 13 \times 6) = 16658/546 = 30$$

$$N(c_1+c_2) \text{ of type-3} = 30$$

Number of blocks of 5 odd numbers that would occur in a cyclical manner-

$$30/5 = 6$$

As mentioned above, in every block of 5 odd numbers there would be 2 such c_1+c_2 combinations which have already been derived while calculating c_1+c_2 combinations of type-2. Hence in present case, their number would be $2 \times 6 = 12$

This implies that 12 out of 30 c_1+c_2 combinations of type-3 formed by 7&13 have already been identified while calculating c_1+c_2 of type-2.

For identifying c_1+c_2 already derived while calculating c_1+c_2 of type-3 for 7&11 the following reasoning is applied:

Here one of the numbers in the prime numbers 7&11 and 7&13 is the number 7. This means 7 would be a factor on one side of the c_1+c_2 combination. Those composites on the other side of the combination of which 11&13 are factors need to be identified since such composites alongwith corresponding composites of which 7 is a factor would have been already identified by the previous prime pair of 7&11. This can be done as follows-

We first identify the first c_1+c_2 of which 7&13 are factors-

$$16658 - 7(6I+1) = (7 \times 2377)$$

$$16658 - 16639 = 19$$

$$(19+42n)/13 = I$$

Here we need to find the value of 'n' where 'I' becomes an integer.

$(19+42n)/13$ becomes a integer for $n=11$

$$\{19+(42 \times 11)\}/13 = 37$$

So $13 \times 37 (= 481) + 7 \times 2311 (= 16177) = 16658$ i.e. $2k$

Thereafter $13 \times (37+42n)$ would be the former term of further c_1+c_2 combinations and the corresponding terms would be $7 \times (2311-78n)$

Adding $42n$ to 37 will give further composites of which 13 would be a factor. If we can identify such a composite of which 11 is also a factor derived by 7×11 prime pair.

i.e. $(37+42n)/11$ should be an integer

This condition is fulfilled for the value of $n=2$ since $37+(42 \times 2) = 121$ which is divisible by 11 .

Therefore 13×121 is the first such c_1+c_2 combination which is divisible by both 13×11 and will be a c_1+c_2 that has already been derived by the prime pair 7×11 .

Thereafter every $(11 \times 42)_{nd}$ number will be such a c_1+c_2

i.e. $121+(11 \times 42) = 462^{nd}$ number

$$121+462n \leq 6I+1 \text{ (i.e. } 13)$$

This would give us the following numbers= $121, 583, 1045$

$13 \times$ these numbers would give c_1+c_2 combinations already derived by the prime pair 7×11

However it needs to be noted that it is possible that such c_1+c_2 identified may have been identified and adjusted while calculating c_1+c_2 of type-2 already derived. These need to be identified and adjusted to avoid double subtraction of the same c_1+c_2 .

This may be identified through the following rationale-

16658 is a SADN(8//8) number which means (c_1+c_2) combinations of type-2 would involve composite odd numbers ending in 3 . Since 13 (the number for which $n(c+c)$ or $n(c_1+c_2)$ is being calculated) ends in 3 its product with numbers ending in 1 and 5 would yield c_1+c_2 combinations of type-2. Similarly the product of 13 with numbers ending in 5 would yield such composites where corresponding number in the c_1+c_2 combination would end in 3 and would qualify therefore to be a c_1+c_2 combination of type-2.

Therefore of the composite numbers identified as common to both 13 and 11, those ending in the digits 1 and 5 need to be identified as these are $c1+c2$ combinations already removed on account of being identified as relevant $c1+c2$ combinations of type-2.

In the above example the 3 such composites were identified-

13x121, 13x583, 13x1045

Of these 13x121 would yield a composite ending in 3 and 13x1045 would yield a composite ending in 5. So these composites may be understood to have formed $c1+c2$ combinations of type-2 which have already been identified and removing those again would result in double-removal and therefore under-reporting of unique $c1+c2$ for the prime pair of 7&13. Therefore here we consider only one composite- 15x583 to form a $c1+c2$ with 7 on the other side that has already been identified while deriving $c1+c2$ combinations of type-3 for the prime pair of 7&11.

Therefore unique $c1+c2$ for the prime pair 7&13 are given as:-

$$30-12[\text{i.e. } n(c1+c2) \text{ of type-2}]-1[\text{i.e. } n'(c1+c2) \text{ of type-3 for 7\&11}]=30-13=17$$

This implies that the prime pair of 7&13 will form 17 unique $c1+c2$ combinations of type-3 for the even number 2k as 16658.

Deriving unique $c1+c2$ for the prime pair 7&17:

$$16658/(7 \times 17 \times 6) = 23$$

As ff of $(23/5) = 4$, there would be 4 blocks of 5 different odd numbers.

So number of $c1+c2$ combinations which have already been derived as type-2 = $4 \times 2 = 8$ [as per mentioned above]

To determine the value of $6I-1$ for even number 16658:

$$I = \text{ff of } [(1/6) \text{ ff of } (16658/17)] = 163$$

$$\text{Hence } 6I-1 = 977$$

$c1+c2$ already derived by previous prime pairs 7&11, 7&13:

$$(19+42n)/17 = \text{Integer for } n \text{ equals to } 4$$

$$\text{First } c1+c2 = 187(\text{i.e. } 11 \times 17) + 16471(\text{i.e. } 7 \times 2353)$$

To identify $c1+c2$ already derived by previous prime pairs 7&11 and 7&13, we need to identify composites common to 11&17 and 13&17 as one part of $c1+c2$ for the prime pair 7&17.

For this we calculate:

$$17 \times [(11+42n)/11] = \text{an integer for a particular value of 'n'}.$$

Here it would be an integer for value of $n=0$. So 187, the first term of the $c1+c2$ combination formed by 7&17 is divisible by 11. Thereafter every $17(11 \times 7 \times 6)^{\text{th}}$ number would be a composite already derived while calculating $c1+c2$ for 7&11.

This would be $7 \times (11 + 42n) \leq 6I-1$

11,473,935

Of these 17x935 has already [*how do we know that?*] been derived while identifying $c1+c2$ type combinations. So we need to remove combinations formed by 11x17 and 17x473

Similarly to identify $c1+c2$ combination already derived by the prime pair 7&13 we need to identify a $c1+c2$ formed by 7&17 in which 17 and 13 form a common composite. This can be derived as:

$$17 \times [(11 \times 42n)/13]$$

Here $(11 \times 42n)/13$ becomes an integer for the value of n as 5 i.e. $[11 + (42 \times 5)]/13 = 221$

Here on every $13 \times 42^{\text{nd}}$ number will also be a composite divisible by 13 and a part of $c1+c2$ already derived by 7&13:

$$17 \times [221 + 42 \times 13n] \text{ i.e. } 17(221 + 546n) \leq 6I-1$$

221, 767, will be two such numbers whose product with 17 will form $c1+c2$ combination for the prime pair 7&17 and which have already been derived while calculating $c1+c2$ for the prime pair 7&13.

Total unique $c1+c2$ formed by the prime pair 7&17 =

23 - 9[i.e. number of $c1+c2$ combinations already derived while calculating $c1+c2$ combinations of type-2 for 17] - 2[i.e. number of $c1+c2$ combinations already derived while calculating $c1+c2$ of type3 for 7&11] - 2[i.e. number of $c1+c2$ combinations already derived while calculating $c1+c2$ of type3 for the prime pair 7&13]

$$\text{i.e. } 23 - (9 + 2 + 2) = 23 - 13 = 10$$

This implies that 17 forms 10 unique $c1+c2$ combinations with 7 upto the even number 16658.

Similarly for calculating number of $c1+c2$ combinations of type-3 for the prime pair 7&19, we first calculate the total number of $c1+c2$ combinations of type-3 using the method:

$$2k/(6 \times 7 \times 19) \text{ which gives us } n(c1+c2) = 12$$

Here again we first identify and remove the number of $c1+c2$ combinations already derived while calculating number of combinations of $c1+c2$ of type-2 by applying the method of dividing $n(c1+c2)$ into as many blocks of 5 odd numbers as possible. Here $n(c1+c2) = 12$, so $\text{ff of } (12/5) = 2$. There are 2 complete blocks of 5 odd numbers of which 2 out of 5 would be already derived while calculating $c1+c2$ combinations of type-2. Therefore $2 \times 2 = 4$ $c1+c2$ combinations would be removed.

We next identify and remove c_1+c_2 already derived while calculating c_1+c_2 for the previous prime pairs 7&11, 7&13 and 7&17. This will first require identification of the first c_1+c_2 combination of type-3 formed by the prime pair 7&19. For this we follow the step-

$$(19+42n)/19=I$$

To identify the value of n where I becomes an integer: Here I becomes an integer for $n=19$. So first c_1+c_2 combination formed by 7&19 would be 817(i.e. 19×43)+15841(i.e. 7×2263)

To identify first c_1+c_2 combination already derived by 7&11, we need to identify the c_1+c_2 combinations where one composite term is divisible by 7 and the other is divisible by both 19 and 11. This requires the following step:

$$19[(43+42n)/11] = \text{an integer for a specific value of } n.$$

Here this condition is satisfied for value of $n=5$. So 19×253 will be the first such composite of which 19&11 are both factors and whose corresponding number in the combination i.e. $2k - (19 \times 253)$ would be divisible by 7. Thereafter all terms those satisfy the condition:

$$19(253 + (11 \times 42)n) \leq 6I+1$$

$$19(253+462n) \leq 6I+1$$

will form further c_1+c_2 combination already derived by the previous prime pairs 7&11 which are now being identified again while deriving c_1+c_2 of type-3 for the prime pairs 7&19.

$$\text{Similarly } 19[(43+42n)/13] = I$$

So if we find value of n for which I becomes an integer for the above equation, we will get the first c_1+c_2 common to the prime pairs 7&13 and 7&19. Here I becomes an integer for value of n equals to 3. This implies 19×169 is the composite form of the first c_1+c_2 combination common to the prime pairs 7&13 and 7&19.

Thereafter all composites that satisfy the condition $19(169+546n) \leq 6I+1$ will be c_1+c_2 combination common to the prime pairs 7&13 and 7&19.

After identifying these c_1+c_2 already obtained for previous prime pairs, the next step would be to identify if any of these have been already derived by the c_1+c_2 earlier or are common to any previous prime pair. These need to be identified and adjusted to avoid double removal of the same c_1+c_2 combination.

Combinations common to any 2 previous prime pairs would be derived for value of n where I would become an integer in the following formula:-

Suppose unique number of c_1+c_2 combinations for 7&19 is calculated and we want to identify c_1+c_2 combinations already derived by 7&11 and 7&13 that are common to both these previous prime pairs then $19[(43+42n)/(11 \times 13)] = I$

The value of integer ' n ' is such that the resultant value is $< 2k$. Thereafter all terms that satisfy the condition $19(143 + (6 \times 143)n) \leq 6I+1$ will form further c_1+c_2 common to the prime pairs 7&11 and 7&13.

Once all unique c_1+c_2 combinations formed by 7 with other prime elements have been derived, we need to identify c_1+c_2 , combinations for other prime pairs like 11&13, 11&17, 11&19 where one of the terms would be common to all prime pairs and would be a prime number > 7 . While calculating unique c_1+c_2 for such a prime pair, we follow the same steps as above.

Only an additional step would be to identify these c_1+c_2 already derived by previous prime pairs in the nature of $7p_1$ i.e. those prime pairs where 7 is a common component of the prime pair. For this we follow the following method:-

Identify total number of c_1+c_2 combinations of type-3 for the prime pair 11&13, i.e. ff of $[16658/(6 \times 11 \times 13)] = 19$

Hence 11&13 prime pair will form 19 c_1+c_2 combinations of type-3 upto $2k$.

The next step will be to divide this $n(c_1+c_2)$ into blocks of 5 odd numbers and calculate number of c_1+c_2 combinations already derived while calculating c_1+c_2 combinations of type-2.

Thereafter we find c_1+c_2 combinations already derived while calculating c_1+c_2 for prime pairs in the nature of $7p_1$. For this we follow the following steps:-

First c_1+c_2 of 11&13:-

$$187(\text{i.e. } 11 \times 17) + 16471(\text{i.e. } 13 \times 1267)$$

Now we find the term

$$11[(17+78n)/7] = \text{an integer, for a particular value of } n$$

Here this expression becomes an integer for value of $n = 4$. So 11×329 is the first number that is a component in a c_1+c_2 combination common to 11&13 and to a particular prime pair $7p_1$. Thereafter all natural numbers that satisfy the condition $11(329+(7 \times 78)n) \leq 6I-1$ will be further c_1+c_2 combinations common to the prime pairs 11&13 and $7p_1$.

Similarly, we need to identify such components common to 7&13 also since they too will constitute c_1+c_2 common to both 11&13 and $7p_1$.

For this we apply the formula to get:-

$$13[(1267-66n)/7]=I$$

and find values of n where I becomes an integer. Here I becomes an integer for value of $n=0$. So 13×1267 itself is a component common to 13&7 and will therefore constitute a c_1+c_2 common to the prime pairs 11&13 and $7p_1$.

Therefore all numbers that satisfy the condition:-

$13(1267-(7 \times 66)n) > 0$ will give other numbers that form part of c_1+c_2 combination common to 11&13 and $7p_1$. Once these common c_1+c_2 combinations have been identified we need to find out if there is any commonality between these c_1+c_2 and those c_1+c_2 identified while calculating c_1+c_2 combinations of type-2. Adjusting for these terms we arrive at unique number of c_1+c_2 combinations of type-3 for the prime pair 11&13.

General method to derive number of unique c_1+c_2 combinations of type-3:-

Step-1:

Step1: calculate total number of $c+c^3$ type combinations for first prime pair p_1p_2 as follows:-

$$2k/6p_1p_2=q.r$$

If fractional part $r=p_1a_1+p_2b_{q-1}$

The $nc+c^3=q+1$

If fractional part $r+6p_1p_2=p_1a_1+p_2b_q$ then

$$Nc+c^3=q$$

Step-2:

Step2: identify those $c+c$ out of $nc+c^3$ that have already been derived while calculating $c+c^2$ type combinations. For this divide $nc+c^3/5$ which gives the number of blocks of 5 odd numbers ending in 5 different possible odd digits. 2 out of every 5 blocks of 5 will be part of a $c+c$ already derived while calculating $c+c^2$ type combinations (as details discussed above)

If $nc+c^3/5=a$

Then number of $c+c$ already derived while calculating $c+c^2=2a$

Note: this number may increase by 1 or maximum 2 depending on 2 factors

- (a) Last digits of the components of first $c+c$ formed by the prime pair p_1p_2
- (b) Remainder of the value of $n(c+c^3)/5=a.r$, where ff of $[N(c+c^3)/5] = a$

This value needs to be calculated from $n(c+c^3)$.

Step3:

For the first prime pair p_1p_2 ; $n(c+c^3)-2a$ will give the number of unique c_1+c_2 combinations of type-3 formed by this prime pair for the given $2k$.

Step4:

For the next prime pair p_1p_3 after calculating $n(c+c^3)$ and $2a$, an additional step required would be to identify c_1+c_2 already derived by the previous prime pair p_1p_2 . Since p_1 is common to both the prime pairs p_1p_2 and p_1p_3 , any c_1+c_2 formed by p_1p_3 wherein one of the two components of the combinations is divisible by p_1 and the other is divisible by both p_2 and p_3 would necessarily be a c_1+c_2 common to both these prime pairs p_1p_2 and p_1p_3 . This common c_1+c_2 may be identified as follows:-

- (a) Find out the first c_1+c_2 of which p_1p_3 are factors on either side of the combination.
 $2k-p_1(6l+1) = W$
 $(W+6np_1)/p_3 = \text{an integer, for a particular value of } n.$
 Find the value of n where this expression becomes an integer. Let us denote this integer as y .
 This implies p_3y is the composite component divisible by p_3 which alongwith $2k-p_3y$ will form the first c_1+c_2 combination for the prime pair p_1p_3 .

- (b) Now we need to find the c_1+c_2 combination for p_1p_3 in which one component would be composite whose factors are p_2 and p_3 . For this we calculate the following:-

$P_3 [(y+n \cdot 6 \cdot p_1)/p_2] = \text{an integer, for a particular value of } n.$

Find value of n which makes this expression as an integer. Let us denote this integer as z . It implies that p_3z is a composite component of which p_2 and p_3 are factors and which is a part of the first such c_1+c_2 combination that is common to both the prime pairs p_1p_2 and p_1p_3 . Thereafter every such combination that satisfies the condition $p_2z+6p_1p_2 \leq 6I+1$ will form further such c_1+c_2 combinations that are common to the c_1+c_2 combinations obtained from prime pairs p_1p_2 and p_1p_3 . Let us call this number of combinations as $n'(c+c_3)$.

Therefore $n'(c+c_3) = [\{(6I+1)-(p_2z)\}/6p_1p_2] + 1$

This $n'(c+c_3)$ will give total number of c_1+c_2 combinations common to both prime pairs p_1p_2 and p_1p_3 .

- (c) The next step would be to identify if any of these $n'(c+c_3)$ combinations include a component whose last digit is 5. If there is/are; then this would indicate c_1+c_2 combinations already removed while counting for repetitions due to c_1+c_2 combinations of type-2. Those need to be identified and adjusted to avoid double removal of the same c_1+c_2 combination. If of $(n'(c+c_3)/5)$ will give the number of such c_1+c_2 already derived while identifying c_1+c_2 combinations of type-2 that are being identified again while identifying common c_1+c_2 combinations for the prime pairs p_1p_2 and p_1p_3 . Let's denote the number of such common c_1+c_2 combinations as $n''(c+c_3)$.

Hence Unique c_1+c_2 combinations for the prime pair $p_1p_3 = n(c+c_3) - 2a - n'(c+c_3) - n''(c+c_3)$

Step 5:

For the next prime pair p_1p_4 , we first calculate $n(c+c_3)$ and $2a$:

$2k/6p_1p_4 = q.r$, where 'r' is the fractional part

If of $(q/5) = a$

we then calculate $n'(c+c_3)$ for identifying c_1+c_2 common to the prime pairs p_1p_2 and p_1p_4 .

First c_1+c_2 for p_1p_4 is obtained as:-

$(W+6np_1)/p_4 = \text{an integer, for a particular value of } n$

Find value of n for which this expression becomes an integer, denoted as y .

p_4y would be part of the first c_1+c_2 formed by p_1p_4

$p_4[(y+6p_1n)/p_2] = I$

Find values of n for which I becomes an integer, denoted as z .

$p_4 \cdot p_2z$ is the first such component which is divisible by both p_2 and p_4 and is a part of c_1+c_2 combinations formed by p_1p_4 . Thereafter all combinations that satisfy the condition $p_2z+6p_1p_2 \leq 6I+1$ will be part of such c_1+c_2 that are common to both prime pairs p_1p_2 and p_1p_4 and would involve a component whose factors are p_2 and p_4 . Such number of c_1+c_2 combinations may be denoted as $n'(c+c_3)$.

Therefore $n'(c+c_3) = [\{(6I \pm 1) - p_2z\} / 6p_1p_2] + 1$

Similarly we derive c_1+c_2 common to the prime pairs p_1p_3 and p_1p_4 . To identify the first such c_1+c_2 the following steps are to be followed:

$$p_4[(y+6p_1)/p_3] = I$$

find value of n for which I becomes an integer denoted as z' .

p_4z' would be a composite divisible by both p_3 and p_4 and would be a component of c_1+c_2 common to both prime pairs p_1p_3 and p_1p_4 . Further such c_1+c_2 combinations common to the prime pairs p_1p_3 and p_1p_4 would be derived as those fulfilling the $p_3z'+6p_1p_2 \leq 6I \pm 1$

$$n'(c+c_3) = [(6I \pm 1) - p_3z'] / 6p_1p_2 + 1$$

in order to avoid double removal of the same c_1+c_2 , we need to derive $n''(c+c_3)$ for both the prime pairs p_1p_2 and p_1p_3 . Here an additional step would be to identify those components that are common to both p_1p_2 and p_1p_3 and removing them while calculating c_1+c_2 for p_1p_4 would result in double removal. These may be identified as those c_1+c_2 combinations where 1 component is divisible by p_2 , p_3 and p_4 and forms part of c_1+c_2 combination for p_1p_4 . The first such c_1+c_2 combination would include a component which satisfies the following condition:-

$$p_4[(y+6p_1n)/p_2p_3] = I$$

Find value of n where I (denoted as z'') becomes an integer. Thereafter further c_1+c_2 that satisfy the condition:

$$p_2p_3z'' + 6p_1p_2p_3 \leq 6I \pm 1$$

This number would be derived as $n'''(c+c_3) = [(6I \pm 1) - p_2p_3z''] / 6p_1p_2p_3 + 1$

Number of Unique composites for the prime pair p_1p_4 would be derived as $n(c+c_3) - 2a - \{n'(c+c_3) - n''(c+c_3) - n'''(c+c_3)\}$

Step6:

Identifying unique c_1+c_2 for the prime pair p_2p_3 while deriving c_1+c_2 for p_2p_3 , two types of repetitions, i.e. c_1+c_2 derived by previous steps, need to be identified. First those that have been already calculated while deriving c_1+c_2 combinations of type-2. Second, those that have been derived by previous prime pairs p_1p_2 , p_1p_3 , p_1p_4 , ..., p_1p_n .

We begin by deriving $n(c+c_3)$ for p_2p_3 prime pair by applying the formula of $(2k/6p_2p_3) = q$. Thereafter we derive c_1+c_2 combinations already identified while calculating c_1+c_2 of type-2.

ff of $(n(c+c_3)/5) = a$ and number of c_1+c_2 identified while calculating c_1+c_2 of type-2 is as $2a$

Find value of n where I becomes an integer denoted as y .

$p_3y + (2k - p_3y)$ will be the first c_1+c_2 combination for the prime pair p_2p_3 . Let us say the two components of this combination are p_2y' and p_3y .

Further combinations would be $p_3(y+6np_2) \leq 6I+1$ while the corresponding components would be:

$$p_2(y' - 6np_3) \leq 6I \pm 1$$

To identify c_1+c_2 already derived by previous prime pairs including p' we follow the following steps:- $(y+6p_2n)/p_1 = I$

Find value of n for which I becomes an integer, denoted as z .

Therefore all $c1+c2$ that satisfy the following conditions would be such $c1+c2$ that have been derived by previous prime pairs:- $z+6np_1p_2 \leq 6I \pm 1$

For all these combinations, p_1 would be a factor and it would be a component of $c1+c2$ already derived by previous prime pairs.

Similarly:-

$$(y'-6np_3)/p_1 = I$$

Find value of n for which I becomes an integer, denoted as z' .

This would also give a component of which p_2 and p_1 are factors and would constitute $c1+c2$ already derived by previous prime pairs. Thereafter all components that satisfy the following condition will constitute $c1+c2$ already derived by previous prime pairs:-

$$z'-6np_1p_3 \leq 6I \pm 1$$

Once these combinations have been identified, the last step would be to identify if there are any common $c1+c2$ combinations that are being removed twice i.e. while calculating $c1+c2$ combinations of type-2 and those derived by previous prime pairs.

This is possible by identifying $c1+c2$ in which any one component ends in the digit 5.

This $c1+c2$ may be denoted as $n'(c+c3)$

$$\text{Unique } c1+c2 \text{ for the prime pair } p_2p_3 = n(c+c3) - 2a - (n'(c+c) - n''(c+c))$$

After deriving the total number of $c1+c2$ combinations of the three steps discussed above, the number of unique $c1+c2$ is to be derived. These would be represented through the following Venn diagram:-

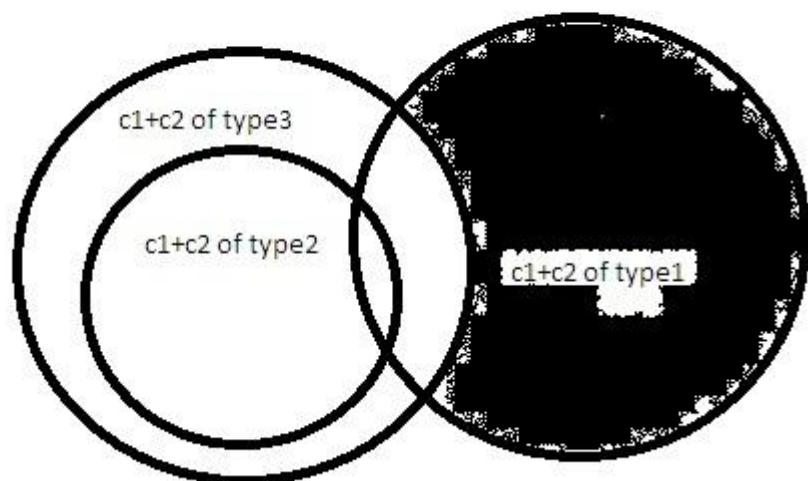


Diagram 11C.1: Venn diagram representing the relation of subset and superset among the $c1+c2$ combinations of types1, 2 and 3

Difference between $c1+c2$ of type 2 and $c1+c2$ of type 3 are:-

$C1+c2$ of type 2 is a special case of $c1+c2$ of type 3. The points of difference between these 2 types of $c1+c2$ combinations may be summarised as follows:-

- I. For deriving $c1+c2$ of type 2, the last digit of $2k$ plays a determining factor, which has no role to play in deriving $c1+c2$ of type 3.
- II. A particular $c1+c2$ of type 2 once derived for an even number having a particular SADN and ending in a particular last digit will be a $c1+c2$ of type 2 for all even numbers of the same SADN ending with the same digit. For example the first $c1+c2$ of type 2 for $2k$ of SADN741//2 would be $77+35 = 112$; since 35 is the first composite ending in 5 on the S5 series and 77 is the first composite ending in 7 on the S5 series. Therefore it would be considered as $c1+c2$ for $2k = 112$. Thereafter 77 will form a $c1+c2$ combination for all even numbers of SADN741//2 infinitely, since for any such number $2k-77$ will be a composite number ending in 5. In case of $c1+c2$ of type 3, this will not hold true as same composite component may form part of $c1+c2$ for a particular $2k$ but may form a $p+c$ combination for some other $2k$.

For example for $2k = 196$, SADN7//6, 77 will form part of $c1+c2$ combination for this $2k$ since the corresponding number would be 119 which is also a composite number but for $2k = 226$, again SADN741//6, $2k-77 = 149$ which is a prime number. Therefore, here 77 will form part of $p+c$ combinations.

- III. For a given $2k$, identifying the first $c1+c2$ of type 2 is predictable. For example for $2k =$ SADN 741//2, a component of $c1+c2$ of type 2 will be identified as $(7+30n) \times (11+30n)$ and this will be universal across the number line. In case of $c1+c2$ of type 3, there is no such predictability. This is because $c1+c2$ of type 3 for any prime pair p_1p_2 is derived as follows:

$$[\{2k-(p_1(6I\pm 1))\}/p_2] + 6np_1 = \text{an integer, for a particular value of 'n'}$$

and the first $c1+c2$ for this prime pair p_1p_2 will be identified for a value of n where I becomes an integer. Since $2k - (p_1 \times (6I \pm 1))$ will be different for different even numbers, the first $c1+c2$ formed by p_1p_2 will be different for different even numbers, the first $c1+c2$ formed by p_1p_2 will also be different for different even numbers.

11D.**Deriving total number of c_1+c_2 combinations for a given $2k$: Some illustrations****Illustration 1:**

Deriving c_1+c_2 combinations for $2k$ of SADN (5,2,8) :-

Consider the even number 1682. The relevant details required for deriving c_1+c_2 combinations for this number are summarized in the following table:

SADN	8
Relevant series	S7 series
Last digit	2
k	841
Nature of k	Composite (29×29)
Total no of combinations	$(841-1)/6=140$
Total no of elements	$2n_{TC}=280$
No of primes	127
No of composites	153

Table 11D.1: Details required for deriving c_1+c_2 combinations for illustration of SADN(8)//2

As mentioned in table 11D.1, $k=841$ which is a composite odd number derived as 29×29 . We begin by identifying c_1+c_2 combinations by following the steps as mentioned above.

Deriving c_1+c_2 combinations of type 1:

As mentioned above these c_1+c_2 combinations are derived as $(k+6n)+(k-6n)$ till we reach a value of n that leads us to the first element of the concerned series. Accordingly the following c_1+c_2 combinations can be identified for $2k=1682$:

- i. $29 \times 29 + 29 \times 29$
- ii. $29 \times 35 + 29 \times 23$
- iii. $29 \times 41 + 29 \times 17$
- iv. $29 \times 47 + 29 \times 11$
- v. $29 \times 53 + 29 \times 5$

Total number of c_1+c_2 combinations of type 1 thus derived are 5.

Deriving c_1+c_2 combinations of type 2:

As mentioned above these c_1+c_2 combinations are derived from the last digit of the even number. Since the even number under consideration ends in 2, composite odd numbers on

the S7 series which end in 7 will form part of c1+c2 combinations since the corresponding number in the combination would end in 5 and would thereby be a multiple of 5. We begin by identifying c1+c2 combinations of type 2 involving the prime number 7.

$N(c+c)$ for 7 = ff of $[(235-31)/30]+1$ = ff of (7.8)

ff of 7.8 = 7

The combinations thus derived would be :-

- i. $7 \times 31 (=217) + 1465$
- ii. $7 \times 61 (=427) + 1255$
- iii. $7 \times 91 (=637) + 1045$
- iv. $7 \times 121 (=847) + 835$
- v. $7 \times 151 (=1057) + 625$
- vi. $7 \times 181 (=1267) + 415$
- vii. $7 \times 211 (=1477) + 205$

$N(c+c)$ for the prime number 11 will be derived as ff of $[(149-17)/30]+1$ = ff of (5.4)

ff of 5.4 = 5

C+C combinations thus derived are:

- i. 11×17 (i.e. 187) + 1495
- ii. 11×47 (i.e. 517) + 1165
- iii. 11×77 (i.e. 847) + 835
- iv. 11×107 (i.e. 1177) + 505
- v. 11×137 (i.e. 1507) + 175

Of these combinations 11×77 (i.e. 847) + 835 has already been derived while identifying c1+c2 combinations for the previous prime number 7, so it will have to be removed from total number of c1+c2 combinations for 11 to avoid double counting. Unique c+c combinations involving 11 would therefore be 4.

$nC+C$ for the prime number 13 will be derived as ff of $[(127-19)/30]+1$ = ff of (4.6)

ff of 4.6 = 4

C+C combinations thus derived would be:-

- i. 13×19 (i.e. 247) + 1435
- ii. 13×49 (i.e. 637) + 1045

- iii. $13 \times 79 (\text{i.e. } 1027) + 655$
- iv. $13 \times 109 (\text{i.e. } 1417) + 265$

Of these, the combination $637+1045$ has already been derived while identifying C+C combinations for the previous prime number 7 so it needs to be subtracted from the total C+C combinations involving the number 13 to avoid double counting. Unique C+C combinations of type 2 for the prime number 13 would therefore be 3.

$n(c+c)$ for the prime number 17 would be derived as ff of $[(95-41)/30]+1 = \text{ff of } (2.8)$

ff of $2.8 = 2$

$c1+c2$ combinations thus derived would be:-

- i. $17 \times 41 (\text{i.e. } 697) + 985$
- ii. $17 \times 71 (\text{i.e. } 1207) + 475$

$n(c+c)$ of type 2 for the prime number 19 would be derived as ff of $[(85-43)/30]+1 = \text{ff of } (2.4)$

ff of $2.4 = 2$

$c1+c2$ combinations thus identified are-

- i. $19 \times 43 (\text{i.e. } 817) + 865$
- ii. $19 \times 73 (\text{i.e. } 1387) + 295$

$n(c+c)$ combinations for the prime number 23 would be derived as ff of $[(71-29)/30]+1 = \text{ff of } (2.4)$ and the combinations thus derived are-

$23 \times 29 (\text{i.e. } 667) + 1015$

$23 \times 59 (\text{i.e. } 1357) + 325$

Of these combinations $667+1015$ has already been counted while identifying $c1+c2$ combinations of type 1 and will not be considered here to avoid double counting. Therefore, effectively the total number of C+C combinations involving the prime number 23 would be 1.

$n(c+c)$ for the prime number 29 would be derived as ff of $[(53-53)/30]+1=1$ and the combination thus derived would be 29×53 (i.e. 1537) + 145

It may be noted here that this combination has already been derived while identifying c_1+c_2 of type-1 and will therefore not be counted here again.

$n(c+c)$ for the next prime number 31 would be derived as ff of $[(49-37)/30]+1=1$ (1.4) and the combination thus derived would be 31×37 (i.e. 1147) + 535

Total number of c_1+c_2 combinations of type 2 identified = 20

Deriving C+C combinations of type 3 for $2k$ as 1682:

As mentioned above C+C combinations of type 3 are identified for prime pairs which satisfy the general condition of $2k/6p_1p_2 \geq 1$. By following the steps discussed above, the following C+C combinations of type 3 may be identified for the $2k$ under consideration:

Deriving C+C combinations of type 3 for the prime pair 7&11:-

Since ff of $[1682/(7 \times 11 \times 6)] = \text{ff of } (1682/462) = \text{ff of } (3.64)$, it may be expected that C+C combinations of type 3 would be identified for this prime pair such that 7 & 11 would be factors on either side of the combination. In order to identify the first such combination we need to first find the value of $2k-(7 \times (6I+1))$. Here this value would be $1682-(7 \times 235)=1682-1645=37$. Thereafter we need to solve for the equation $(37+42n)/11=\text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Here $(37+(42 \times 2))/11=11$ which implies that the first $c+c$ combination involving the prime pair 7&11 would be $(11 \times 11)+(7 \times 223)$ i.e. $121+1561$. Thereafter we can derive further combinations by adding (and subtracting) $7 \times 11 \times 6$ (i.e. 462) to the first (and second) term of the combination. The combinations thus derived would be as follows-

- i. $121+1561$
- ii. $583+1099$
- iii. $1045+637$
- iv. $1507+175$

Of these, the third and fourth combinations have already been derived while identifying c_1+c_2 combinations of type 2 so these need to be removed from the total count of 4 to avoid double counting of the same combination. Unique combinations of type 3 for the prime pair 7&11 would therefore be 2.

Deriving C+C combinations for the prime pair 7&13:

Here again to identify the first C+C combination involving the prime pair 7&13 we solve for the equation $(37+42n)/13 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Since $(37+42 \times 5)/13 = 19$, the first C+C combination for the prime pair 7&13 would be $(13 \times 19) + (7 \times 205) = 247 + 1435$. Further combinations can be derived by adding 546 (i.e. $7 \times 6 \times 13$) to the first term and subtracting the same from the second term. The C+C combinations thus derived would be :-

- i. 247+1435
- ii. 793+889
- iii. 1339+343

Of these, the first combination has already been identified while deriving C+C combinations of type 2 so it will be subtracted from the total count to avoid double counting and thus the total unique C+C combinations for the prime pair 7&13 would be 2.

Deriving c1+c2 combinations for the prime pair 7&17

To identify the first C+C combination involving the prime pair 7&17 we solve for the equation $(37+42n)/17 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Here $(37+42 \times 6)/17 = 17$ and the C+C combinations thus derived would be as follows-

- i. 289+1393
- ii. 1003+679

Since neither of these combinations has appeared earlier while identifying C+C of type 1 or type 2, both of them would be considered as unique C+C combinations of type 3 for the prime pair of 7-17.

Deriving C+C combinations for the prime pair 7&19

In order to identify the C+C combinations for the prime pair 7&19 we solve for the equation $(37+42n)/19 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Since $(37+42 \times 5)/19 = 13$, the C+C combinations thus derived are 247+1435 and 1045+637. Since both these combinations have already been identified while deriving C+C combinations of type 2; the unique C+C combinations of type 3 for the prime pair would be 0.

In the same manner C+C combinations for the prime pair 7&23 would be identified as 667+1015 and 1633+49 of which the former combination has already been identified while deriving C+C combinations of type 1 and therefore the number of unique combinations for this prime pair would be 1. Furthermore, the combination 667+1015 would be derived as $23 \times 29 + 7 \times 145$ and this is the only combination involving the prime pair 7-29 so effectively the unique C+C combinations of type 3 for this prime pair would be 0. Derived on similar lines, the C+C combination for the prime pair 7-31 would be 961(i.e. 31×31)+721(i.e. 7×103) and the number of unique C+C combinations of type 3 for this prime pair would be 1. Likewise, the combination 1591(i.e. 37×43)+91(i.e. 7×13) would be derived for the prime pair 7&37 and the total number of unique C+C combinations of type 3 for this prime pair would be 1.

Deriving unique C+C combinations of type 3 involving the prime number 11:-

The prime pairs involving the prime number 11 which satisfy the general condition $2k/6p_1p_2 \geq 1$ would be 11&13, 11&17, 11&19 and 11&23. In order to identify the combinations we need to first find the value of $2k-11 \times (6I-1)$. This would be $1682 - (11 \times 149) = 1682 - 1639 = 43$. Accordingly C+C combinations for the relevant prime pairs would be derived as follows -

For the prime 11&13:

In order to find the first C+C combination for this prime pair we solve for the equation $(43+66n)/13 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Since $(43+66 \times 9)/13 = 49$, the combinations thus derived would be 637+1045 and 1495+187. Since both these combinations have already been derived while identifying C+C combinations of type 2; unique C+C of type 3 for the prime pair 11&13 would be 0. Derived on similar lines, the C+C combinations of type 3 for the prime pair 11-17 would be 901(17×53)+781(11×71) since this combination has not been derived earlier, it would be considered as a unique C+C combination of type 3 for the prime pair 11&17. Likewise, the unique C+C combination of type 3 for the prime pair 11-19 would be 703(i.e. 19×37)+979(i.e. 11×89). C+C combination of 1495(i.e. 23×65)+187 (i.e. 11×17) for

the prime pair 11&23 has already been derived while identifying C+C of type 2 and therefore, the unique combinations of type 3 for this prime pair would be 0.

Deriving C+C combinations of type 3 for the prime pairs involving 13:

The general condition of $2k/6p_1p_2 \geq 1$ for identifying C+C combinations of type 3 involving the prime number 13 would be satisfied for the prime pairs 13&17 and 13&19. In order to identify these combinations, we first find the value of $2k-13 \times 6I+1= 1682-13 \times 127=1682-1651=31$. Further, to identify the C+C combination/s for the prime pair 13&17 we solve for the equation $(31+78n)/17=I$ to find the value of n where I becomes an integer. Here, $31+78 \times 2/17=11$ so the combinations thus derived would be 187(i.e. 17×11)+1495(i.e. 13×115) and 1513(i.e. 17×89)+169(i.e. 13×13). Of these, the first combination 187+1495 has already been derived while identifying C+C combinations of type 2 and therefore the unique C+C combination of type 3 for the prime pair 13&17 would be 1. Derived similarly, the C+C combination for the prime pair 13&19 would be 637(i.e. 13×49)+1045(i.e. 19×55). Since this combination has already been derived earlier while identifying C+C combinations of type 2, the unique C+C combinations of type 3 for the prime pair 13&19 would be 0.

Total number of C+C combinations for the even number 1682 would be given as summation of number of combinations of type1, number of combinations of type 2 and number of combinations of type 3. These would be $5+20+12=37$. This implies that as many as 37 C+C combinations would be identified for $2k=1682$.

Illustration 2:

Deriving C+C combinations for 2k of SADN(7,4,1):

Consider the even number 1498. The relevant details required to identify C+C combinations for this number are summarized in the following table:-

SADN	4
Relevant series	S5 series
Last digit	8
K	749
Nature of k	Composite (7x107)
Total no of combinations	$(749+1)/6=125$
Total no of elements	249
No of primes	121
No of composites	128

Table 11D.2: Details required for deriving c1+c2 combinations for illustration of SADN(4)//8

Deriving c1+c2 combinations of type 1 for 2k as 1498:-

As mentioned in the table 11D.2, mid-point of the 2k under consideration $1498/2=749$ is a composite odd number derived as 7×107 . Therefore, C+C combinations of type 1 would be derived as $7 \times (107+6n) + 7 \times (107-6n)$ where value of n ranges from 0 to such an integer that leads us to the first element of the S-5 series. The combinations thus derived would be as follows-

$$\begin{aligned} &7 \times 107 + 7 \times 107 \\ &7 \times 113 + 7 \times 101 \\ &7 \times 119 + 7 \times 95 \\ &\vdots \\ &7 \times 5 + 7 \times 209 \end{aligned}$$

The total number of combinations thus derived would be 18.

Deriving C+C combinations of type 2 for 2k as 1498:

The even number under consideration 1498 ends in the digit 8 so all composite odd numbers ending in 3 lying on the S-5 series would form part of C+C combinations of type 2 since the corresponding number in the combination would necessarily end in 5 and would therefore be a multiple of 5.

We begin by noting here that all c1+c2 combinations of type 2 involving the prime number 7 have already been derived above while identifying c1+c2 combinations of type 1 so deriving them again here would amount to double counting. Therefore, we will begin the calculation of c1+c2 of type 2 from the next prime number which is 11.

Hence $n(c+c)$ of type 2 for the prime number 11 would be derived as ff of $[(133-13)/30]+1] = 5$ and the combinations thus derived would be:

- i. $11 \times 13(\text{i.e. } 143) + 1355$
- ii. $11 \times 43(\text{i.e. } 473) + 1025$
- iii. $11 \times 73(\text{i.e. } 803) + 695$
- iv. $11 \times 103(\text{i.e. } 1133) + 365$
- v. $11 \times 133(\text{i.e. } 1463) + 35$

Of these, the fifth combination $1463+35$ has already been derived while identifying $c1+c2$ combinations of type 1 and therefore will be removed to avoid double counting. The total number of $c1+c2$ combinations of type 2 for the prime number 11 would therefore be 4.

$C1+c2$ combinations of type 2 for the prime number 13 would be derived as ff of $[(113-41)/30]+1$ = ff of $(3.4) = 3$ and the combinations thus derived would be as follows-

- i. 13×41 (i.e. 533) + 965
- ii. 13×71 (i.e. 923) + 575
- iii. 13×101 (i.e. 1313) + 185

Since none of these combinations have been identified earlier, all these 3 would be considered as unique combinations for the prime number 13.

Now $n(c+c)$ for the prime number 17 would be derived as ff of $[(85-19)/30]+1$ = ff of $(3.2) = 3$ and the combinations thus derived would be:

- i. 17×19 (i.e. 323) + 1175
- ii. 17×49 (i.e. 833) + 665
- iii. 17×79 (i.e. 1343) + 155

Of these, the second combination $833+665$ has already been derived earlier while identifying $c1+c2$ combinations of type 1 and will therefore be removed to avoid double counting. The unique $c1+c2$ combinations of type 2 for the prime number 17 would thus be 2.

Derived similarly, unique $c1+c2$ combinations of type 2 for the prime number 19 would be $893+605$ since the other combination 19×77 (i.e. 1463) + 35 involving the prime number 19 has already been derived while identifying $c1+c2$ combinations of type 1 and will therefore not be counted again. Unique $c1+c2$ of type 2 for the prime number 19 would therefore be 1.

Likewise, $c1+c2$ combinations of type 2 for the prime number 23 would be $713+785$ and $1403+95$. Both these combinations would be considered as unique to the prime number

23 since they have not been identified earlier. c_1+c_2 of type 2 for the prime number 29 would be $1073+425$ and it would be considered unique since it has not been identified earlier.

Hence total number of c_1+c_2 combinations of type 2 for $2k$ as 1498 would be 13.

Deriving c_1+c_2 combinations of type 3 for $2k$ as 1498:

Here again we begin by noting that prime pairs that satisfy the general condition for identification of c_1+c_2 combinations of type 3 involving the prime number 7 would already have been derived earlier while identifying c_1+c_2 combinations of type 1 and therefore these will not be derived again. We will begin by identifying c_1+c_2 combinations of type 3 for the prime pairs involving the next prime number 11. In order to identify these combinations we begin by finding the value of $2k-11x(6I+1)$ which would be $1498-(11 \times 133)=1498-35$. To identify the c_1+c_2 combinations involving the prime pair 11&13, we solve for the equation $(35+66n)/13 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Since $(35+66 \times 4)/13=23$, the c_1+c_2 thus derived would be:-

- i. $299+1199$
- ii. $1157+341$

Since neither of these combinations has been identified earlier, these would be considered as unique to the prime pair 11-13. Derived on similar lines, the $C+C$ combination/s for the prime pair 11-17 would be $17 \times 37(=629) + 11 \times 79(=869)$ and this combination would be considered as unique to this prime pair since it has not been identified earlier. $C+C$ combination for the prime pair 11-19 which is $893(19 \times 77)+605 (11 \times 55)$ has already been identified while deriving $C+C$ combinations of type 1 and will therefore not be considered unique to the prime pair 11-19.

c_1+c_2 of type 3 for prime pairs involving 13:-

The prime pairs 13&17 and 13&19 satisfy the general condition of $2k/6p_1p_2 \geq 1$. In order to identify the combinations we begin by finding the value of $2k-13x(6I-1) = 1498-13 \times 113=1498-1469=29$

To identify the c_1+c_2 combination of type 3 for the prime pair 13&17 we solve for the equation $(29+78n)/17 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer. Here $(29+78 \times 9)/17=43$

Therefore the c_1+c_2 combination thus derived would be $731(17 \times 43)+767(13 \times 59)$. Since this combination has not been derived earlier, it would be considered unique to the prime pair 13&17. Derived on similar lines a unique combination for the prime pair 13&19 would be identified as $1121(19 \times 59)+377(13 \times 29)$. Since this combination has not been identified earlier it would be considered unique to the prime pair 13&19.

Total number of c_1+c_2 combinations of type 3 for $2k$ as 1498 would be 5.

From the above discussion it may be derived that total number of c_1+c_2 combinations for the even number 1498 would be summation of number of c_1+c_2 combinations of type 1; and number of combinations of c_1+c_2 combinations of type 2; and number of c_1+c_2 combinations of type 3 which would be $18+13+5=36$. This implies that the total number of C+C combinations for the even number 1498 would be 36.

Illustration 3:

Deriving C+C combinations for $2k$ of SADN(3,6,9)

Consider the even number 1596. The relevant details required for deriving c_1+c_2 combinations for this number are summarized in the following table:-

SADN	3
Relevant series	S5 and S7 series
Last digit	6
k	798
Nature of k	Composite ($7 \times 19 \times 6$)
Total no of combinations	$(1596)/6=266$
Total no of elements	$2TC=532$
No of primes	249
No of composites	283

Table 11D.3: Details required for deriving c_1+c_2 combinations for illustration of SADN(3)//6

It is important to note here that $c1+c2$ combinations for even numbers of $SADN(3,6,9)$ are different from even numbers of $SADN(2,5,8)$ and $SADN(7,4,1)$ on two accounts. Firstly, since mid-point of even numbers of $SADN(3)$, $SADN(6)$ or $SADN(9)$ would be of $SADN 6$, $SADN 3$ or $SADN 9$ respectively, the mid-point would essentially lie on the $S3$ series whereas the relevant series for $2k$ of $SADN(3,6,9)$ are both the $S5$ and $S7$ series. Therefore, in this case, $c1+c2$ combinations of type 1 which are derived from the mid-point will be identified in a different manner. Secondly, since the combinations for these numbers are such that one component lies on the $S5$ series and the other lies on the $S7$ series while identifying $c1+c2$ combinations; composites on both the series need to be considered. This will be evident from the following discussion.

Deriving $C+C$ combinations of type1 for $2k$ as 1596

Mid-point for the even number 1596 is 798 which is of $SADN 6$ and its factors are 7, 19 and 6. $c1+c2$ combinations of type 1 will be derived as follows:

As $1596/7=228$, combinations of elements of the $S5$ and $S7$ series which can be summed up to 228, needs to be identified here to derive $c1+c2$ combinations of type 1. For this, the method that can be applied is summing up $(6n+1)+[228-(6n+1)]$ where value of n ranges from 1 to such an integer such that $6n+1$ would be < 228 . These combinations would be derived as $7+221$ (i.e. $228-7$); $13+215$ (i.e. $228-13$); $19+209$ (i.e. $228-19$), , 223 (i.e. value of n is 37)+5(i.e. $228-223$). By multiplying both terms of the addition function by 7 we can derive $c1+c2$ combinations of type 1. The combinations thus derived would be as follows-

$$\begin{aligned} &7 \times 7 \text{ (i.e. } 49) + 7 \times 221 \text{ (i.e. } 1547) \\ &7 \times 13 \text{ (i.e. } 91) + 7 \times 215 \text{ (i.e. } 1505) \\ &7 \times 19 \text{ (i.e. } 133) + 7 \times 209 \text{ (i.e. } 1463) \\ &\vdots \\ &7 \times 223 \text{ (i.e. } 1561) + 7 \times 5 \text{ (i.e. } 35) \end{aligned}$$

In all, 37 such combinations can be identified.

Similarly, since another factor of the even number under consideration is 19, $c1+c2$ combinations of type 1 will be identified for this factor as well. These may be derived as follows:

As $1596/19=84$, applying the same rationale as above, $c1+c2$ combinations of type 1 can be identified as :

$19 \times 7(\text{i.e.}133)+19 \times 77(\text{i.e.}1463)$
 $19 \times 13(\text{i.e.}247)+19 \times 71(\text{i.e.}1349)$
 $19 \times 19(\text{i.e.}361)+19 \times 65(\text{i.e.}1235)$
 \vdots
 $19 \times 79(\text{i.e.}1501)+19 \times 5(\text{i.e.}95)$

In this manner 13 combinations have been identified. However, 2 of these combinations $19 \times 7(\text{i.e.}133)+19 \times 77(\text{i.e.}1463)$ and $19 \times 49(\text{i.e.}931)+19 \times 35(\text{i.e.}665)$ have been identified while deriving $c1+c2$ combinations of type 1 for the earlier factor 7. Therefore, these combinations will not be considered here to avoid double counting. The unique $c1+c2$ combinations derived for the prime factor 19 would therefore be $13-2=11$.

Thus total number of $c1+c2$ combinations of type 1 for the even number 1596 would be $37+11=48$.

Deriving $c1+c2$ combinations of type 2 for $2k$ as 1596

Since the even number under consideration ends in the digit 6, odd composite numbers ending in 1 on the relevant series (both the S5 and S7 series) would form part of $c1+c2$ combinations of type 2 since the corresponding number in the combination would essentially end in 5 and would therefore be a multiple of 5. It may be noted here that the first relevant prime number, 7, need not be considered while identifying $c1+c2$ combinations of type 2 since these would have already been derived while identifying $c1+c2$ combinations of type 1. Therefore, we begin by identifying $c1+c2$ combinations of type 2 formed by the prime number 11 on the S5 series. These combinations can be derived as ff of $[(145-31)/30]+1 = \text{ff of } (4.8) = 4$; and the combinations thus derived would be:

- i. $11 \times 31(\text{i.e.}341)+1255$
- ii. $11 \times 61(\text{i.e.}671)+925$
- iii. $11 \times 91(\text{i.e.}1001)+595$
- iv. $111 \times 121(\text{i.e.}1331)+265$

Of these, the third combination $1001+595$ has already been derived as $7 \times 143(\text{i.e.}1001)+595$ while identifying $c1+c2$ combinations involving the previous prime number 7 so this combination will not be considered here . Therefore, the total number of unique $c1+c2$ combinations in which 11 is a factor of the component on the S5 series would be 3.

c1+c2 combinations in which 11 is a factor of the component on the S7 series can be derived as ff of $[(143-11)/30+1] = \text{ff of } (5.4) = 5$ and the combinations thus derived are:

- i. $11 \times 11 (\text{i.e. } 121) + 1475$
- ii. $11 \times 41 (\text{i.e. } 451) + 1145$
- iii. $11 \times 71 (\text{i.e. } 781) + 815$
- iv. $11 \times 101 (\text{i.e. } 1111) + 485$
- v. $11 \times 131 (\text{i.e. } 1441) + 155$

All these combinations are unique to the prime number 11 and therefore the total number of c1+c2 combinations involving the number 11 can be derived as 3 (in which 11 is a factor of the component on the S5 series) and 5 (in which 11 is a factor of the component on the S7 series). Thus total number of combinations would be $3+5=8$.

n(c+c) of type 2 for prime number 13:

c1+c2 combinations where 13 is a factor of the component on the S5 series can be derived as ff of $[(119-17)/30+1] = \text{ff of } (4.4) = 4$ and the combinations thus derived are-

- i. $13 \times 17 (\text{i.e. } 221) + 1375$
- ii. $13 \times 47 (\text{i.e. } 611) + 985$
- iii. $13 \times 77 (\text{i.e. } 1001) + 595$
- iv. $13 \times 107 (\text{i.e. } 1391) + 205$

Of these, the third combination $1001+595$ has already been derived earlier and will not be considered here. Total number of c1+c2 in which 13 is a factor of the component on the S5 series would therefore be 3.

c1+c2 combinations where 13 is a factor of the component on the S7 series would be derived as ff of $[(121-37)/30+1] = \text{ff of } (3.8) = 3$ and the combinations thus derived would be

- i. $13 \times 37 (\text{i.e. } 481) + 1115$
- ii. $13 \times 67 (\text{i.e. } 871) + 725$
- iii. $13 \times 97 (\text{i.e. } 1261) + 335$

All these combinations are unique to the number 13 so the total number of unique combinations would be 3(in which 13 is a factor of the component on the S5 series) and 3 (in which 13 is a factor of the component on the S7 series). The total number of combinations would therefore be $3+3=6$.

Deriving c1+c2 combinations of type 2 for the prime number 17:

c1+c2 combinations where 17 is a factor of the component on the S5 series would be derived as ff of $[(91-43)/30]+1 = \text{ff of } (2.6) = 2$ and the combinations thus derived would be

- i. $17 \times 43 (\text{i.e. } 731) + 865$
- ii. $17 \times 73 (\text{i.e. } 1241) + 355$

c1+c2 combinations where 17 is a factor of the component on the S7 series can be derived as ff of $[(89-23)/30]+1 = \text{ff of } (3.2) = 3$ and the combinations thus derived would be:

- i. $17 \times 23 (\text{i.e. } 391) + 1205$
- ii. $17 \times 53 (\text{i.e. } 901) + 695$
- iii. $17 \times 83 (\text{i.e. } 1411) + 185$

Total number of combinations would be $2 + 3 = 5$.

Deriving c1+c2 combinations for the prime number 19:

Here again it may be noted that c1+c2 combinations in which 19 is a factor of the composite component would have already been derived while identifying c1+c2 combinations of type 1 therefore, deriving them again would result in double counting.

Deriving c1+c2 combinations of type 2 for the prime number 23:

c1+c2 combinations where 23 is a factor of the component on the S5 series can be derived as ff of $[(67-37)/30]+1 = \text{ff of } (2) = 2$ and the combinations thus derived are:

- i. $23 \times 37 (\text{i.e. } 851) + 745$
- ii. $23 \times 67 (\text{i.e. } 1541) + 55$

c1+c2 combinations where 23 is a factor of the component on the S7 series can be derived as ff of $[(65-47)/30]+1 = \text{ff of } (1.6) = 1$ and the combination thus derived would be $1081+515$. Total number of c1+c2 combinations in which 23 is a factor would therefore be $2+1 = 3$.

Deriving c1+c2 combinations of type 2 for prime number 29

Where 29 is a factor of the component on the S-5 series can be derived as $55-49/30+1=1.2$ and the combination thus derived would be $1421+175$. Since this combination has already been derived earlier it will not be considered here.

Where 29 is a factor of the components on the S-7 series can be derived as $[(53-29)/30]+1=1.8$ and the combination thus derived would be $29 \times 29 (=841) + 755$. Total number of combinations for the number 29 would therefore be 1.

Deriving c1+c2 combinations of type 2 for the number 31:

c1+c2 combinations where 31 is a factor of the component on the S5 series can be derived as ff of $[(47-41)/30]+1 = \text{ff of } (1.2) = 1$ and the combination thus derived would be $31 \times 41 (\text{i.e. } 1271) + 325$

c1+c2 combinations where 31 is a factor of the component on the S7 series can be derived as ff of $[(49-31)/30]+1 = \text{ff of } (1.6) = 1$ and the combination thus derived would be $31 \times 31 (\text{i.e. } 961) + 635$. Total number of combinations would be 2.

The total number of c1+c2 combinations of type 2 for 2k as 1596 would be 44.

Deriving c_1+c_2 combinations of type 3 for $2k$ as 1596:

While deriving c_1+c_2 combinations of type 3 for even numbers of SADN(3,6,9), it is important to note that two types of combinations would be identified. Firstly, those in which p_1 is a factor of the component on the S5 series and p_2 is a factor of the component on the S7 series and secondly in the form where p_1 is a factor of the component on the S7 series while p_2 is a factor of the component on the S5 series. Further, it may be noted that since 7 is a factor of the even number under consideration, all c_1+c_2 combinations involving 7 would be derived as c_1+c_2 combinations of type 1. Therefore, while identifying c_1+c_2 combinations of type 3, we begin with the prime pair 11&13.

Deriving c_1+c_2 combinations of type 3 for the prime pair 11&13:

$c_1 + c_2$ combination where 11 is a factor of the composite component on the S7 series and 13 is a factor of the composite component on the S5 series:

We begin by first finding the value of $2k-11x(6I-1) = 1596-(11x143) = 1596-1573 = 23$. Thereafter we solve for the equation $(23+66n)/13 = \text{Int (an Integer)}$, to find the value of n where Int becomes an integer. Here $(23+66x3)/13 = 17$ which leads us to the first c_1+c_2 combination $13x17(\text{i.e.} 221)+11x125(\text{i.e.} 1375)$. Further combinations would be derived by adding $11x13x6(\text{i.e.} 858)$ to the first term and subtracting the same from the second term. The combination/s thus derived would be $1079(13x83)+517(11x47)$. Of these, the first combination $221+1375$ has already been derived while identifying c_1+c_2 combinations of type 2 so it will not be considered here. Number of unique c_1+c_2 combinations of type 3 for the prime pair 11&13 would therefore be 1.

Deriving c_1+c_2 combinations for p_1p_2 as 11&13 wherein 11 is a factor of the composite component on the S5 series and 13 is a factor of the composite component on the S7 series. Here we begin by identifying the value of $2k-11(6I+1)=1596-(11x145)=1596-1595=1$. Thereafter we solve for the equation $(1+66n)/13 = \text{Int (an Integer)}$, to find the value of n where Int becomes an integer. Here $(1+66x12)/13 = 61$ and the combination thus derived would be $793(\text{i.e.} 13x61)+803(\text{i.e.} 11x73)$. Since this combination has not been derived earlier, it would be considered as a unique c_1+c_2 combination for the prime pair 11-13. Total number of c_1+c_2 combinations for this prime pair would be $1+1=2$.

Deriving C+C combinations of type 3 for the prime pair 11&17:

The combinations would be derived as $(23+66n)/17 = \text{Integer}$ and $(1+66n)/17 = \text{Int}$ (an Integer), to find the value of n where Int becomes an integer; $(23+66 \times 3)/17 = 13$. The c1+c2 combination thus derived would be 221+1375. Further combination/s would be derived by adding $11 \times 17 \times 6$ (i.e. 1122) to the first term and subtracting the same from the second term. This leads us to the combination 1343 (i.e. 17×79) + 253 (i.e. 11×23). Of these, the first combination 221+1375 has already been identified while deriving C+C combinations of type 2 and will not be considered here and accordingly only 1 c1+c2 combination can be identified.

Likewise $(1+66 \times 9)/17 = 35$ which leads us to the combination 1001 (i.e. 11×91) + 595 (i.e. 17×85). This is the only c1+c2 combination derived for the prime pair 11&17 in which the former is a factor of the composite component on the S5 series and the latter is a factor of the component on the S7 series. Since this combination has already been derived earlier while identifying c1+c2 combinations of type 1; it will not be considered here. In sum, the total number of unique c1+c2 combinations for the prime pair 11&17 would be 1.

c1+c2 combinations for the prime pair 11&19 would have already been derived while identifying c1+c2 combinations of type 1 so these need not be derived again.

Deriving c1+c2 combinations of type 3 for the prime pair 11&23:

c1 + c2 combination where 11 is a factor of the component on the S5 series while 23 is a factor of the component on the S7 series. In order to find such a combination we will first solve the equation $1+66n/23=I$ to find the value of n where I becomes an integer. Here, $1+66 \times 8 / 23 = 23$ and the combination thus derived would be 529 (i.e. 23×23) + 1067 (i.e. 11×97). Likewise in order to identify the combination in which 11 is a factor of the component on the S7 series while 23 is a factor of the component on the S5 series we solve for the equation $(23+66n)/23=I$ to find the value of n where I becomes an integer. Here, $(23+66 \times 23)/23 = 67$ and the combination thus derived would be 1541 (i.e. 23×67) + 55 (i.e. 11×5). Since this combination has already been derived while identifying c1+c2 combinations of type 2, it will not be considered here. In effect the total number of unique c1+c2 combinations of type 3 for the prime pair 11&23 will be 1.

Deriving c1+c2 combinations of type 3 for the prime pair 13&17:

$c_1 + c_2$ combination where 13 is a factor of the component on the S5 series while 17 is a factor of the component on the S7 series

We begin by first finding the value of $2k - 13 \times (6I + 1)$ which is $1596 - (13 \times 121) = 1596 - 1573 = 23$. Thereafter we solve the equation $(23 + 78n)/17 = \text{Int (an Integer)}$, to find the value of n where Int becomes an integer. Here $[23 + (78 \times 13)]/17 = 61$ and the C+C combination thus derived would be $1037(17 \times 61) + 559(13 \times 43)$.

Likewise in order to find the combination where 13 is a factor of the component on the S-5 series while 17 is a factor of the component on the S-7 series we first find the value of $2k - 13 \times (6I - 1)$. This would be $1596 - 13 \times 119 = 1596 - 1547 = 49$. Thereafter we solve the equation $(49 + 78n)/17 = \text{Int (an Integer)}$, to find the value of n where Int becomes an integer. Here $(49 + 78 \times 7)/17 = 35$ and the combination thus derived would be $595(\text{i.e. } 17 \times 35) + 1001(\text{i.e. } 13 \times 77)$. Since this combination has already been derived earlier it will not be considered again. The total number of unique $c_1 + c_2$ combinations for the prime pair 13&17 would be 1.

Thus total number of $c_1 + c_2$ combinations of type 3 for the even number 1596 would be 5.

Therefore total number of $c_1 + c_2$ combinations for $2k$ as 1596 would be summation of number of $c_1 + c_2$ combinations of type 1; and number of $c_1 + c_2$ combinations of type 2; and number of $c_1 + c_2$ combinations of type 3. These would be $48 + 44 + 5 = 97$

Therefore, total number of $c_1 + c_2$ combinations for 1596 would be 97.

11E.

PYTHON Codes used to compute for verification in case of any given even number (2k) of SADN(5,2,8), SADN(7,4,1) or SADN(6,3,9)

```

#program for finding number of composites<EN, TotalCombi.(TC),
CC, PE composites for an even no. of SADN(5,2,8)
t1 = {25}
t2={91}
# composites smaller than the even number are in set compo_st_
en defined below
compo_st_en={25}
# the even number is denoted as 'en' below
for n in range (521,522):
    en=(6*n)-4
    en_set={en}
    u_limit=(en-5)//30
#    u_limit=100
    diff=0
    count = 0
# generating set 't1'
    for x in range(1, u_limit+1):
        for y in range(0, u_limit+1):
            t1.add((6*x-1)*(6*(x+y)-1))
# generating set 't2'
    for x in range(1, u_limit+1):
        for y in range(0, u_limit+1):
            t2.add((6*x+1)*(6*(x+y)+1))
# taking union of t1 and t2 and calling it t1
    t1.update(t2)
    l1=list(t1)
    print("Even number (EN) in consideration is: " + str(en))
    result_t1=sorted(l1)
#    print(result_t1)
    count_k=0
    for k in result_t1:
        if k < en :
#            print (k)
            count_k=count_k+1
    print ("Number of composites smaller than EN on relevant s
eries:" + str(count_k))
    for number in result_t1:
        if number < en :
#            print(number)
            diff=en-number

```

```

#         print(diff)
        if diff in result_t1:
            count=count+1
        cc_combi=count//2
        tot_combi=((en//2)-1)//6
        print("Number of TOTAL combinations:" + str(tot_combi))
        if en/2 in result_t1:
            cc_combi=cc_combi+1
        print("Number of CC combinations:" + str(cc_combi))
        pe_composites=count_k-(2*cc_combi)
        if en/2 in result_t1:
            pe_composites=pe_composites+1
        print("Number of PRIME-
EATER composites:" + str(pe_composites))
        print("...XXX...XXX...XXX...XXX...XXX...XXX...XXX...XXX...
XXX...XXX...XXX...XXX...XXX...XXX...XXX...XXX...")

```

Python code 1: Code for finding number of composites less than given Even Number(2k), Number of Total Combinations(TC), Number of Composite(CC) combinations, Number of Prime Eater (PE) composites for an even no. of SADN(5,2,8)

```

#program for finding number of composites<EN, TotalCombi.(TC),
CC, PE composites for an even no. of SADN(7,4,1) WHERE n>22
t3 = {35}
# composites smaller than the even number are in set compo_st_
en2 defined below
compo_st_en2={35}
# the even number is denoted as 'en' below
for n in range (175,195):
    en=(6*n)+4
    en_set={en}
    u_limit2=(en-35)//30
    diff=0
    count = 0
# generating set 't3'
    for x in range(1, u_limit2+1):
        for y in range(0, u_limit2+1):
            t3.add((6*x-1)*(6*(y)+7))
    l3=list(t3)
    print("Even number (EN) in consideration is: " + str(en))
    result_t3=sorted(l3)
#     print(result_t3)
    count_k=0
    for k in result_t3:
        if k < en :
#             print (k)

```



```

        count_k=count_k+1
    print ("Number of composites smaller than EN on relevant s
eries:" + str(count_k))
    for number in result_t3:
        if number < en :
#           print(number)
            diff=en-number
#           print(diff)
            if diff in result_t3:
                count=count+1
#           print("Did you notice new count of CC:" + str(c
ount))
        cc_combi=count//2
#        tot_combi=((en//2)-1)//6
#        print("Number of TOTAL combinations:" + str(tot_combi))
        if en/2 in result_t3:
            cc_combi=cc_combi+1
        print("Number of CC combinations:" + str(cc_combi))
#        pe_composites=count_k-(2*cc_combi)
#        if en/2 in result_t3:
#            pe_composites=pe_composites+1
#        print("Number of PRIME-
EATER composites:" + str(pe_composites))
        print("...XXX...XXX...XXX...XXX...XXX...XXX...XXX...XXX...
XXX...XXX...XXX...XXX...XXX...XXX...XXX...XXX...")

```

Python code 2: Code for finding number of composites less than given Even Number(2k), Number of Total Combinations(TC), Number of Composite(CC) combinations, Number of Prime Eater (PE) composites for an even no. of SADN(7,4,1)

```

#program for finding number of composites<EN, TotalCombi.(TC),
CC, PE composites for an even no. of SADN(6,3,9)
t1={25}
t2={91}
t3={35}
# composites smaller than the even number are in set compo_st_
en defined below
compo_st_en={25}
# composites smaller than the even number are in set compo_st_
en2 defined below
compo_st_en2={35}
# the even number is denoted as 'en' below
for n in range (160,162):
    en=6*n
    en_set={en}
    u_limit=(en-9)//30

```

```

    u_limit2=(en-31)//30
    diff=0
    count = 0
# generating set 't1'
    for x in range(1, u_limit+1):
        for y in range(0, u_limit+1):
            t1.add((6*x-1)*(6*(x+y)-1))
# generating set 't2'
    for x in range(1, u_limit+1):
        for y in range(0, u_limit+1):
            t2.add((6*x+1)*(6*(x+y)+1))
# taking union of t1 and t2 and calling it t1
    t1.update(t2)
    # generating set 't3'
    for x in range(1, u_limit2+1):
        for y in range(0, u_limit2+1):
            t3.add((6*x-1)*(6*(y)+7))
    t1.update(t3)
    print("Even number (EN) in consideration is: " + str(en))
    l1=list(t1)
    result_t1=sorted(l1)
#     print(result_t1)
    count_k=0
    for k in result_t1:
        if k < en :
#             print (k)
            count_k=count_k+1
    print ("Number of composites smaller than EN on relevant s
eries:" + str(count_k))
    for number in result_t1:
        if number < en :
#             print(number)
            diff=en-number
#             print(diff)
        if diff in result_t1:
            count=count+1
#             print("Did you notice new count of CC:" + str(c
ount))
    cc_combi=count//2
#     tot_combi=en//6
#     print("Number of TOTAL combinations:" + str(tot_combi))
    print("Number of CC combinations:" + str(cc_combi))
#     pe_composites=count_k-(2*cc_combi)
#     print("Number of PRIME-
EATER composites:" + str(pe_composites))
    print("...XXX...XXX...XXX...XXX...XXX...XXX...XXX...XXX...
XXX...XXX...XXX...XXX...XXX...XXX...XXX...XXX...")

```

Python code 3: Code for finding number of composites less than given Even Number(2k), Number of Total Combinations(TC), Number of Composite(CC) combinations, Number of Prime Eater (PE) composites for an even no. of SADN(6,9,3)

12

Number of c_1+c_2 combinations: minimum vis-à-vis actual number of c_1+c_2 combinations**Concepts of minimum number of c_1+c_2 combinations and actual number of c_1+c_2 combinations for even numbers if $n_{TC} < n_c$:**

For a given even number (given as $2k$) if total number of acceptable combinations is greater than total number of unique composites by even 1, it directly follows that even if all the composites are prime-eaters, there would still remain a p_1+p_2 combination. Suppose total number of acceptable combinations is denoted as n_{TC} and total number of unique composites is denoted as n_c , then if $n_{TC} > n_c$ by atleast 1 then even if all composites are prime-eaters; p_1+p_2 combinations would still be identified and their number would be $n_{TC} - n_c$. In this case, $n_{TC} - n_c$ would be the minimum number of p_1+p_2 for the given $2k$. Since we have not explored the possibility of c_1+c_2 combinations so far, it may be noted that with identification of c_1+c_2 combinations, difference between n_{TC} and number of combinations absorbed by composites in the nature of c_1+c_2 and $p+c$ combinations will go on increasing and thereby the number of p_1+p_2 combinations will also increase.

For a given $2k$ if $n_{TC} < n_c$ then it is not possible to directly arrive at the possibility of the existence of p_1+p_2 combinations. Consider the following situations:-

Number of total acceptable combinations = 50 and no. of unique composites = 62 which implies that $(50 \times 2) - 62$ i.e. $100-62$ i.e. 38 would be the number of prime numbers (on the relevant series in case of given $2k$), which would be smaller than $2k$. If all composites are considered to be prime-eaters then number of $p+c$ combinations would be 38 since number of primes is 38 and maximum number of $p+c$ combinations can be equal to the number of primes. Since $n_{TC} = 50$, it follows that the remaining combinations (i.e. n_{TC} - number of combinations of type $(p+c)$) i.e. $n_{TC} - n_{PC} = 50 - 38 = 12$ will be number of c_1+c_2 combinations. This is because TC comprises of 3 components:- $p+c$, c_1+c_2 and p_1+p_2 . Here all primes have been absorbed by composites so there would be no p_1+p_2 combinations. Therefore the remaining combinations will be in the nature of c_1+c_2 combinations. In the above example 38 would be the maximum number of $p+c$ combinations and 12 would be the minimum number of c_1+c_2 combinations.

From this we can define the concept of minimum number of c_1+c_2 as the number of c_1+c_2 combinations that would be identified if $n_{TC} > n_p$ and all primes are absorbed by $p+c$ type of combinations.

In general terms min_{c+c} (i.e. number of c_1+c_2 combinations if all primes are theoretically absorbed by $p+c$ combinations) can be derived as follows:-

$$\text{Min}_{c+c} = n_{TC} - n_p \text{ which is equal to } n_c - n_{TC} \text{ which is equal to } (n_c - n_p)/2$$

Maximum and minimum values of $\text{minc}+c$:

An important question is what could be the minimum and maximum value of minimum number of c_1+c_2 type of combinations (i.e. $\text{minc}+c$). Since minimum c_1+c_2 may be defined as the number of c_1+c_2 that would be identified if all primes are absorbed by $p+c$ combinations, its value will depend on number of primes vis-a-vis number of composites. For a given $2k$ if $n_{TC} = n_p$ (i.e. number of primes) it follows that $n_p=n_c$ (i.e. number of composites). This is because $2n_{TC} = n_c+n_p$, so if $n_{TC}= n_p$ then $n_{TC}=n_c$ as well i.e. $n_{TC}=n_p=n_c$. Here minimum c_1+c_2 will be 0 (zero) i.e. if all prime elements are absorbed by $p+c$ combinations (and $n_{TC}=n_p$) then only one type of combinations will be identified i.e. $p+c$ type of combinations. For example consider the even number 800 (i.e. $\text{SADN}8//0$); for this number $n_{TC} = 66$ and the number of primes as well as the number of composites = 66. In this case minimum $c_1+c_2 = 0$.

Similarly if $2k=806$ (i.e. $\text{SADN}5//6$); for this number $n_{TC}=67$ and n_c as well as $n_p = 67$, here again minimum $c_1+c_2 = 0$. If instead for a given $2k$ all primes form part of $p+c$ combinations, then minimum c_1+c_2 will become equal to actual c_1+c_2 . So it follows that the maximum value of minimum c_1+c_2 will be the number of actual c_1+c_2 derived by following the steps mentioned in an earlier section, i.e. section 11.

Derivation of number of combinations of $p+c$ type:

Correspondingly we can derive the number of prime&composite combinations as follows:-

If the number of total combinations is greater than the number of prime for a given $2k$, the maximum possible number of prime-eater composites would be the number of primes less than $2k$. This situation would occur if all prime numbers combine with composite numbers to form $p+c$ type of combinations. The actual number of $p+c$ type of combinations would be equal to the number of composite elements that remain after absorption of composite numbers by c_1+c_2 combinations. Every c_1+c_2 combination will absorb two composite numbers.

Therefore total number of composites absorbed by c_1+c_2 combination would be twice of n_{c+c} . After we calculate this number of composites absorbed by c_1+c_2 combinations, the remaining composites would combine with prime numbers to form $p+c$ combinations.

Number of combinations of type $p+c = n_c - 2n_{c+c}$

Where n_c = number of composites,

n_{c+c} = number of actual composite combinations or c_1+c_2 combinations.

Concept of minimum number of required combinations of c_1+c_2 type:

We would now return to the above example where minimum c_1+c_2 was derived to be 12.

By applying the steps to calculate c_1+c_2 discussed in section 11, if we can identify even 1 more than the minimum c_1+c_2 , then the composition of TC would change as follows:-

$$\begin{array}{llll} n_{TC}=50; & n_{c_1+c_2}=13; & n_{p+c}=36; & n_{p_1+p_2}=1 \\ n_c=62; & n_{c_1+c_2}=13 \times 2=26; & n_{p+c}=62-26=36 & \end{array}$$

It therefore follows that if number of c_1+c_2 is greater than minimum $c+c$ (considering all composites to be prime-eaters) by even 1 then a p_1+p_2 combination would be identified. In general terms the minimum number of c_1+c_2 required to identify at least 1 p_1+p_2 for a given $2k$ if $n_{TC} < n_c$ is $n_{TC} - n_p + 1$

i.e. minimum number of c_1+c_2 required to identify atleast one p_1+p_2 combination is denoted as $\text{minreq } c+c$

$$\text{minreq } c+c = n_{TC} - n_p + 1$$

Relation between actual and minimum c_1+c_2 and identification of p_1+p_2 combinations:

As mentioned earlier if $TC > np$, then minimum $c+c$ will be a positive number and if $TC = np$ then minimum $c+c = 0$. Further the concept of minimum required $c+c$ has also been defined which says that if actual number of combinations of type c_1+c_2 (i.e. actual $c+c$) is greater than minimum $c+c$ by even 1 (one), then p_1+p_2 combinations would be identified.

Consider the following situation:-

$$T_c = 100; \quad n_p = 80; \quad n_c = 120.$$

$$\text{Also minimum } c_1+c_2 = 100 - 80 = 20$$

$$\text{Therefore minimum required } c_1+c_2 = 21$$

If actual c_1+c_2 is now calculated, its value can range from value of minimum $c_1+c_2 = t_c - n_p = 100 - 80 = 20$ to maximum value of c_1+c_2 which is equal to number of composites $< k$ which is equal to 59. So value of actual number of combinations of type c_1+c_2 can range from 20 to 59.

Now upon calculating number of combinations of type c_1+c_2 , if its value = 21, then out of the 100 t_c , 21 would be c_1+c_2 in nature. As defined earlier actual $p+c$ would be $= n_c - 2n_{cc}$ i.e. $120 - 2 \times 21 = 120 - 42 = 78$

Therefore, composition of n_{TC} will be as follows:-

$$n_{TC}(100); \quad n_{c+c}=21; \quad n_{p+c}=78; \quad n_{p+p}=1$$

If value of actual number of combinations of c_1+c_2 type is derived as 22 then accordingly the composition of n_{TC} will emerge as follows:-

$$n_{TC}(100); \quad n_{c+c} = 22; \quad n_{p+c}=76; \quad n_{p+p}=2$$

Likewise if actual $c1+c2 = 30$ then

Composition of nTC will be nTC(100); $nc+c = 30$; $np+c=60$; $np+p=10$

If actual $c+c$ attains its maximum value of 59, then composition of nTC will be as follows:-

nTC(100); $nc+c=59$; $np+c=2$; $np+p=39$

It therefore follows that the number of $p1+p2$ will be derived as the difference between minimum $c+c$ and actual $c+c$.

Behaviour of number of $c1+c2$ combinations over a range of even numbers:

In order to understand the behaviour of $c1+c2$ combinations over a range of numbers, we now consider the number of such combinations for 2k of different SADN at fixed intervals. Specifically we will consider 2k of SADN 8//2 beginning from the number 242 (*why 242?*) at an interval of 720 (*why 720?*) natural numbers which will be representative (*why representative?*) of even numbers of SADN(2,5,8). Similarly we will consider 2k of SADN 4//8 beginning from the number 238 again at an interval of 720 natural numbers which would be representative of even numbers of SADN(1,4,7). Likewise we will consider 2k of SADN 6//0 beginning from the number 240 again at an interval of 720 numbers which would be representative of even numbers of SADN(3,6,9). The numbers considered here are consecutive even numbers to ensure that the beginning point remains more or less the same and the intervals have been kept the same to ensure that the behaviour remains comparable over the range of numbers under consideration. The following table shows the number of $c1+c2$ combinations as a proportion of number of composites on the relevant series as also the correspondence between number of $c1+c2$ combinations and number of total combinations which increase at a fixed number of 60 (*why 60?*) combinations.

For SADN8//2:

2k	nTC	Nc	nC+C	nC+C as % of nTC	nC+C as % of nc
242	20	15	3	15	20
962	80	83	19	23.75	22.89157
1682	140	153	37	26.42857	24.18301
2402	200	225	60	30	26.66667
3122	260	304	88	33.84615	28.94737
3842	320	378	107	33.4375	28.30688
4562	380	457	129	33.94737	28.22757
5282	440	535	155	35.22727	28.97196
6002	500	616	177	35.4	28.73377
6722	560	691	200	35.71429	28.94356
7442	620	775	229	36.93548	29.54839

8162	680	858	283	41.61765	32.98368
8882	740	934	282	38.10811	30.19272
9602	800	1015	292	36.5	28.76847
10322	860	1095	334	38.83721	30.50228

Table 12.1: Behaviour of number of c1+c2 combinations over a range of even numbers (2k) of SADN8//2

For SADN4//8:

2k	nTC	Nc	nC+C	nC+C as % of nTC	nC+C as % of nc
238	20	14	4	20	28.57143
958	80	76	18	22.5	23.68421
1678	140	145	36	25.71429	24.82759
2398	200	220	58	29	26.36364
3118	260	293	74	28.46154	25.25597
3838	320	371	109	34.0625	29.38005
4558	380	446	125	32.89474	28.02691
5278	440	526	171	38.86364	32.50951
5998	500	602	174	34.8	28.90365
6718	560	684	203	36.25	29.67836
7438	620	764	226	36.45161	29.58115
8158	680	839	246	36.17647	29.32062
8878	740	921	275	37.16216	29.85885
9598	800	1001	298	37.25	29.77023
10318	860	1080	359	41.74419	33.24074

Table 12.2: Behaviour of number of c1+c2 combinations over a range of even numbers (2k) of SADN4//8

For SADN6//0:

2k	nTC	Nc	nC+C	nC+C as % of nTC	nC+C as % of nc
240	40	29	8	20	27.58621
960	160	159	44	27.5	27.67296
1680	280	298	101	36.07143	33.89262
2400	400	444	135	33.75	30.40541
3120	520	597	195	37.5	32.66332
3840	640	749	237	37.03125	31.64219
4560	760	904	291	38.28947	32.19027
5280	880	1061	357	40.56818	33.6475

6000	1000	1218	396	39.6	32.51232
6720	1120	1374	494	44.10714	35.95342
7440	1240	1539	509	41.04839	33.07342
8160	1360	1698	576	42.35294	33.92226
8880	1480	1855	631	42.63514	34.01617
9600	1600	2017	678	42.375	33.61428
10320	1720	2176	733	42.61628	33.68566

Table 12.3: Behaviour of number of c1+c2 combinations over a range of even numbers (2k) of SADN6//0

From abovementioned tables 12.1-12.3, it is evident that the ratio of *number of c1+c2 combinations* to *total number of combinations* as well as ratio of *number of c1+c2 combinations* to *total number of composites* goes on increasing as the even number increases. It would be important to identify the reasons for this behavioural pattern of number of c1+c2 combinations over the range of even numbers. This is as discussed below.

Consider the even number 494. Since the number ends in 4, composite odd numbers ending in 9 would form part of c1+c2 combinations of type 2. These can be derived as follows:-

- (i) $2k = 494$, i.e. of SADN 8, so the relevant series would be the S7 series
- (ii) Composite numbers on the S7 series would be derived as products of intraseries elements of the S5 and S7 series. We begin by deriving composite odd numbers on the S7 series ending in 9 of which 7 is a factor. This can be calculated as follows:-

$$7(7+30n) \leq 2k$$

The numbers thus derived would be 49, 259, 469 and correspondingly the c1+c2 combinations would be:-

$$49+445$$

$$259+235$$

$$469+25$$

Similarly c1+c2 formed by 11 can be derived as follows:-

$$11(29+30n) \leq 2k$$

The only composite; of which 11 is a factor, ends in 9, and is < 494 ; is given as $11 \times 29 = 319$. The c1+c2 combination formed by this number would be:-

$$319+175$$

Similarly c_1+c_2 formed by 13 would be:

As $13 \times 13 = 169$; hence c_1+c_2 combination is given as $169+325$

Similarly c_1+c_2 formed by 17 = $289+205$

These are the c_1+c_2 of type 2; for the even number 494; which are 6 in number.

Apart from these c_1+c_2 of type 2; some c_1+c_2 of type 1 can also be identified in the following manner:

As $494/2=247$

And $247= 13 \times 19$ so the following c_1+c_2 can be derived

$$19 \times 7 + 19 \times 19 \text{ i.e. } 133 + 361$$

$$13 \times 7 + 13 \times 31 \text{ i.e. } 91 + 403$$

$$13 \times 19 + 13 \times 19 \text{ i.e. } 247 + 247$$

Further, the condition for deriving c_1+c_2 of type3 is $2k/6p_1p_2 \geq 1$

This condition is fulfilled by prime pair 7&11 since $6p_1p_2$ i.e. $6 \times 7 \times 11 = 462$ which is < 494 . So it may be expected that there would be 1 c_1+c_2 of which 7 & 11 are factors on either side of the combination.

Now if we double the value of $2k$ and consider the closest approximate even number of SADN(2,5,8) that ends in 4, say 974, consider the c_1+c_2 combination for this number .

c_1+c_2 of type 2 formed by 7:-

49, 259, 469, 679, 889

c_1+c_2 formed by 11:-

319, 649, 979

c_1+c_2 formed by 13:-

169, 559, 949

c_1+c_2 formed by 17:-

289, 799

It is important to note here that c_1+c_2 of type 2 were formed by those numbers for 494 as well. The total number of c_1+c_2 combinations formed by these numbers (viz. 7, 11, 13 and 17) for 974 = 13 which was 6 for the number 494.

However c_1+c_2 combinations of type 2 will be formed by some other numbers as well. These can be derived as:-

$$19 \times (31+30n) \leq 2k$$

$$589$$

$$23 \times (23+30n) \leq 2k$$

$$529$$

Therefore the actual number of c_1+c_2 of type 2 for 974= 15

It is important to note here that the increase in number of c_1+c_2 of type 2 occurs on two accounts. Firstly numbers which formed c_1+c_2 for 494 would form twice as many c_1+c_2 of type 2 for 974.

In this example number of c_1+c_2 combinations formed by 7, 11, 13 and 17 for 494 = 6 and this number more than doubled to 13 with approximate doubling of $2k$ to 974. This may be referred to as an intensive increase in the number of c_1+c_2 combinations of type 2 wherein prime numbers forming c_1+c_2 of type 2 for a given $2k$ can be observed to form a larger number of c_1+c_2 combinations for even numbers of greater value.

Secondly, some numbers wherein very first composite odd number of which these numbers are a factor is greater than the earlier $2k$ and therefore were not relevant will now come into the picture. For example the first composite formed by 19 on the S7 series is 589 (19×31 i.e. 589) which itself is > 494 . So this composite is not relevant for the even number 494. But when we consider $2k=974$; 589 will play a role in forming c_1+c_2 of type 2. Similarly, the first composite number on the S7 series ending in 9 of which 23 is a factor would be 529 (i.e. 23×23). Since 529 is greater than 494, it will not form a c_1+c_2 combination for 494 but since 529 is less than 974, it would participate in formation of c_1+c_2 combination of type 2 for 974. This takes the total number of c_1+c_2 of type 2 to 15. This may be referred to as an extensive increase in number of c_1+c_2 combinations wherein odd composite numbers formed by prime numbers which were earlier irrelevant will now have a role to play for even numbers of greater value.

Due to this intensive and extensive increase in number of c_1+c_2 combinations of type 2, the overall number of c_1+c_2 combinations of type 2 will increase more than proportionately with a given increase in value of $2k$ i.e.: with a doubling of $2k$, c_1+c_2 of type 2 will increase by more than double.

This trend in number of c_1+c_2 combinations wherein with a given increase in values of even number, the number of c_1+c_2 combinations increases more than proportionately can be observed in c_1+c_2 combination of type 3 as well. Continuing with the same example given above, it was mentioned that the earlier prime pairs that satisfy the condition $2k/6p_1p_2 \geq 1$ for the number 494 was the prime pair 7&11. Since $6 \times 7 \times 11 = 462$ is ≤ 494 .

However when we consider the number 974, number of $c1+c2$ combinations will increase on two accounts. Firstly, since $494/462 = 1$ and $974/462 = 2$, it may be expected that at least 2 $c1+c2$ combinations of the prime pair 7&11 will be derived. Secondly, a few more prime pairs will now satisfy the condition $2k/6p1p2 \geq 1$. These are 7&13=546, 7&17=714, 7&19=798, 7&23=966, 11&13=858.

This implies that while $c1+c2$ combination of type3 in which 7&11 are factors on either side of the combinations were formed for $2k=494$, for the even number 974, this number of $c1+c2$ combinations for prime pair 7&11 will double. Additionally, 5 more prime pairs will now satisfy the condition of type 3. The increase in number of $c1+c2$ formed by the prime pair 7&11 may be referred to as an intensive increase in number of $c1+c2$ combinations of type 3. The increase in number of $c1+c2$ combinations of type 3 because of additional prime pairs satisfying the condition for identification of $c1+c2$ combination of type 3 may be considered as an extensive increase.

The above discussion shows that with a given increase in the value of $2k$ due to the intensive and extensive nature of increase in number of $c1+c2$ combinations, the overall number of $c1+c2$ will increase more than proportionately. This is evident from the abovementioned tables 12.1-12.3. It is important to note here that this behaviour may be attributed to the intensive and extensive rise in the number of $c1+c2$ combinations over a range of even numbers and this behavioural pattern can be observed for any range of even numbers lying anywhere on the number-line.

Behaviour of minimum number of $c1+c2$ combinations over a range of even numbers:-

As discussed above, minimum number of $c1+c2$ combinations can be derived as $nTC-np$ or $nc - nTC$ or $(nc - np)/2$. Therefore in order to understand the behavioural pattern of minimum number of $c1+c2$ combinations, we need to first examine the behaviour of the underlined factors of np and nc over a range of even numbers. The following tables present the minimum number of $c1+c2$ over the same range of even numbers for which the actual number of $c1+c2$ combinations was discussed above.

For even numbers of SADN8//2:

2k	nTC	nc	np	min C+C
242	20	15	25
962	80	83	77	3
1682	140	153	127	13
2402	200	225	175	25
3122	260	304	216	44
3842	320	378	262	58
4562	380	457	303	77
5282	440	535	345	95
6002	500	616	384	116

6722	560	691	429	131
7442	620	775	465	155
8162	680	858	502	178
8882	740	934	546	194
9602	800	1015	585	215
10322	860	1095	625	235

Table 12.4: Behaviour of Minimum c+c over a range of Even numbers for SADN 8//2**For even numbers of SADN4//8:**

2k	nTC	nc	np	min C+C
238	20	14	26	...
958	80	76	84	...
1678	140	145	135	5
2398	200	220	180	20
3118	260	293	227	33
3838	320	371	269	51
4558	380	446	314	66
5278	440	526	354	86
5998	500	602	398	102
6718	560	684	436	124
7438	620	764	476	144
8158	680	839	521	159
8878	740	921	559	181
9598	800	1001	599	201
10318	860	1080	640	220

Table 12.5: Behaviour of Minimum c+c over a range of Even numbers for SADN 4//8**For even numbers of SADN6//0:**

2k	nTC	nc	np	min C+C
240	40	29	51	...
960	160	159	161	...
1680	280	298	262	18
2400	400	444	356	44
3120	520	597	443	77
3840	640	749	531	109
4560	760	904	616	144
5280	880	1061	699	181
6000	1000	1218	782	218

6720	1120	1374	866	254
7440	1240	1539	941	299
8160	1360	1698	1022	338
8880	1480	1855	1105	375
9600	1600	2017	1183	417
10320	1720	2176	1264	456

Table 12.6: Behaviour of Minimum $c+c$ over a range of Even numbers for SADN 6//0

It can be observed from the above tables 12.4-12.6 that there would be a steady increase in the minimum number of $c1+c2$ combinations but the rate of increase would be less than the rate of increase in actual number of $c1+c2$ combinations. In order to understand the reason for this pattern, we need to understand the nature of increase in number of composites.

The method for calculating total number of unique composites for a given $2k$ has been discussed in detail in section 5B. In a nutshell composite numbers on the $S7$ series are derived as follows:-

Composites formed by elements of the $S5$ series

$(6n-1) \times 5$ onwards every $(6n-1) \times 6n$ th number; for example composites formed by 5 would be derived as 5×5 onwards $5 \times 6n$ th numbers i.e. 25, 55, 85, Similarly, composites formed by 11 would be derived as 11×5 onwards as $11 \times 6n$ th number i.e. 55, 121, 187, 253,

However since 5×11 has clearly been derived while calculating composites formed by 5, the first effective composite formed by 11 is 121. Similarly composites formed by 17 can be observed as 17×5 onwards $17 \times 6n$ th number i.e. 85, 187, 289, 391, ...

Here again 17×5 has already been derived while calculating composites for 5 and 17 i.e. calculating composites for 11. So the first effective composite formed by 17=289. This shows that the first effective composite on the $S7$ series by a given prime number would be its square since all earlier composites of which the number would be a factor would have been derived by earlier prime numbers on the series. The same rationale can be applied to imply that the first effective composite formed by elements of the $S7$ series on the $S7$ series would be the square of the number and thereafter the number multiplied by $6n$ would give further composites. The question now is how the number of composites on the series behaves over a range of even numbers. Let us consider the same numbers discussed above for understanding the behaviour of $c1+c2$ combinations.

There are 38 composites on the $S7$ series < 494 and this number increases to 83 composites, < 974 . This increase in number of composites may be attributed to two factors:-

Prime numbers for which unique composites were identified for 494, the number of composites formed by these numbers between 494 and 974 would be greater. This may be extended as follows:

This shows that number that were relevant for 494 while deriving number of composites now form a higher number of composites for 974. This may be referred to as an intensive increase in number of composites.

However a few more composites can be observed while moving from $2k=494$ to $2k=974$ because squares of some more numbers will now become relevant. For example $23^2 = 529$, which is greater than 494. So number of unique composites formed by 23 is relevant for 494 but for 974, 23^2 will be counted as a unique composite as also other composites formed by 23 whose value would be < 974 . In case of 23, as many as 3 composites formed by 23 will be counted while calculating number of unique composites for 974.

Besides 23, $29^2 = 841$ and $31^2 = 962$ will also be counted while calculating unique composites for 974.

This increase in number of unique composites with an increase in value of $2k$ may be referred to as extensive increase in number of composites. Due to this extensive and intensive increase in number of composites the total number of composites increases with an increase in value of $2k$.

Stated differently, tables 12.1-12.6 show that the density of composites increases as we move along the number line in positive direction. However it is important to note that the rate of increase in number of composites would be slow due to the following reasons. Firstly, while calculating number of unique composites we found that composites formed by prime numbers with other prime numbers greater than its own value and composites of which its previous prime elements are not factors would be identified. For instance while calculating number of composites formed by 11, we found that unique composites of which 11 is a factor are the prime numbers on the S5 series that are > 11 and the composite numbers on the S5 series of which 5 is not a factor.

Similarly while calculating unique composites of 17, we found that only composites would be considered which 17 forms with prime numbers greater than 17 on the S5 series as also with the composites on the S5 series of which 5 and 11 are not factors.

Since elements on the S5 and S7 series comprise of both the prime and composite numbers, a rise in composite numbers would result in a corresponding decrease in the number of primes. This in turn will cause a decrease in the rate at which composite numbers would be formed.

Secondly, the first relevant composite number on the S7 series would be the square of a prime element after which the number under consideration multiplied by $6n$ would give further composites. Since the gap between consecutive squares goes on increasing as we progress along the number line, the numbers which come into the picture increase at a decreasing rate.

Secondly, as discussed above; the first unique composite formed by a number would be its square. This implies that if the square of a certain prime number p_1 i.e. p_1^2 is such that it is greater than $2k$ and less than $2k'$, then p_1 would have a role to play in identifying composites

for $2k'$ which was not the case for $2k$. As mentioned above, it causes an extensive rise in the number of composites over a range of numbers.

Due to this the extrinsic increase in number of composites occurs at a decreasing rate.

Over the range of numbers where an extensive increase does not occur, only the intensive increase in number of composites plays a role.

Another important factor here is that as composites are derived as $5 \times (6n-1)$ onwards each $6(6n-1)$ th number on the S5 series, for every successive numbers on the S5 series, value of starting point and the subsequent numbers would go on increasing. For instance, $17 \times 5 = 85$ and $17 \times 6 = 102$ while $23 \times 5 = 115$ and $23 \times 6 = 138$. So the $(6n-1) \times 6$ th number goes on increasing with successive numbers.

So even in case of an intensive increase, the rate at which successive elements of the series contributes to formation of composites goes on decreasing.

Due to the operation of these factors, the overall number of composites increases at a decreasing rate.

Since elements on the series comprise of either prime elements or composite elements, an increase in number of composites at a decreasing rate will cause an increase in number of primes also at a decreasing rate.

As mentioned above, minimum number of $c+c$ combinations depends on the number of composites and primes. While both TC and np increase; TC increases at a fixed rate whereas np increases at a decreasing rate. Therefore the resultant variable would increase at a rather slow pace. This is evident from the above tables and this pattern can be observed over any range of even numbers.

Zeno's Achilles and tortoise paradox..

Zeno's Achilles and tortoise paradox associated with the Greek philosopher Zeno presents a very interesting prospective on what would transpire in the race between Achilles and tortoise under certain conditions. The paradox says that even though the tortoise runs at a speed evidently slower than that of Achilles, the latter would never be able to win the race with tortoise under two conditions- firstly that the tortoise has a headstart, in the race, of a finite distance and secondly, both the Achilles and tortoise run at a constant speed.

The reasoning goes as follows:-

Achilles runs at a speed of x km/h while tortoise runs at a speed of y km/h such that $y = x/2$ i.e. Achilles runs twice the speed of the tortoise. Further suppose Achilles gives tortoise a headstart of z meters where z is a finite positive number.

Once the race commences, in order to beat the tortoise in the race, Achilles needs to first catch up with the tortoise. Achilles would cover the distance z at its speed of x km/h and reach where the tortoise started from. But by that time the tortoise has moved ahead at its speed of y km/h and tortoise needs to cover this distance in order to catch up. But by the time Achilles covers this distance, tortoise has travelled further. Even though the distance between Achilles and tortoise goes on decreasing and tends to become zero, it remains a non-zero quantity. If we consider any distance that is even infinitesimally greater than zero, it is a nonzero distance nevertheless. The tortoise remains ahead of the Achilles infinitely. This makes the Zeno's Achilles and tortoise paradox conceptually sound.

Now consider what would happen to this if we introduce two changes to the situation. First, both Achilles' and tortoise's speeds increase over the race, but the rate of increase in Achilles' speed is greater than the rate of increase in tortoise's speed. Second, it is Achilles and not the tortoise that gets a headstart in the race. Effectively what would happen is tortoise running the race at a speed less than that of Achilles throughout and beginning the race at a distance behind the Achilles. The question is: Will the tortoise ever be able to catch up with the Achilles. Conceptually, if the fast runner is not able to catch up with the slow runner if the latter gets a headstart then the slow runner starting the race at a disadvantageous position would never be able to catch up with the fast runner. Consider the following situation. Achilles runs at a speed of x km/h and begins the race at point z . Tortoise runs at a speed of y km/h and begins the race at point z' . Note that value of x and y may change but $x > y$ at all times and z is ahead of z' by a finite distance.

In this situation if tortoise has to catch up with Achilles, it needs to first cover the distance between z' and z . But by the time tortoise reaches point z , Achilles has covered a distance of $z-z''$ and as Achilles' speed is greater than tortoise's speed, $z'' - z$ would be definitely greater than $z'-z$. This means at this stage the gap between Achilles and tortoise is now greater than what it was at the beginning of the race. By the time tortoise reaches point z'' , Achilles would have covered a greater distance which further widens the gap between Achilles and tortoise. This would logically widen the gap between Achilles and tortoise continuously and infinitely.

Who is Achilles and who is the tortoise?

The behaviour of actual $c1+c2$ and minimum $c1+c2$ has been discussed above alongwith the reasoning for their corresponding behaviour. It would be useful to now compare their behaviour. The following table shows the difference between the minimum and actual number of $c1+c2$ combinations:

For SADN 8//2:-

2k	act cc	min cc	act- min cc
242	3
962	19	3	16
1682	37	13	24

2402	60	25	35
3122	88	44	44
3842	107	58	49
4562	129	77	52
5282	155	95	60
6002	177	116	61
6722	200	131	69
7442	229	155	74
8162	283	178	105
8882	282	194	88
9602	292	215	77
10322	334	235	99

Table 12.7: Difference between actual and minimum number of c1+c2 combinations for even numbers of type SADN 8//2

For SADN 4//8:-

2k	act cc	min cc	act- min cc
238	4
958	18
1678	36	5	31
2398	58	20	38
3118	74	33	41
3838	109	51	58
4558	125	66	59
5278	171	86	85
5998	174	102	72
6718	203	124	79
7438	226	144	82
8158	246	159	87
8878	275	181	94
9598	298	201	97
10318	359	220	139

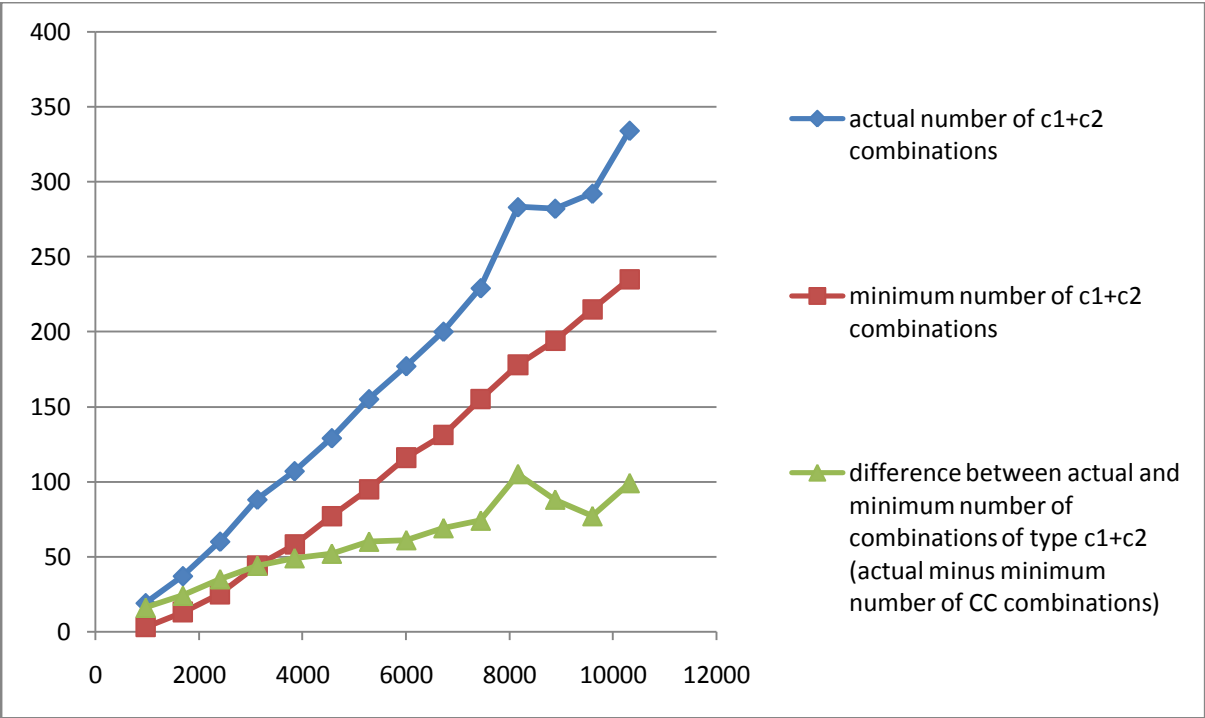
Table 12.8: Difference between actual and minimum number of c1+c2 combinations for even numbers of type SADN 4//8

For SADN 6//0:-

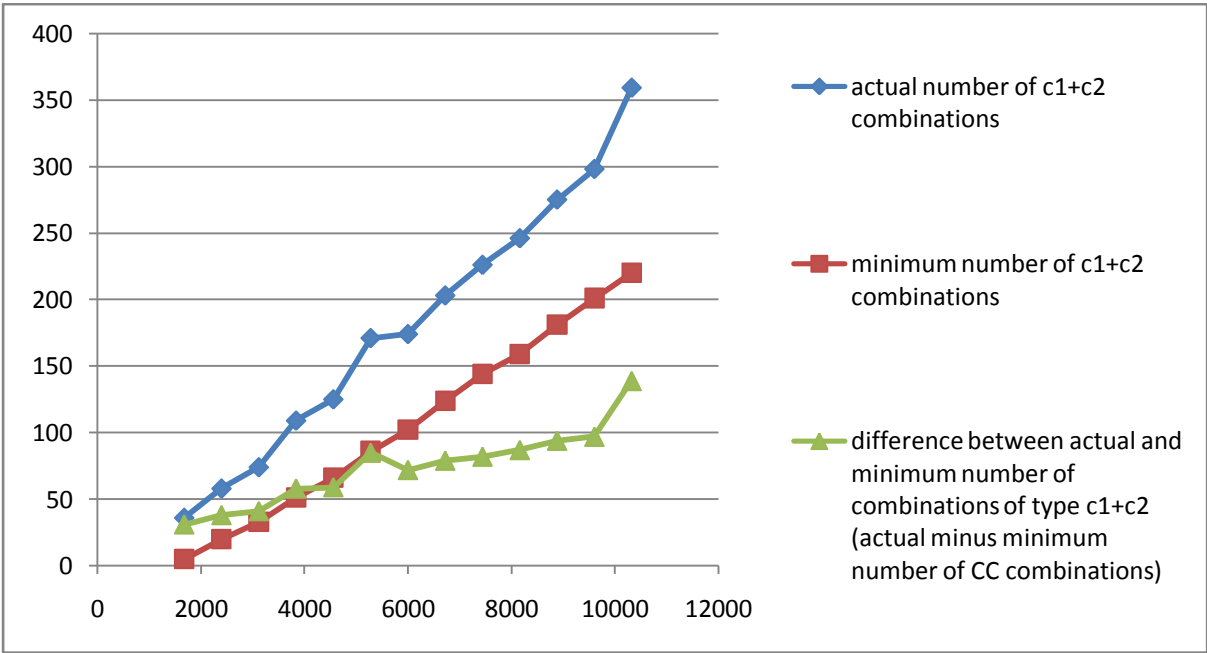
2k	act cc	min cc	act- min cc
240	8

960	44
1680	101	18	83
2400	135	44	91
3120	195	77	118
3840	237	109	128
4560	291	144	147
5280	357	181	176
6000	396	218	178
6720	494	254	240
7440	509	299	210
8160	576	338	238
8880	631	375	256
9600	678	417	261
10320	733	456	277

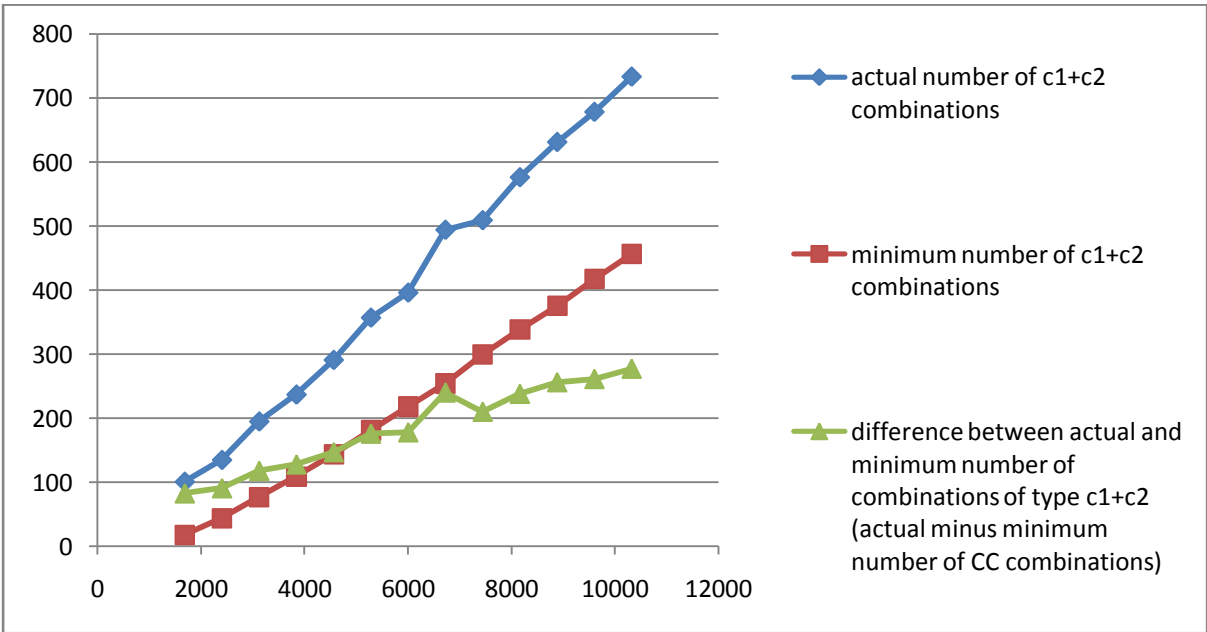
Table 12.9: Difference between actual and minimum number of c1+c2 combinations for even numbers of type SADN 6//0



Graph 12.1: Depiction of actual number of c1+c2 combinations, minimum number of c1+c2 combinations and difference between actual vis-à-vis minimum number of c1+c2 combinations for even numbers of SADN 6//0



Graph 12.2: Depiction of actual number of c1+c2 combinations, minimum number of c1+c2 combinations and difference between actual vis-à-vis minimum number of c1+c2 combinations for even numbers of SADN4//8



Graph 12.3: Depiction of actual number of c1+c2 combinations, minimum number of c1+c2 combinations and difference between actual vis-à-vis minimum number of c1+c2 combinations for even numbers of SADN6//0

It is evident from the tables 12.7-12.9 as well as the graphs 12.1-12.3 that the difference between the actual and minimum number of c_1+c_2 combinations goes on increasing thereby causing a continuous divergence between the actual and minimum c_1+c_2 functions.

From this discussion it is concluded that actual c_1+c_2 is in the role of the Achilles and min c_1+c_2 is in the role of the tortoise.

Both actual c_1+c_2 and minimum c_1+c_2 increase over a range of numbers but the former increases at a rate higher than that of the latter which causes a divergence between the distance covered by the tortoise and the Achilles.

An important question here is that where do these functions begin from. An examination of all even numbers of SADN(2,5,8) shows that for all numbers less than 800, $TC > \text{number of composites}$. 800 is the first even number where $TC = \text{number of composites} = \text{number of primes} = 67$.

Here minimum $c_1+c_2 = TC - np = 67 - 67 = 0$

So minimum required $c_1+c_2 = 0+1 = 1$

In case of any even number of SADN(5,2,8); if we calculate the actual number of c_1+c_2 for 800 we find that there are 20 c_1+c_2 combinations for the number 800 whereas the minimum required $c_1+c_2 = 1$. This implies that at the point on the number line where the actual and minimum c_1+c_2 functions become relevant, actual c_1+c_2 is observed to be substantially greater than minimum required c_1+c_2 . This may be interpreted as the Achilles having a headstart of 19 in the race with the tortoise.

An examination of all even numbers of SADN(7,4,1) shows that for all numbers less than 1144, $nTC > \text{number of composites}$. 1144 is the first even number where $nTC = \text{number of composites} = \text{number of primes} = 95$.

Here minimum $c_1+c_2 = tc - np = 95 - 95 = 0$

So minimum required $c_1+c_2 = 0+1 = 1$

In case of any even number of SADN(7,4,1); if we calculate the actual number of c_1+c_2 for 1144 we find that there are 24 c_1+c_2 for the number 1144 whereas the minimum required $c_1+c_2 = 1$. This implies that at the point on the number line where the actual and minimum c_1+c_2 functions become relevant, actual c_1+c_2 is observed to be substantially greater than minimum required c_1+c_2 . This may be interpreted as the Achilles having a headstart of 23 in the race with the tortoise.

An examination of all even numbers of SADN(6,3,9) shows that for all numbers less than 966, $TC > \text{number of composites}$. 966 is the first even number where $tc = \text{number of composites} = \text{number of primes} = 161$.

Here minimum $c_1+c_2 = tc-np = 161-161=0$

So minimum required $c_1+c_2 = 0+1 = 1$

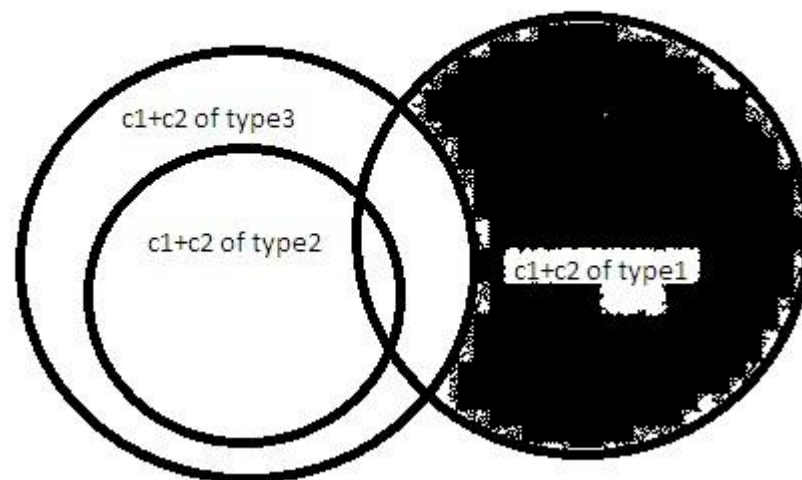
If we calculate the actual number of c_1+c_2 for 966 we find that there are 45 c_1+c_2 for the number 966 whereas the minimum required number of combinations of type $c_1+c_2 = 1$. This implies that at the point on the number line where the actual and minimum c_1+c_2 functions become relevant, actual c_1+c_2 is observed to be substantially greater than minimum required c_1+c_2 . This may be interpreted as the Achilles having a headstart of 44 in the race with the tortoise.

In a nutshell therefore, the relation between the actual and minimum c_1+c_2 functions is analogous to the race between the Achilles and the tortoise where the former has a headstart over the latter and also the speed of the former increases at a rate faster than the increase in the speed of the latter. This will evidently cause a divergence between the two functions continuously and infinitely.

Two exceptions to the pattern of divergence will be found. Firstly, the actual c_1+c_2 function will appear to dip for an even number where mid-point k is a prime and such an even number follows one whose mid-point is composite.

Consider an even number $2k$ of SADN2 whose mid-point k is a prime. The previous even number on this series would be $2k-6$. If the mid-point of $2k-6$ is a composite number, it will appear that $nc+c$ for $2k$ is less than $nc+c$ for $2k-6$. Since $2k-6$ appears before $2k$ on the number line, it will appear that the actual c_1+c_2 function is dipping. This may be attributed to the correlation between the number of c_1+c_2 combinations for a given $2k$ and the nature of k ; i.e. whether k is prime or composite.

As mentioned earlier, c_1+c_2 combinations can be derived across three steps for a given even number. However since c_1+c_2 of type 1 are derived from the factors of mid-point k , such combinations would exist for even numbers if mid-point is composite in nature. However for even numbers whose mid-point is a prime, c_1+c_2 of type 1 will not exist and only c_1+c_2 combinations of type 2 and type 3 will exist. The Venn diagram described in section 11C, and reproduced below, may be referred in this context.



Reproduced here Diagram 11C.1: Venn diagram representing the relation of subset and superset among the $c1+c2$ combinations of types1, 2 and 3

It is evident from the abovementioned Venn diagram that some $c1+c2$ of type 1 will overlap with $c1+c2$ of types 2 and 3 whereas some $c1+c2$ are derived uniquely only by $c1+c2$ of type1.

These $c1+c2$ represented by the shaded portion will be derived only for $2k$ where k is composite. Therefore if k is prime then $c1+c2$ of type 1 will not exist thereby bringing the number of $c1+c2$ down. Therefore for even numbers ($2k$) where k is prime, $nc+c$ will be less than $nc+c$ for even numbers where k is composite. This also explains the dependence of number of $p1+p2$ combinations on the nature of mid-point k ; whether k is prime or composite. This pattern has been reported as an observation by Watanabe [12].

Another exception where actual $c1+c2$ function will appear to converge towards the minimum $c1+c2$ function is for numbers that immediately follow even numbers ending in zero(0) i.e. consider an even number $2k$ of SADN 2 ending in 0 and having 'B' number of $nc+c$. For the next even number which would be of SADN 8, the last digit would be 6 and generally $nc+c$ for this number would be less than 'B'. Here also it appears that the actual and minimum $c1+c2$ functions converge towards one another. This dip in $nc+c$ can be attributed to the following reasoning:-

For even numbers ending in zero(0), all composite odd numbers ending in 5 whose value is less than $2k$ will form part of $c1+c2$ combinations of type 2. Since composites of which 5 is a factor occur at the highest frequency i.e. at a gap of 30 natural numbers, the number of composites of which 5 is a factor would be the highest. Since all these composites would form part of $c1+c2$ combinations of type 2, the overall number of $c1+c2$ combinations for these numbers ending in zero(0) would be higher as compared to even numbers ending in other digits. Due to this $nc+c$ for $2k//0$ would be in the nature of local maxima and $nc+c$ for

2k immediately following these numbers would be less in number as compared with the number of c_1+c_2 combinations for the previous even number ending in 0. This causes the actual c_1+c_2 function to appear to converge towards the minimum c_1+c_2 function.

However it is logically reasoned to note that these convergences are temporary (relative to the location on the number line) in nature and the general nature of relation between the actual and minimum c_1+c_2 function is one of continuous/resultant divergence.

Will the tortoise ever be able to catch up with the Achilles?

The tortoise would be able to catch up with the Achilles only if its speed abruptly rises and rises to an extent that it covers the earlier divergence between the two that has been created during the race. In terms of the actual and minimum c_1+c_2 it implies that the minimum c_1+c_2 will rise and intersect the actual c_1+c_2 function if the prime gap at that point is so large that it causes the minimum c_1+c_2 function to rise sharply.

It may be noted that since the speed at which the tortoise runs the race increases during the course of the race, it will attain that much speed in due course of the race but by then the required speed to catch up with the Achilles would have become significantly greater (Since in the mean time the distance between Achilles and the tortoise would have increased further).

Consider the following hypothetical situation:-

Achilles begins the race at a speed of 11km/hour while tortoise begins at 5km/hour. Further let us assume this speed increases after every 1 km distance and Achilles begins the race one km ahead of the tortoise. Also assume that the speed of the Achilles increases at rate of 1km/h after every 1 km distance while speed of the tortoise increases by 0.5 km/hour after every 1 km distance.

For the tortoise to catch up Achilles in the race, it has to cover the distance for which Achilles has a headstart and also the distance which Achilles will travel in the meantime.

Tortoise will cover the initial distance of 1 km at its speed of 5km/hour in 12 minutes. By this time Achilles will cover an additional distance of a bit more than 2 kms at its initial speed of 11km/hour causing the distance between Achilles and tortoise to widen.

At the beginning of the race, in order to beat Achilles, the tortoise needs to run at a constant speed of 12km/hour to catch Achilles; also running at a constant speed, after 1 hour.

It is important to note that the speed of the tortoise would, at a point be 12km/hour since its speed rises at the rate of 0.5 km/hour after every 1 km distance. But by the time the tortoise's speed rises to this much rate, by that time the speed of Achilles has increased upto 25 km/hour, which is greater than the tortoise's speed of 12 km/hour at this moment of time.

An analogy may be derived here to the role of prime gaps. The prime gap required to cause minimum c_1+c_2 to rise and catch up with the actual c_1+c_2 function will be identified at some location on the number line, but at the location on the number line where such a required

prime gap would be identified the divergence between the minimum and actual $c1+c2$ functions would have increased causing the required prime gap for minimum $c1+c2$ to rise further to the level of actual $c1+c2$.

By the time this gap would be reached, the divergence between minimum and actual $c1+c2$ will rise further causing the required prime gap to rise further. This brings us to the argument that not only is the magnitude of the prime gap important but also the location at number line where this prime gap occurs. For example, a prime gap of 50 numbers at a value of $2k=600$ would be significant in its implications as compared to a prime gap of 50 numbers at the value of $2k = 100000$.

It may be inferred from the above discussion that due to two factors, firstly the rate at which the actual $c1+c2$ function rises is greater than the rate at which the minimum $c1+c2$ function rises. Second, the actual $c1+c2$ function has a headstart over the minimum $c1+c2$ function at the value of $2k$ where these functions become relevant. Due to these two factors, there would be a continuous divergence between the actual and minimum $c1+c2$ functions and thus minimum $c1+c2$ function would never be able to intersect the actual $c1+c2$ function.

As discussed earlier the actual $c1+c2$ function has a headstart over the minimum $c1+c2$ function and increases at a rate faster than the rate of increase in minimum $c1+c2$ function causing a continuous divergence between the two functions. Since the difference between minimum $c1+c2$ and actual $c1+c2$ is equal to the number of $p1+p2$ combinations, or divergence between minimum $c1+c2$ and actual $c1+c2$ function indicates that the number of $p1+p2$ combinations continuously increases over a range of numbers.

What does this imply?

As discussed above, the difference between actual and minimum numbers of $c1+c2$ combinations indicates the existence of $p1+p2$ combinations. Once again we return to the above table 12.7-12.9 which presented the difference between these two variables. Accordingly we can now disintegrate TC into its three components by applying the following reasoning:-

$$nTC = (nc+c) + (np+c) + (np+p)$$

$nc+c$ has been derived by following the steps mentioned in section 11.

$np+c$ is equal to $nc - 2x(nc+c)$ i.e. nc minus twice the number of combinations of type $c1+c2$.

$np+p$ is equal to $nTC - (nc+c) - (np+c)$

The relation between these three components for the range of even numbers mentioned above is presented in the following table:-

For SADN 8//2:

2k	NC	NP	NTC	NCC	NCP	NPP
242	15	25	20	3	9	8

962	83	77	80	19	45	16
1682	153	127	140	37	79	24
2402	225	175	200	60	105	35
3122	304	216	260	88	128	44
3842	378	262	320	107	164	49
4562	457	303	380	129	199	52
5282	535	345	440	155	225	60
6002	616	384	500	177	262	61
6722	691	429	560	200	291	69
7442	775	465	620	229	317	74
8162	858	502	680	283	292	105
8882	934	546	740	282	370	88
9602	1015	585	800	292	431	77
10322	1095	625	860	334	427	99

Table 12.10: Relation between components of TC for even numbers of type SADN 8//2

For SADN 4//8:

2k	NC	NP	NTC	NCC	NCP	NPP
238	14	26	20	4	6	10
958	76	84	80	18	40	22
1678	145	135	140	36	73	31
2398	220	180	200	58	104	38
3118	293	227	260	74	145	41
3838	371	269	320	109	153	58
4558	446	314	380	125	196	59
5278	526	354	440	171	184	85
5998	602	398	500	174	254	72
6718	684	436	560	203	278	79
7438	764	476	620	226	312	82
8158	839	521	680	246	347	87
8878	921	559	740	275	371	94
9598	1001	599	800	298	405	97
10318	1080	640	860	359	362	139

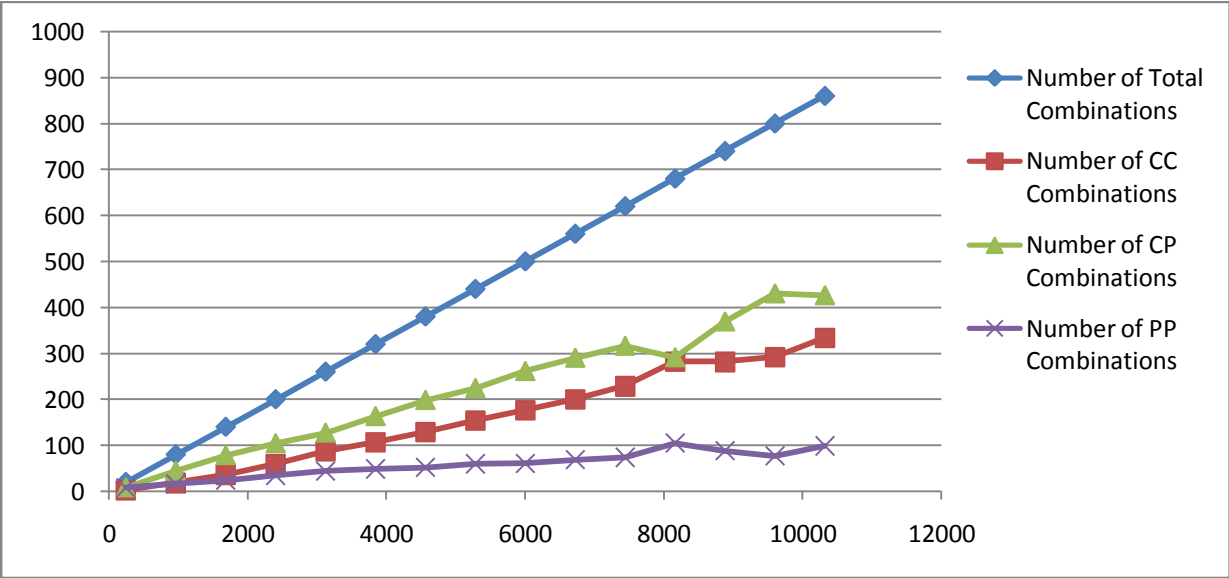
Table 12.11: Relation between components of TC for even numbers of type SADN 4//8

For SADN 6//0:

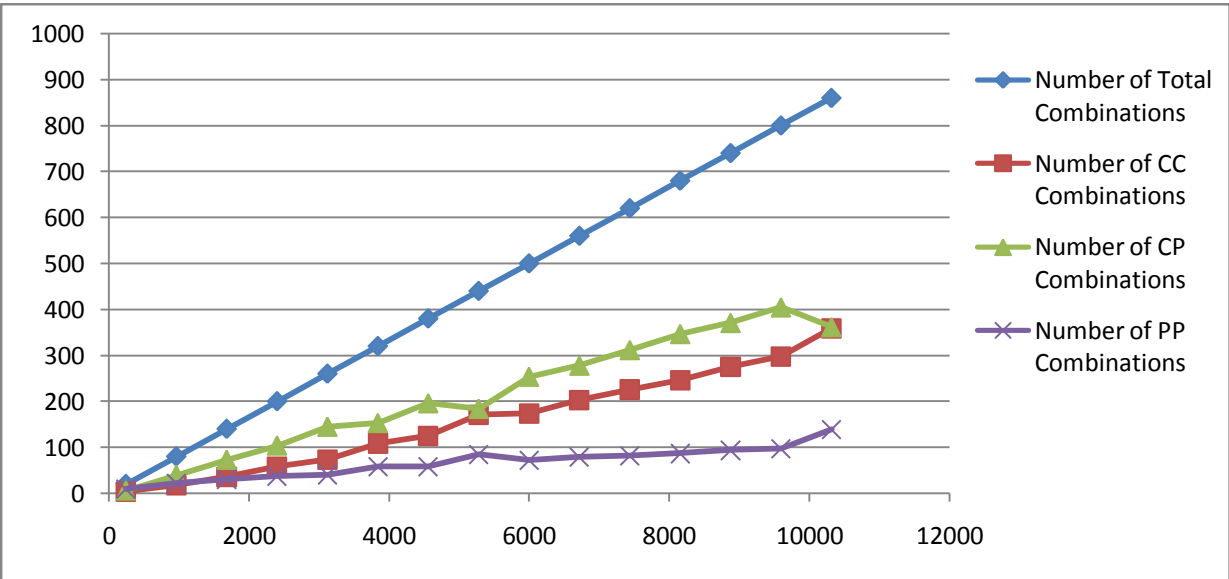
2k	NC	NP	NTC	NCC	NCP	NPP
240	29	51	40	8	13	19

960	159	161	160	44	71	45
1680	298	262	280	101	96	83
2400	444	356	400	135	174	91
3120	597	443	520	195	207	118
3840	749	531	640	237	275	128
4560	904	616	760	291	322	147
5280	1061	699	880	357	347	176
6000	1218	782	1000	396	426	178
6720	1374	866	1120	494	386	240
7440	1539	941	1240	509	521	210
8160	1698	1022	1360	576	546	238
8880	1855	1105	1480	631	593	256
9600	2017	1183	1600	678	661	261
10320	2176	1264	1720	733	710	277

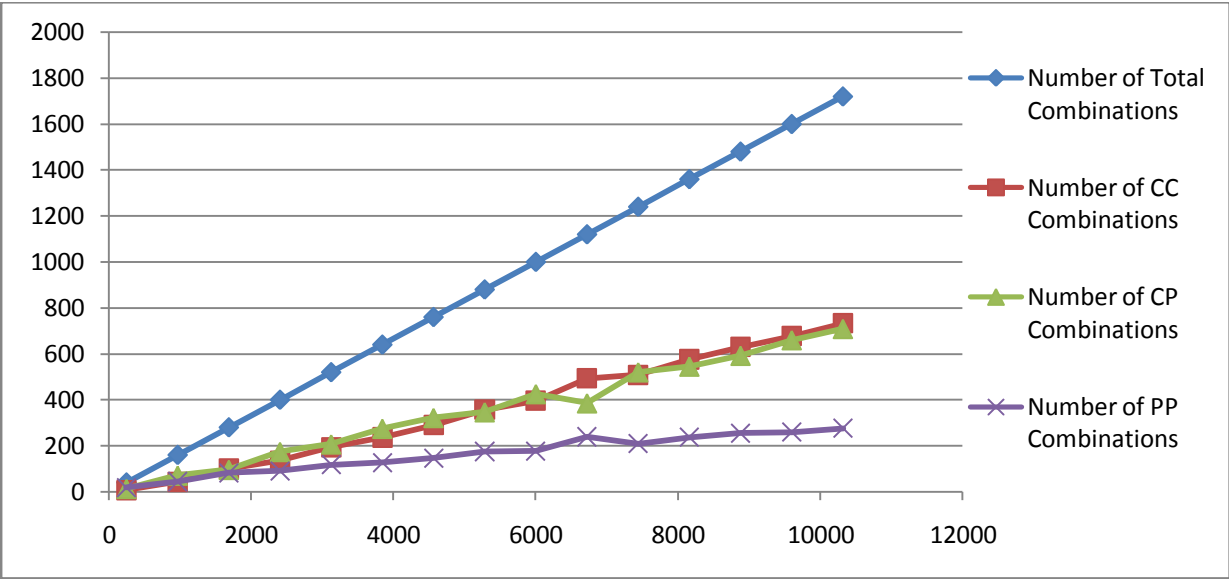
Table 12.12: Relation between components of TC for even numbers of type SADN 6//0



Graph 12.4: Depiction of number of total combinations, number of c1+c2 combinations, number of c+p combinations and number of p1+p2 combinations for even numbers of SADN8//2



Graph 12.5: Depiction of number of total combinations, number of c1+c2 combinations, number of c+p combinations and number of p1+p2 combinations for even numbers of SADN6//8



Graph 12.6: Depiction of number of total combinations, number of c1+c2 combinations, number of c+p combinations and number of p1+p2 combinations for even numbers of SADN4//0

It is deduced from above tables 12.10-12.12 and graphs 12.4 -12.6 that the number of pp combinations (i.e. p1+p2 combinations) steadily increases over the given range of even numbers. Here again it is important to note that the behaviour of actual and minimum c1+c2 observed in the above range of numbers can be observed across any range of even numbers lying anywhere on the number line. Therefore the divergence between these two functions would be continuous and increasing, which implies that number of p1+p2 combinations would correspondingly and continuously increase over the number line.

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Conclusion**Conclusion: Implications of the above analysis for the Goldbach conjecture**

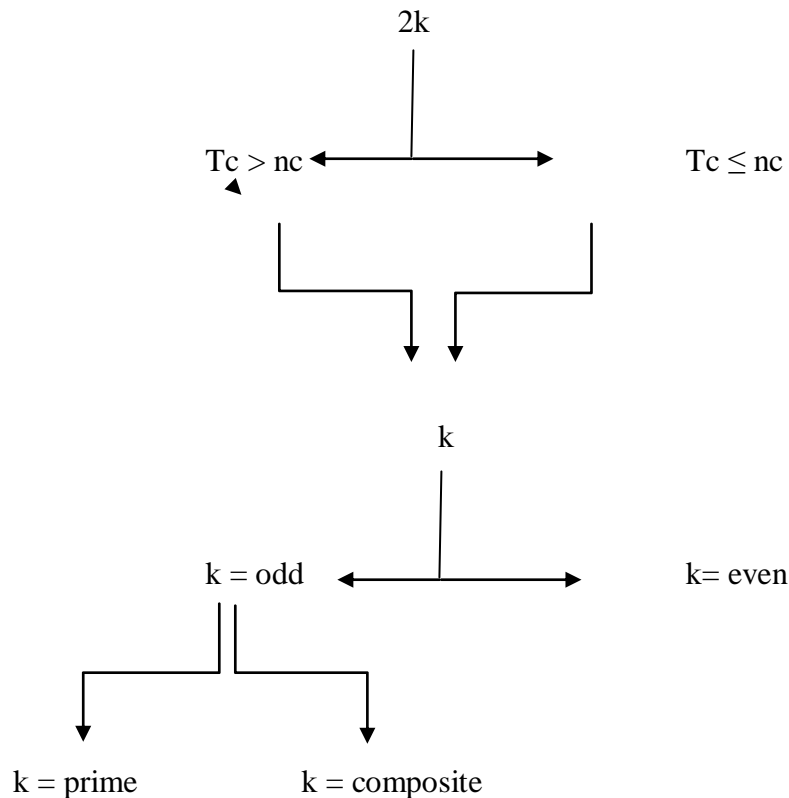
The much celebrated Goldbach conjecture states that all even numbers can be expressed as summation of two prime numbers i.e. $2k = p_1 + p_2$ where p_1 and p_2 are both prime numbers. Threads of the above discussion lead us to the following broad conceptual framework:-

SADN of any natural number, odd or even, may range from 1 to 9, while SADN of prime numbers can be 1,2,4,5,7 or 8. Odd numbers, primes or composites, that are of $6n-1$ type are of SADN (5,2,8) and occur in a cyclic order infinitely along the number line while primes or composites of $6n+1$ type are of SADN(7,4,1) and occur in a cyclic order infinitely along the number line. This allows us to classify odd numbers into three series- S1, S5 and S3 series- of which the S3 series comprises only of composite numbers while the S1 and S5 series comprise of both prime and composite numbers.

If we consider the possible SADN of prime numbers that add upto a particular even number we find that prime combinations that add upto even numbers of SADN (5,2,8) can be found on the S1 series of odd numbers. Prime combinations for even numbers of SADN (7,4,1) can be found on the S5 series while prime combinations for even numbers of SADN (3,6,9) can be found such that one component lies on the S1 series while the other component lies on the S5 series. We define this as the relevant series for even numbers in that relevant series for $2k$ of SADN (5,2,8) is the S1 series, for $2k$ of SADN (7,4,1) is the S5 series and that for $2k$ of SADN (3,6,9) are both the S1 and S5 series. Thereafter we derive the total number of combinations on the relevant series for a given $2k$ which would include all elements of the relevant series, either primes or composites, whose value is less than $2k$ i.e. all elements on the relevant series whose value is less than $2k$ will find a place in one combination or the other. Since these elements can be of either prime or composite in nature, these combinations can be of three types wherein both components of the combination are primes, both components are composites or one component of the combination is prime while the other component is composite i.e. if we denote primes as p_1 and p_2 and composites as c_1 and c_2 , the three combinations for an even number can be in the nature of:-

- i. $p_1 + p_2$ where both components are prime in nature
- ii. $c_1 + c_2$ where both components are composites in nature
- iii. $p_1 + c_1$ (or $p + c$) where one component is prime in nature while the other component is composite.

Based on the relation between total number of acceptable combinations n_{TC} and total number of composites n_c , all even numbers can be classified as those where $n_{TC} > n_c$ and those where $n_{TC} \leq n_c$. Within these two categories a further classification is where mid-point can be either odd or even and if odd then mid-point may be either prime or composite.



For even numbers where $n_{TC} > n_c$ by atleast one, even if all composites are prime-eaters i.e. form part of $p+c$ combinations, there would still be atleast one combination of type p_1+p_2 . This is because if $n_{TC} > n_c$ then considering $n_p = 2n_{TC} - n_c$, $n_p > n_c$ and the difference between n_p and n_c would be $2x(n_{TC} - n_c)$. Since prime elements will form part of either $p+c$ or p_1+p_2 combinations, prime elements remaining after being absorbed by composites to form $p+c$ combinations will form p_1+p_2 combinations i.e. where $n_{TC} > n_c$, n_p would be $> n_c$ and $n_p - n_c = 2x(n_{TC} - n_c)$

In this case, the maximum possible number of $p+c$ combinations would be equal to n_c and number of p_1+p_2 combinations would be $(n_p - n_c)/2$. Therefore for even numbers where $n_{TC} > n_c$ by atleast one, it directly follows that p_1+p_2 combination is existing. For even numbers where midpoint k is prime, $k+k$ will form a combination in which both components of the combination would be prime.

In both the above cases where $n_{TC} > n_c$ and where k is prime, it directly follows that a prime combination that adds up to the given $2k$ is existing and therefore the Goldbach Conjecture holds good in these cases.

In section 12, we have introduced the concepts of actual and minimum c_1+c_2 and discussed the relation between these two in detail. Actual c_1+c_2 refers to the total number of c_1+c_2

combinations derived by following the steps discussed in section 11 and broadly includes c_1+c_2 of following three types:-

- i. c_1+c_2 derived from mid-point k which is applicable only where k is a composite number.
- ii. c_1+c_2 derived from last digit of $2k$ which would be in the nature of a combination where one component would be a multiple of 5.
- iii. c_1+c_2 formed by prime pairs p_1p_2 which satisfy the general condition of $2k/6p_1p_2 \geq 1$

Minimum c_1+c_2 refers to the number of c_1+c_2 that would be identified if all primes would form part of $p+c$ combinations and this would be derived as $n_{TC} - np$ or $nc - T_c$ or $(nc-np)/2$

The discussion in section 12 leads us to the solution that though both actual and minimum c_1+c_2 increase continuously as we move forward along the number line, due to the nature of behaviour of the underlying factors determining the number of actual and minimum c_1+c_2 , the former increases at a rate faster than the latter. Also, at the point at which these functions become relevant in the analysis i.e. where $n_{TC} = nc$ and $n_{TC} < nc$, actual c_1+c_2 is found to be greater in number as compared to the latter. These concepts put together lead us to the conclusion that actual c_1+c_2 is bound to be greater than minimum c_1+c_2 at the point of relevance and thereafter due to the difference in the rate of increase in the two functions, there would be a continuous divergence between them except of two situations as described with reasons in section 12. This divergence would be continuous and infinite in nature.

It has also been shown in section 12 that the difference between minimum c_1+c_2 and actual c_1+c_2 indicates the number of p_1+p_2 combinations for a given $2k$ i.e.

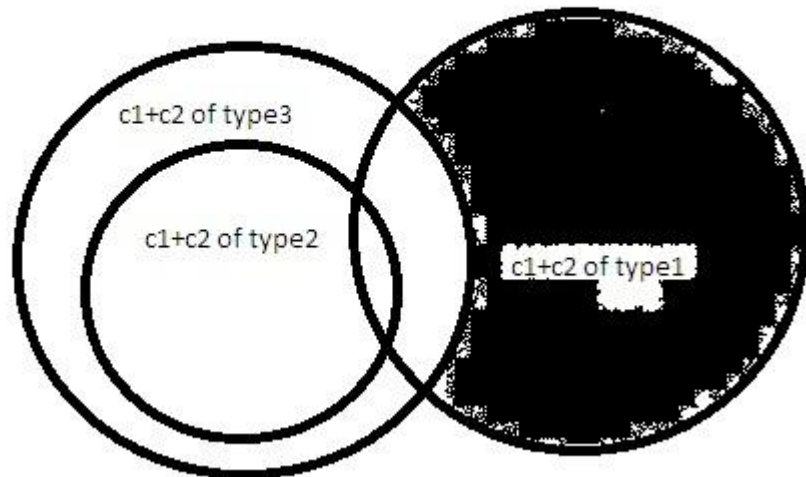
$$np+p = \text{actual } c+c - \text{minimum } c+c$$

considering that the divergence between the actual and minimum c_1+c_2 functions is continuous and the difference between the two is the number of p_1+p_2 combinations, it follows that where $n_{TC} \leq nc$, due to the gap between minimum c_1+c_2 and actual c_1+c_2 ; p_1+p_2 combinations would be identified. This confirms the existence of p_1+p_2 combinations for $2k$ where $n_{TC} \leq nc$ and therefore proves the Goldbach conjecture for those numbers as well.

Another noteworthy point here is the relation between the nature of k and the number of p_1+p_2 combinations i.e. the dependence of number of p_1+p_2 combinations on whether k is prime or composite. Watanabe [12] has observed that in case of $2k$ if k is prime, the number of p_1+p_2 combinations identified would be less as compared to $np+p$ for $2k$ if k is composite in nature. This condition can be derived from the Venn diagram presented in section 11C.

c_1+c_2 combinations of all three types exist for numbers where k is composite in nature while in case of numbers where k is a prime, c_1+c_2 combinations of only types 2 and 3 are existing. This is because c_1+c_2 of type 1 are derived from the composite midpoint (k) by the factors of the midpoint. So when k is a prime, these c_1+c_2 combinations would not be identified at all. Further, it is evident from the Venn diagram referred here that there would be some overlap

between c_1+c_2 of type 1 with c_1+c_2 of types 2 and 3 but some c_1+c_2 of type 1 will be unique in nature.



Reproduced here Diagram 11C.1: Venn diagram representing the relation of subset and superset among the c_1+c_2 combinations of types 1, 2 and 3

The shaded portion of c_1+c_2 of type 1, in the above venn diagram shows the c_1+c_2 of type 1 that will not be identified by methods of type 2 or type 3. Therefore if k is prime; c_1+c_2 combinations to the extent of the shaded portion will not be identified and this will cause the overall number of c_1+c_2 to be less as compared to number of c_1+c_2 identified for $2k$ if k is composite in nature. As number of composites participating in c_1+c_2 combinations decreases, the number of $p+c$ combinations would correspondingly increase. Due to this reason, number of p_1+p_2 combinations for $2k$ where k is prime will be less as compared to number of p_1+p_2 for even number $2k$ where k is composite in nature.

Since even numbers can be of SADN 1 to 9 and the relation between T_c and n_c for all even numbers can either be of $T_c > n_c$ or $T_c \leq n_c$, the above discussion which shows that the Goldbach conjecture is true for both these categories of even numbers, is totally inclusive of all even numbers in general terms and since analysis of every even number is common in methodology but unique in compilation, this analysis apart from being totally inclusive, is also mutually exclusive in nature.

This proves that the Goldbach conjecture which states that all even numbers can be expressed as atleast one combination of two prime numbers holds true for all even numbers, across all categories possible. Additionally this approach based on conceptual framework of SADN proves that the identification of p_1+p_2 combinations which would validate the Goldbach conjecture lies in the identification of c_1+c_2 combinations.

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