Understanding nuclear stability range with $A_s \cong (Z+2.95)^{1.2\pm0.015}$

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Abstract. With reference to our 4G model of final unification, strong coupling constant and strong interaction charge of magnitude $e_s\cong 2.95e$, it is possible to understand the nuclear stability range with a simple power law of the form, $A_s\cong \left(Z+2.95\right)^{1.2\pm0.015}$

Keywords: 4G model of final unification; strong coupling constant; strong elementary charge; nuclear stability range;

1. Introduction

We would like to emphasize the fact that, physics and mathematics associated with fixing of the energy coefficients of semi empirical mass formulae (SEMF) [1,2,3,4] are neither connected with residual strong nuclear force nor connected with strong coupling constant. Since nuclear force is mediated via quarks and gluons, it is necessary and compulsory to study the nuclear binding energy scheme in terms of nuclear coupling constants. In this direction, N. Ghahramany and team members have taken a great initiative in exploring the secrets of nuclear binding energy and magic numbers [5,6]. Very interesting point of their study is that - nuclear binding energy can be understood with two or three terms having a single (variable) energy coefficient. In this direction, based on three unified assumptions connected with gravity and atomic interactions, in a semi empirical approach, we have developed a very simple formula for nuclear binding energy with single energy coefficient having four simple terms [7-14]. Starting from Z=3 to 118, corresponding relations can be expressed as,

$$\begin{aligned} A_s &\cong 2Z + 0.0064Z^2 \\ &\cong \text{Estimated mass number close to proton-neutron mean stability line.} \end{aligned} \tag{1}$$

$$\begin{aligned} B_A &\cong \left\{ A - \left(1 + 0.00189A\sqrt{ZN}\right) - A^{1/3} - \frac{\left(A_s - A\right)^2}{A_s} \right\} \left(B_0 \cong 10.1 \text{ MeV}\right) \\ &\cong \text{Estimated nuclear binding energy} \end{aligned} \tag{2}$$

Here, we would like to appeal that,

1) The two numbers, 0.0064 and 0.00189 can be considered to be associated with fine structure ratio and strong coupling constant [7,9,15].

2) $\left(1+0.00189A\sqrt{ZN}\right)$ can be called as the number of free or unbound nucleons.
3) $A^{1/3}$ can be called as radial factor associated with nucleons.
4) $\frac{\left(A_s-A\right)^2}{A_s}$ can be called as isotopic asymmetric term associated with mean stable mass number.
5) Binding energy coefficient $B_0 \cong 10.1$ MeV seems to be associated with nuclear radius, strong coupling constant and fine structure ratio.

Here, interesting point to be noted is that, in current semi empirical mass formulae, most stable atomic nuclide is associated with maximum binding energy and asymmetry and Coulombic energy constants. Its approximate relation is,

$$Z \cong \frac{A}{2.0+\left(a_c/2a_a\right)A^{2/3}} \cong \frac{A}{2.0+0.0153A^{2/3}} \tag{3}$$

where $a_c \approx 0.71$ MeV and $a_a \approx 23.2$ MeV

One de-merit of this relation is that its input is a 'mass number' whereas in our proposal, input is a 'proton number'. In our recent papers, we have proposed that [7,8,9],

$$0.0153A^{2/3} \approx \sqrt{0.0064A+1}-1 \tag{4}$$

$$Z \cong \frac{A}{1+\sqrt{0.0064A+1}} \tag{5}$$

In this short paper, we try to simplify the relations (1), (3) and (5) further.

2. List of symbols

Newtonian gravitational constant $= G_N$
Electromagnetic gravitational constant $= G_e$
Nuclear gravitational constant $= G_s$
Weak gravitational constant $= G_W$
Fermi's weak coupling constant $= G_F$
Strong coupling constant $= \alpha_s$
Fine structure ratio $= \alpha$
Electroweak fermion $= M_w$
Reduced Planck's constant $= \hbar$
Speed of light $= c$
Elementary charge $= e$
Strong nuclear charge $= e_s$

Mass of proton $= m_p$
Mass of neutron $= m_n$
Mass of electron $= m_e$
Charge radius of nucleus $= R_0$
Proton number $= Z$
Neutron number $= N$
Mass number $= A$
Estimated mass number close to stability $= A_s$
Proton-Neutron stability coefficient $= k_s$
Free nucleon coefficient $= k_f$
Nuclear binding energy coefficient $= B_0$
Ratio of $\ln\left(A_s\right)$ and $\ln(Z) = \beta$

3. Basic assumptions

1) There exists a characteristic electroweak fermion of rest energy, $M_w c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.

2) There exists a strong interaction elementary charge (e_s) in such a way that, its squared ratio with normal elementary charge is close to reciprocal of the strong coupling constant.
3) Each atomic interaction is associated with a characteristic gravitational coupling constant.

Motivation for assumption-3: When mass of any elementary particle is extremely small/negligible compared to macroscopic bodies, highly curved microscopic space-time can be addressed with large gravitational constants and magnitude of elementary gravitational constant seems to increase with decreasing squared mass and increasing interaction range. Based on this logic, we consider the possibility of existence of three large gravitational constants assumed to be associated with electromagnetic, strong and weak interactions. Approximate background relation is, $G_x m_x^2 \approx \hbar c$.

Based on these assumptions, in our recently published paper [14], we have developed a semi empirical scheme for deriving the important results. Readers are encouraged to refer it for further analysis. Quantitatively,

$$\left.\begin{aligned} &G_e \cong 2.374335\times10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\ &G_s \cong 3.329561\times10^{28}\text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\ &G_w \cong 2.909745\times10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\ &G_N \cong 6.679855\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\ &G_F \cong 1.4402105\times10^{-62}\ \text{J.m}^3\\ &\alpha_s \cong 0.1151937\\ &\text{and } e_s \cong 2.9463591e \end{aligned}\right\}$$

Since our model is associated with 3 atomic gravitational constants and one celestial gravitational constant, we call our model as 4G model of Final Unification. Three important results are [14,15,16,17],

$$G_N \cong \frac{16\pi^4}{\alpha^2}\left(\frac{m_e}{m_p}\right)^{14}\left(\frac{\hbar c}{m_p^2}\right) \tag{6}$$

$$\hbar c \cong G_w M_w^2 \cong G_s M_w m_e \tag{7}$$

$$\begin{aligned} &G_F \cong G_w M_w^2 R_w^2\\ &\text{where, } R_w \cong \left(2G_w M_w / c^2\right) \end{aligned} \tag{8}$$

4. Procedure for developing the relation, $A_s \cong (Z+2.95)^{1.2\pm0.015}$

Based on relation (1), starting from Z = 2 to 118, we try to find the ratio of $\frac{\ln(A_s)}{\ln(Z)}$.

$$\frac{\ln(A_s)}{\ln(Z)} \cong \beta \approx (2.0 \text{ to } 1.21) \tag{9}$$

$$A_s \approx Z^{(2.0 \text{ to } 1.21)} \tag{10}$$

Based on relations (8) and (10) and assumption (2), we try to modify relation (10) as follows. Starting from Z= 10 to 118,

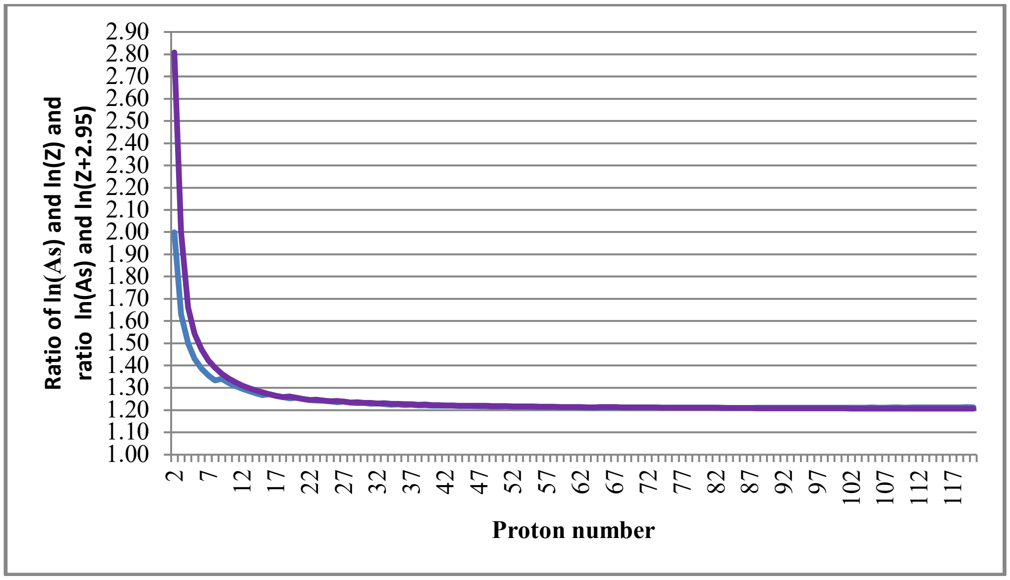
$$A_s \cong \left[Z + \left(\frac{e_s}{e}\right)\right]^{1.20} \cong \left[Z + 2.95\right]^{1.20} \tag{11}$$

where, $\left(\frac{e_s}{e}\right) \cong 2.946359 \cong 2.95$

$$\frac{\ln(A_s)}{\ln[Z + 2.95]} \cong 1.20 \tag{12}$$

See Figure 1 for the comparison of relations (9) and (12). Blue curve represents relations (9) and purple curve represents relation (12). Starting from Z=10 to 118, in a macroscopic view, both curves seems to be identical.

Figure 1: Graphical comparison of relations (9) and (12)



Here interesting point to be noted is that, from Z=83, onwards, A_s estimated with relation (11) is slowly decreasing compared to A_s estimated with relation (1). Here, our aim is,

1) To understand the range of stable atomic nuclides for medium and heavy atoms [18].
2) To understand the lower mass limits of stable super heavy elements [18,19,20].

Considering the medium atomic nuclides and observed super heavy atomic nuclides, we try to consider a stable mass number range with a power factor of $\beta \cong (1.2 \pm 0.015)$. Then,

$$(A_s)_{lower} \cong A_s \cong [Z + 2.95]^{1.185} \tag{13}$$

$$(A_s)_{mean} \cong A_s \cong [Z + 2.95]^{1.20} \tag{14}$$

$$(A_s)_{upper} \cong A_s \cong [Z + 2.95]^{1.215} \tag{15}$$

It needs further study. See Table-1.

Table 1: To estimate the lower and upper mass limits of Z = 9 to 118 and to compare with observed isotopic range

Proton number	Mass number close to mean stability estimated with relation (1)	Mass number close to mean stability estimated with relation (14)	Observed full range of isotopes	Observed range of main isotopes	Stability range estimated with relations (13,15)
9	19	20	14 – 31	18 – 19	19 – 20
10	21	22	16 – 34	20 – 22	21 – 22
11	23	24	18 – 37	22 – 24	23 – 25
12	25	26	19 – 40	24 – 26	25 – 27
13	27	28	19 – 42	26 – 27	27 – 29
14	29	30	22 – 44	28 – 32	29 – 31
15	31	32	24 – 46	31 – 33	31 – 33
16	34	34	26 – 49	32 – 36	33 – 36
17	36	36	28 – 51	35 – 37	35 – 38
18	38	38	30 – 53	36 – 42	37 – 40
19	40	41	32 – 56	39 – 41	39 – 43
20	43	43	34 – 57	40 – 48	41 – 45
21	45	45	37 – 60	44 – 48	43 – 47
22	47	47	38 – 63	44 – 50	45 – 50
23	49	50	40 – 65	48 – 51	47 – 52
24	52	52	42 – 67	50 – 54	50 – 55
25	54	54	44 – 69	52 – 55	52 – 57
26	56	57	45 – 72	54 – 60	54 – 60
27	59	59	47 – 75	56 – 60	56 – 62
28	61	61	48 – 78	58 – 64	58 – 65
29	63	64	52 – 80	63 – 67	61 – 67
30	66	66	54 – 83	64 – 72	63 – 70
31	68	69	56 – 86	66 – 73	65 – 72
32	71	71	58 – 89	68 – 76	67 – 75
33	73	74	73 – 75	73 – 75	70 – 78
34	75	76	65 – 94	72 – 82	72 – 80
35	78	79	66 – 101	79 – 81	74 – 83
36	80	81	69 – 101	78 – 86	77 – 86
37	83	84	71 –102	83 – 87	79 – 88
38	85	86	80 – 93	82 – 90	81 – 91
39	88	89	82 –100	87 – 91	84 – 94
40	90	91	84 –102	88 – 96	86 – 96
41	93	94	86 – 104	90 – 96	88 – 99
42	95	96	88 – 106	92 – 100	91 – 102
43	98	99	90 – 108	95 – 99	93 – 105
44	100	101	92 – 110	96 – 106	96 – 107
45	103	104	95 – 112	99 – 105	98 – 110
46	106	107	97 – 114	100 – 110	101 – 113
47	108	109	99 – 123	105 – 111	103 – 116
48	111	112	100 – 125	106 – 116	105 – 119
49	113	114	104 – 127	111 – 115	108 – 121
50	116	117	107 – 134	112 – 126	110 – 124
51	119	120	110 – 136	121 – 125	113 – 127
52	121	122	109 – 138	120 – 130	115 – 130
53	124	125	115 – 140	123 – 135	118 – 133
54	127	128	113 – 142	124 – 136	120 – 136
55	129	131	116 – 144	131 – 137	123 – 139
56	132	133	119 – 146	130 – 138	125 – 142
57	135	136	125 – 148	137 – 139	128 – 145
58	138	139	129 – 150	134 – 144	130 – 147
59	140	141	132 – 151	141 – 143	133 – 150
60	143	144	134 – 154	142 – 150	135 – 153
61	146	147	137 – 154	145 – 147	138 – 156
62	149	150	137 – 157	144 – 154	141 – 159
63	151	152	139 – 160	150 – 155	143 – 162
64	154	155	142 – 162	148 – 160	146 – 165
65	157	158	145 – 164	157 – 159	148 – 168
66	160	161	147 – 167	154 – 164	151 – 171
67	163	164	150 – 170	163 – 167	153 – 174

68	166	166	150 – 173	160 – 172	156 – 177
69	168	169	151 – 176	167 – 171	159 – 180
70	171	172	154 – 178	166 – 177	161 – 183
71	174	175	155 – 180	173 – 176	164 – 187
72	177	178	168 – 185	172 – 182	167 – 190
73	180	181	169 – 186	177 – 183	169 – 193
74	183	183	170 – 190	180 – 186	172 – 196
75	186	186	172 – 192	185 – 187	175 – 199
76	189	189	172 – 196	184 – 194	177 – 202
77	192	192	171 – 198	188 – 194	180 – 205
78	195	195	174 – 201	190 – 198	183 – 208
79	198	198	175 – 199	195 – 199	185 – 211
80	201	201	177 – 206	194 – 204	188 – 214
81	204	204	184 – 210	203 – 205	191 – 218
82	207	207	185 – 214	202 – 214	193 – 221
83	210	209	189 – 215	207 – 210	196 – 224
84	213	212	193 – 218	208 – 210	199 – 227
85	216	215	194 – 221	209 – 211	201 – 230
86	219	218	213 – 226	210 – 224	204 – 233
87	222	221	215 – 229	212 – 223	207 – 237
88	226	224	217 – 230	223 – 228	209 – 240
89	229	227	219 – 232	225 – 227	212 – 243
90	232	230	221 – 238	227 – 234	215 – 246
91	235	233	223 – 240	229 – 234	218 – 249
92	238	236	226 – 242	232 – 238	220 – 253
93	241	239	228 – 244	235 – 239	223 – 256
94	245	242	232 – 247	238 – 244	226 – 259
95	248	245	232 – 249	241 – 243	229 – 262
96	251	248	236 – 252	242 – 250	232 – 266
97	254	251	236 – 254	245 – 249	234 – 269
98	257	254	240 – 256	248 – 254	237 – 272
99	261	257	240 – 258	252 – 255	240 – 276
100	264	260	241 – 260	252 – 257	243 – 279
101	267	263	248 – 260	256 – 260	245 – 282
102	271	266	250 – 262	253 – 259	248 – 285
103	274	269	254 – 266	254 – 266	251 – 289
104	277	272	253 – 270	261 – 267	254 – 292
105	281	275	255 – 270	262 – 270	257 – 295
106	284	278	258 – 271	265 – 271	259 – 299
107	287	281	260 – 274	267 – 278	262 – 302
108	291	285	263 – 277	269 – 271	265 – 305
109	294	288	266 – 278	274 – 282	268 – 309
110	297	291	267 – 281	279 – 281	271 – 312
111	301	294	272 – 282	279 – 286	274 – 315
112	304	297	277 – 285	283 – 286	277 – 319
113	308	300	278 – 286	278 – 290	279 – 322
114	311	303	284 – 289	284 – 290	282 – 326
115	315	306	287 – 290	287 – 290	285 – 329
116	318	309	290 – 294	290 – 294	288 –332
117	322	312	293 – 294	293 – 294	291 – 336
118	325	316	294 – 295	294 – 295	294 – 339

5. Discussion

1) The coefficients 0.0064 and 0.00189 can be understood with the trial-error relation.

$$\left[\frac{(1-\alpha_s)^n}{2n-1}\right]\alpha \cong \left(k_s \cong 0.006457, k_f \cong 0.0019\right) \tag{16}$$

where $\alpha_s \cong 0.1151937$, $\alpha \cong 0.007297353$ and $n = 1$ and 2.

2) Binding energy coefficient can be understood with the following relations.

$$B_0 \cong \frac{1}{\alpha_s}\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong \left(\frac{e_s^2}{4\pi\varepsilon_0 R_0}\right) \cong 10.08 \text{ MeV} \tag{17}$$

where, $\alpha_s \cong 0.1152$ and $R_0 \cong 1.24$ fermi

In our approach, $\left\{ \frac{1}{\alpha_s} \cong \left(\frac{e_s}{e}\right)^2 \cong \left(\frac{G_s m_p^2}{\hbar c}\right)^2 \text{ and } R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.24 \text{ fermi} \right\}$

Hence,

$$B_0 \cong \frac{1}{2}\left(\frac{e e_s}{4\pi\varepsilon_0 \hbar c}\right)\left(m_p c^2\right) \cong \frac{1}{2}\sqrt{\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)\left(\frac{e_s^2}{4\pi\varepsilon_0 \hbar c}\right)}\left(m_p c^2\right) \cong 10.09 \text{ MeV} \tag{18}$$

where, $\left(\frac{e_s^2}{4\pi\varepsilon_0 \hbar c}\right) \cong 0.06334854$ can be called as 'nuclear fine structure ratio'.

$\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right) \cong \alpha$ is the 'fine structure ratio'.

Based on relation (18), quantitatively, $2B_0 \cong -20.2$ MeV can be considered as the Proton potential and kinetic energy of proton can be expressed as, $\left(2B_0 - B_0\right) \cong +10.1$ MeV.

3) We are working on understanding the origin of the factor, $\beta \cong (1.2 \pm 0.015)$. Based on figure 1 and data presented in table 1, relations (13) to (15) can be recommended for further research to narrow down the best possible range of atomic nuclides starting from Z=10 to 118.

6. Conclusion

Even though our approach is differing with mainstream physical research, as our concepts and results are having some workability and as current research methodology is not showing a way to implement strong coupling constant in low energy nuclear physics, proposed relations (1 to 18) pertaining to nuclear binding energy and stability can be given a partial consideration in understanding and combining high energy and low energy nuclear physical concepts.

Acknowledgments

Author Seshavatharam is indebted to professors Shri M. Nagaphani Sarma, Chairman;, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

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