Inversion of distance and magnetic permeability based on material-independent and lift-off insensitive algorithms using eddy current sensor

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Abstract — Eddy current sensors can be used to test the characteristics and measure the parameters of the conductive samples. As the main obstacle of the multi-frequency eddy current sensor, the lift-off distance affects the effectiveness and accuracy of the measurement. In this paper, a material-independent algorithm has been proposed for the restoration of the lift-off distance when using the multi-frequency eddy current sensor, which is based on the approximation under the thin-skin effect. Experiment testing on the performance of the proposed method is presented. Results show that from the dual-frequency inductance, the lift-off distance could be restored with a maximum error of 0.24 mm for the distance up to 12 mm. Besides, the derived lift-off distance is used for the inversion of the magnetic permeability. Based on a lift-off insensitive inductance (LII) feature, the magnetic permeability of steels can be inversed in an iterative manner, with an error of less than 0.6 % for the lift-off distance up to 12 mm.

Index Terms — Eddy current; lift-off; material-independent; permeability measurement; non-destructive testing.

I. INTRODUCTION

 $E_{\mathrm{the}}^{\mathrm{DDY}}$ current techniques are widely used in interrogating the conductive materials in diverse industrial nondestructive testing (NDT) [1-9]. Owing to its merits (including high adaptability and sensitivity), the eddy current sensor has been used for the testing of the material characteristics, detection of structural integrity, the inspection of surface crack fatigue, and measurement of material properties including thickness and electromagnetic (EM) properties (electrical conductivity and magnetic permeability). The EM properties of materials are directly linked to the phase fractions of alloys [4]. To increase material homogeneity, and thus improve consistency in the mechanical properties, significant advances in materials characterisation would be obtained if the EM property information could be determined online during steel production in a non-destructive and remote manner [4]. However, like other eddy current techniques (including the single-frequency eddy current testing and pulsed eddy current testing), the multi-frequency eddy current (MEC) testing is sensitive to the lift-off distance between the sensor and test piece (particularly for the surface-defected sample), which

To address the lift-off issue, strategies including optimization of the coil structure, signal processing techniques, and novel measurement methods have been proposed. By analysing the signature of receiving coils, Giguere et al. have found a lift-off point of intersection (LOI) feature using the PEC method [16]. The exploited LOI feature does not vary significantly with variation in coupling or increase in probe liftoff. Researchers have further optimized, polished, and implemented the LOI feature for the measurement of materials using PEC techniques [17]. Abu-Nabah has proposed a semiquadratic system to reduce the lift-off effect in high-frequency apparent eddy current conductivity spectroscopy [18]. Moreover, a phase signature has been used by Yin et al and Pinotti et al from the multi-frequency inductance [19-21]. With the proposed phase signature, the inductance change caused by the test sample is less affected by the lift-off distance. However, it has been found the inductance phase is still sensitive to the lift-off around the inflection point (near to the peak frequency feature) [22]. Therefore, an algorithm has been proposed for compensating the lift-off noise of the impedance/inductance phase [22]. However, compared to the impedance or inductance, its phase is less sensitive to the test piece. Therefore, it is necessary to explore an alternative feature from the swept inductance instead of its phase.

Previously, to reduce the lift-off effect, approaches involve planar sensor designs, multi-frequency features (including zerocrossing feature, and peak frequency features), the lift-off invariant phenomenon (conductivity and permeability invariant phenomena), and the phase signature [22-30]. In this paper, an alternative method based on the material-independent phase term under the eddy-current thin-skin effect has been proposed, which has improved and extended the previous measurement range of the lift-off distance (from 6 mm to 12 mm) without sacrificing the accuracy. Experiment testing on different magnetic steels has been carried out. The lift-off distance can be restored from the inductance of dual high frequencies, which is shown independent of different materials. Moreover, an identical lift-off insensitive inductance (LII) feature has been found on the swept frequency inductance of different magnetic

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could influence the accuracy of the measurement [10-15].

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steels. That is, multi-frequency inductance curves of different lift-offs nearly intersect on one point. By referring to the restored lift-off and the corresponding frequency of LII, the relative magnetic permeability of ferromagnetic steels has been retrieved.

II. MATERIAL-INDEPENDENT ALGORITHM AND LIFT-OFF INSENSITIVE INDUCTANCE FEATURE

A. Original analytical formulation





As shown in Fig. 1, the eddy current sensor is composed of three co-axial coil windings. Since the magnetic flux generated from the transmitter diverges for a considerable distance (the gap, g between the transmitter and receiver, as shown in Fig. 1 and Table 1), particularly for a relatively large lift-off distance between the sensor and sample, the radius of the receiver and reference coil is designed larger than that of the transmitter to fully catch the reflected magnetic flux.

In recent years, the Dodd-Deeds formula [31] has been widely used for the inductance analysis of the air-core coil above the conductive plates [32-37]. For the designed eddy current sensor shown in Fig. 1, the inductance change caused by the test magnetic steel is given by the expression.

$$L_1 = K \int_0^\infty M_1 e^{-2\alpha l_0} \Phi d\alpha \tag{1}$$

$$L_2 = K \int_0^\infty M_2 \, e^{-2\alpha l_0} \Phi d\alpha \tag{2}$$

 L_1 and L_2 are the inductance change from the transmitterreceiver and transmitter-reference sensing pairs. K is a constant related to the cross-section of the coil.

$$K = \frac{\pi \mu_0 N^2 (r_{r_2} + r_{r_1})}{2h^2 (r_{r_2} - r_{r_1}) (r_{t_2} - r_{t_1})}$$
(3)

In (3), μ_0 is the magnetic permeability of the free space (or vacuum magnetic permeability). Three coils have identical coil heights h, winding turns N, and separation distance g. The inner and outer radius of the transmitter coil is r_{t1} and r_{t2} . The receiver and reference coil have the same inner and outer radius r_{r1} and r_{r2} . The integrand consists of the coil-dependent magnitude part (M_1 and M_2) with the lift-off decay term ($e^{-2\alpha l_0}$), and the material-dependent phase term (Φ).

$$M_{1} = \frac{P_{t}P_{r}}{\alpha^{6}}e^{-\alpha(h+g)}(e^{-\alpha h} - 1)^{2}$$
(4)

$$M_{2} = \frac{P_{t}P_{r}}{\alpha^{6}}e^{-3\alpha(h+g)}(e^{-\alpha h} - 1)^{2}$$
(5)

where P_t and P_r are the integral of the Bessel series; α is related to the wavenumber of the incident transverse electric

(TE) plane EM wave (in the free space) [31,37].

$$P_{t} = \int_{\alpha r_{t1}}^{\alpha r_{t2}} \tau J_{1}(\tau) d\tau$$
(6)

$$P_{\rm r} = \int_{\alpha r_{\rm r1}}^{\alpha r_{\rm r2}} \tau J_1(\tau) d\tau \tag{7}$$

 J_{1} denotes the first-order Bessel function of the first kind.

The material-dependent phase term (Φ) is defined as the real part of a complex fractional function.

$$\Phi = \operatorname{Re}\left(\frac{(\alpha_1 + \mu_1 \alpha)(\alpha_1 - \mu_1 \alpha) - (\alpha_1 + \mu_1 \alpha)(\alpha_1 - \mu_1 \alpha)e^{2\alpha_1 c}}{-(\alpha_1 - \mu_1 \alpha)(\alpha_1 - \mu_1 \alpha) + (\alpha_1 + \mu_1 \alpha)(\alpha_1 + \mu_1 \alpha)e^{2\alpha_1 c}}\right)$$
(8)

 μ_1 and c are the relative magnetic permeability and thickness of the magnetic plate. α_1 is the square root of a complex term, which is related to the wavenumber of the transverse electric (TE) plane EM wave (in the steel) [31,38].

$$\alpha_1 = \sqrt{\alpha^2 + j2\pi\sigma\mu_1\mu_0}f \tag{9}$$

f is the working frequency of the exciting current.

B. Integration version of material-independent algorithm – inversion of lift-off distance

Generally, for a magnetic steel slab (unlike the non-magnetic materials), the eddy current is restrained around the surface of the sample even under the working frequency of 100 Hz (referring to the skin depth formula $(\pi\sigma\mu_0 f)^{-1/2}$). Owing to the eddy current skin effect, the magnetic slab can be treated as a conductive half-space. Therefore, $\text{Re}(e^{2\alpha_1 c}) \gg 1$ satisfies. Besides, Φ in (8) can be expressed as

$$\Phi = \operatorname{Re}\left(\frac{\mu_1 \alpha - \alpha_1}{\mu_1 \alpha + \alpha_1}\right) \tag{10}$$

For an eddy current sensor with moderate size (e.g. Table 1) the effective range of α is limited (according to Fig. 3, from 0 to 180 for the sensor in Table 1). That is, $2\pi\sigma\mu_1\mu_0 f \gg \alpha^2$ for the whole frequency range. Thus, (10) can be simplified as



Fig. 2 Φ and simplified linear function T under high working frequencies

As shown in Fig. 2, for the relatively high working frequencies (particularly frequencies exceed 400 kHz), it has been found Φ can be approximated with a linear function T.

$$T = \sqrt{\frac{\mu_1}{\pi \sigma \mu_0 f}} \alpha - 1 \tag{12}$$

In (12), the slope is related to material-dependent parameters μ_1 , σ , and the working frequency f. Thus, the inductance

formula in (1) and (2) can be expressed as two terms, with one dependent on the material and working frequency.

$$L_1 = K \sqrt{\frac{\mu_1}{\pi \sigma \mu_0 f}} \int_0^\infty \alpha M_1 e^{-2\alpha l_0} \, d\alpha - K \int_0^\infty M_1 e^{-2\alpha l_0} \, d\alpha \qquad (13)$$

$$L_2 = K \sqrt{\frac{\mu_1}{\pi \sigma \mu_0 f}} \int_0^\infty \alpha M_2 e^{-2\alpha l_0} \, d\alpha - K \int_0^\infty M_2 e^{-2\alpha l_0} \, d\alpha \qquad (14)$$

From (13) and (14), the material-dependent term (first term of right sides) can be eliminated as,

$$Y_2(L_1 + KF_1) - Y_1(L_2 + KF_2) = 0$$
(15)

In (15), F_1 , F_2 , Y_1 , and Y_2 are only dependent on the parameter of the sensor and lift-off variation l_0 .

$$F_1 = \int_0^\infty M_1 e^{-2\alpha l_0} \, d\alpha \tag{16}$$

$$F_2 = \int_0^\infty M_2 e^{-2\alpha l_0} \, d\alpha \tag{17}$$

$$Y_1 = \int_0^\infty \alpha M_1 e^{-2\alpha l_0} \, d\alpha \tag{18}$$

$$Y_2 = \int_0^\infty \alpha M_2 e^{-2\alpha I_0} \, d\alpha \tag{19}$$

Theoretically, with the measured inductance from transmitter-receiver L_1 and transmitter-reference L_2 sensing coils, the solution of the lift-off variation can be derived by solving the integration equation (15). However, due to the calculation burden caused by the integral from (16) to (19), the processing time is considerable and cannot be applied for the real-time measurement. Besides, the exponential term is illposed to l_0 , which could influence the convergence and accuracy.

C. Simplified material-independent algorithm

In Fig. 3, to simplify the integrand and work out the integral, it has been found M_2 in F_2 and Y_2 can be well estimated as a sinusoidal function $e^{-\alpha(4h)} \sin^2(\alpha\pi/2\alpha_0)$. α_0 controls the peak of the sinusoidal function, which is related to the parameter of the sensor (including the radius, coil height h, and separation distance g, according to equation 5)



Fig. 3 Estimation of M2 with a sinusoidal function

Since M_1 cannot be perfectly fitted by the sinusoidal function, only the inductance from the transmitter-reference sensing winding - L_2 is considered for the simplified algorithm of lift-off reconstruction.

From the approximation of M_2 using the sinusoidal function,

the inductance change from the transmitter-reference sensing winding - L_2 becomes,

$$L_{2}(f) = KG \int_{0}^{2\alpha_{0}} e^{-\alpha(2l_{0}+4h+3g)} \sin^{2}\left(\frac{\alpha\pi}{2\alpha_{0}}\right) \left(\sqrt{\frac{\mu_{1}}{\pi\sigma\mu_{0}f}\alpha-1}\right) d\alpha$$
$$= K\left(\sqrt{\frac{\mu_{1}}{\pi\sigma\mu_{0}f}}Y_{2} - F_{2}\right)$$
(20)

In (20), G is the normalization term between M_2 and the sinusoidal function.

$$G = \frac{P_t P_r}{\alpha_0^6} e^{\alpha h} (e^{-\alpha h} - 1)^2$$
(21)

Thus, the simplified version of (17) and (19) for (20) becomes,

$$F_2 = G \int_0^{2\alpha_0} e^{-\alpha(2l_0 + 4h + 3g)} \sin^2\left(\frac{\alpha\pi}{2\alpha_0}\right) d\alpha$$
(22)

$$Y_2 = G \int_0^{2\alpha_0} \alpha e^{-\alpha(2l_0 + 2h + g)} \sin^2\left(\frac{\alpha\pi}{2\alpha_0}\right) d\alpha$$
(23)

To eliminate the material-dependent term $\sqrt{\frac{\mu_1}{\pi\sigma\mu_0 f}} Y_2$, the dual working frequencies f_1 and f_2 are considered.

$$\sqrt{f_2}L_2(f_2) - \sqrt{f_1}L_2(f_1) = K(\sqrt{f_1} - \sqrt{f_2})F_2$$
(24)

In (22) and (23), assign,

$$X_2 = \alpha_0(2l_0 + 4h + 3g)$$
 (25)

Then, after the integration, F₂ becomes,

$$F_2 = \frac{\pi^2 \alpha_0 G(1 - e^{-2X_2})}{2X_2 (X_2^2 + \pi^2)}$$
(26)

Since $X_2 \gg 1$, the exponential term $e^{-2X_2} \ll 1$. Thus, F_2 becomes,

$$F_2 = \frac{\pi^2 \alpha_0 G}{2X_2 (X_2^2 + \pi^2)}$$
(27)

Substitute (27) into (24), a simplified equation (without integration) for X_2 can be derived.

$$\frac{\pi^2 \alpha_0 G}{2X_2 (X_2^2 + \pi^2)} = \frac{\sqrt{f_2} L_2(f_2) - \sqrt{f_1} L_2(f_1)}{K (\sqrt{f_1} - \sqrt{f_2})}$$
(28)

Assume the solution of X_2 in the simplified equation (28) is X_s , then the lift-off distance can be derived according to equation (2).

$$l_0 = \frac{X_s}{2\alpha_0} - \frac{4h + 3g}{2}$$
(29)

D. Lift-off insensitive inductance feature - measurement of relative magnetic permeability







Fig. 4 Analytical inductance change from the transmitter-receiver sensing pair versus lift-off distance under different working frequencies a) DP 600 b) DP 800 c) Duplex stainless steel (in Table 2)

Since the transmitter-receiver sensing pair is closer and sensitive to the test material (compared to transmitterreference), its inductance value is used for the measurement of the magnetic permeability of the magnetic slab. As can be observed in Fig. 4, the inductance change L_1 is found insensitive to the lift-off variation under a certain working frequency. Besides, the corresponding inductance is independent of different materials. For the sensor listed in Table 1, the corresponding inductance, termed as the lift-off insensitive inductance, is -3.97×10^{-10} H. Therefore, the magnetic permeability can be measured by combining the derived lift-off distance from the transmitter-reference coil via (29) with the corresponded frequency at the lift-off insensitive inductance. The relative magnetic permeability can be restored from iterative loops [24].

$$\mu_1 = \Delta \mu_1 + \mu_r \tag{30}$$

In (30), μ_r is the reference relative magnetic permeability. The change of the relative permeability in each iterative loop is,

$$\Delta \mu_1 = J^{-1}(L_0 - L_1(\mu_r, f_{L_0}))$$
(31)

 L_1 is the analytical inductance with the input of μ_r and the corresponded frequency f_{L_0} at L_0 . That is, the input frequency of L_1 can be referred from the frequency (according to the swept-frequency inductance spectrum) closest to the lift-off insensitive inductance L_0 . J is the inductance sensitivity around μ_r .

$$J = \frac{L_1(\mu_r, f_{L_0}) - L_1(\mu_r - \lambda \mu_r, f_{L_0})}{\lambda \mu_r}$$
(32)

 λ in (32) is a residual value, which is defined as 0.01 for the restoration loop.

III. EXPERIMENT

The predictions algorithms in (29) and (30) have been tested with measurements on three different materials of magnetic steels. The eddy current sensor is designed as three co-axial coil windings (structure shown in Fig. 1). The transmitter is enwound between the receiver (bottom) coil and reference (top) coil with an identical separation gap g. As listed in Table 1, the radius of the receiver and the reference coil is designed larger than that of the transmitter coil to fully receive the reflected magnetic flux from the test piece. The measured inductance from the transmitter-reference sensing pair is used for the inversion of the lift-off distance via the simplified materialindependent algorithm in (28) and (29). Besides, the corresponding frequency of the measured inductance (from transmitter-receiver sensing pair) closest to the benchmark (liftoff insensitive inductance) is used for the inversion of the magnetic permeability of the samples.

TABLE I Parameters of the eddy current sensor



Fig. 5 Measurement system

Δ

$$Z_s = R_s + R_c + j2\pi f L_s + Z_c$$
(33)

$$Z_a = R_c + j2\pi f L_a + Z_c$$
(34)

$$L = L_{s} - L_{a} = \frac{Im(Z - Z_{a})}{2\pi f}$$
 (35)

In Fig. 5, the designed eddy current sensor is connected to the impedance analyser. The measured inductance data is exported to the PC via the USB interface cable. In equations from (33) to (35), Z_s and Z_a denote the measured impedance with and without (in the free space) the sample, respectively; L_s and L_a are the experimental inductance with and without (in the free space) the sample, respectively; R_s and R_c are the mutual resistance (real part of the impedance) caused by the sample and coils, respectively; By using the inductance change equation in (35) (which is corresponding to the analytical equations in 1 and 2), the ambient noise signals (including the mutual resistance R_s and R_c , or potentially high-frequency parasitic impedance of the coils Z_c [35]) are excluded from the experimental inductance change. Besides, the working frequency of the

impedance analyser is from 1 kHz to 5 MHz, which is much lower than the resonance frequency. Consequently, the proximity effect (with parasitic capacitance) barely exists during the measurement. Frequencies lower than 1 kHz will result in a relatively poor Signal-to-Noise Ratio (SNR).

As listed in Table 2, the magnetic steels are (ferrite-austenite) alloys with different ferrite fractions. Since the thickness of the steel slab is much larger than the skin depth, the samples can be treated as the half-space (and the skin effect exists) under the whole frequency range. Therefore, the phase term Φ in (8) can be approximated by (10), which is independent of the sample thickness.

TABLE II
PARAMETERS OF MAGNETIC STEELS

	DP 600	DP 800	Duplex stainless steel	
Thickness (mm) c	4.0	4.3	6.45	
Relative permeability μ_1	222	144	45	
Electrical conductivity σ (MS/m)	4.13	3.80	1.30	

IV. RESULT AND DISCUSSION

A. Swept-frequency inductance

The swept-frequency inductance change (due to the test steel) from both the transmitter-receiver and transmitterreference sensing pairs are shown in Fig. 6 a) and b). As the frequency increases, the inductance curve begins with a positive value then gradually decreases to a negative one. Besides, the inductance curve of one sample with different lift-off distances will converge at one point, where the measured inductance is shown less affected by the lift-off distance. As the frequency further increases, the inductance curve of one sample with different lift-off variations will gradually diverge. However, the inductance curve for one lift-off distance but different samples will gradually converge, where the inductance is shown sensitive to the lift-off distance but less sensitive to the test sample due to the restrained eddy current under the skin effect. Thus, the lift-off distance of the sensor is inversed from the inductance under high working frequencies.





Fig. 6 Swept-frequency inductance for the sensor above the magnetic slab with lift-off distance of 2, 4, and 6 mm a) transmitter-receiver sensing pair b) transmitter-reference sensing pair

B. Inversion of lift-off distance using simplified materialindependent algorithm



Fig. 7 Inductance for the sensor above the magnetic slab with different lift-off distances under the working frequency of 5 MHz a) transmitter-receiver sensing pair b) transmitter-reference sensing pair

Fig. 7 shows the inductance change from both the transmitter-receiver and transmitter-reference sensing pairs at different lift-off distances under the working frequency of 5 MHz. It can be observed that, due to the significantly restrained eddy current under the skin effect, the inductance of different lift-off distances is less influenced by the test steel, particularly when the lift-off distance reaches 12 mm. As shown in Fig. 2,

under the high working frequencies, the material-dependent phase term Φ approaches -1. Thus, with the increased working frequency, the test steel gradually becomes a pure inductive material. Besides, the inductance is less affected by the parameters of the material, as can be referred to equation (12).



Fig. 8 Inversion of the lift-off distance from the inductance of different dualfrequency combinations when the test piece is a) DP 600 b) DP 600 c) Duplex stainless steel

As only the coil-dependent magnitude term M_2 for transmitter-reference sensing pair can be well fitted by the sinusoidal function, the lift-off distance is inversed from L_2 using the simplified algorithm in (28) and (29). As shown in Fig. 8, with the inductance change of dual working frequencies, the inversed lift-off distance is shown less affected by different samples, particularly for the dual-frequency combinations of





Fig. 9 Error of the inversion for the lift-off distance from the inductance of different dual-frequency combinations when the test piece is a) DP 600 b) DP 600 c) Duplex stainless steel

In Fig. 9, the lift-off inversion is shown to be more affected by different samples under low dual-frequency combinations, especially for the dual-frequency combination of 50.06 kHz – 112.05 kHz, where the phase term Φ in (12) (Fig. 2) is more influenced by the parameter μ_1 and σ . Considering the accuracy, the inversed lift-off under the dual-frequency combinations of 3.16 MHz – 5.00 MHz is selected for the further inversion of the magnetic permeability (with a maximum error of 0.24 mm).





Fig. 10 Inductance for the transmitter-receiver sensing pair above the magnetic slab versus lift-off distance under different working frequencies



Fig. 11 a) Inversion of the relative magnetic permeability versus lift-off distance under the corresponded frequency of the lift-off insensitive inductance b) error of the inversion

As the transmitter-receiver sensing pair is closer and more sensitive to the test steel, the inductance L_1 is used for the inversion of the relative magnetic permeability. Fig. 10 depicts L_1 versus lift-off distances under different working frequencies. It can be observed that for one sample, the inductance change is shown to be less affected by the lift-off distance, termed as the lift-off insensitive inductance. Moreover, inductance curves of different samples share the same lift-off insensitive inductance L_0 (around -3.97×10^{-10} H). From the swept-

frequency inductance in Fig. 6 a), the corresponding frequency of the inductance closest to the lift-off insensitive inductance L_0 are 46.63, 32.87, and 27.78 kHz for DP 600, DP 800, and Duplex stainless steel.

Parameters including the inversed lift-off distance, lift-off insensitive inductance L_0 and its corresponded frequency are used for the inversion of the relative magnetic permeability of the steel using iterative equations from (30) to (32). In Fig. 11, with the inversed lift-off distance (material-independent), lift-off insensitive inductance benchmark ($L_0 = -3.972 \times 10^{-1}$ H – data tips in in Fig. 10), and corresponded frequencies f_{L_0} (46.63, 32.87, 27,78 kHz for DP 600, DP 800, and Duplex stainless steel – legend of Fig. 10 or data tips in Fig. 6), the relative magnetic permeability can be accurately restored with a maximum error of 0.6 % at the lift-off distance of 12 mm. Besides, the error is nearly independent of different ferrite-austinite steels.

V. CONCLUSION

In this paper, two algorithms have been proposed for the inversion of both the lift-off distance and magnetic permeability of the steel. For the inversion of the lift-off distance, a simplified algorithm (without redundant integration) has been explored, which therefore can be used for the online measurement. With the inductance of transmitter-reference sensing pair, the inversed lift-off distance is shown to be material-independent. From the experiments on three different magnetic alloys, the lift-off distance is verified can accurately restore the lift-off distance, with a maximum error of 0.24 mm at 12 mm. Moreover, the restored lift-off distance is implemented for the inversion of magnetic permeability using iterative algorithms. Based on the lift-off insensitive inductance feature (which is independent of the test sample and less affected by the lift-off) of the transmitter-receiver sensing pair (closer and sensitive to the test piece) and its corresponded frequency (sensitive to sample and insensitive to lift-off distance), the error of the restored permeability can be controlled within 0.6 % for the lift-off distance up to 12 mm.

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