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Extension of the Stoney Equation for a Taiko Wafer (Si and SiC)

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Abstract

An extension of the Stoney formula for the case of a back side metallized 8" silicon taiko wafer has been developed, in the elastic regime, within the frame of the theory of elasticity. A good correlation between the calculated warpage, determined by the stress released by a given back side metallization (BSM), and the corresponding experimental warpages of the same thick metal layers deposited on an 8" silicon taiko wafer provides evidences of the correctness of the developed theory. This development suggests the possibility to extend this approach to the case of 8" taiko wafers based on a wide band gap semiconductor such as silicon carbide (SiC).

Keywords: Taiko Wafer, Warpage, Silicon, Silicon Carbide, Stoney Formula, Backside Metallization, Ultrathin Chips.

Introduction

Developments in vertical power MOSFETs devices based on silicon [1] and silicon carbide [2] require, specifically in the automotive sector, a significant cutdown of the switching losses as the technology evolves from one generation to another [3] [4]. A strategy to achieve this goal consists in modifying some key parameters which affect the operations of the device. For example, an improvement of the heat dissipation, along with a lowering of the drain-source on resistance (Rdson) of the device, allows to improve their performances. A large contribution to Rdson and the thermal resistance results from the thickness of the substrate, since the electrical current goes through it. To reduce this quantity a method consists in thinning the semiconductor wafer. Moreover, to improve the thermal coupling as well as the electrical and mechanical properties, an appropriate back side metallization (BSM) is usually required in high power devices [5]. A BSM consists of a multiplicity of metal layers with thicknesses ranging from hundreds of nanometers to microns However, the combination of thinned semiconductor wafer with a metalized multilayer determines that the whole wafer undergoes mechanical stress conditions which results in the warpage of the structure hindering, sometimes severely, the performances and the following manufacturing processes of the devices.

In general, gaining a control on the warpage determined by the BSM residual stress can benefit the whole semiconductor industry and disclose future developments. And indeed, the trend of the increase of the wafer size, which is also occurring in the field of SiC [6] [7], requires a more fundamental control on the warpage effects in the whole wafer as well as an understanding of the resulting warpage in the singled die.
Moreover, it is known that with the increase of the size, the handling of a thinned wafer becomes more and more critical. For this reason, in 2008 DISCO proposed the patented taiko process [8] [9] [10], which consists in a back-grinding method that leaves an annular region around the whole wafer (see figure 1) . This solution, which is now a standard, allows an easier handling of the wafer itself and a reduction of the warpage.
Indeed, because of the intrinsic structure of the taiko wafer, for a given metal stress and metal film thickness, the resulting warpage of a BSM thinned wafer is mitigated with respect to the case of a canonical (flat) BSM thinned one (see figure 1b).

According to the literature, extended reviews on the use and applications of ultrathin chips (UTC) have been reported , with a focus ranging from UTC for flexible electronics [11] to conventional ones [12] which included also power electronics [13] [14] [15] [16].
Moreover, an attempt to extend the Stoney formula for a non-uniform wafer thickness has been considered in [17] for the case of a wafers having a slight step change in the peripheral region. However, the considered step is treated as a small perturbation with respect to the thickness of the wafer. Therefore, this work does not provide a general solution for the case of a taiko wafer.

Approaches based on the Finite Elements Analysis (FEA) have been also pursued, but for the case of flat canonical substrates by employing standard tools for structural analysis such as ANSYS [18] [19] and ABAQUS [20]. From these works it emerges that it is difficult to predict the final shape of the warped flat substrate. Indeed, the numerical and stable solution usually is not the physical one, unless a small perturbative displacement which will lead to the curl shape [18] is applied. Moreover, simulations have been also reported for the case of a flat and patterned substrate [21]
However, and at the best of our knowledge, no extensive essays have been reported for the case of the taiko wafer, in the general case. It is hence unknown how the change of the warpage occurs for these kinds of wafers. In this work a rational and analytical description, developed according to the theory of the elasticity of the resulting warpage, induced by a stressing metal thin film, in the elastic or linear regime, in a taiko wafer, is provided. The calculated warpage for a BSM thinned taiko wafer is benchmarked and

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assessed according to experimental results gained investigating thick BSM 8” Si taiko wafers. Finally, the utility of a modified Stoney formula is proved and extended for the case of an 8” SiC taiko wafer.

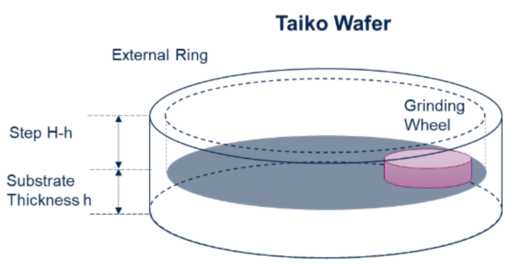

Figure 1a. Schematic of the taiko back grinding process.

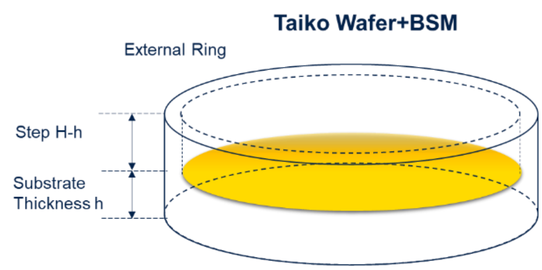

Figure 1b. Schematic of the taiko wafer with a back side metal (BSM) layer.

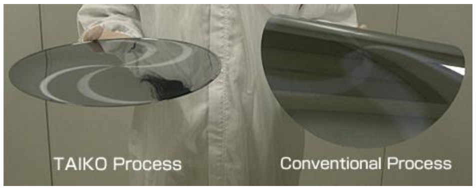

Figure 1c. Taiko process vs conventional process [9] (12” wafers)

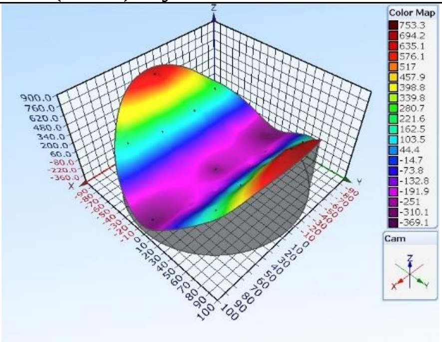

Figure 1d. Typical curl-shape observed, in a thick BSM stressed 8” taiko wafer, by means of warpage measurements.

1) Background: the Stoney formula

Internal stresses in a thin BSM film deposited onto the back side of a semiconducting substrate cause the whole set of film and substrate pair to warp until the thermo-mechanical equilibrium is reached. This condition is reached when both the net forces and the resulting bending moments acting on the system reach a null value, at the given operating temperature T. A disk-shaped wafer substrate allows a simplification of the theory. Indeed, if the substrate thickness h is constant and negligible with respect to the diameter D of the wafer, whenever the thickness of the film, t_f, is uniform and small compared to that of the substrate, the average film stress, σ_f can be determined from the curvature of the elastically deformed coated substrate. In 1909 Stoney published a simple formula [22] which provides a straightforward and rational relationship between the measured average curvature κ and the average film stress σ_f:

$$\sigma_f = \frac{Eh^2}{6t_f(1-\sigma)}\kappa\,, \qquad (1.1)$$

where E, and σ are the Young’s modulus and Poisson’s ratio of the substrate. Equation (1.1) constitutes the standard formula which is exploited daily in manufacturing sites, labs and plentifully reported in the literature to infer how film stresses relates with the experimental measurement of the system’s curvature (2). After more than a hundred years [23] the Stoney’s formula is still the benchmark or reference for the thin film induced stress investigation.

However, eq. 1.1 is not valid for the case of a taiko wafer, because it does not consider the presence of the ring. In this work steps to rationally modify eq. 1.1 for the case of a taiko wafer have been considered. The premises of these developments are the following. Though the abrasive removal of silicon or SiC during the back-grinding process leads to a layer of damaged silicon crystal structure which has high compressive stress [24], we will treat the taiko wafer as an idealized, symmetry driven structure, whose influence on the warpage is determined by the presence of the ring. As in the case of the flat wafer, after thinning we consider that the wafer has approximately a constant curvature. Another important aspect is the influence of the gravitational force. In fact, as the final thickness decreases, the wafer becomes progressively less able to support its weight. However, corrections to the warpage can be applied in order to discriminate the influence of the stress

From an instructive point of view the Stoney formula is usually recovered in textbooks in the context of the classical Euler-Bernoulli beam theory [25]. If we consider the cross section of an ultrathin wafer as a beam, its curvature κ is determined by the ratio between the bending moment M_s applied to the substrate, which is generated by the stressing metal film acting on the wafer, and the flexural rigidity EI_s

$$\kappa = \frac{M_s}{EI_s} \quad (1.2)$$

where E is the modulus of elasticity of the substrate and I_s is the moment of inertia of the section of the beam with respect to the neutral axis, which results from the intersection between the neutral plane of the wafer and the plane containing the examined cross section. The bending moment M_s is determined by the product of the force applied to the substrate F_s, at the interface between the film and the substrate, times the distance $\frac{h}{2}$ between the surface of the substrate and the neutral axis, that in this case matches the gravity center axis. Because of the mechanical equilibrium, the resulting forces, acting on the film F_f and on the substrate F_s, as well as the bending moments, on the film M_f and in the substrate M_s, are null: that is $F_f + F_s = 0$, $M_f + M_s = 0$. This means that $M_s = -\sigma_f t_f \frac{h}{2}$. Finally, the moment of inertia of a section of the beam whose height is h and an infinitesimal width dw, perpendicular to the height, with respect to the neutral axis, located at $\frac{h}{2}$ from the interface, is equal to $\frac{h^3}{12}$. By combining these quantities, it is easy to recover eq. 1.1.

In the case of a taiko wafer the neutral plane is shifted upward, because of the annular region. This has two outcomes. The first occurs on the bending moments, which changes because of the new position of the neutral axis. The second is that the moment of inertia of the section with respect to the neutral axis will increase. The modified Stoney formula, can be hence written as:

$$\kappa_{taiko} = \frac{M_{taiko}}{EI_{taiko}} \quad (1.3)$$

In order to determine the quantities that appear in eq. 1.3, a more rigorous point of view must be adopted and the whole dissertation must be developed within the theory of elasticity [26], [27].

2) Taiko wafer According to the theory of elasticity

According to the theory of elasticity, (see Landau-Lifshitz, [26]) , if the taiko wafer is not subject to the force of gravity, the vertical displacement ζ with respect to the neutral surface satisfies the biharmonic equation $\Delta^2\zeta = 0$, which holds both in the ring region as well as in the thinned wafer region.

In the ring region, $R_{int} < r \leq R_{ext}$, the simplest solution of the biharmonic equation, that does not consider the dependence on the angle θ is:

$$\zeta_{ext} = ar^2 + b + cr^2 ln\left(\frac{r}{R_{ext}}\right) + dln\left(\frac{r}{R_{ext}}\right) \quad (2.1),$$

where a, b, c and d are four coefficients that must be determined from the boundary conditions.
Whereas in the internal and thinned substrate region, $0 \leq r \leq R_{int}$, the solution of the biharmonic equation is

$$\zeta_{int} = a_{sub} r^2 + b_{sub} \quad (2.2),$$

with a_{sub} and b_{sub} two coefficients to be determined, such that to match with eq. 2.1.

In general, the biharmonic equation has the Michell's solution [28], which takes into account also of the dependence on the polar angle θ. In the case of a taiko wafer, in the experimental practice it results that these wafers warp preferentially in one direction and less in the perpendicular one, for the case of a thick BSM. In the hypothesis that the profile of warpage is parabolic, a simple mathematical solution that takes into account of the dependence on the angle ϑ, in the internal region that can be considered is the following:

$$\zeta_{int}(r, \theta) = r^2 \left(\frac{a_{sub} + a_{sub}^{\perp}}{2} + \frac{a_{sub} - a_{sub}^{\perp}}{2} cos(2\theta) \right) + b_{sub} \quad (2.3).$$

where a_{sub} and $a_{sub}^{\perp}$ are related with the curvature in the preferential warpage direction and in the perpendicular direction, respectively.
On the other hand, in the ring region, a solution which considers the dependence on the angle θ, compatible with eq. 2.3 is:

$$\zeta_{ext}(r, \theta) = \left(\frac{a + a^{\perp}}{2} + \frac{a - a^{\perp}}{2} cos(2\theta) \right) r^2 + b + cr^2 ln\left(\frac{r}{R_{ext}}\right) + dln\left(\frac{r}{R_{ext}}\right) \quad (2.4),$$

where a and $a^{\perp}$ are related to the curvatures in the preferential warpage direction and in the perpendicular direction of the ring region, respectively.
By considering the average value of equation (2.3) and (2.4) along the ϑ angles, the resulting equations are still solutions of the biharmonic equation, but of the simplest form:

$$\begin{cases} \langle \zeta_{int}(r, \theta) \rangle = \left(\frac{a_{sub} + a_{sub}^{\perp}}{2} \right) r^2 + b_{sub} \\ \langle \zeta_{ext}(r, \theta) \rangle = \left(\frac{a + a^{\perp}}{2} \right) r^2 + b + cr^2 ln\left(\frac{r}{R_{ext}}\right) + dln\left(\frac{r}{R_{ext}}\right) \end{cases} \quad (2.5)$$

Hereafter, it is considered that the average warpage of the real taiko wafer does not depend on the angle ϑ, and that the behavior of the taiko wafer can be investigated by studying the behavior of the warpage on the radial distance r, only, by means of the set of equations 2.5 .

In figure 2 a schematic of half of the vertical cross section of a Taiko wafer (supposed fully cylindrical symmetric) having a thickness of the substrate h, a height of the rim H, an internal radius R_{int} , and external radius R_{ext} , respectively, is reported. A reference frame is fixed at the center of the taiko wafer. The origin is placed at the neutral plane, such that the front of the wafer is at $z = z_B$ with respect to the neutral plane,

with $z_B = -|z_B|$. Onto the thinned back side substrate surface a thin metal film having a modulus of elasticity E_f and thickness t_f , indicated in yellow, is deposited. The neutral axis results from the intersection of the neutral plane and the cross-section plane. Once the reference is fixed, $h + z_B$ is the distance between the neutral axis (plane) and the back side substrate surface, $h + z_B + t_f$ is the distance between the neutral axis and the BSM thin film surface, whereas $H + z_B$ is the distance of the ring region surface of the taiko wafer with respect to the neutral axis.
At the interface between the BSM thin film and the wafer substrate, at a distance r from the center, the taiko wafer is subject to a radial and a circumferential force, whose value per unit length is set equal to F_r and F_θ, respectively.
The wafer is supported at the external ring R_{ext} , whereas N_r and N_θ are the reaction forces evaluated per unit length.

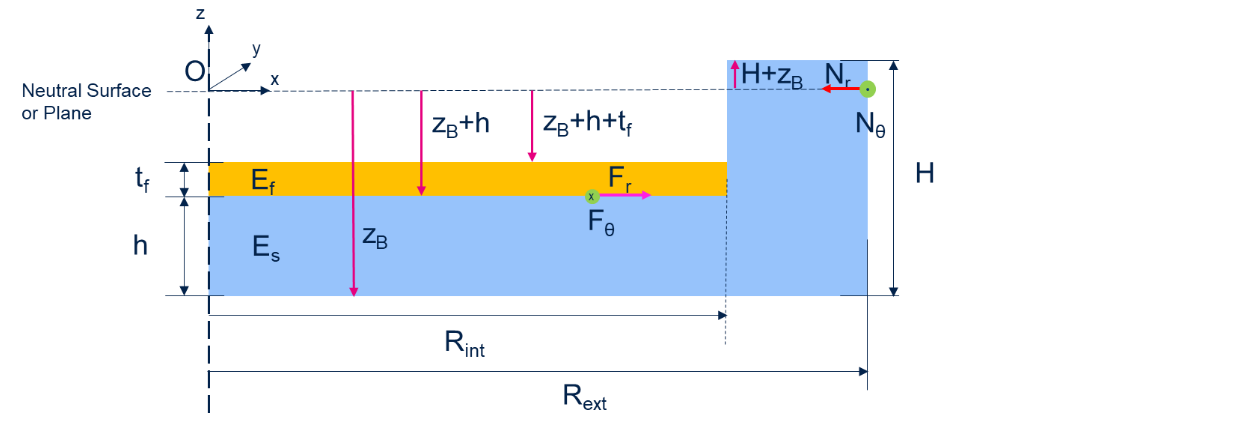

Fig.2. Vertical cross section of a Taiko wafer having internal radius R_{int} , external radius R_{ext} , thickness of the substrate h and of the rim H, respectively. The taiko wafer is subject to a radial force at the internal step edge of the ring, whose value per unit length is F_r. The wafer is supported at the external ring R_{ext} and N_r is the reaction force evaluated per unit length. The front of the wafer is at $z = z_B$ with respect to the neutral plane, with $z_B = -|z_B|$.

If this is the case, since the taiko wafer is in mechanical equilibrium, the resultant of the forces and the moments must be equal to zero. In particular, the equilibrium of forces and moments holds locally for the thin film deposited in the substrate region:

$$\begin{cases} F_{r,\theta_{sub}} + F_{r,\theta_{film}} = 0 \\ M_{r,\theta_{sub}} + M_{r,\theta_{film}} = 0 \end{cases} \quad (2.6)$$

And the reaction moments acting on the ring at R_{ext} are in equilibrium with those of the substrate.

$$M_{r,\theta_{ring}}\Big|_{R_{ext}} = 0 \quad (2.7)$$

The moments per unit length in the substrate region and ring region are the following:

$$\begin{cases} M_{r,sub}(r) = \frac{EI_{sub}}{(1-\sigma^2)}\left(\frac{\partial^2\zeta_{int}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial\zeta_{int}}{\partial r}\right) = F_{r\,sub}(z_B + h) \\ M_{\theta,sub}(r) = \frac{EI_{sub}}{(1-\sigma^2)}\left(\sigma\frac{\partial^2\zeta_{int}}{\partial r^2} + \frac{1}{r}\frac{\partial\zeta_{int}}{\partial r}\right) = F_{\theta\,sub}(z_B + h) \\ M_{r,ring}(R_{ext}) = \frac{EI_{ring}}{(1-\sigma^2)}\left(\left.\frac{\partial^2\zeta_{ext}}{\partial r^2}\right|_{R_{ext}} + \frac{\sigma}{R_{ext}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{ext}}\right) = 0 \\ M_{\theta,ring}(R_{ext}) = \frac{EI_{ring}}{(1-\sigma^2)}\left(\sigma\left.\frac{\partial^2\zeta_{ext}}{\partial r^2}\right|_{R_{ext}} + \frac{1}{R_{ext}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{ext}}\right) = 0 \end{cases} \quad (2.8)$$

Where EI_{sub} and EI_{ring} are the flexural rigidity of the substrate and ring regions, respectively. Whereas I_{sub} and I_{ring} are the moments of inertia of a section of the taiko wafer, considered as a beam of width $dw = rd\alpha$, with respect to the neutral axis [29].
Being $F_{r,\theta\,sub}$ the forces per unit length acting on the length $dw = rd\alpha$, the moment per unit length equals $F_{r,\theta\,sub}(z_B + h)$.

Since $\frac{\partial^2\zeta_{int}}{\partial r^2} = 2a_{sub}$, *and* $\frac{\partial\zeta_{int}}{\partial r} = 2a_{sub}r$, in the limit $H \to h$, $M_{r,sub}(r) = M_{\theta,sub}(r) = \frac{Eh^3}{12(1-\sigma\ )}\frac{\partial^2\zeta_{int}}{\partial r^2}$, which implies that $F_{r\,sub} = F_{\theta\,sub}$. Moreover, in the limit $H \to h$ according to the Stoney formula $\frac{Eh^3}{12(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}\frac{h}{2}$, which implies that $F_{r\,sub} = F_{\theta\,sub} = -\sigma_{film}h_{film}$. In the event, the set of four equations for the moments can be written as:

$$\begin{cases} M_{r,sub}(r) = \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}(h + z_B) \\ M_{\theta,sub}(r) = \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}(h + z_B) \\ M_{r,ring}(R_{ext}) = \frac{EI_{ring}}{(1-\sigma^2)}\left(\left.\frac{\partial^2\zeta_{ext}}{\partial r^2}\right|_{R_{ext}} + \frac{\sigma}{R_{ext}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{ext}}\right) = 0 \\ M_{\theta,ring}(R_{ext}) = \frac{EI_{ring}}{(1-\sigma^2)}\left(\sigma\left.\frac{\partial^2\zeta_{ext}}{\partial r^2}\right|_{R_{ext}} + \frac{1}{R_{ext}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{ext}}\right) = 0 \end{cases} \quad (2.9)$$

In general, I_{sub} and I_{ring} are functions of z_B, such that in the limit $H \to h$, $I_{sub} = I_{ring} = \frac{h^3}{12}$.

The equation

$$\frac{EI_{sub}(z_B)}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}(h + z_B) \quad (2.10)$$

is the extension of the Stoney formula for the case of a taiko wafer. It can be solved once that $I_{sub}(z_B)$ and z_B are known.

3) Determination of the moment of inertia

By following reference [30] along with the schematic of figure 2, it is possible to gain an expression of the flexural rigidities $EI_{sub}(z_B)$ and $EI_{ring}(z_B)$. Indeed, the total bending moment per unit length M_{sub}^T acting in the substrate region covered with the BSM film can be expressed as:

$$\frac{M_{sub}^T}{-\kappa} = E_f \int_{|h+z_B|}^{|h+z_B+t_f|} z^2 dz + E \int_{z_B}^{|h+z_B|} z^2 dz = \frac{E_f}{3}\left[\left(\left|h + z_B + t_f\right|\right)^3 - (|h + z_B|)^3\right] + \frac{E}{3}[|h + z_B|^3 - {z_B}^3] \quad (3.1)$$

Hence, the flexural rigidity of the substrate region, corrected by the Poisson coefficient σ of the thin film and of the substrate is (being $-z_B = |z_B|$) :

$$\frac{EI_{sub}}{1-\sigma} = \frac{E_f}{3(1-\sigma_f)}\left[\left|h + z_B + t_f\right|^3 - |h + z_B|^3\right] + \frac{E}{3(1-\sigma)}[|h + z_B|^3 + |z_B|^3] \qquad (3.2)$$

If $t_f = 0$, $\frac{EI_{sub}}{1-\sigma} = \frac{E}{3(1-\sigma)}[|h + z_B|^3 + |z_B|^3]$. In the limit $H \to h$, since $h + z_B = h/2$, I_{sub} becomes $\frac{h^3}{12}$, which is what we expect.

Analogously, in the ring region it holds

$$\frac{E}{(1-\sigma^2)}\left(\left.\frac{\partial^2 \zeta_{ext}}{\partial r^2}\right|_{R_{ext}} + \frac{\sigma}{R_{ext}}\left.\frac{\partial \zeta_{ext}}{\partial r}\right|_{R_{ext}}\right)\int_{z_B}^{H+z_B} z^2 dz = \frac{E}{3(1-\sigma^2)}\left(\left.\frac{\partial^2 \zeta_{ext}}{\partial r^2}\right|_{R_{ext}} + \frac{\sigma}{R_{ext}}\left.\frac{\partial \zeta_{ext}}{\partial r}\right|_{R_{ext}}\right)[(H + z_B)^3 - {z_B}^3] \quad (3.4)$$

and the flexural rigidity in this region becomes

$$\frac{E}{1-\sigma^2} I_{ring}(z_B) = \frac{E\left[(H+z_B)^3 + |z_B|^3\right]}{3(1-\sigma^2)} \quad (3.5)$$

4) Evaluation of z_B

In order to gain the stress σ_{film} from measurement of warpage, the evaluation of z_B, is required. This quantity can be calculated from the equilibrium of the forces.

Substrate region. By following still [30] we can write the total force acting in the substrate region in the radial direction as

$$\frac{E_f}{1-\sigma_f}\int_{h_1-t_f}^{h_1} z dz + \frac{E}{1-\sigma}\int_{-h_2}^{h_1-t_f} z dz = 0 \quad (4.1)$$

where $h_1 = \left|h + z_B + t_f\right|$ and $h_2 = |z_B|$.

At a distance r we can write,

$$\frac{\partial^2 \zeta_{int}}{\partial r^2}\left[\frac{E_f}{1-\sigma_f}\int_{|h+z_B|}^{|h+z_B+t_f|} z dz + \frac{E}{1-\sigma}\int_{z_B}^{|h+z_B|} z dz\right] dw \quad (4.2)$$

and the total force acting on the substrate region is

$$d\theta \frac{\partial^2 \zeta_{int}}{\partial r^2}\left[\frac{E_f}{1-\sigma_f}\int_{|h+z_B|}^{|h+z_B+t_f|} zdz + \frac{E}{1-\sigma}\int_{z_B}^{|h+z_B|} zdz\right]\int_0^{R_{int}} rdr \quad (4.3)$$

being $dw = rdrd\theta$.

Ring region. In the ring region we need to evaluate the quantity:

$$d\theta \frac{E}{(1-\sigma^2)}\int_{z_B}^{H+z_B} zdz \int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2 \zeta_{ext}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext}}{\partial r}\right) rdr \quad (4.4)$$

Taiko wafer. Hence, the total force acting on the wafer, in the radial direction, equals zero, being in equilibrium:

$$\left.\frac{\partial^2 \zeta_{int}}{\partial r^2}\right|_{R_{int}}\left[\frac{E_f}{1-\sigma_f}\int_{|h+z_B|}^{|h+z_B+t_f|} zdz + \frac{E}{1-\sigma}\int_{z_B}^{|h+z_B|} zdz\right]\int_0^{R_{int}} rdr + \frac{E}{(1-\sigma^2)}\int_{z_B}^{H+z_B} zdz \int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2 \zeta_{ext}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext}}{\partial r}\right) rdr = 0 \quad (4.5)$$

In the limit $H \to h$, $R_{ext} = R_{int}$, equation (4.5) reduces to the canonical case of a flat disk wafer:

$$\left[\frac{E_f}{1-\sigma_f}\int_{|h+z_B|}^{|h+z_B+t_f|} zdz + \frac{E}{1-\sigma}\int_{z_B}^{|h+z_B|} zdz\right]\int_0^{R_{int}} rdr = 0 \quad (4.6)$$

where z_B is equal to

$$z_B = \frac{-\frac{E_f t_f(2h+t_f)}{1-\sigma_f \quad 2} - \frac{E}{1-\sigma}\frac{h^2}{2}}{\frac{E_f t_f}{1-\sigma_f} + \frac{Eh}{1-\sigma}} \quad (4.7)$$

which is the value of the limit $\lim_{H\to h} z_B$ if $H \to h$. Moreover, if $t_f = 0$, $\lim_{H\to h} z_B = -\frac{h}{2}$.

To evaluate z_B in the general case, it is necessary to combine, the modified Stoney formula for the taiko wafer (eq 2.7) and calculate the value of the integral (see appendix C) according to the function $\zeta_{ext}(r)$ evaluated in appendix A. By doing so a linear equation is obtained, whose solution provides the value of the neutral axis z_B.

$$z_B = -\frac{\left(\frac{E_f t_f(2h+t_f)}{1-\sigma_f \quad 2} + \frac{E}{1-\sigma}\frac{h^2}{2}\right)\frac{R_{int}^2}{2} + \frac{E}{(1-\sigma^2)}\frac{H^2}{2}\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2 \zeta_{ext,Norm}}{\partial r^2} + \frac{\sigma \partial \zeta_{ext,Norm}}{r \quad \partial r}\right) rdr}{\left(\frac{E_f t_f}{1-\sigma_f} + \frac{Eh}{1-\sigma}\right)\frac{R_{int}^2}{2} + \frac{E}{(1-\sigma^2)} H \int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2 \zeta_{ext,Norm}}{\partial r^2} + \frac{\sigma \partial \zeta_{ext,Norm}}{r \quad \partial r}\right) rdr} \quad (4.8)$$

Where $\zeta_{ext,Norm}$ is the normalized function reported in eq. C1 of Appendix C.

If $t_f = 0$, this expression is simplified as

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$$z_B = -\frac{\frac{E}{1-\sigma}\frac{h^2}{2}\frac{R_{int}^2}{2}+\frac{E}{(1-\sigma^2)}\frac{H^2}{2}\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{\sigma}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr}{\frac{Eh}{1-\sigma}\frac{R_{int}^2}{2}+\frac{E}{(1-\sigma^2)}H\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{\sigma}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr} =$$

$$-\frac{1}{2}\frac{h^2+H^2\frac{1}{1+\sigma}\frac{2}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{\sigma}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr}{h+H\frac{1}{1+\sigma}\frac{2}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{\sigma}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr} \quad (4.9).$$

The normalized integral I_1

$$I_1 = \frac{1}{1+\sigma}\frac{2}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{\sigma}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr \quad (4.10)$$

has been evaluated and its value is also reported in Appendix C.

Besides the equilibrium of the forces in the radial direction, also the net forces along the circumferential θ direction must be considered. The net forces in this direction must satisfy eq. 4.11:

$$\left.\frac{\partial^2\zeta_{int}}{\partial r^2}\right|_{R_{int}}\left[z_B\left(\frac{E_f t_f}{1-\sigma_f}+\frac{Eh}{1-\sigma}\right)+\frac{E_f t_f}{1-\sigma_f}\frac{(2h+t_f)}{2}+\frac{E}{1-\sigma}\frac{h^2}{2}\right]\frac{R_{int}^2}{2}+\frac{E}{(1-\sigma^2)}\frac{H(H+2z_B)}{2}\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext}}{\partial r}\right)rdr = 0 \quad (4.11)$$

which leads to an additional value of the z_B

$$z_B = -\frac{\left(\frac{E_f t_f}{1-\sigma_f}\frac{(2h+t_f)}{2}+\frac{E}{1-\sigma}\frac{h^2}{2}\right)\frac{R_{int}^2}{2}+\frac{E}{(1-\sigma^2)}\frac{H^2}{2}\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr}{\left(\frac{E_f t_f}{1-\sigma_f}+\frac{Eh}{1-\sigma}\right)\frac{R_{int}^2}{2}+\frac{E}{(1-\sigma^2)}H\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr} \quad (4.12)$$

that can be simplified, if $t_f = 0$, as

$$z_B = -\frac{1}{2}\frac{h^2+\frac{1}{(1+\sigma)}H^2\frac{2}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr}{h+\frac{1}{(1+\sigma)}H\frac{2}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr} \quad (4.13)$$

Also in this case the normalized integral I_2

$$I_2 = \frac{2}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr \quad (4.14).$$

has been evaluated and reported in appendix C.

In the event, it results that there are two values of z_B, a value determined by eq. 4.8, hereafter indicated as $z_{B,r}$, which determines a neutral axis along the radial direction, because of the radial net force, and a value determined by eq. 4.11, hereafter indicated with $z_{B,\theta}$, which determines an additional neutral axis in the radial direction determined by the equilibrium of the net circumferential forces.
Indeed, according to this description, eq. 2.7 splits into a pair of equations:

$$\begin{cases} \frac{EI_{sub}(z_{B,r})}{(1-\sigma)} \frac{\partial^2 \zeta_{int}}{\partial r^2}\Big|_{z_{B,r}} = -\sigma_{film} h_{film}(h + z_{B,r}) \\ \frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)} \frac{\partial^2 \zeta_{int}}{\partial r^2}\Big|_{z_{B,\theta}} = -\sigma_{film} h_{film}(h + z_{B,\theta}) \end{cases} \quad (4.15)$$

or also

$$\begin{cases} \kappa_{rr,z_{B,r}} = -\frac{(1-\sigma)\sigma_{film} h_{film}(h+z_{B,r})}{EI_{sub}(z_{B,r})} \\ \kappa_{rr,z_{B,\theta}} = -\frac{(1-\sigma)\sigma_{film} h_{film}(h+z_{B,\theta})}{EI_{sub}(z_{B,\theta})} \end{cases} \quad (4.16).$$

5) Determination of warpages

For the case of a standard and flat wafer, the warpage can be evaluated from the variation $\Delta\zeta$ of the value of the measured z-coordinate between the center and the peripheral region. If $\zeta(r) = \frac{\kappa}{2} r^2$, where κ is the curvature, the $\Delta\zeta$ between the value at $r = R_{ext}$ and $r = 0$ is the warpage, which is equal to $\frac{\kappa}{2} {R_{ext}}^2$.
In the case of a taiko wafer, there are two radiuses that must be considered, R_{int} and R_{ext}. In the substrate region, according to details reported in appendix A, the $\zeta_{int}(r)$ function (see eq. A.35) varies with respect to the neutral axis and assumes the value $\zeta_{int}(R_{int}) = a_{sub} {R_{int}}^2 + b_{sub}$ at $r = R_{int}$ and $\zeta_{int}(0) = b_{sub} at$ $r = 0$. On the other hand, being $\zeta_{int}(R_{int}) = \zeta_{ext}(R_{int})$ it results that

$$\zeta_{int}(R_{int}) = \zeta_{ext}(R_{int}) = -\frac{\sigma_{film} h_{film}(z_B+h){R_{ext}}^2}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}\right]} \left[\frac{{R_{int}}^2}{{R_{ext}}^2} - 1 - \left(\frac{{R_{int}}^2}{{R_{ext}}^2} + 1\right) ln\left(\frac{R_{int}}{R_{ext}}\right)\right] \quad (5.1)$$

and being $a_{sub} = -\frac{1}{2}\frac{\sigma_{film} h_{film}(z_B+h)}{\frac{EI_{sub}}{(1-\sigma)}}$, the warpage $\zeta_{int}(0)$with respect to the neutral plane is

$$\zeta_{int}(0) = b_{sub} = \zeta_{int}(R_{int}) - a_{sub}{R_{int}}^2 = \kappa_{rr,sub}\left[-\frac{{R_{int}}^2}{2} + {R_{ext}}^2 \frac{\frac{{R_{int}}^2}{{R_{ext}}^2} - 1 - \left(\frac{{R_{int}}^2}{{R_{ext}}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}}\right] \quad (5.2)$$

However, the warpage of the taiko wafer, is evaluated with respect to the surface of the ring region, which lies at a distance of $H + z_B$ with respect to the neutral axis. Hence, by considering the case of a prevalent warping direction, $a_{sub}^{\perp} \approx 0$, with respect to the surface of the external ring, the warpage of the taiko wafer is:

$$warpage\_taiko = 2\zeta_{int}(0) + H + z_B = 2\kappa_{rr,sub}\left[-\frac{{R_{int}}^2}{2} + {R_{ext}}^2 \frac{\frac{{R_{int}}^2}{{R_{ext}}^2} - 1 - \left(\frac{{R_{int}}^2}{{R_{ext}}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}}\right] + H + z_B \quad (5.3)$$

Since there are two values of z_B, $z_{B,r}$ and $z_{B,\theta}$, respectively, we have two warpages

$$warpage\_taiko\_radial\_from\ z_{B,r} = 2\kappa_{rr,sub,z_{B,r}}\left[-\frac{{R_{int}}^2}{2} + {R_{ext}}^2 \frac{\frac{{R_{int}}^2}{{R_{ext}}^2} - 1 - \left(\frac{{R_{int}}^2}{{R_{ext}}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}}\right] + H + z_{B,r} \quad (5.4)$$

$$warpage\_taiko\_radial\ from\ z_{B,\theta} = 2\kappa_{rr,sub,z_{B,\theta}} \left[-\frac{R_{int}^2}{2} + R_{ext}^2 \frac{\frac{R_{int}^2}{R_{ext}^2} - 1 - \left(\frac{R_{int}^2}{R_{ext}^2} + 1\right) ln\left(\frac{R_{int}}{R_{ext}}\right)}{1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}} \right] + H + z_{B,\theta} \ (5.2)$$

In table 1 the whole set of quantities determined for the case of a taiko wafer are reported.

Direction	Curvatures substrate region, $\kappa_{rr,sub,z_{B,r}}$, $\kappa_{rr,sub,z_{B,\theta}}$	Moments of Inertia $I_{sub}(z_{B,r}), I_{sub}(z_{B,\theta})$	Neutral axis $z_{B,r}, z_{B,\theta}$	Warpages
Radial	$\frac{-\sigma_{film}h_{film}(h+z_{B,r})}{\frac{EI_{sub}(z_{B,r})}{(1-\sigma)}}$	$\frac{\lvert h+z_{B,r}\rvert^3 + \lvert z_{B,r}\rvert^3}{3}$	$-\frac{1}{2}\frac{h^2+\frac{H^2I_1}{1+\sigma}}{h+\frac{HI_1}{1+\sigma}}$	$2\zeta_{int,z_{B,r}}(0) + H + z_{B,r}$
Circumferential	$\frac{-\sigma_{film}h_{film}(h+z_{B,\theta})}{\frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}}$	$\frac{\lvert h+z_{B,\theta}\rvert^3 + \lvert z_{B,\theta}\rvert^3}{3}$	$-\frac{1}{2}\frac{h^2+\frac{H^2I_2}{(1+\sigma)}}{h+\frac{HI_2}{(1+\sigma)}}$	$2\zeta_{int,z_{B,\theta}}(0) + H + z_{B,\theta}$

Table 1. Quantities determined for the case of a taiko wafer.

6) Results and discussion

In the standard practice, a taiko wafer typically has a constant step ($H - h$ according to figure 2) of hundreds of microns. In the case of an 8" taiko Si wafer it holds that $step + h \leq 725\ \mu m$, being $725\ \mu m$ the thickness of an 8" Si wafer. The thickness of the substrate region typically ranges below $100\ \mu m$ for the case of ultrathin chips (UTC).

In table 2, the values of the main parameters used in this work, have been reported. In order to test our findings we have calculated the resulting curvatures in the substrate region and the warpages for the case of an 8" Si wafer, having a ring width of 3.7 mm and a constant step of $450\ \mu m$, as a function of the thickness h of the substrate region, with $h \leq 295\ \mu m$. The properties of the BSM metallization have been summarized in terms of a stress σ_f, which is of the order of hundreds of MPa, and a thickness of the metal film, which ranges between 100 nm and 2000 nm in the case of thick metal layers.

R_{ext} (mm)	H (µm)	*Ring width (mm)*	R_{int} (mm)	$Step = H - h$ (µm)	Poisson's coefficient	E Young Modulus of Si (GPa)	I_1	I_2
100	725	3.7	96.3	450	0.27	131	-1.58943	5.51814

Table 2. Values of the parameters for the external radius R_{ext}, H, ring width, internal radius R_{int}, step height, Poisson's coefficient and Young modulus E for an 8" Si taiko wafer. In the last two columns the values of the normalized integrals I_1 and I_2 is reported.

Experimental

The warpages of six thick BSM 8" Si taiko wafers 90 µm thin, all having a step of *450* µm and a ring width of 3.7 mm, have been measured with an MX-204 equipment (E+H Metrology), at room temperature, and the measurement have been reported in Fig.3a-3d. The BSMs consist of a multilayer of several metals

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which result in a global metal stress σ_f and film thickness that ranges from 230 MPa to 450 MPa and from 1300 nm to 2450 nm, whereas the warpage ranges from 500 µm to about 1300 µm, respectively. The thick BSMs have been deposited by one of our suppliers and the values of the metal stress has been determined independently on thicker chip size substrates, by exploiting the canonical Stoney formula.

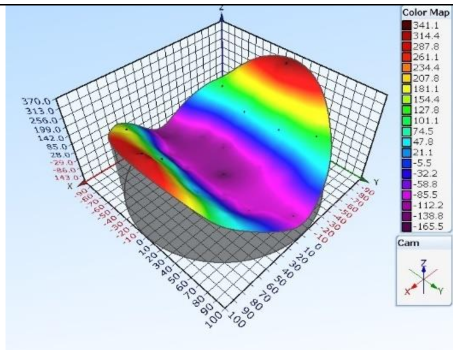

Figure 3a. Measured warpage, for the case of an 8” taiko Si wafer, 90 µm thick, having a measured stress of 230 MPa and metal film of 1500 nm, respectively. The measured warpage is 506.57 µm.

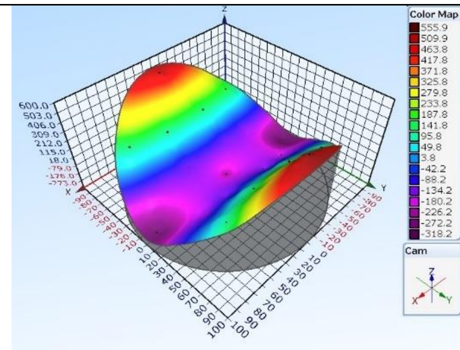

Figure 3b. Measured warpage of 874.07 µm for an 8” taiko Si wafer, 90 µm thick, having a BSM of 450 MPa and a thickness of 1300 nm.

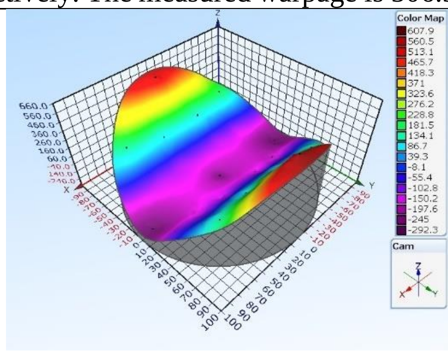

Figure 3c. Measured warpage of **900.19** µm for an 8” taiko Si wafer, 90 µm thick, having a BSM of 250 MPa and a thickness of 2000 nm.

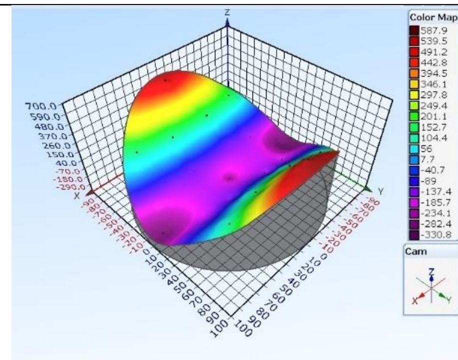

Figure 3d. Measured warpage of **918.64** µm for an 8” taiko Si wafer, 90 µm thick, having a BSM of 400 MPa and a thickness of 1500 nm.

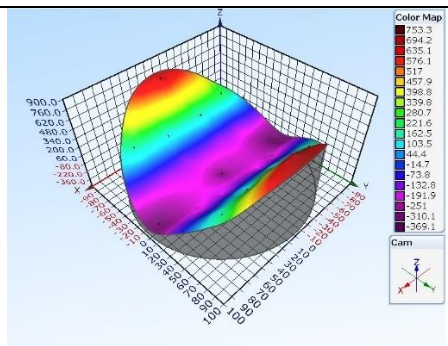

Figure 3e. Measured warpage of **1122.44** µm for an 8” taiko Si wafer, 90 µm thick, having a BSM of 310 MPa and a thickness of 2450 nm.

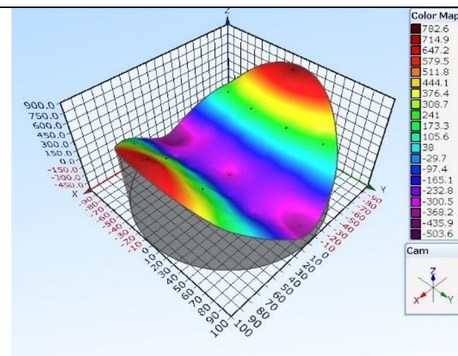

Figure 3d. Measured warpage of **1286.18** µm for an 8” taiko Si wafer, 90 µm thick, having a BSM of 446 MPa and a thickness of 2000 nm.

It is evident that all the wafers show a preferential direction of warpage which determine a curl-shape. Table 3, summarizes and compares the measured warpages for the cases reported in figure 3 from a to d.

Step (µm)	*Substrate thickness (µm)*	σ_f *Metal stress (MPa)*	*Film thickness* t_f*(nm)*	*Measured warpage (µm)*
450	*90*	230	1500	506.57
450	*90*	450	1300	874.07
450	*90*	250	2000	900.19
450	*90*	400	1500	918.64
450	*90*	310	2450	1122.44
450	*90*	446	2000	1286.18

Table 3. Measured warpages for the case of an 8" Si taiko wafers 90 µm thin, deposited with thick metal layers whose thickness and stress values have been obtained by measurements on equivalent thicker non-taiko wafers. On the last column it is reported the evaluation of the warpage according to the proposed model.

Calculations

The warpages $\zeta_{int}(0)$ determined by the neutral axis $z_{B,r}$ and $z_{B,\theta}$ respectively, have been calculated according to eq. 5.2, by exploiting the values of the curvatures $\kappa_{rr,z_{B,r}}$ and $\kappa_{rr,z_{B,\theta}}$ in the substrate region gained from the modified Stoney's equation 4.15, for the six cases of BSM investigated experimentally.
Fig. 4a reports the warpages determined by the neutral axis $z_{B,r}$ as a function of the substrate thickness (in µm), for the six cases of BSMs 8" Si taiko wafer, with thick metallization. The warpages depend on the product $\sigma_f h_f$ and decreases as the substrate thickness increases. The values of the warpages for a substrate thickness of 90 µm, have been extracted for a comparison with the measured values of the warpages. Analogously, in Fig. 4b the warpages determined by the neutral axis $z_{B,\theta}$ have been reported as a function of the substrate thickness, for the six cases of BSMs 8" Si taiko wafer, with thick metallization. Also in this case the warpages increase as the product $\sigma_f h_f$ increase and decreases as the substrate thickness increases. Similarly, also in this case, the values of the warpages for a substrate thickness of 90 µm, have been extracted for a comparison with the measured values of the warpages.

In table 4 the extracted values of the warpages determined for the cases of the 90 µm thin substrates have been reported for the $z_{B,r}$ and $z_{B,\theta}$, respectively.

Measured warpage (µm)	Calculated warpage $\zeta_{int,z_{B,r}}(0)$ in µm, for 90 µm thin BSM 8" Si taiko wafer.	$H + z_{B,r}$ in µm, for a 90 µm thin substrate	Calculated warpage $\zeta_{int,z_{B,\theta}}(0)$ in µm, for 90 µm thin BSM 8" Si taiko wafer.	$H + z_{B,\theta}$ in µm, for a 90 µm thin substrate	*Calculated warpage (µm) according to equation 5.4.*
506.57	211.6	243.6	263.6	276.6	666.9
874.07	358.8	243.6	446.9	276.6	961.3
900.19	306.7	243.6	381.9	276.6	857.0
918.64	368.0	243.6	458.4	276.6	979.9
1122.44	465.8	243.6	580.2	276.6	1175.3
1286.18	547.1	243.6	681.4	276.6	1337.9

Table 4. Extracted values of the warpages determined for the cases of the 90 µm thin substrates by the neutral planes $z_{B,r}$ and $z_{B,\theta}$, respectively. The values of the surface of the ring with respect to the neutral axis $H + z_{B,r}$ and $H + z_{B,\theta}$ are reported for the case of 90 µm thin BSM 8" Si taiko wafer.

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In fig. 5a the linear correlation between the calculated warpages resulting from the neutral axis $z_{B,\mathrm{r}}$ and the measured warpages referred to the neutral axis, that is the value $Measured\ warpage - (H + z_{B,r})$, for the six measured 8” taiko wafers, 90 µm thin, has been determined. A best fit line corresponding to $y = 0.53x$, very close to the theoretical dependence reported in eq. 5.4, has been determined, with an $R^2 = 0.99$.
Very similarly, in Fig. 5b, the linear correlation of the calculated warpage determined by the neutral axis $z_{B,\theta}$ and the measured warpage with respect to the neutral axis ($Measured\ warpage - (H + z_{B,\theta})$) for the six measured 8” taiko wafers, 90 µm thin. The calculated data are correlated with the measured ones, though the best fit line $y = 0.69x$ is a slightly further than the theoretical value of y=0.5x.

By considering the good agreement with the hypothesized dependence of the warpage, for the $z_{B,r}$ neutral axis, the linear correlation between the calculated warpages according to eq. 5.4 and the measured warpages has been reported in figure 6. It results that the best fit line has a slope of 1.05, with an interval of confidence of 95% ranging between 0.97 and 1.13. The two quantities show a good linear correlation, being the $R^2 = 0.99$.

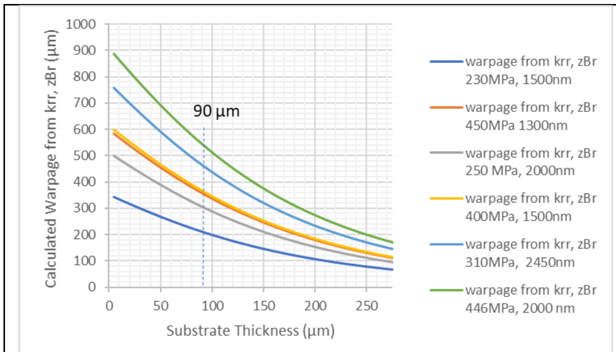

Figure 4a. Calculated warpages determined by the neutral axis $z_{B,r}$ according to eq. 5.2, as a function of the substrate thickness (in µm), for the six cases of BSMs, experimentally tested, 8” Si taiko wafer, with thick metallization. The thickness of 90 µm is reported and the values of the warpages have been extracted.

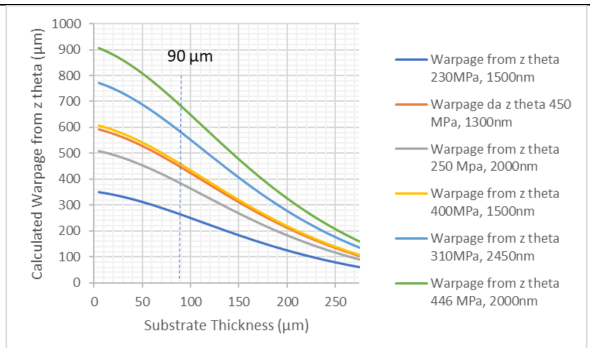

Figure 4b. Calculated warpages determined by the neutral axis determined by $z_{B,\theta}$, according to eq. 5.2, as a function of the substrate thickness (in µm), for the six cases of BSMs , experimentally tested, 8” Si taiko wafer , with thick metallization. The thickness of 90 µm is reported and the values of the warpages have been extracted.

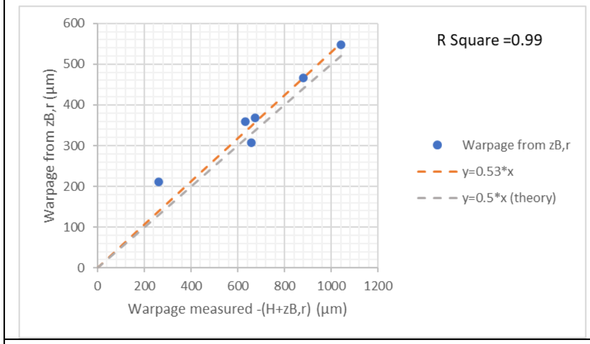

Figure 5a. Linear correlation between the calculated warpage determined by the neutral axis $z_{B,r}$ and the measured warpage with respect to the neutral axis, for the six measured 8” taiko wafers, 90 µm thin. The best fit line y=0.53x results very close to the theoretical value of y=0.5x.

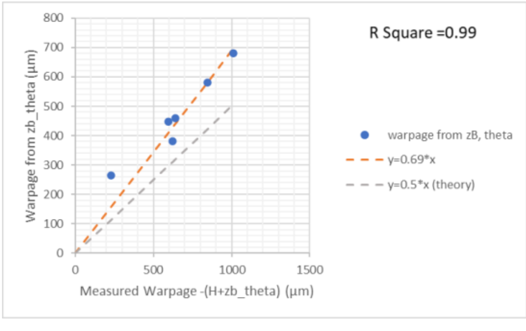

Figure 5b. Linear correlation between the calculated warpage determined by the neutral axis $z_{B,\theta}$ and the measured warpage with respect to the neutral axis, for the six measured 8” taiko wafers, 90 µm thin. The calculated data are correlated with

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	the measured ones, though the best fit line y=0.69x is a slightly further the theoretical value of y=0.5x.

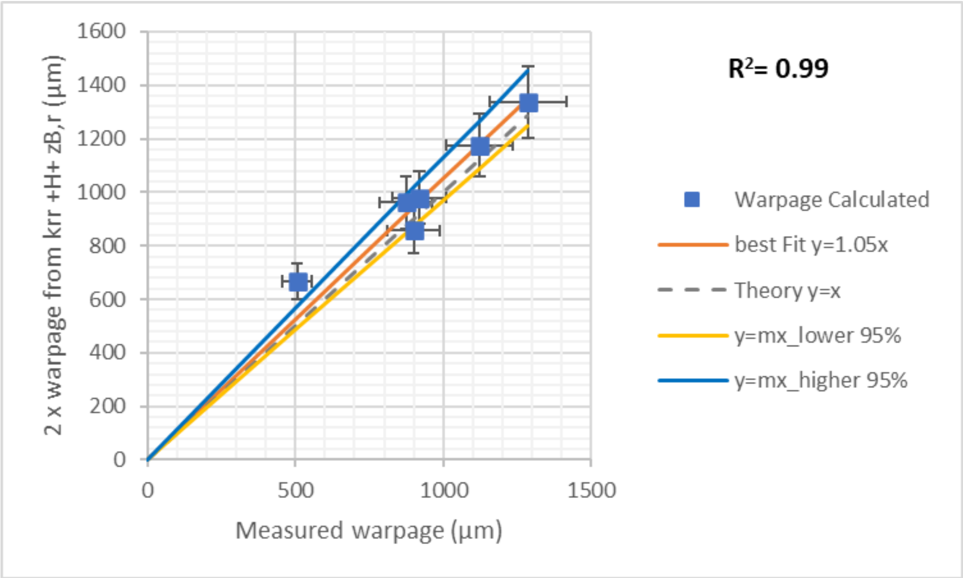

Figure 6. Linear correlation between the calculated warpage determined by the neutral axis $z_{B,r}$ according to equation 5.4 and the measured warpage with respect to the neutral axis, for the six measured 8" taiko wafers, 90 µm thin. The calculated warpage shows a good linear correlation with the measured warpage.

7) Extension to the case of the 8" SiC taiko wafer.

It is possible to extend the determination of the warpages, for the case of an 8" SiC taiko wafer. In particular we consider a taiko wafer having the same geometrical characteristics of the 8" Si taiko wafer, that is a constant step of 450 µm a ring width of 3.7 mm, a radius of 100 mm. The parameters exploited in this calculation are reported in table 5.

R_{ext} (mm)	H (µm)	*Ring width (mm)*	R_{int} (mm)	$Step = H - h$ (µm)	Poisson's coefficient	E Young Modulus of Si (GPa)	I_1	I_2
100	775	3.7	96.3	450	0.36	370	-1.48257	6.864387

Table 5. Values of the parameters for the external radius R_{ext}, H, ring width, internal radius R_{int}, step height, Poisson's coefficient and Young modulus E for an 8" SiC taiko wafer. In the last two columns the values of the normalized integrals I_1 and I_2 is reported.

In figure 7, the calculated warpage according to eq. 5.4 determined by the neutral axis $z_{B,r}$, for an 8" SiC taiko wafer, as a function of the substrate thickness (in µm), for the six cases of BSMs, examined for the 8" Si taiko wafer, with thick metallization, has been reported.

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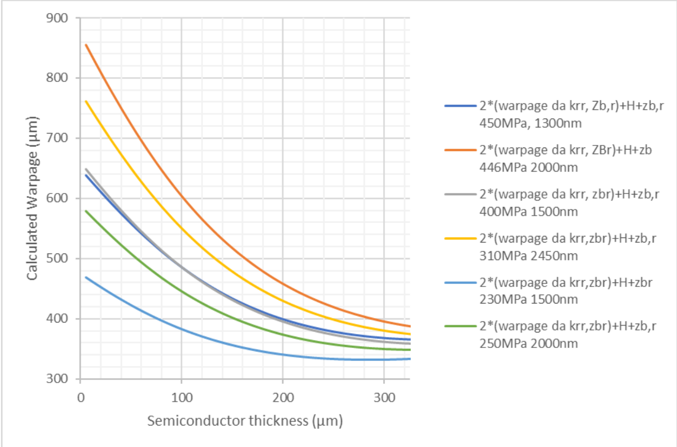

Fig.7. Calculated warpages determined by the neutral axis $z_{B,r}$ according to eq. 5.4, for an 8" SiC taiko wafer, as a function of the substrate thickness (in µm), for the six cases of BSMs, examined for the 8" Si taiko wafer, with thick metallization.

With respect to the case of the 8" Si wafer, the warpages are lower in the case of the 8" SiC taiko wafer. However, it occurs to consider that being SiC a brittle material, defects induced by e.g. the back grinding process can result into cracks or fractures at wafer or die level, that can results in low flexural stresses.

8) Discussion: the utility of the extended Stoney equation

The extended Stoney equation 2.10 and eq. 5.4 can be usefully combined in order to determine the total force $\sigma_{film}h_{film}$, determined by the BSM, acting on the substrate:

$$\sigma_{film}h_{film} = -\frac{warpage\_taiko\_measured-(H+z_{B,r})}{2\left[-\frac{R_{int}^2}{2R_{ext}^2}+\frac{\frac{R_{int}^2}{R_{ext}^2}-1-\left(\frac{R_{int}^2}{R_{ext}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}}\right]}\frac{EI_{sub}(z_{B,r})}{(1-\sigma)(h+z_{B,r})R_{ext}^2} \quad (8.1)$$

In table 6 the values of the stresses have been determined for the six cases of BSM examined for an 8" Si taiko wafer *90 µm* thin, having a step of *450 µm.*

Step (µm)	*Substrate thickness (µm)*	*Measured warpage (µm)*	σ_f *Metal stress (MPa)*	*Film thickness* t_f*(nm)*	$\sigma_f t_f$ *(Pa × m) Experimental*	$\sigma_f t_f$ *(Pa × m) Calculated*
450	*90*	506.57	230	1500	345	294
450	*90*	874.07	450	1300	585	704
450	*90*	900.19	250	2000	500	733
450	*90*	918.64	400	1500	600	754
450	*90*	1122.44	310	2450	759.5	981
450	*90*	1286.18	446	2000	892	1164

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Table 6. Summary of the properties of the BSM 8” taiko wafers 90 µm thin, having a step of 450 µm. the last two columns report the experimental values of the product $\sigma_{film}h_{film}$ and the calculated values $\sigma_{film}h_{film}$ according to equation eq. 8.1 .

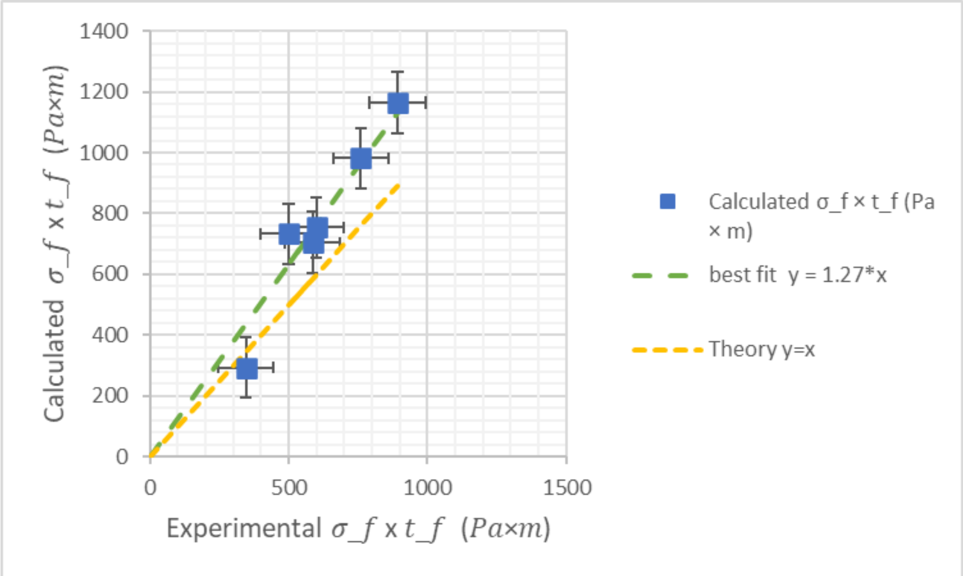

Figure 8. Plot of the calculated $\sigma_{film}h_{film}$ from eq. 8.1 as a function of the experimental $\sigma_{film}h_{film}$ and evaluation of the linear correlation. *An error of 100 Pa x m has been attributed to both quantities.* The experimental and calculated values of the product $\sigma_{film}h_{film}$ are correlated with an R^2=0.99.

In figure 8 the calculated $\sigma_{film}h_{film}$ quantity is compared with the experimental $\sigma_{film}h_{film}$ quantity of the corresponding value for the six BSM examined cases of the 8” Si taiko wafer 90 µm thin. The best fit analysis suggests a linear and positive correlation with a slope of 1.27. the discrepancy with respect to the expected theoretical value of 1 of this quantity can also be determined by the different conditions of measurements exploited. Indeed, the experimental value of the stress is determined by exploiting the canonical Stoney formula in a thicker Si flat wafer. Clearly the two corresponding substrates can differ and determine this discrepancy. Until now, in the normal practice, a further measurement was needed in order to have an estimate of the stress, instead by exploiting eq. 8.1 it is possible to gain a value of the stress determined by the BSM film on the taiko substrate.

In fact, from the values reported in table 8 we can determine a set of corresponding values for the stress, directly calculated for the case of the taiko wafers. In table 7 we report the calculated values of the stress determined by the BSM film in MPa. In figure 9, the correlation between the experimental and calculated values of the stresses is reported. In particular, a slope of 1.24 can be determined with an R^2 value of 0.98.

σ_f *Metal stress (MPa) Experimental*	*Film thickness* t_f*(nm)*	σ_f *Metal stress (MPa) Calculated*
230	1500	196
450	1300	542
250	2000	367
400	1500	502
310	2450	400
446	2000	582

Table 7. Calculated metal stresses in MPa, calculated according to the extended Stoney equation, for the six 8” BSM Si taiko wafers examined.

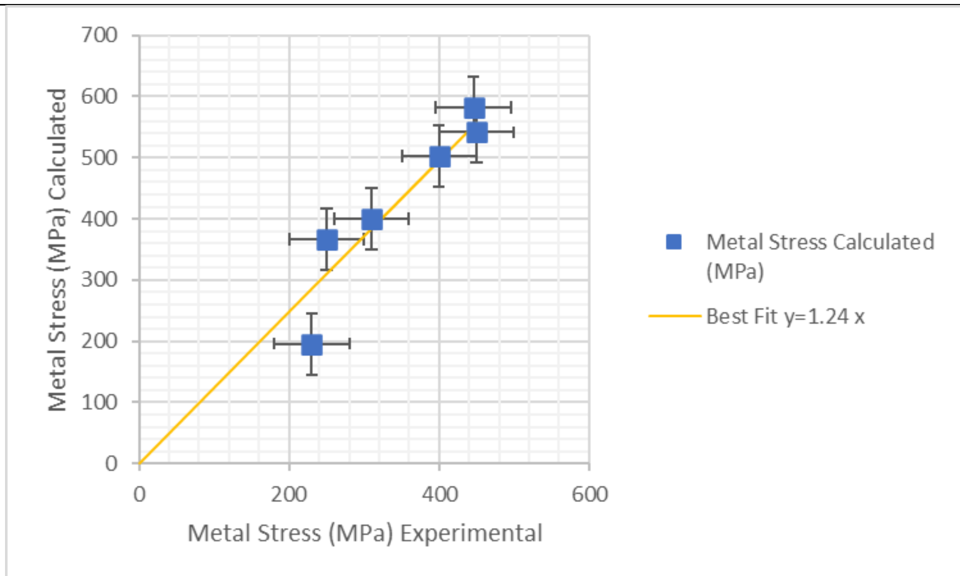

Fig. 9. Plot of the calculated Metal stress (MPa) vs. the experimental stress value for the six 8” BSM Si taiko wafers examined.

Finally, the value of the warpage at a die level can be also easily calculated. Indeed, once the ring is removed and the wafer diced into LxL die, the corresponding warpage can be determined according to eq.8.2.

$$Warpage\ x,y\ die\ = \frac{3}{8}\frac{warpage\_taiko\_radial\_from\ z_{B,r}-(H+z_{B,r})}{\left[-\frac{R_{int}^{\ 2}}{2}+R_{ext}^{\ 2}\frac{\frac{R_{int}^{\ 2}}{R_{ext}^{\ 2}}-1-\left(\frac{R_{int}^{\ 2}}{R_{ext}^{\ 2}}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^{\ 2}}{R_{int}^{\ 2}}}\right]}\frac{\frac{I_{sub}(z_{B,r})}{h^3}}{\frac{(h+z_{B,r})}{h}}L_{x,y}^2 \quad (8.2)$$

9) Conclusions

In conclusion, an extension of the Stoney formula has been developed for the case of an 8” taiko wafer, according to the theory of elasticity. It has been shown that the taiko wafer mitigates the warpage, because of the presence of the annular region. On one hand the ring region increases the moment applied to the substrate. In fact, the neutral axis is shifted upwardly because of the presence of the ring. On the other hand, the higher moment applied to the substrate is mitigated by the higher moment of inertia of the section of the taiko wafer. Therefore, the resulting warpage, despite a thicker BSM is mitigated and lowered with respect to a canonical and flat substrate.

The developed model has been validated according to experimental values benchmarked for six case of 8” BSM Si taiko wafer, having a substrate thickness of 90 µm and a step of 450 µm. It has been shown that there is a good agreement between the developed theory and the experimental values of the warpage. The modified Stoney equation has been usefully exploited to gain an estimate of the BSM metal stress. The comparison between the measured valued of the stress and the calculated value of the stress shows a linear correlation and states the utility of this formulation. Finally, an extension of the model has been considered for the case of an 8” SiC BSM taiko wafer.

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Contributions

A.L. proposed the case to V.V. and elaborated an earliest explanation according to the beam theory. V.V. elaborated the model of warpage according to the theory of elasticity and drafted the initial version of the paper. A.L. conducted and provided the warpage measurements on the six thick BSM wafers. A.L. and V.V. have elaborated the results section and both edited the final version of the paper.

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Supporting information

Appendix A: Determination of $\zeta_{ext}(r)$

If the taiko wafer is supported at $r = R_{ext}$ by following Landau, the boundary conditions tell us that the there is no vertical displacement at the edge $r = R_{ext}$ and that the bending moment is zero:

$$\begin{cases} \zeta_{ext}(R_{ext}) = 0 \\ M_{r,ring}(R_{ext}) = \frac{EI_{ring}}{(1-\sigma^2)}\left(\frac{\partial^2\zeta_{ext}}{\partial r^2}\Big|_{R_{ext}} + \frac{\sigma}{R_{ext}}\frac{\partial\zeta_{ext}}{\partial r}\Big|_{R_{ext}}\right) = 0 \end{cases} \quad (A.1)$$

In order to determine the four constants a, b, c, and d, two additional equations are required, to solve alongside with the boundary conditions for ζ_{ext}.

$$\begin{cases} a{R_{ext}}^2 + b = 0 \\ \frac{EI_{ring}}{(1-\sigma^2)}\left(\frac{\partial^2\zeta_{ext}}{\partial r^2}\Big|_{R_{ext}} + \frac{\sigma}{R_{ext}}\frac{\partial\zeta_{ext}}{\partial r}\Big|_{R_{ext}}\right) = 0 \\ \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}(z_B + h) \\ \frac{EI_{ring}}{(1-\sigma^2)}\left(\sigma\frac{\partial^2\zeta_{ext}}{\partial r^2}\Big|_{R_{ext}} + \frac{1}{R_{ext}}\frac{\partial\zeta_{ext}}{\partial r}\Big|_{R_{ext}}\right) = 0 \end{cases} \quad (A.2)$$

By calculating the first and the second derivative of $\zeta_{ext}(r)$, we obtain:

$$\frac{\partial\zeta_{ext}}{\partial r} = 2ar + 2crln\left(\frac{r}{R_{ext}}\right) + cr^2\frac{\frac{1}{R_{ext}}}{\frac{r}{R_{ext}}} + d\frac{\frac{1}{R_{ext}}}{\frac{r}{R_{ext}}} = 2ar + 2crln\left(\frac{r}{R_{ext}}\right) + cr + d\frac{1}{r} \quad (A3)$$

$$\frac{\partial^2\zeta_{ext}}{\partial r^2} = 2a + c + 2cln\left(\frac{r}{R_{ext}}\right) + 2cr\frac{\frac{1}{R_{ext}}}{\frac{r}{R_{ext}}} - \frac{d}{r^2} = 2a + 3c + 2cln\left(\frac{r}{R_{ext}}\right) - \frac{d}{r^2} \quad (A4)$$

$$\frac{\partial^2\zeta_{ext}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial\zeta_{ext}}{\partial r} = 2a + 3c + 2cln\left(\frac{r}{R_{ext}}\right) - \frac{d}{r^2} + \frac{\sigma}{r}\left(2ar + 2crln\left(\frac{r}{R_{ext}}\right) + cr + d\frac{1}{r}\right) = 2a(1+\sigma) + 2cln\left(\frac{r}{R_{ext}}\right)(1+\sigma) + c(3+\sigma) - \frac{d}{r^2}(1-\sigma) \quad (A5)$$

$$\sigma\frac{\partial^2\zeta_{ext}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext}}{\partial r}=2a\sigma+3c\sigma+2c\sigma ln\left(\frac{r}{R_{ext}}\right)-\frac{d\sigma}{r^2}+2a+2cln\left(\frac{r}{R_{ext}}\right)+c+\frac{d}{r^2}=2a(1+\sigma)+c(3\sigma+1)+2cln\left(\frac{r}{R_{ext}}\right)(1+\sigma)+\frac{d}{r^2}(1-\sigma)\ (A6)$$

$$\begin{cases} aR_{ext}{}^2+b=0 \\ 2a(1+\sigma)+c(3+\sigma)-\frac{d}{R_{ext}{}^2}(1-\sigma)=0 \\ \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2}=-\sigma_{film}h_{film}(z_B+h) \\ 2a(1+\sigma)+c(3\sigma+1)+\frac{d}{R_{ext}{}^2}(1-\sigma)=0 \end{cases}\ (A.7)$$

$$\begin{cases} 2a(1+\sigma)+c(3+\sigma)-\frac{d}{R_{ext}{}^2}(1-\sigma)=0 \\ 2a(1+\sigma)+c(3\sigma+1)+\frac{d}{R_{ext}{}^2}(1-\sigma)=0 \end{cases}\ (A.8)$$

$$\begin{cases} c(3+\sigma)-\frac{d}{R_{ext}{}^2}(1-\sigma)=-2a(1+\sigma) \\ c(3\sigma+1)+\frac{d}{R_{ext}{}^2}(1-\sigma)=-2a(1+\sigma) \end{cases}\ (A.9)$$

$$\Delta=\begin{vmatrix}(3+\sigma) & -\frac{1}{R_{ext}{}^2}(1-\sigma) \\ (3\sigma+1) & \frac{1}{R_{ext}{}^2}(1-\sigma)\end{vmatrix}=(3+\sigma)\frac{1}{R_{ext}{}^2}(1-\sigma)+(3\sigma+1)\frac{1}{R_{ext}{}^2}(1-\sigma)=\frac{1}{R_{ext}{}^2}[(3+\sigma)(1-\sigma)+(3\sigma+1)(1-\sigma)]=\frac{1}{R_{ext}{}^2}[3-3\sigma+\sigma-\sigma^2+3\sigma-3\sigma^2+1-\sigma]=\frac{1}{R_{ext}{}^2}[4-4\sigma^2]\ (A10)$$

$$c=\frac{\begin{vmatrix}-2a(1+\sigma) & -\frac{1}{R_{ext}{}^2}(1-\sigma) \\ -2a(1+\sigma) & \frac{1}{R_{ext}{}^2}(1-\sigma)\end{vmatrix}}{\frac{4(1-\sigma^2)}{R_{ext}{}^2}}=\frac{-2a(1+\sigma)(1-\sigma)-2a(1+\sigma)(1-\sigma)}{4(1-\sigma^2)}=-a\ (A11)$$

$$d = \frac{\begin{vmatrix} (3+\sigma) & -2a(1+\sigma) \\ (3\sigma+1) & -2a(1+\sigma) \end{vmatrix}}{\frac{4(1-\sigma^2)}{{R_{ext}}^2}} = \frac{2a(1+\sigma)[-(3+\sigma)+(3\sigma+1)]}{\frac{4(1-\sigma^2)}{{R_{ext}}^2}} = \frac{2a(1+\sigma)2[-1+\sigma]}{\frac{4(1-\sigma^2)}{{R_{ext}}^2}} = -a{R_{ext}}^2 \quad (A12)$$

$$\begin{cases} c(3+\sigma) - \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a(1+\sigma) \\ c(3\sigma+1) + \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a(1+\sigma) \end{cases} \quad (A13)$$

$$\begin{cases} c(4+4\sigma) = -4a(1+\sigma) \\ c(3\sigma+1) + \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a(1+\sigma) \end{cases} \quad (A14)$$

$$\begin{cases} c = -a \\ c(3\sigma+1) + \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a(1+\sigma) \end{cases} \quad (A15)$$

$$\begin{cases} c = -a \\ -a(3\sigma+1) + \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a(1+\sigma) \end{cases} \quad (A16)$$

$$\begin{cases} c = -a \\ \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a(1+\sigma) + a(3\sigma+1) \end{cases} \quad (A17)$$

$$\begin{cases} c = -a \\ \frac{d}{{R_{ext}}^2}(1-\sigma) = -2a - 2a\sigma + 3a\sigma + a \end{cases} \quad (A18)$$

$$\begin{cases} c = -a \\ \frac{d}{{R_{ext}}^2}(1-\sigma) = -a + a\sigma \end{cases} \quad (A19)$$

$$\begin{cases} c = -a \\ \frac{d}{{R_{ext}}^2} = -a \\ c = \frac{d}{{R_{ext}}^2} \end{cases} \quad (A20)$$

$$\begin{cases} \zeta_{ext}(r) = ar^2 + b + cr^2 ln\left(\frac{r}{R_{ext}}\right) + dln\left(\frac{r}{R_{ext}}\right) \\ b = -a{R_{ext}}^2 \\ c = -a \\ d = -a{R_{ext}}^2 \\ \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2 \zeta_{int}}{\partial r^2} = -\sigma_{film} h_{film}(z_B + h) \end{cases} \quad (A21)$$

$$\frac{\partial^2 \zeta_{ext}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext}}{\partial r} = 2a(1+\sigma) - 2aln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - a(3+\sigma) + \frac{a{R_{ext}}^2}{r^2}(1-\sigma) \quad (A22)$$

$$\frac{\partial^2 \zeta_{ext}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext}}{\partial r} = a\left[2(1+\sigma) - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - (3+\sigma) + \frac{{R_{ext}}^2}{r^2}(1-\sigma)\right] (A23)$$

$$\sigma\frac{\partial^2 \zeta_{ext}}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta_{ext}}{\partial r} = a\left[2(1+\sigma) - (3\sigma+1) - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - \frac{{R_{ext}}^2}{r^2}(1-\sigma)\right] (A24)$$

$$\begin{cases} \zeta_{ext}(r) = a\left[r^2 - {R_{ext}}^2 - r^2 ln\left(\frac{r}{R_{ext}}\right) - {R_{ext}}^2 ln\left(\frac{r}{R_{ext}}\right)\right] \\ \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2 \zeta_{int}}{\partial r^2} = -\sigma_{film} h_{film}(z_B + h) \end{cases} \quad (A.25)$$

In the internal and thinned substrate region, $0 \leq r \leq R_{int}$, the solution of the biharmonic equation is

$$\zeta_{int} = a_{sub} r^2 + b_{sub} \quad (A.26)$$

by imposing the continuity of the functions and of the first derivatives in $r = R_{int}$ for $\zeta_{int}(r)$ and $\zeta_{ext}(r)$, we obtain:

$$\begin{cases} \zeta_{ext}(R_{int}) = \zeta_{int}(R_{int}) \\ \left.\frac{\partial \zeta_{ext}}{\partial r}\right|_{R_{int}} = \left.\frac{\partial \zeta_{int}}{\partial r}\right|_{R_{int}} \end{cases} \quad (A.27)$$

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Moreover, being $\left.\frac{\partial^2\zeta_{int}}{\partial r^2}\right|_{R_{int}} = \frac{1}{R_{int}}\left.\frac{\partial\zeta_{int}}{\partial r}\right|_{R_{int}}$, we gain a condition on $\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{int}}$. In fact $\frac{1}{R_{int}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{int}} = \left.\frac{\partial^2\zeta_{int}}{\partial r^2}\right|_{R_{int}}$

$$\frac{\partial\zeta_{ext}}{\partial r} = 2ar + 2crln\left(\frac{r}{R_{ext}}\right) + cr + d\frac{1}{r} \quad (A28)$$

$$\frac{1}{R_{int}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{int}} = 2a - 2aln\left(\frac{R_{int}}{R_{ext}}\right) - a - \frac{aR_{ext}^{\ 2}}{R_{int}^{\ 2}} \quad (A29)$$

$$\frac{1}{R_{int}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{int}} = a\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^{\ 2}}{R_{int}^{\ 2}}\right] \quad (A30)$$

$$\begin{cases} \zeta_{ext}(r) = a\left[r^2 - R_{ext}^{\ 2} - r^2 ln\left(\frac{r}{R_{ext}}\right) - R_{ext}^{\ 2} ln\left(\frac{r}{R_{ext}}\right)\right] \\ \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}(z_B + h) \\ \frac{1}{R_{int}}\left.\frac{\partial\zeta_{ext}}{\partial r}\right|_{R_{int}} = \left.\frac{\partial^2\zeta_{int}}{\partial r^2}\right|_{R_{int}} = a\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^{\ 2}}{R_{int}^{\ 2}}\right] \end{cases} \quad (A.31)$$

Hence

$$a = \frac{\left.\frac{\partial^2\zeta_{int}}{\partial r^2}\right|_{R_{int}}}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^{\ 2}}{R_{int}^{\ 2}}\right]} = \frac{-\sigma_{film}h_{film}(z_B + h)}{\frac{EI_{sub}}{(1-\sigma)}\left[1 - 2log\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^{\ 2}}{R_{int}^{\ 2}}\right]} \quad (A.32)$$

In conclusion,

$$\begin{cases} \zeta_{ext}(r) = -\frac{\sigma_{film}h_{film}(z_B + h)}{\frac{EI_{sub}}{(1-\sigma)}\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^{\ 2}}{R_{int}^{\ 2}}\right]}\left[r^2 - R_{ext}^{\ 2} - r^2 ln\left(\frac{r}{R_{ext}}\right) - R_{ext}^{\ 2} ln\left(\frac{r}{R_{ext}}\right)\right] \\ \frac{EI_{sub}}{(1-\sigma)}\frac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film}h_{film}(z_B + h) \end{cases} \quad (A.34)$$

Or also

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$$\begin{cases} \zeta_{ext}(r) = -\dfrac{\sigma_{film} h_{film}(z_B+h) R_{ext}{}^2}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]}\left[\dfrac{r^2}{R_{ext}{}^2}-1-\left(\dfrac{r^2}{R_{ext}{}^2}+1\right)ln\left(\dfrac{r}{R_{ext}}\right)\right] \\ \dfrac{EI_{sub}}{(1-\sigma)}\dfrac{\partial^2\zeta_{int}}{\partial r^2} = -\sigma_{film} h_{film}(z_B+h) \end{cases} \quad \text{(A.35)}$$

Appendix B: Curvatures

According to equation A.35 of appendix A, by considering that there are the two values of z_B , two curvatures, indicated with the subscript $z_{B,r}$ and $z_{B,\theta}$, will result in the substrate region:

$$\begin{cases} \dfrac{EI_{sub}(z_{B,r})}{(1-\sigma)}\dfrac{\partial^2\zeta_{int}}{\partial r^2}\bigg|_{z_{B,r}} = -\sigma_{film} h_{film}(h+z_{B,r}) \\ \dfrac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}\dfrac{\partial^2\zeta_{int}}{\partial r^2}\bigg|_{z_{B,\theta}} = -\sigma_{film} h_{film}(h+z_{B,\theta}) \end{cases} \quad \textit{(4.17)}$$

which allow us to calculate two functions $\zeta_{ext}(r)$, evaluated along the two neutral axes:

$$\begin{cases} \zeta_{ext,z_{B,r}}(r) = -\dfrac{\sigma_{film} h_{film}(z_{B,r}+h) R_{ext}{}^2}{\frac{EI_{sub}(z_{B,r})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]}\left[\dfrac{r^2}{R_{ext}{}^2}-1-\left(\dfrac{r^2}{R_{ext}{}^2}+1\right)ln\left(\dfrac{r}{R_{ext}}\right)\right] \\ \dfrac{EI_{sub}(z_{B,r})}{(1-\sigma)}\dfrac{\partial^2\zeta_{int}}{\partial r^2}\bigg|_{z_{B,r}} = -\sigma_{film} h_{film}(z_{B,r}+h) \\ \zeta_{ext,z_{B,\theta}}(r) = -\dfrac{\sigma_{film} h_{film}(z_{B,\theta}+h) R_{ext}{}^2}{\frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]}\left[\dfrac{r^2}{R_{ext}{}^2}-1-\left(\dfrac{r^2}{R_{ext}{}^2}+1\right)ln\left(\dfrac{r}{R_{ext}}\right)\right] \\ \dfrac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}\dfrac{\partial^2\zeta_{int}}{\partial r^2}\bigg|_{z_{B,\theta}} = -\sigma_{film} h_{film}(z_{B,\theta}+h) \end{cases} \quad \text{(B.1)}$$

In particular, in the ring region $R_{int} < r \leq R_{ext}$ the curvatures are the following:

$$\begin{cases}
\kappa_{rr,ring,z_{B,r}} = \frac{\partial^2 \zeta_{ext,z_{B,r}}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext,z_{B,r}}}{\partial r} = \frac{-\sigma_{film} h_{film}(z_{B,r}+h)}{\frac{EI_{sub}(z_{B,r})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}\right]}\left[\sigma - 1 - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) + \frac{R_{ext}^2}{r^2}(1-\sigma)\right] \\
\kappa_{\theta\theta,ring,z_{B,r}} = \sigma\frac{\partial^2 \zeta_{ext,z_{B,r}}}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta_{ext,z_{B,r}}}{\partial r} = \frac{-\sigma_{film} h_{film}(z_{B,r}+h)}{\frac{EI_{sub}(z_{B,r})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}\right]}\left[1 - \sigma - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - \frac{R_{ext}^2}{r^2}(1-\sigma)\right] \\
\kappa_{rr,sub,z_{B,r}} = \kappa_{\theta\theta,sub,z_{B,r}} = \frac{-\sigma_{film} h_{film}(z_{B,r}+h)}{\frac{EI_{sub}(z_{B,r})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}\right]}\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right] \\
\kappa_{rr,ring,z_{B,\theta}} = \frac{\partial^2 \zeta_{ext,z_{B,\theta}}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext,z_{B,\theta}}}{\partial r} = \frac{-\sigma_{film} h_{film}(z_{B,\theta}+h)}{\frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}\right]}\left[\sigma - 1 - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) + \frac{R_{ext}^2}{r^2}(1-\sigma)\right] \\
\kappa_{\theta\theta,ring,z_{B,\theta}} = \sigma\frac{\partial^2 \zeta_{ext,z_{B,\theta}}}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta_{ext,z_{B,\theta}}}{\partial r} = \frac{-\sigma_{film} h_{film}(z_{B,\theta}+h)}{\frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}\right]}\left[1 - \sigma - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - \frac{R_{ext}^2}{r^2}(1-\sigma)\right] \\
\kappa_{rr,sub,z_{B,\theta}} = \kappa_{\theta\theta,sub,z_{B,\theta}} = \frac{-\sigma_{film} h_{film}(z_{B,\theta}+h)}{\frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}\right]}\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]
\end{cases} \quad (B.2)$$

By exploiting also the canonical Stoney equation eq (1.1):

$$\kappa_{Stoney} = \frac{-\sigma_{film} h_{film}\, h/2}{\frac{Eh^3}{12(1-\sigma)}}$$

The relative curvatures can be expressed as:

$$\begin{cases}
\frac{\kappa_{rr,ring,z_{B,r}}}{\kappa_{Stoney}} = \frac{z_{B,r}+h}{h/2}\frac{h^3}{12I_{sub}(z_{B,r})}\left[\frac{\sigma-1-2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma)+\frac{R_{ext}^2}{r^2}(1-\sigma)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}}\right] \\
\frac{\kappa_{\theta\theta,ring,z_{B,r}}}{\kappa_{Stoney}} = \frac{z_{B,r}+h}{h/2}\frac{h^3}{12I_{sub}(z_{B,r})}\left[\frac{1-\sigma-2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma)-\frac{R_{ext}^2}{r^2}(1-\sigma)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}}\right] \\
\frac{\kappa_{rr,sub,z_{B,r}}}{\kappa_{Stoney}} = \frac{\kappa_{\theta\theta,sub,z_{B,r}}}{\kappa_{Stoney}} = \frac{-\sigma_{film} h_{film}(z_{B,r}+h)}{\frac{EI_{sub}(z_{B,r})}{(1-\sigma)}} \Big/ \frac{-\sigma_{film} h_{film} h/2}{\frac{Eh^3}{12(1-\sigma)}} = \frac{z_{B,r}+h}{h/2}\frac{h^3}{12I_{sub}(z_{B,r})} \\
\frac{\kappa_{rr,ring,z_{B,\theta}}}{\kappa_{Stoney}} = \frac{z_{B,\theta}+h}{h/2}\frac{h^3}{12I_{sub}(z_{B,\theta})}\left[\frac{\sigma-1-2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma)+\frac{R_{ext}^2}{r^2}(1-\sigma)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}}\right] \\
\frac{\kappa_{\theta\theta,ring,z_{B,\theta}}}{\kappa_{Stoney}} = \frac{z_{B,\theta}+h}{h/2}\frac{h^3}{12I_{sub}(z_{B,\theta})}\left[\frac{1-\sigma-2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma)-\frac{R_{ext}^2}{r^2}(1-\sigma)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}^2}{R_{int}^2}}\right] \\
\frac{\kappa_{rr,sub,z_{B,\theta}}}{\kappa_{Stoney}} = \frac{\kappa_{\theta\theta,sub,z_{B,\theta}}}{\kappa_{Stoney}} = \frac{-\sigma_{film} h_{film}(z_{B,\theta}+h)}{\frac{EI_{sub}(z_{B,\theta})}{(1-\sigma)}} \Big/ \frac{-\sigma_{film} h_{film} h/2}{\frac{Eh^3}{12(1-\sigma)}} = \frac{z_{B,\theta}+h}{h/2}\frac{h^3}{12I_{sub}(z_{B,\theta})}
\end{cases} \quad (B.3)$$

Appendix C: evaluation of the integrals

The integral appearing in the equation of the z_B *can be easily evaluated from the function of* $\zeta_{ext,Norm}(r)$

$$\zeta_{ext,Norm}(r) = \frac{r^2 - R_{ext}^2 - r^2 ln\left(\frac{r}{R_{ext}}\right) - R_{ext}^2 ln\left(\frac{r}{R_{ext}}\right)}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} \quad (C1)$$

Indeed, being

$$\frac{\partial^2 \zeta_{ext,Norm}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext,Norm}}{\partial r} = \frac{\left[2(1+\sigma) - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - (3+\sigma) + \frac{R_{ext}^2}{r^2}(1-\sigma)\right]}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} \quad (C2)$$

The value of the integral is

$$\int_{R_{int}}^{R_{ext}} \left(\frac{\partial^2 \zeta_{ext,Norm}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext,Norm}}{\partial r}\right) rdr = \frac{\int_{R_{int}}^{R_{ext}} \left[2(1+\sigma) - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - (3+\sigma) + \frac{R_{ext}^2}{r^2}(1-\sigma)\right] rdr}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} =$$

$$\frac{(\sigma-1)\frac{1}{2}(R_{ext}^2 - R_{int}^2) + R_{ext}^2(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right) - (1+\sigma)\int_{R_{int}}^{R_{ext}} 2ln\left(\frac{r}{R_{ext}}\right) rdr}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} =$$

$$\frac{(\sigma-1)\frac{1}{2}(R_{ext}^2 - R_{int}^2) + R_{ext}^2(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right) - (1+\sigma)\left[ln\left(\frac{R_{ext}}{R_{ext}}\right)R_{ext}^2 - \frac{R_{ext}^2}{2} - ln\left(\frac{R_{int}}{R_{ext}}\right)R_{int}^2 + \frac{R_{int}^2}{2}\right]}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} =$$

$$\frac{(\sigma-1)\frac{1}{2}(R_{ext}^2 - R_{int}^2) + R_{ext}^2(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right) - (1+\sigma)\left[\frac{R_{int}^2}{2} - \frac{R_{ext}^2}{2} - ln\left(\frac{R_{int}}{R_{ext}}\right)R_{int}^2\right]}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} \quad (C3)$$

$$\int_{R_{int}}^{R_{ext}} \left(\frac{\partial^2 \zeta_{ext,Norm}}{\partial r^2} + \frac{\sigma}{r}\frac{\partial \zeta_{ext,Norm}}{\partial r}\right) rdr = \frac{(\sigma-1)\frac{1}{2}(R_{ext}^2 - R_{int}^2) + R_{ext}^2(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right) - (1+\sigma)\left[\frac{R_{int}^2}{2} - \frac{R_{ext}^2}{2} - ln\left(\frac{R_{int}}{R_{ext}}\right)R_{int}^2\right]}{\left[1 - 2ln\left(\frac{R_{int}}{R_{ext}}\right) - \frac{R_{ext}^2}{R_{int}^2}\right]} \quad (C4)$$

Analogously, we can calculate the integral

$$\int_{R_{int}}^{R_{ext}} \left(\sigma\frac{\partial^2 \zeta_{ext,Norm}}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta_{ext,Norm}}{\partial r}\right) rdr \quad (C5)$$

$$\sigma\frac{\partial^2 \zeta_{ext}}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta_{ext}}{\partial r} = a\left[2(1+\sigma) - (3\sigma+1) - 2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma) - \frac{R_{ext}^2}{r^2}(1-\sigma)\right] \quad (C6)$$

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With

$$a = \frac{\left.\frac{\partial^2 \zeta_{int}}{\partial r^2}\right|_{R_{int}}}{\left[1-2log\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]} = \frac{-\sigma_{film}h_{film}(z_B+h)}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2log\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]} \quad (C7)$$

$$\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr$$

$$= \int_{R_{int}}^{R_{ext}}\frac{1-\sigma\ -2ln\left(\frac{r}{R_{ext}}\right)(1+\sigma)-\frac{R_{ext}{}^2}{r^2}(1-\sigma)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}}rdr$$

$$= \frac{(1-\sigma)\int_{R_{int}}^{R_{ext}}rdr-(1+\sigma)\int_{R_{int}}^{R_{ext}}2ln\left(\frac{r}{R_{ext}}\right)rdr-(1-\sigma)R_{ext}{}^2\int_{R_{int}}^{R_{ext}}\frac{1}{r}dr}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}}$$

$$= \frac{(1-\sigma)\frac{1}{2}\left(R_{ext}{}^2-R_{int}{}^2\right)-(1+\sigma)\int_{R_{int}}^{R_{ext}}2ln\left(\frac{r}{R_{ext}}\right)rdr-(1-\sigma)R_{ext}{}^2ln\left(\frac{R_{ext}}{R_{int}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}}$$

$$= \frac{(1-\sigma)\frac{1}{2}\left(R_{ext}{}^2-R_{int}{}^2\right)-(1+\sigma)\left(ln\left(\frac{R_{ext}}{R_{ext}}\right)R_{ext}{}^2-\frac{R_{ext}{}^2}{2}-ln\left(\frac{R_{int}}{R_{ext}}\right)R_{int}{}^2+\frac{R_{int}{}^2}{2}\right)-(1-\sigma)R_{ext}{}^2ln\left(\frac{R_{ext}}{R_{int}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}}$$

(C8)

$$\int_{R_{int}}^{R_{ext}}\left(\sigma\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{1}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr$$

$$= \frac{(1-\sigma)\frac{1}{2}\left(R_{ext}{}^2-R_{int}{}^2\right)-(1+\sigma)\left(ln\left(\frac{R_{ext}}{R_{ext}}\right)R_{ext}{}^2-\frac{R_{ext}{}^2}{2}-ln\left(\frac{R_{int}}{R_{ext}}\right)R_{int}{}^2+\frac{R_{int}{}^2}{2}\right)-(1-\sigma)R_{ext}{}^2ln\left(\frac{R_{ext}}{R_{int}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}}$$

(C10)

$$\frac{1}{R_{int}^2}\int_{R_{int}}^{R_{ext}}\left(\frac{\partial^2\zeta_{ext,Norm}}{\partial r^2}+\frac{\sigma}{r}\frac{\partial\zeta_{ext,Norm}}{\partial r}\right)rdr$$
$$=\frac{1}{R_{int}^2}\frac{(\sigma-1)\left({R_{ext}}^2-{R_{int}}^2\right)+{R_{ext}}^2(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right)-(1+\sigma)\left[\frac{{R_{int}}^2}{2}-\frac{{R_{ext}}^2}{2}-ln\left(\frac{R_{int}}{R_{ext}}\right){R_{int}}^2\right]}{\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}\right]}$$
$$=\frac{(\sigma-1)\left(\frac{{R_{ext}}^2}{R_{int}^2}-1\right)+\frac{{R_{ext}}^2}{R_{int}^2}(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right)-(1+\sigma)\left[\frac{1}{2}-\frac{{R_{ext}}^2}{2R_{int}^2}-ln\left(\frac{R_{int}}{R_{ext}}\right)\right]}{\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}\right]}$$
$$=\left(\frac{{R_{ext}}^2}{R_{int}^2}-1\right)\frac{(\sigma-1)+\frac{\frac{{R_{ext}}^2}{R_{int}^2}(1-\sigma)ln\left(\frac{R_{ext}}{R_{int}}\right)}{\left(\frac{{R_{ext}}^2}{R_{int}^2}-1\right)}-(1+\sigma)\frac{\left[\frac{1}{2}-\frac{{R_{ext}}^2}{2R_{int}^2}-ln\left(\frac{R_{int}}{R_{ext}}\right)\right]}{\left(\frac{{R_{ext}}^2}{R_{int}^2}-1\right)}}{\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}\right]}$$

(C11)

Appendix D: Warpages

Because of eq. 4.17, *since* $2a_{sub}=\frac{\partial^2\zeta_{int}}{\partial r^2}$, *it results that* $\zeta_{int}(r)=-\frac{\sigma_{film}h_{film}(z_B+h)r^2}{2\frac{EI_{sub}}{(1-\sigma)}}+b_{sub}$.

Being (see eq. A.11 and A.14)

$$\zeta_{int}(R_{int})=\zeta_{ext}(R_{int})=-\frac{\sigma_{film}h_{film}(z_B+h){R_{ext}}^2}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}\right]}\left[\frac{{R_{int}}^2}{{R_{ext}}^2}-1-\left(\frac{{R_{int}}^2}{{R_{ext}}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)\right] \text{(D1)}$$

where

$$\zeta_{int}(R_{int})=-\frac{\sigma_{film}h_{film}(z_B+h){R_{int}}^2}{2\frac{EI_{sub}}{(1-\sigma)}}+b_{sub} (D2)$$

and

$$b_{sub}=\zeta_{int}(R_{int})+\frac{\sigma_{film}h_{film}(z_B+h){R_{int}}^2}{2\frac{EI_{sub}}{(1-\sigma)}} \text{(D3)}$$

Finally, it is obtained

$$\zeta_{int}(r)=-\frac{\sigma_{film}h_{film}(z_B+h)r^2}{2\frac{EI_{sub}}{(1-\sigma)}}+\zeta_{int}(R_{int})+\frac{\sigma_{film}h_{film}(z_B+h){R_{int}}^2}{2\frac{EI_{sub}}{(1-\sigma)}} \text{(D4)}$$

That can be expressed also as,

$$\zeta_{int}(r)=-\frac{\sigma_{film}h_{film}(z_B+h)(r^2-{R_{int}}^2)}{2\frac{EI_{sub}}{(1-\sigma)}}+\zeta_{ext}(R_{int})=-\frac{\sigma_{film}h_{film}(z_B+h){R_{ext}}^2\left(\frac{r^2}{{R_{ext}}^2}-\frac{{R_{int}}^2}{{R_{ext}}^2}\right)}{2\frac{EI_{sub}}{(1-\sigma)}}-$$
$$\frac{\sigma_{film}h_{film}(z_B+h){R_{ext}}^2}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{{R_{ext}}^2}{{R_{int}}^2}\right]}\left[\frac{{R_{int}}^2}{{R_{ext}}^2}-1-\left(\frac{{R_{int}}^2}{{R_{ext}}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)\right] (D5)$$

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The $\zeta_{int}(r)$ *value at* $r = 0$ *is*

$$\zeta_{int}(0) = \frac{\sigma_{film} h_{film}(z_B+h) R_{int}{}^2}{2\frac{EI_{sub}}{(1-\sigma)}} - \frac{\sigma_{film} h_{film}(z_B+h) R_{ext}{}^2}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]}\left[\frac{R_{int}{}^2}{R_{ext}{}^2} - 1 - \left(\frac{R_{int}{}^2}{R_{ext}{}^2}+1\right) ln\left(\frac{R_{int}}{R_{ext}}\right)\right] \text{(D6)}$$

Or also

$$\zeta_{int}(0) = \frac{\sigma_{film} h_{film}(z_B+h) R_{int}{}^2}{2\frac{EI_{sub}}{(1-\sigma)}} - \frac{\sigma_{film} h_{film}(z_B+h) R_{ext}{}^2}{\frac{EI_{sub}}{(1-\sigma)}\left[1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}\right]}\left[\frac{R_{int}{}^2}{R_{ext}{}^2} - 1 - \left(\frac{R_{int}{}^2}{R_{ext}{}^2}+1\right) ln\left(\frac{R_{int}}{R_{ext}}\right)\right] \text{(D7)}$$

Which can be expressed also as

$$\zeta_{int}(0) = \kappa_{rr,sub}\left[-\frac{R_{int}{}^2}{2} + R_{ext}{}^2\,\frac{\frac{R_{int}{}^2}{R_{ext}{}^2}-1-\left(\frac{R_{int}{}^2}{R_{ext}{}^2}+1\right)ln\left(\frac{R_{int}}{R_{ext}}\right)}{1-2ln\left(\frac{R_{int}}{R_{ext}}\right)-\frac{R_{ext}{}^2}{R_{int}{}^2}}\right] \text{(D8)}$$

This is the Warpage with respect to the neutral axis.