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Article

QCD theory of the hadrons and filling the Yang-Mills mass gap

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Abstract: The rank-3 antisymmetric tensors which are the magnetic monopoles of SU(N) Yang-Mills gauge theory dynamics, unlike their counterparts in Maxwell's U(1) electrodynamics, are non-vanishing, and do permit a net flux of Yang-Mills analogs to the magnetic field through closed spatial surfaces. When electric source currents of the same Yang-Mills dynamics are inverted and their fermions inserted into these Yang-Mills monopoles to create a system, this system in its unperturbed state contains exactly 3 fermions due to the monopole rank-3 and its 3 additive field strength gradient terms in covariant form. So to ensure that every fermion in this system occupies an exclusive quantum state, the Exclusion Principle is used to place each of the 3 fermions into the fundamental representation of the simple gauge group with an SU(3) symmetry. After the symmetry of the monopole is broken to make this system indivisible, the gauge bosons inside the monopole become massless, the SU(3) color symmetry of the fermions becomes exact, and a propagator is established for each fermion. The monopoles then have the same antisymmetric color singlet wavefunction as a baryon, and the field quanta of the magnetic fields fluxing through the monopole surface have the same symmetric color singlet wavefunction as a meson. Consequently, we are able to identify these fermions with colored quarks, the gauge bosons with gluons, the magnetic monopoles with baryons, and the fluxing entities with mesons, while establishing that the quarks and gluons remain confined and identifying the symmetry breaking with hadronization. Analytic tools developed along the way are then used to fill the Yang-Mills mass gap.

Keywords: hadrons; baryons; mesons; quarks; gluons; QCD; hadronization; quark-gluon plasma; Yang-Mills mass gap

Introduction

After the discovery of the muon in 1936, Rabi is said to have exclaimed: "who ordered that?" But to this day, the same question can still be asked of the proton and neutron which are at the nuclear heart of the observed material universe, and of the other baryons. We do have a very good understanding that the proton and neutron and other baryons are composed of three confined "colored" quarks in the fundamental representation of SU(3), with highly non-linear gluonic interactions among these quarks, wherein baryons interact with one another by exchanging a variety of mesons. But we still do not have a good dynamic answer, rooted in fundamental physics principles, to Rabi's very basic question: who ordered the baryons? Nor is there a good understanding of the dynamic origin of quark and gluon confinement.

One of the most notable features of Maxwell's U(1) differential and integral equations $\nabla \cdot \mathbf{B} = 0$ and $\oiint \mathbf{B} \cdot d\mathbf{S} = 0$ is that magnetic monopoles do not exist and that there is never a net flux of magnetic fields across any closed two-dimensional spatial surface. But in the dynamic Maxwell equations for an SU(N) Yang-Mills [1] theory of non-commuting gauge fields these monopoles *do exist*, and *there is a net flux* of the analogs to magnetic fields through closed surrounding surfaces. Given the extraordinary success of Yang-Mills gauge theory in describing weak and strong interactions, and its presumably-significant role in any "grand unified theory" which might become generally accepted in the future, the question how the magnetic monopoles of Yang-Mills might manifest themselves in the natural world must be given due consideration.

What is shown here is that when the Yang-Mills (YM) analog of Maxwell's electric charge equation is inverted, then inserted into the analog of Maxwell's magnetic equation for what are now

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non-vanishing monopoles – effectively combining both of Maxwell's covariant equations into a single indivisible equation – the non-perturbative state of these monopoles contains exactly 3 fermions arising from the monopole density being an antisymmetric tensor of rank 3. Treating the monopole as a "system" to which the Exclusion Principle must be applied and so using SU(3) to enforce an exclusive quantum state for each of these three fermions, and following a form of spontaneous symmetry breaking which moves a degree of freedom from gauge bosons to fermions, makes the bosons massless and renders SU(3) an exact symmetry, these YM magnetic monopoles acquire the same SU(3) antisymmetric color singlet wavefunction as baryons, and the magnetic field analogs which flow through the monopole surfaces obtain the same symmetric color singlet wavefunction as mesons. This enables the three fermion states inside the monopole to be identified as confined colored quarks, the gauge bosons to be identified as confined colored gluons, the YM magnetic monopoles to be identified as baryons, the mesons to be identified as quanta of the YM magnetic fields which net flow in and out of these monopoles, and the symmetry breaking to be identified with ultra-high-energy hadronization from a plasma of free quarks and gluons.

From this we also answer Rabi's question: baryons were ordered by Maxwell, Gauss and Faraday together with Yang and Mills, and by Weyl via gauge theory itself, with an assist from Fermi-Dirac-Pauli via Dirac's quantum theory of the electron and the Exclusion Principle, and with credit to Hamilton for pioneering non-commuting quaternions which later became the foundation of YM gauge theory in the form of Pauli matrices and their extension to SU(N).

Finally, we employ the foregoing development to fill the Yang-Mills Mass Gap [2].

1. A brief review of Maxwell's Equations using duality and differential forms

We start with Maxwell's equations in covariant form, in flat spacetime. A gauge field / vector potential A^ν with dimensionality of energy/charge is used to first define a field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ which in turn is used in the two differential Maxwell equations:

$$c\mu_0 j^\nu = \partial_\sigma F^{\sigma\nu} = \partial_\sigma \partial^\sigma A^\nu - \partial_\sigma \partial^\nu A^\sigma = \left(g^{\mu\nu}\partial_\sigma\partial^\sigma - \partial^\mu\partial^\nu\right)A_\mu , \tag{1.1a}$$

$$c\mu_0 p^{\sigma\mu\nu} = \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu} = 0 . \tag{1.1b}$$

Also, from (1.1a) we obtain the continuity equation:

$$c\mu_0 \partial_\nu j^\nu = \partial_\nu \partial_\sigma \partial^\sigma A^\nu - \partial_\nu \partial_\sigma \partial^\nu A^\sigma = 0 . \tag{1.2}$$

Above, $j^\nu = \left(c\rho, \mathbf{j}\right)$ is a current density four-vector in which ρ has dimensions of charge per volume (charge density), μ_0 is the vacuum permeability of free space, and $p^{\sigma\mu\nu}$ is a third-rank antisymmetric tensor defining a magnetic charge (monopole) current density.

In flat spacetime where $g_{\mu\nu} = \eta_{\mu\nu}$ and the Riemann tensor $R_{\mu\nu\alpha\sigma} = 0$, the commutator $\left[\partial_{;\mu}, \partial_{;\nu}\right]A_\alpha = R_{\mu\nu\alpha\sigma}A^\sigma$ for covariant derivatives simplifies to $\left[\partial_\mu, \partial_\nu\right] = 0$ with partial derivatives commuting. As a result, in (1.1a) we obtain the configuration space operator $g^{\mu\nu}\partial_\sigma\partial^\sigma - \partial^\mu\partial^\nu$ operating on A_μ; and in (1.1b) we find that the magnetic monopole density $p^{\sigma\mu\nu} = 0$, by identity, which is understood to mean there are no isolated magnetic charges in nature (setting aside the monopoles theorized as a possibility by Dirac in [3]). Likewise, (1.2) is also true by identity and governs the conservation of electric charge sources. If we impose the covariant gauge condition $\partial_\sigma A^\sigma = 0$ then (1.1a) simplifies to $c\mu_0 j^\nu = \partial_\sigma \partial^\sigma A^\nu$. If, instead, we introduce a Proca mass $m \neq 0$ so (1.1a) becomes $c\mu_0 j^\nu = \left(g^{\mu\nu}\left(\partial_\sigma\partial^\sigma + m^2\right) - \partial^\mu\partial^\nu\right)A_\mu$, then because of (1.2) we find that $m^2 \partial_\sigma A^\sigma = 0$ thus $\partial_\sigma A^\sigma = 0$, which is no longer a gauge condition but a continuity requirement.

Duality and differential forms express the differential Maxwell's equations (1.1) in a compact form which simplifies obtaining the integral Maxwell's equations. Using duality reviewed e.g. at pp. 87-89 of [4], we may rewrite (1.1) as:

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$$c\mu_0 * j^{\sigma\mu\nu} = \partial^\sigma * F^{\mu\nu} + \partial^\mu * F^{\nu\sigma} + \partial^\nu * F^{\sigma\mu}, \tag{1.3a}$$

$$c\mu_0 p^{\sigma\mu\nu} = \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\nu} = 0. \tag{1.3b}$$

Then, via differential forms reviewed e.g. in chapter 4 of [4] and pp. 218-220 of [5], we compact:

$$c\mu_0 * j = d * F = d * dA, \tag{1.4a}$$

$$c\mu_0 p = dF = ddA = 0. \tag{1.4b}$$

Likewise, the continuity equation (1.2) becomes:

$$c\mu_0 d * j = dd * F = dd * dA = 0. \tag{1.5}$$

Both (1.4b) and (1.5) apply the differential forms relation $dd = 0$ that the exterior derivative of the exterior derivative is zero, see e.g. §4.6 of [4].*

The form relations (1.4) are ideal for casting Maxwell's equations into integral form. Specifically, for any *p*-form *H* on a $p+1$-dimensional manifold *M* with boundary ∂M:

$$\int_M dH = \int_{\partial M} H, \tag{1.6}$$

see e.g. pp. 218-220 of [5]. So, using (1.4) in (1.6), we arrive at the integral Maxwell's equations:

$$c\mu_0 \iiint * j = \iiint d * F = \oiint * F = \oiint * dA, \tag{1.7a}$$

$$c\mu_0 \iiint p = \iiint dF = \oiint F = \oiint dA = \iiint 0 = 0. \tag{1.7b}$$

In Section 18.3 of [6], Close uses Gauss' theorem for electric charge contained in (1.7a) to "consider the chromodynamics case which is analogous to the above." Similarly (1.7b) contains Gauss' law for magnetism, $\oiint \mathbf{B} \cdot d\mathbf{S} = 0$. The reason these surface integrals are of interest, is because one way to state the confinement of net color charge inside a baryon is via the schematic expression $\oiint_B \text{net color} = 0$ over the baryon *B* surface. Likewise, with only color-neutral objects (e.g. mesons) net flowing across baryon surfaces, we may write $\oiint_B \text{color-neutral} \neq 0$. Using Gauss' theorem, what can and cannot flow across a baryon surface becomes of keen interest.

2. Maxwell's Yang-Mills canonic equations

Maxwell's electrodynamics is a U(1) abelian gauge theory, so-named because its gauge fields are commuting, $\left[A_\mu, A_\nu\right] = 0$. Yang-Mills gauge theory, on the other hand, uses gauge fields we denote generally as G^ν to distinguish from A^ν which are non-commuting, $\left[G_\mu, G_\nu\right] \neq 0$. Specifically, for a simple group SU(N) with traceless NxN Hermitian generators $\tau_i = \tau_i^\dagger$ with $i = 1 \ldots N^2 - 1$ normalized to $\mathrm{tr}\left(\tau_i^2\right) = \frac{1}{2}$ for each τ_i, and a commutator $\left[\tau_i, \tau_j\right] = i f_{ijk} \tau_k$, these gauge fields are constructed via $G^\mu = \tau_i G_i^\mu = G^{\mu\dagger}$ and so are likewise NxN Hermitian matrices. We may use the commutator to find that $\left[G_\mu, G_\nu\right] = \left[\tau_i, \tau_j\right] G_{i\mu} G_{j\nu} = i f_{ijk} \tau_k G_{i\mu} G_{j\nu}$. Likewise, while $\mathrm{tr}\left(G^\mu\right) = \mathrm{tr}\left(\tau_i G_i^\mu\right) = 0$, it can be generally shown that $\mathrm{tr}\left(AB\right) = \mathrm{tr}\left(\tau_i \tau_j A_i B_j\right) = \frac{1}{2} A_i B_i$ for any $A = \tau_i A_i$, $B = \tau_j B_j$, which means that $\mathrm{tr}\left(G_\mu G_\nu\right) = \mathrm{tr}\left(\tau_i \tau_j G_{i\mu} G_{j\nu}\right) = \frac{1}{2} G_{i\mu} G_{i\nu}$ for a product of two gauge fields. Although weak interactions use SU(2) and strong interactions SU(3), at the outset we shall not examine any particular gauge groups. Our immediate interest is to develop the counterparts to Maxwell's equations generally, for *any* SU(N) Yang-Mills (YM) gauge theory.

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* Above, we adopt the convention that the indexes of a dual object match those on the left side of the Levi-Civita tensor, whereby $*X^{\sigma\mu\nu} = \varepsilon^{\sigma\mu\nu\alpha} X_\alpha$, $*Y^{\mu\nu} = \frac{1}{2!}\varepsilon^{\mu\nu\alpha\beta} Y_{\alpha\beta}$ and $*Z^\nu = \frac{1}{3!}\varepsilon^{\nu\alpha\beta\gamma} Z_{\alpha\beta\gamma}$ for any vector X_α and antisymmetric tensors $Y_{\alpha\beta}$ and $Z_{\alpha\beta\gamma}$. This is to ensure that (1.3a) and (1.4a) maintain the same relative sign between $*j$ and $d*F$.

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In general, a gauge-covariant derivative is defined by $\hbar c D^{\mu} = \hbar c \partial^{\mu} - igG^{\mu}$, where g is a charge strength. We then use these to define a field strength tensor in natural units $\hbar = c = 1$ by:

$$F^{\mu\nu} = D^{\mu}G^{\nu} - D^{\nu}G^{\mu} = \left(\partial^{\mu} - igG^{\mu}\right)G^{\nu} - \left(\partial^{\nu} - igG^{\nu}\right)G^{\mu} = \partial^{\mu}G^{\nu} - \partial^{\nu}G^{\mu} - ig\left[G^{\mu}, G^{\nu}\right]. \quad (2.1)$$

Were these gauge fields to commute, that is, were we to have $\left[G^{\mu}, G^{\nu}\right] = 0$, this would reduce to $F^{\mu\nu} = \partial^{\mu}G^{\nu} - \partial^{\nu}G^{\mu}$ which recovers the template $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ for U(1) abelian electrodynamics. But, of course, these $\left[G_{\mu}, G_{\nu}\right] = if_{ijk}\tau_k G_{i\mu}G_{j\nu}$ are non-commuting. Using differential forms, the above compacts to:

$$F = dG - igG^2 \quad (2.2)$$

In Yang-Mills gauge theory we obtain dynamic equations starting with a canonic replacement $\partial_{\mu} \rightarrow D_{\mu}$ of ordinary with gauge-covariant derivatives. Likewise promoting electric and magnetic charge densities $j^{\nu} \mapsto J^{\nu}$ and $p^{\sigma\mu\nu} \mapsto P^{\sigma\mu\nu}$ to capitalization, (1.1) become*:

$$c\mu_0 J^{\nu} = D_{\sigma}F^{\sigma\nu} = \partial_{\sigma}F^{\sigma\nu} - igG_{\sigma}F^{\sigma\nu} = D_{\sigma}D^{\sigma}G^{\nu} - D_{\sigma}D^{\nu}G^{\sigma} = \left(g^{\mu\nu}D_{\sigma}D^{\sigma} - D^{\mu}D^{\nu}\right)G_{\mu}, \quad (2.3a)$$

$$c\mu_0 P^{\sigma\mu\nu} = D^{\sigma}F^{\mu\nu} + D^{\mu}F^{\nu\sigma} + D^{\nu}F^{\sigma\mu} = \left(\partial^{\sigma} - igG^{\sigma}\right)F^{\mu\nu} + \left(\partial^{\mu} - igG^{\mu}\right)F^{\nu\sigma} + \left(\partial^{\nu} - igG^{\nu}\right)F^{\sigma\mu} = 0. \quad (2.3b)$$

Likewise, the continuity equation (1.2) generalizes to:

$$\begin{aligned} c\mu_0 D_{\nu}J^{\nu} &= c\mu_0\left(\partial_{\nu} - igG_{\nu}\right)J^{\nu} = D_{\nu}D_{\sigma}F^{\sigma\nu} = D_{\nu}D_{\sigma}D^{\sigma}G^{\nu} - D_{\nu}D_{\sigma}D^{\nu}G^{\sigma} \\ &= \partial_{\nu}\partial_{\sigma}F^{\sigma\nu} - \left(ig\left(G_{\nu}\partial_{\sigma} + \partial_{\nu}G_{\sigma}\right) + g^2G_{\nu}G_{\sigma}\right)F^{\sigma\nu} = \left(\partial_{\nu}\partial_{\sigma} - V_{\nu\sigma}\right)F^{\sigma\nu} = 0 \end{aligned}, \quad (2.4)$$

which in momentum space $i\partial_{\mu} \mapsto p_{\mu}$ becomes $\left(p_{\nu} + gG_{\nu}\right)J^{\nu} = 0$. Unlike (1.1b) and (1.2) the zeros above arise not from derivative commutation, but from the Jacobian identity $\left[D^{\sigma},\left[D^{\mu},D^{\nu}\right]\right] + \left[D^{\mu},\left[D^{\nu},D^{\sigma}\right]\right] + \left[D^{\nu},\left[D^{\sigma},D^{\mu}\right]\right] = 0$ and $\left[D_{\nu},\left[D_{\sigma},\left[D^{\sigma},D^{\nu}\right]\right]\right] = 0$, in view of the further identities $-igF_{\mu\nu}\phi = \left[D_{\mu}, D_{\nu}\right]\phi = D_{\mu}\left(D_{\nu}\phi\right) - D_{\nu}\left(D_{\mu}\phi\right)$ and $\left[D_{\sigma}, F_{\mu\nu}\right]\phi = D_{\sigma}F_{\mu\nu}\phi$ operating on any field $\phi(t, \mathbf{x})$, see e.g., [7], with careful attention given to the product rule. In the bottom line of (2.4), we define $V_{\nu\sigma} \equiv ig\left(G_{\nu}\partial_{\sigma} + \partial_{\nu}G_{\sigma}\right) + g^2G_{\nu}G_{\sigma}$ to be a "perturbation tensor," so-named because its trace $V = V^{\sigma}{}_{\sigma} = ig\left(G^{\sigma}\partial_{\sigma} + \partial^{\sigma}G_{\sigma}\right) + g^2G^{\sigma}G_{\sigma}$ is the standard expression for the perturbation in the Klein-Gordon (relativistic Schrödinger) equation, and houses the difference $-V = D_{\sigma}D^{\sigma} - \partial_{\sigma}\partial^{\sigma}$ between ordinary and gauge-covariant Laplacians.

3. Maxwell's Yang-Mills dynamic equations

One of the major lessons of the General Theory of Relativity [8] which Hermann Weyl later adapted to gauge theory [9], [10], [11] is the *canonic* prescription of invariantly maintaining the original form of a field equation or Lagrangian density while merely replacing all ordinary derivatives with suitable covariant derivatives. Then, by separating the original equation or Lagrangian and its ordinary derivatives from the new terms arising via covariant derivatives, we ascertain the physical, *dynamic* impacts of this prescription. For example, in General Relativity the covariant promotion of derivatives in the flat spacetime commutator $\left[\partial_{\mu}, \partial_{\nu}\right] = 0$ produces $\left[\partial_{;\mu}, \partial_{;\nu}\right]A_{\alpha} = R_{\mu\nu\alpha\sigma}A^{\sigma}$ with the Riemann tensor; and Newton's first law of motion written as

* Note that $c\mu_0$ remains the constant factor in (2.3) as it was in (1.3) et seq. In U(1) electrodynamics $c\mu_0 = 4\pi\hbar\alpha_e / e^2$ is the ratio between the running fine structure coupling $\alpha_e(\mu = 0) = 1/137.036...$ and charge strength e. For SU(N) it remains the ratio $c\mu_0 = 4\pi\hbar\alpha / g^2$ between dimensionless running couplings α and charge strengths g generally.

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$du^\alpha / d\tau = 0$ with a four velocity $u^\alpha = dx^\alpha / d\tau$ produces $Du^\alpha / D\tau = du^\beta / d\tau + \Gamma^\beta{}_{\mu\nu} u^\mu u^\nu = 0$ i.e. $du^\alpha / d\tau = -\Gamma^\alpha{}_{\mu\nu} u^\mu u^\nu$, which is the gravitational geodesic equation. And in gauge theory, the Klein-Gordon equation $\left(\partial_\sigma \partial^\sigma + m^\sigma\right)\phi = 0$ promotes to $\left(D_\sigma D^\sigma + m^\sigma\right)\phi = 0$, i.e., $\left(\partial_\sigma \partial^\sigma + m^\sigma\right)\phi = V\phi$ with the perturbation $V = ig\left(\partial_\sigma G^\sigma + G_\sigma \partial^\sigma\right) + g^2 G_\sigma G^\sigma$ noted above; while Dirac's equation $\left(i\gamma^\sigma \partial_\sigma - m\right)\psi = 0$ promotes to $\left(i\gamma^\sigma D_\sigma - m\right)\psi = 0$, i.e., $\left(i\gamma^\sigma \partial_\sigma - m\right)\psi = -\gamma^0 V_D \psi$ with a Dirac (D) perturbation $\gamma^0 V_D \equiv g\gamma^\sigma G_\sigma$. In all instances, this canonic prescription is to 1) replace ordinary with covariant derivatives, then 2) segregate the original equation with ordinary derivatives to see the dynamic impact, whereby what was originally a "zero" becomes a "non- zero." In Section 2 we took the first step of promoting ordinary derivatives in Maxwell's equations to covariant derivatives to obtain canonic equations. Now, we segregate the original Maxwell equations to see the dynamic Yang-Mills content, and what the zeros become as non-zeros.

To do so, we first define lowercase-denoted electric and magnetic charge densities by $c\mu_0 j^\nu \equiv \partial_\sigma F^{\sigma\nu}$ and $c\mu_0 p^{\sigma\mu\nu} \equiv \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu}$ exactly as in the Maxwell equations (1.1), using *ordinary* derivatives of $F^{\mu\nu}$, with the field strength now being the Yang-Mills (2.1). Then, we use $c\mu_0 J^\nu = D_\sigma F^{\sigma\nu}$ and $c\mu_0 P^{\sigma\mu\nu} = D^\sigma F^{\mu\nu} + D^\mu F^{\nu\sigma} + D^\nu F^{\sigma\mu} = 0$ in (2.3) which have exactly the same form but for $\partial \mapsto D$ and $j, p \mapsto J, P$, together with $D^\mu = \partial^\mu - igG^\mu$ in natural units, to separate the *ordinary* derivatives of $F^{\mu\nu}$, as such:

$$\begin{aligned} c\mu_0 j^\nu &\equiv \partial_\sigma F^{\sigma\nu} = \partial_\sigma\left(D^\sigma G^\nu - D^\nu G^\sigma\right) = \left(g^{\mu\nu}\partial_\sigma D^\sigma - \partial^\mu D^\nu\right)G_\mu \\ &= \left(g^{\mu\nu}\left(\partial_\sigma \partial^\sigma - ig\partial_\sigma G^\sigma\right) - \left(\partial^\mu \partial^\nu - ig\partial^\mu G^\nu\right)\right)G_\mu \end{aligned} , \tag{3.1a}$$

$$\begin{aligned} c\mu_0 p^{\sigma\mu\nu} &\equiv \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu} = igG^\sigma F^{\mu\nu} + igG^\mu F^{\nu\sigma} + igG^\nu F^{\sigma\mu} \\ &= \partial^\sigma\left(D^\mu G^\nu - D^\nu G^\mu\right) + \partial^\mu\left(D^\nu G^\sigma - D^\sigma G^\nu\right) + \partial^\nu\left(D^\sigma G^\mu - D^\mu G^\sigma\right) \\ &= -ig\left(\partial^\sigma\left[G^\mu, G^\nu\right] + \partial^\mu\left[G^\nu, G^\sigma\right] + \partial^\nu\left[G^\sigma, G^\mu\right]\right) \neq 0 \end{aligned} . \tag{3.1b}$$

Comparing (3.1) with (2.3) we see the foregoing implies definitions $c\mu_0 j^\nu \equiv c\mu_0 J^\nu + igG_\sigma F^{\sigma\nu}$ and $c\mu_0 p^{\sigma\mu\nu} \equiv c\mu_0 P^{\sigma\mu\nu} + igG^\sigma F^{\mu\nu} + igG^\mu F^{\nu\sigma} + igG^\nu F^{\sigma\mu}$ between lowercase and uppercase source densities, also mindful that $P^{\sigma\mu\nu} = 0$. In (3.1b) the ordinary derivative commutator $\left[\partial_\mu, \partial_\nu\right] = 0$ cancels terms just as in Maxwell's monopole equation (1.1b). What is very important is that the Yang-Mills dynamic equation (3.1b) contains a non-vanishing magnetic monopole density $p^{\sigma\mu\nu} \neq 0$, versus $p^{\sigma\mu\nu} = 0$ in (1.1b) for Maxwell's U(1) electrodynamics. The "zero" which here has turned into a "nonzero," is the magnetic monopole density $p^{\sigma\mu\nu}$.

Cast into differential forms (3.1) may be compacted to (compare (1.4)):

$$c\mu_0 {*j} = d{*F} = d{*DG} = d{*}\left(dG - igG^2\right) = d{*dG} - igd{*G^2}, \tag{3.2a}$$

$$c\mu_0 p = dF = dDG = d\left(dG - igG^2\right) = ddG - igdG^2 = -igdG^2 \neq 0 . \tag{3.2b}$$

These are Maxwell's dynamic equations in differential forms, for any Yang-Mills gauge group SU(N). Although we continue to apply the exterior calculus relation $dd = 0$ to remove $ddG = 0$, again, these magnetic monopoles do not zero out entirely. There remains a residual term $-igdG^2$ which arises directly out of the two-form $G^2 = \tfrac{1}{2}\left[G_\mu, G_\nu\right]dx^\mu dx^\mu \neq 0$, that is, directly from the non-commuting nature of Yang-Mills gauge theories.

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Another reason it is important to segregate ordinary derivatives is because the d in $dH=(-1)^p\partial_\mu H dx^\mu$ in (1.6) remains an ordinary, geometric, calculus derivative and does not change for Yang-Mills theory. So, having placed (3.1) into the compact form (3.2) with segregated ordinary derivatives, we may use (1.6) to recast these into integral form:

$$c\mu_0\iiint *j=\iiint d*F=\oiint *F=\iiint d*\left(dG-igG^2\right)=\oiint *dG-ig\oiint *G^2\,, \tag{3.3a}$$

$$c\mu_0\iiint p=\iiint dF=\oiint F=-ig\iiint dG^2=-ig\oiint G^2\neq 0\,. \tag{3.3b}$$

These are the Yang-Mills counterparts to (1.7). The YM "electric" equation (3.3a) contains a new term $-ig\oiint *G^2$ which does not appear in (1.7a), again, because $G^2\neq 0$. The YM magnetic equation written as $\oiint F=-ig\oiint G^2$ indicates something quite unique in contrast to Maxwell's electrodynamics: Whereas (aside perhaps from Dirac's [3]) there is no net flux of any U(1) magnetic fields through any closed two-dimensional surface, we learn from (3.3b) that Yang-Mills SU(N) "magnetic" fields can and do exhibit a net flux through such closed surfaces. This is because (3.2b) does provide Yang-Mills theory with non-vanishing magnetic monopoles.

Yang-Mills gauge theories have proved to be very successful for understanding the natural world. Weak interactions are correctly understood using SU(2), strong using SU(3), and electroweak using SU(2)xU(1). It is plausible grand unification will eventually start with some larger SU(N) and spontaneously break symmetry in stages down to the phenomenological SU(3)xSU(2)xU(1). In short, we take Yang-Mills gauge theories very seriously for their ability to render what we observe in nature. Accordingly, if Yang-Mills theories also predict magnetic monopoles as in (3.1b), (3.2b) and (3.3b), we must ask equally-serious questions about these monopoles. Most importantly: in the natural world, in what form do we observe these YM monopoles? And, what are the G^2 objects which in (3.3b) net flow across the monopole surface?

As we shall see, these monopoles are observed as baryons and these G^2 objects which net flow across surfaces of these monopoles are observed as mesons. Together, they are the hadrons.

4. Populating Yang-Mills magnetic monopoles with source currents, by inverting the Yang-Mills electric source equation and then combining both Maxwell equations into one

It is common practice to start with an electric equation of the form (1.1a), or, presently, (3.1a) in which the source density j^ν is a function of the gauge field A_μ or G_μ, then invert the configuration space operator to obtain the gauge field as a function of the source current. It is well known, however, that the inverse of (1.1a) is infinite and cannot be obtained without removing some of the gauge freedom, typically through the gauge condition $\partial_\sigma A^\sigma=0$. Alternatively, a finite inverse can be obtained with a Proca mass m added by hand, whereby (1.1a) is written as $c\mu_0 j^\nu=\left(g^{\mu\nu}\left(\partial_\sigma\partial^\sigma+m^2\right)-\partial^\mu\partial^\nu\right)A_\mu$. Of course, a theory with a vector boson mass added by hand is no longer renormalizable, but it is also known how to cure this by revealing a mass through spontaneous symmetry breaking of the sort which underlies electroweak theory. With this in mind, we now add a mass by hand to (3.1a), and with $\hbar=c=1$ write:

$$\begin{aligned} c\mu_0 j^\nu=\partial_\sigma F^{\sigma\nu}&=\left(g^{\mu\nu}\left(\partial_\sigma D^\sigma+m^2\right)-\partial^\mu D^\nu\right)G_\mu \\ &=\left(g^{\mu\nu}\left(\left(\partial_\sigma\partial^\sigma+m^2\right)+g^2G_\sigma G^\sigma\right)-\left(\partial^\mu\partial^\nu-ig\partial^\mu G^\nu\right)\right)G_\mu \end{aligned} \tag{0.19}$$

With this same m, (2.3a) becomes $c\mu_0 J^\nu=\left(g^{\mu\nu}\left(D_\sigma D^\sigma+m^2\right)-D^\mu D^\nu\right)G_\mu$. Then we find from (2.4) that $m^2 D_\sigma G^\sigma=0$ is now a required covariant condition – not merely an optional gauge condition – through which the scalar degree of freedom is removed from G^σ. So, for a massive vector boson in Yang-Mills theory:

$$D_\sigma G^\sigma = 0 \quad \text{i.e.} \quad \partial_\sigma G^\sigma = igG_\sigma G^\sigma , \tag{4.2}$$

which we already used in going from (3.1a) to the bottom line of (4.1). It is interesting – and a precursor to solving the Yang-Mills Mass Gap problem [2] in Section 15 – that this produces a correctly-signed term $+g^2 G_\sigma G^\sigma$ in the configuration space operator of (4.1), because this is the form in which the electroweak Lagrangian $\mathfrak{L} = D^\sigma * \phi * D_\sigma \phi = \partial^\sigma \phi * \partial_\sigma \phi + g^2 G^\sigma G_\sigma \phi * \phi$ reveals renormalizable gauge boson masses following spontaneous symmetry breaking at the Fermi vev.

From here the inverse calculation is straightforward: Via (4.1) we define a tensor $I_{\alpha\nu}$ by:

$$c\mu_0 I_{\alpha\nu} j^\nu = I_{\alpha\nu}\left(g^{\mu\nu}\left(\partial_\sigma D^\sigma + m^2\right) - \partial^\mu D^\nu\right) G_\mu \equiv \delta^\mu{}_\alpha G_\mu = G_\alpha . \tag{4.3}$$

Because $I_{\alpha\nu}\left(g^{\mu\nu}\left(\partial_\sigma D^\sigma + m^2\right) - \partial^\mu D^\nu\right) \equiv \delta^\mu{}_\alpha$, we see that $I_{\alpha\nu}$ is the left-side inverse of the configuration space operator $g^{\mu\nu}\left(\partial_\sigma D^\sigma + m^2\right) - \partial^\mu D^\nu$. And because the inverse M^{-1} of any N-dimensional square matrix M must commute with M, i.e. $M^{-1}M = MM^{-1} = \mathrm{Id}$ where Id is a like-dimensioned identity matrix, the full specification of this inverse for operation on either side is:

$$I_{\alpha\nu\text{LEFT}}\left(g^{\mu\nu}\left(\partial_\sigma D^\sigma + m^2\right) - \partial^\mu D^\nu\right) = \left(g^{\mu\nu}\left(\partial_\sigma D^\sigma + m^2\right) - \partial^\mu D^\nu\right) I_{\alpha\nu\text{RIGHT}} \equiv \delta^\mu{}_\alpha , \tag{4.4}$$

with the further requirement that $I_{\alpha\nu\text{LEFT}} = I_{\alpha\nu\text{RIGHT}} \equiv I_{\alpha\nu}$. Then, once we have an $I_{\alpha\nu}$ which satisfies (4.4), then we may insert (4.3) written as $G^\mu = c\mu_0 I^{\mu\sigma} j_\sigma$ into (3.1b) with suitable index renaming, and use $c^2\mu_0\varepsilon_0 = 1$ where ε_0 is the free space vacuum permittivity, to obtain:

$$ic\varepsilon_0 p^{\alpha\mu\nu} = g\left(\partial^\alpha\left[I^{\mu\sigma} j_\sigma, I^{\nu\tau} j_\tau\right] + \partial^\mu\left[I^{\nu\tau} j_\tau, I^{\alpha\gamma} j_\gamma\right] + \partial^\nu\left[I^{\alpha\gamma} j_\gamma, I^{\mu\sigma} j_\sigma\right]\right). \tag{4.5}$$

This is important for two reasons: First, the U(1) Maxwell equations (1.1) are two distinct equations because there are no magnetic monopoles. But in (4.5) – courtesy of the non-vanishing YM monopole (3.1b) and the YM current sources (3.1a) with Proca mass in (4.1), the latter inverted using (4.3) – the resulting YM monopole *combines both Yang-Mills Maxwell equations into a single equation*. Second, this YM monopole $p^{\alpha\mu\nu}$ has now been populated with a triplet of YM source currents j_σ, j_τ, j_γ having three distinct indexes. So when expanded with $j^\nu = \tau_i j_i^{\ \nu}$, and with each source being related to fermion wavefunctions courtesy of Dirac's [12], the net result of (4.5) is that *we have populated the Yang-Mills monopole with a triplet of fermions*. Via the exclusion principle following symmetry breaking these will end up in the fundamental representation of an exact SU(3) group, and thus have the same character as a quark triplet. This is the route to discovering that these YM monopoles possess all the QCD properties of baryons.

5. Nonlinear recursive interactions contained in the inverse Yang-Mills electric equation

The next step is to explicitly calculate the inverse using (4.4), then insert this in (4.3) and (4.5). This inverse calculation is carried out in detail in Appendix A. The result including the $+i\varepsilon$ prescription is (A.12). Before proceeding, however, let us establish notation conventions for representing the energy-momentum of a particle four-vector: Whenever the energy-momentum is that of a fermion we shall use the notation p^μ. For a massive vector boson we use k^μ. And for a massless boson such as a photon or gluon we use q^μ. Accordingly, with c restored, because (A.12) is for a massive vector boson, with a parenthetical $\left(G_\alpha ...\right)$ for highlighting reasons to be momentarily reviewed, and using the quoted "denominator" from (A.12) whereby we represent an inverse by $M^{-1} \equiv 1/"M"$ with that inverse placed at the $_\vee$, we shall write this result as:

$$I_{\alpha\nu} = \frac{{}_\vee -g_{\alpha\nu} + \dfrac{k_\nu k_\alpha + gk_\nu\left(G_\alpha ...\right)/c}{m^2c^2}}{"k_\sigma k^\sigma - m^2c^2 - g^2\left(G_\sigma ...\right)\left(G^\sigma ...\right)/c^2 + i\varepsilon"}, \tag{5.1}$$

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Next, we use (5.1) in (4.3) to write:

$$G_\alpha\left(j^\nu,G_\alpha\right)=c\mu_0 I_{\alpha\nu} j^\nu = c\mu_0 \frac{{}_\vee{-g_{\alpha\nu}}+\dfrac{k_\nu k_\alpha + g k_\nu\left(G_\alpha\ldots\right)/c}{m^2c^2}}{"k_\sigma k^\sigma - m^2c^2 - g^2\left(G_\sigma\ldots\right)\left(G^\sigma\ldots\right)/c^2+i\varepsilon"} j^\nu . \tag{5.2}$$

When we contract the above from the left with another j^α to form the Lagrangian density term $j^\alpha G_\alpha$ the result represents the propagator for a YM vector boson mediating between two source currents, which interactions are routinely represented with Feynman diagrams.

Now, were it not for the Yang-Mills terms with $\left(G_\alpha\ldots\right)$ in the above, we could say that (5.2) inverts (4.1a) to obtain G_α as a function $G_\alpha\left(j^\nu\right)$ exclusively of j^ν. But, G^σ is in (5.2), so that is not what (5.2) does. Rather, in (5.2) $G_\alpha\left(j^\nu,G_\alpha\right)$ is a *recursive* function of j^ν *and of itself*. For example, at the first recursion we substitute G_α on the left into the $\left(G_\alpha\ldots\right)$ to obtain:

$$G_\alpha = c\mu_0 \frac{{}_\vee{-g_{\alpha\nu}}+\dfrac{k_\nu k_\alpha + g k_\nu\left(c\mu_0 \dfrac{{}_\vee{-g_{\alpha\beta}}+\dfrac{k_\beta k_\alpha + g k_\beta\left(G_\alpha\ldots\right)/c}{m^2c^2}}{"k_\sigma k^\sigma - m^2c^2 - g^2\left(G_\sigma\ldots\right)\left(G^\sigma\ldots\right)/c^2+i\varepsilon"} j^\beta\right)/c}{m^2c^2}}{"k_\sigma k^\sigma - m^2c^2 - g^2\left(c\mu_0 \dfrac{{}_\vee{-g_{\sigma\tau}}+\dfrac{k_\tau k_\sigma + g k_\tau\left(G_\sigma\ldots\right)/c}{m^2c^2}}{"k_\sigma k^\sigma - m^2c^2 - g^2\left(G_\sigma\ldots\right)\left(G^\sigma\ldots\right)/c^2+i\varepsilon"} j^\tau\right)^2/c^2+i\varepsilon"} j^\nu . \tag{5.3}$$

In the bottom denominator we use the shorthand $G_\sigma G^\sigma \equiv G^{\sigma 2}$. And this type of substitution can and must be done indefinitely, approaching an infinite number of substitutions, before we truly have $G_\alpha\left(j^\nu\right)$ and not $G_\alpha\left(j^\nu,G_\alpha\right)$. We may symbolically represent this infinite recursive series by $G_\alpha\left(j^\nu,G_\alpha\left(j^\nu,G_\alpha\left(j^\nu,G_\alpha\left(j^\nu,G_\alpha\left(\ldots\right)\right)\right)\right)\right)$, and by the $\left(G_\alpha\ldots\right)$ in (5.1) to (5.3). Obviously, the Lagrangian term $j^\alpha G_\alpha$ has similarly recursive character.

Likewise, this introduces an infinite number of occurrences of j^ν into (5.3), amplifying the nonlinearity of interactions amongst the j^ν themselves. This should not be surprising, because it is well-known that Yang-Mills gauge theories are highly non-linear: Using (2.1), the Lagrangian density $\mathfrak{L} = -\tfrac{1}{4}\partial^{[\mu}G_i^{\nu]}\partial_{[\mu}G_{i\nu]} - \tfrac{1}{2}gf_{ijk}\partial^{[\mu}G_i^{\nu]}G_{j\mu}G_{k\nu} - \tfrac{1}{4}g^2 f_{ijk}f_{ilm}G_j^{\mu}G_k^{\nu}G_{l\mu}G_{mv}$ in which $G^\mu = \tau_i G_i^\mu$ and $\left[\tau_i,\tau_j\right] = if_{ijk}\tau_k$, provides the basis for three- and four-gluon vertices. This is often used to illustrate the inherent non-linearity of particle interactions in Yang-Mills gauge theory, versus linear behaviors in QED where photons do not interact among themselves. So, the recursion in (5.3) is just another manifestation of this Yang-Mills non-linearity.

6. Introducing the inverse Yang-Mills electric source equation into the Yang-Mills magnetic monopoles, then populating these monopoles with Dirac fermions

Next, we substitute the inverse (5.1) with renamed indexes as needed and recursion-highlighting parentheses removed, into the monopoles (4.5), to obtain:

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$$ic\varepsilon_0 p^{\alpha\mu\nu} = g\left(\begin{array}{l} \partial^\alpha\left[\dfrac{-g^{\mu\sigma}+\dfrac{k^\sigma k^\mu + gk^\sigma G^\mu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}j_\sigma, \dfrac{-g^{\nu\tau}+\dfrac{k^\tau k^\nu + gk^\tau G^\nu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}j_\tau\right] \\ +\partial^\mu\left[\dfrac{-g^{\nu\tau}+\dfrac{k^\tau k^\nu + gk^\tau G^\nu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}j_\tau, \dfrac{-g^{\alpha\gamma}+\dfrac{k^\gamma k^\alpha + gk^\gamma G^\alpha/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}j_\gamma\right] \\ +\partial^\nu\left[\dfrac{-g^{\alpha\gamma}+\dfrac{k^\gamma k^\alpha + gk^\gamma G^\alpha/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}j_\gamma, \dfrac{-g^{\mu\sigma}+\dfrac{k^\sigma k^\mu + gk^\sigma G^\mu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}j_\sigma\right] \end{array}\right). \quad (6.1)$$

This provides a complete, explicit description of the way in which the YM monopoles are populated with the three source currents j_σ, j_τ, j_γ, keeping in mind that every G^μ needs to be filled with an unlimited-approaching-infinite number of recursions as in (5.3).

And when we further expand using $j^\nu = g\overline{\psi}\gamma^\nu\psi + \kappa^\nu$ obtained from the Yang-Mills continuity equation as reviewed in (B.4) of Appendix B, we can directly populate (6.1) with Dirac fermions, as such:

$$ic\varepsilon_0 p^{\alpha\mu\nu} =$$

$$g\left(\begin{array}{l} \partial^\alpha\left[\dfrac{-g^{\mu\sigma}+\dfrac{k^\sigma k^\mu + gk^\sigma G^\mu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}\begin{pmatrix} g\overline{\psi}\gamma_\sigma\psi \\ +\kappa_\sigma \end{pmatrix}, \dfrac{-g^{\nu\tau}+\dfrac{k^\tau k^\nu + gk^\tau G^\nu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}\begin{pmatrix} g\overline{\psi}\gamma_\tau\psi \\ +\kappa_\tau \end{pmatrix}\right] \\ +\partial^\mu\left[\dfrac{-g^{\nu\tau}+\dfrac{k^\tau k^\nu + gk^\tau G^\nu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}\begin{pmatrix} g\overline{\psi}\gamma_\tau\psi \\ +\kappa_\tau \end{pmatrix}, \dfrac{-g^{\alpha\gamma}+\dfrac{k^\gamma k^\alpha + gk^\gamma G^\alpha/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}\begin{pmatrix} g\overline{\psi}\gamma_\gamma\psi \\ +\kappa_\gamma \end{pmatrix}\right] \\ +\partial^\nu\left[\dfrac{-g^{\alpha\gamma}+\dfrac{k^\gamma k^\alpha + gk^\gamma G^\alpha/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}\begin{pmatrix} g\overline{\psi}\gamma_\gamma\psi \\ +\kappa_\gamma \end{pmatrix}, \dfrac{-g^{\mu\sigma}+\dfrac{k^\sigma k^\mu + gk^\sigma G^\mu/c}{m^2c^2}}{\text{"}k_\sigma k^\sigma - m^2c^2 - g^2G_\sigma G^\sigma/c^2 + i\varepsilon\text{"}}\begin{pmatrix} g\overline{\psi}\gamma_\sigma\psi \\ +\kappa_\sigma \end{pmatrix}\right] \end{array}\right). \quad (6.2)$$

Then, by an indefinite-number-approaching-infinity of recursions as reviewed at (5.2) and (5.3), the fermion and boson interactions inside these monopoles are seen to be highly-nonlinear to infinite order. The final step is to show that (6.2) indeed represents QCD properties of a baryon with three quarks and highly-nonlinear gluon interactions among these quarks.

7. The Yang-Mills "signal" magnetic monopole, without perturbative "noise"

It is well established that protons and neutrons, which are the two most important baryons insofar as they form the nuclei of the observed material universe, are teeming with non-linear interactions. For example, 2019 PDG data [13] informs us that the free proton and neutron rest masses are $M_p = 938.272081 \pm 0.000006\ \text{MeV}$ and $M_n = 939.565413 \pm 0.000006\ \text{MeV}$, but that the up and down current quark masses in an $\overline{\text{MS}}$ renormalization scheme at a scale $\mu \approx 2\,\text{GeV}$ are $m_u = 2.16^{+0.49}_{-0.26}\ \text{MeV}$ and $m_d = 4.67^{+0.48}_{-0.17}\ \text{MeV}$, all respectively. With quark content p(duu) and n(udd), these "current quark" masses contribute about 1% to the overall rest energies of these baryons, with the other 99% arising from nonlinear gluon-mediated interactions among these quarks and from internal kinetic energies. These are distinguished from "constituent quark" masses which stem from attributing about 1/3 of the total rest energy of a nucleon (≈ 313 GeV) arising from their

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quark and gluon energies, to each quark. So, for a baryon, we may similarly coin "current baryon" to refer to the bare quark structure of the baryon with all nonlinearity stripped away, and "constituent baryon" to refer to the baryon including all its nonlinear behaviors. Here, going forward, we shall borrow the electrical engineering terms "signal" and "noise," and use the term "signal baryon" to refer to a baryon with all non-linear behaviors stripped away (the "current baryon"), and use "signal-plus-noise baryon" to refer to the entire observed baryon in all its nonlinear glory (the "constituent baryon"). Using this language, with each G^{μ} in (6.2) treated recursively in the manner of (5.3), what we have in (6.2) is clearly a "signal-plus-noise" monopole density. To explore the underlying QCD behaviors of these monopoles, we now shall study just the "signal" monopole density with all "noise" removed.

It can be fairly said that Yang-Mills gauge theory is a theory of perturbations added to Maxwell's linear electrodynamics in Section 1. For example, in the opening paragraph of Section 3 we find this in $V = ig\left(G^{\sigma}\partial_{\sigma} + \partial^{\sigma}G_{\sigma}\right) + g^2 G^{\sigma}G_{\sigma} = \partial_{\sigma}\partial^{\sigma} - D_{\sigma}D^{\sigma}$ arising from the second-order structure of the Klein-Gordon equation, and in $V_D = g\gamma^0\gamma^{\sigma}G_{\sigma}$ arising from the first-order Dirac equation. This is the "noise" of Yang-Mills theory. This is what Jaffe and Witten in [2] refer to as "excitations of the vacuum." If we wish to study just the "signal" without "excitations," the way to do so is to set the perturbations to zero and see what is left. So we do just that: Working from (6.1), the sources j^{μ} can be related to Dirac fermions via $j^{\mu} = g\overline{\psi}\gamma^{\mu}\psi + \kappa^{\mu}$ obtained in (B.4). Then we set $V_D = 0$ which likewise means we have set $G_{\sigma}\left(t, \mathbf{x}\right) = 0$ thus $V_{\nu\sigma} = 0$. This removes from (6.1), all recursive non-linearity reviewed at (5.3), and it turns "denominators" into regular denominators without quotes. Still remaining are objects of the form $k^{\sigma} j_{\sigma}$. But as seen following (B.4), when $V_{\nu\sigma} = 0$ the continuity equation is $p_{\nu} j^{\nu} = 0$ which in notation reviewed at the start of Section 5 means that $k^{\sigma} j_{\sigma} = 0$. So, all terms of the form $k^{\sigma}k^{\mu} j_{\sigma} / m^2c^2 = 0$ can be removed from (6.1). At this point all that remains are six numerator terms for which indexes can be raised via $-g^{\mu\sigma} j_{\sigma} = -j^{\mu}$, and signs cancelled. We finally segregate commutators in the numerator, so the pure "signal monopole" density inside the "noisy" (6.1) simply becomes:

$$ic\varepsilon_0 p^{\sigma\mu\nu} = g\frac{\partial^{\sigma}\left[j^{\mu}, j^{\nu}\right] + \partial^{\mu}\left[j^{\nu}, j^{\sigma}\right] + \partial^{\nu}\left[j^{\sigma}, j^{\mu}\right]}{\left(k_{\tau}k^{\tau} - m^2c^2 + i\varepsilon\right)^2}. \tag{7.1}$$

It is helpful to contrast (7.1) with (3.1b) from which it originated: All that has changed, is that the gauge boson commutators $\left[G^{\mu}, G^{\nu}\right]$ have been replaced by the electric source commutators $\left[j^{\mu}, j^{\nu}\right]$, with the newly-appearing denominators reflecting the inversion of Maxwell's Yang-Mills electric equation (4.1a) with mass *m* into $G_{\alpha}\left(j^{\nu}\right)$ for the non-recursive signal monopole. Because the monopole is itself an NxN matrix for SU(N), with generators each normalized to $\mathrm{tr}\left(\tau_i^{\,2}\right) = \tfrac{1}{2}$ we may additionally use the relation $\mathrm{tr}\left(AB\right) = \mathrm{tr}\left(\tau_i\tau_j A_i B_j\right) = \tfrac{1}{2} A_i B_i$ for $A = \tau_i A_i$ and $B = \tau_i B_i$, thus $\mathrm{tr}\left[j^{\mu}, j^{\nu}\right] = \tfrac{1}{2}\left[j_i^{\,\mu}, j_i^{\,\nu}\right]$, to take the trace of both sides of (7.1), with the result that:

$$ic\varepsilon_0 \mathrm{tr}\, p^{\sigma\mu\nu} = \tfrac{1}{2} g\frac{\partial^{\sigma}\left[j_i^{\,\mu}, j_i^{\,\nu}\right] + \partial^{\mu}\left[j_i^{\,\nu}, j_i^{\,\sigma}\right] + \partial^{\nu}\left[j_i^{\,\sigma}, j_i^{\,\mu}\right]}{\left(k_{\tau}k^{\tau} - m^2c^2 + i\varepsilon\right)^2}. \tag{7.2}$$

8. Populating the Yang-Mills "signal" magnetic monopole with Dirac fermions: two alternatives, each of which shows that the signal monopole contains exactly three fermions

The next step is to populate the "signal" monopole trace (7.2) with Dirac fermions, similarly to what we did going from (6.1) to (6.2). The general relation $j^\nu = g\overline{\psi}\gamma^\nu\psi + \kappa^\nu$ between each j^ν and its fermion wavefunctions ψ is (B.4). But since (7.1) is a signal monopole in which we have set $V_{\nu\sigma} = 0$ thus $\kappa^\nu = 0$, (B.4) becomes $j^\nu = g\overline{\psi}\gamma^\nu\psi$. Because j^ν is an NxN matrix, the expansion of this is $j^\nu = \tau_i j_i^{\ \nu} = g\tau_i\overline{\psi}\tau_i\gamma^\nu\psi$ with $j_i^{\ \nu} = g\overline{\psi}\tau_i\gamma^\nu\psi$. For SU(N) these ψ are Nx4 column vectors, with N arising from the Yang-Mills and 4 from the Dirac structure of each fermion. Because these sit in the fundamental representation of SU(N) we need to have N distinct SU(N) state labels for each ψ and adjoint $\overline{\psi}$. There are two possibilities:

First, because $j_i^{\ \mu} = g\overline{\psi}\tau_i\gamma^\mu\psi$, for example, has a μ index, we may assign the label μ to this fermion and its adjoint and write $j_i^{\ \mu} = g\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_{(\mu)}$. Likewise for $j_i^{\ \nu}$ and $j_i^{\ \sigma}$. We may also label $m_{(\mu)}$ and $\varepsilon_{(\mu)}$ and $k_\mu k^\mu$ (the last with indexes doubling as labels) in the denominator, ditto for ν and σ. Doing this, and expanding commutators in (7.2), we obtain:

$$ic\varepsilon_0 \mathrm{tr}\, p^{\sigma\mu\nu} = \tfrac{1}{2}g^3 \begin{pmatrix} \partial^\sigma \dfrac{\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_{(\mu)}\overline{\psi}_{(\nu)}\tau_i\gamma^\nu\psi_{(\nu)} - \overline{\psi}_{(\nu)}\tau_i\gamma^\nu\psi_{(\nu)}\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_{(\mu)}}{\left(k_\mu k^\mu - m_{(\mu)}{}^2c^2 + i\varepsilon_{(\mu)}\right)\left(k_\nu k^\nu - m_{(\nu)}{}^2c^2 + i\varepsilon_{(\nu)}\right)} \\ +\partial^\mu \dfrac{\overline{\psi}_{(\nu)}\tau_i\gamma^\nu\psi_{(\nu)}\overline{\psi}_{(\sigma)}\tau_i\gamma^\sigma\psi_{(\sigma)} - \overline{\psi}_{(\sigma)}\tau_i\gamma^\sigma\psi_{(\sigma)}\overline{\psi}_{(\nu)}\tau_i\gamma^\nu\psi_{(\nu)}}{\left(k_\nu k^\nu - m_{(\nu)}{}^2c^2 + i\varepsilon_{(\nu)}\right)\left(k_\sigma k^\sigma - m_{(\sigma)}{}^2c^2 + i\varepsilon_{(\sigma)}\right)} \\ +\partial^\nu \dfrac{\overline{\psi}_{(\sigma)}\tau_i\gamma^\sigma\psi_{(\sigma)}\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_{(\mu)} - \overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_{(\mu)}\overline{\psi}_{(\sigma)}\tau_i\gamma^\sigma\psi_{(\sigma)}}{\left(k_\sigma k^\sigma - m_{(\sigma)}{}^2c^2 + i\varepsilon_{(\sigma)}\right)\left(k_\mu k^\mu - m_{(\mu)}{}^2c^2 + i\varepsilon_{(\mu)}\right)} \end{pmatrix}. \tag{8.1}$$

These fermions as well as objects in the denominators are now labelled with the index of the $j_i^{\ \mu}$, $j_i^{\ \nu}$ or $j_i^{\ \sigma}$ which, colloquially speaking, "brought them to the monopole dance" when we inserted fermions via source currents at (6.2), prior to removing the "noise."

Second, alternatively, although the fermions were introduced into the signal-plus-noise monopole at (6.2), once introduced, they have become part and parcel of a monopole *system* merging both Yang-Mills-Maxwell dynamic equations (3.1), with (3.1a) given mass at (4.1) then inverted at (5.2). So, once this monopole system is established, we can change the labeling in (8.1) so each fermion and related denominator objects are labelled, not by the index of the source which "brought them to the dance," but the index of the partial derivative acting on the fermion in the monopole system, thus rendering the system *indivisible*. In this alternative (7.2) becomes:

$$ic\varepsilon_0 \mathrm{tr}\, p^{\sigma\mu\nu} = \tfrac{1}{2}g^3 \begin{pmatrix} \partial^\sigma \dfrac{\overline{\psi}_{(\sigma)}\tau_i\gamma^\mu\psi_{(\sigma)}\overline{\psi}_{(\sigma)}\tau_i\gamma^\nu\psi_{(\sigma)} - \overline{\psi}_{(\sigma)}\tau_i\gamma^\nu\psi_{(\sigma)}\overline{\psi}_{(\sigma)}\tau_i\gamma^\mu\psi_{(\sigma)}}{\left(k_\sigma k^\sigma - m_{(\sigma)}{}^2c^2 + i\varepsilon_{(\sigma)}\right)^2} \\ +\partial^\mu \dfrac{\overline{\psi}_{(\mu)}\tau_i\gamma^\nu\psi_{(\mu)}\overline{\psi}_{(\mu)}\tau_i\gamma^\sigma\psi_{(\mu)} - \overline{\psi}_{(\mu)}\tau_i\gamma^\sigma\psi_{(\mu)}\overline{\psi}_{(\mu)}\tau_i\gamma^\nu\psi_{(\mu)}}{\left(k_\mu k^\mu - m_{(\mu)}{}^2c^2 + i\varepsilon_{(\mu)}\right)^2} \\ +\partial^\nu \dfrac{\overline{\psi}_{(\nu)}\tau_i\gamma^\sigma\psi_{(\nu)}\overline{\psi}_{(\nu)}\tau_i\gamma^\mu\psi_{(\nu)} - \overline{\psi}_{(\nu)}\tau_i\gamma^\mu\psi_{(\nu)}\overline{\psi}_{(\nu)}\tau_i\gamma^\sigma\psi_{(\nu)}}{\left(k_\nu k^\nu - m_{(\nu)}{}^2c^2 + i\varepsilon_{(\nu)}\right)^2} \end{pmatrix}. \tag{8.2}$$

As we shall see, (8.1) and (8.2) are related through a form of symmetry breaking analogous in some ways to what is used in electroweak theory, which will become identified with hadronization.

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Most importantly – for both (8.1) and (8.2) – is that this signal baryon is now seen to contain exactly <u>*three fermions*</u> $\psi_{(\sigma)}$, $\psi_{(\mu)}$, $\psi_{(\nu)}$ which arise from the rank-3 antisymmetric tensor which is the monopole $c\mu_0 p^{\sigma\mu\nu} = \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu}$ of (3.1b) and its three additive terms. This "three-ness" is structurally fundamental to the covariant representation of Maxwell's magnetic equations whether in U(1) Maxwell or in SU(N) Yang-Mills gauge theory. Consequently, the non-perturbative signal monopoles (8.1), (8.2) each describe a *system* – and (8.2) an *indivisible* system – built upon three distinct fermions. Just like a baryon.

9. Using the gauge group SU(3) to establish three distinct quantum states for the three fermions populating a Yang-Mills magnetic monopole

The Fermi-Dirac-Pauli Exclusion Principle mandates that the fermions contained in any *system* of more than one fermion, e.g., an atom, nucleus, nucleon or baryon must be distinguishable from all other fermions in that system by assignment of an *exclusive quantum state*. Accordingly, each of the fermions in (8.1), (8.2) – being part of the monopole *system* – must have an exclusive quantum state. Because both (8.1) and (8.2) contain three fermions, the YM SU(N) gauge group used to provide this exclusion must have $\mathrm{N} \geq 3$, so it *cannot* be SU(2). On the other hand, because there are exactly three fermions in both (8.1) and (8.2), there is no need for $\mathrm{N} > 3$. Accordingly, we now use the group SU(3) to enforce Exclusion on the three fermions $\psi_{(\sigma)}$, $\psi_{(\mu)}$, $\psi_{(\nu)}$ in these alternative signal monopole *systems* (8.1), (8.2). (We note without further detail here, that for these monopoles to be topologically stable, we must eventually employ $\mathrm{SU(3)}\times\mathrm{U(1)}$ following spontaneous symmetry breaking from a larger group, see Cheng and Li [14] at 472-473 and Weinberg [15] at 442. This U(1) generator provides the foundation for introducing hadron *flavor*, which is the next developmental step following the results in this article regarding hadron *color*.)

So, for what has heretofore been τ_i, we now use the 3x3 SU(3) Gell-Mann generators via $\tau_i = \frac{1}{2}\lambda_i$ with $i = 1 \ldots 8$, normalized to $\mathrm{tr}(\tau_i)^2 = \frac{1}{2}$. We use $\mathrm{diag}(\tau_8) = \frac{1}{2\sqrt{3}}(2,-1,-1)$, so that the $i = 1,2,3$ SU(2) subset is embedded in the lower-right portion of τ_i. We may of course choose any labels we wish for these three exclusive states, so we may as well call these Red, Green and Blue, then see whether and how these can be made synonymous with the colored quark states of Quantum Chromodynamics (QCD). With *T* denoting the transpose, we place these in the fundamental SU(3) representation with the explicit column vectors and consequent adjoints:

$$\begin{aligned}
\psi_{(\sigma)} &\equiv \left|\tau_8 = +\tfrac{1}{2}\tfrac{2}{\sqrt{3}}; \tau_3 = 0\right\rangle = \begin{pmatrix}\psi_R & 0 & 0\end{pmatrix}^T; &\quad \overline{\psi}_{(\sigma)} &= \begin{pmatrix}\overline{\psi}_R & 0 & 0\end{pmatrix} \\
\psi_{(\mu)} &\equiv \left|\tau_8 = -\tfrac{1}{2\sqrt{3}}; \tau_3 = +\tfrac{1}{2}\right\rangle = \begin{pmatrix}0 & \psi_G & 0\end{pmatrix}^T; &\quad \overline{\psi}_{(\mu)} &= \begin{pmatrix}0 & \overline{\psi}_G & 0\end{pmatrix}. \\
\psi_{(\nu)} &\equiv \left|\tau_8 = -\tfrac{1}{2\sqrt{3}}; \tau_3 = -\tfrac{1}{2}\right\rangle = \begin{pmatrix}0 & 0 & \psi_B\end{pmatrix}^T; &\quad \overline{\psi}_{(\nu)} &= \begin{pmatrix}0 & 0 & \overline{\psi}_B\end{pmatrix}
\end{aligned} \tag{9.1}$$

Each of these ψ is now a 3x4 column vector (ket) and each $\overline{\psi}$ a 3x4 row vector (bra), with the 3 owing to the YM SU(3) internal symmetry and the 4 owing to the four-components of Dirac wavefunctions and spinors. From here we will use (9.1) in (8.1), then in (8.2).

It is profoundly important that the Exclusion Principle taken together with the aforementioned "three-ness" of magnetic monopoles, combine to put exactly three fermions into the signal monopole for SU(N), and so lead directly to SU(N=3) as the symmetry group required to establish three exclusive fermion states. Normally, SU(3) with R, G, B states is the starting point upon which QCD is founded. Here, in contrast, we can be entirely agnostic *a priori* about the N in SU(N), until we find that *the inherent structure of a Yang-Mills signal magnetic monopole* <u>*requires*</u> *that fermions inside the monopole be placed into the fundamental representation of SU(3)*. This raises the prospect that QCD has its dynamic physical origins in the non-vanishing magnetic monopoles of Yang-Mills gauge theory. But again, for now, R, G and B are just labels: $\mathrm{SU(3)_{QCD}}$ is an *exact* symmetry because gluons in the adjoint representation are massless. But, for example, early theories of baryon flavor similarly placed

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(u, d, s) into the fundamental representation of SU(3) with an *approximate* flavor symmetry which is distinct from the *exact* color symmetry of SU(3)QCD. So we must establish that this SU(3) group arising from the monopoles is truly synonymous with the exact SU(3) group of QCD, not some independent SU(3).

10. The Yang-Mills signal magnetic monopole prior to symmetry breaking

Proceeding, we observe that at the center of each numerator in (8.1) are terms in which a wavefunction is immediately to the left of a *differently-labelled* adjoint e.g. $\psi_{(\mu)}\overline{\psi}_{(\nu)}$, while in (8.2) we see the *same-labelled* e.g. $\psi_{(\sigma)}\overline{\psi}_{(\sigma)}$. In U(1) gauge theory these are 4x4 Dirac matrices, and in view of (9.1) these are 3x3 Yang-Mills matrices of 4x4 Dirac matrices. Specifically, focusing on the $\psi_{(\mu)}\overline{\psi}_{(\nu)}$ etc. "backbone" centering each numerator in (8.1), and carrying through antisymmetric signage between any pair of σ,μ,ν indexes, we first observe that:

$$\psi_{(\mu)}\overline{\psi}_{(\nu)}-\psi_{(\nu)}\overline{\psi}_{(\mu)}+\psi_{(\nu)}\overline{\psi}_{(\sigma)}-\psi_{(\sigma)}\overline{\psi}_{(\nu)}+\psi_{(\sigma)}\overline{\psi}_{(\mu)}-\psi_{(\mu)}\overline{\psi}_{(\sigma)}=\begin{pmatrix}0 & \psi_R\overline{\psi}_G & -\psi_R\overline{\psi}_B\\ -\psi_G\overline{\psi}_R & 0 & \psi_G\overline{\psi}_B\\ \psi_B\overline{\psi}_R & -\psi_B\overline{\psi}_G & 0\end{pmatrix}. \quad (10.1)$$

Then, we use (8.1) to flesh out the above, obtaining:

$$2ic\varepsilon_0 \mathrm{tr}\, p^{\sigma\mu\nu}/g^3 =$$

$$\begin{pmatrix} 0 & \frac{\partial^\nu\left(\overline{\psi}_{(\sigma)}\tau_i\gamma^\sigma\psi_R\overline{\psi}_G\tau_i\gamma^\mu\psi_{(\mu)}\right)}{\left(k_{\sigma,\mu}k^{\sigma,\mu}-m_{(\sigma,\mu)}{}^2c^2+i\varepsilon_{(\sigma,\mu)}\right)^2} & \frac{-\partial^\mu\left(\overline{\psi}_{(\sigma)}\tau_i\gamma^\sigma\psi_R\overline{\psi}_B\tau_i\gamma^\nu\psi_{(\nu)}\right)}{\left(k_{\{\nu,\sigma\}}k^{\{\nu,\sigma\}}-m_{(\nu,\sigma)}{}^2c^2+i\varepsilon_{(\nu,\sigma)}\right)^2}\\ \frac{-\partial^\nu\left(\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_G\overline{\psi}_R\tau_i\gamma^\sigma\psi_{(\sigma)}\right)}{\left(k_{\sigma,\mu}k^{\sigma,\mu}-m_{(\sigma,\mu)}{}^2c^2+i\varepsilon_{(\sigma,\mu)}\right)^2} & 0 & \frac{\partial^\sigma\left(\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_G\overline{\psi}_B\tau_i\gamma^\nu\psi_{(\nu)}\right)}{\left(k_{\{\mu,\nu\}}k^{\mu}-m_{(\mu,\nu)}{}^2c^2+i\varepsilon_{(\mu,\nu)}\right)^2}\\ \frac{\partial^\mu\left(\overline{\psi}_{(\nu)}\tau_i\gamma^\nu\psi_B\overline{\psi}_R\tau_i\gamma^\sigma\psi_{(\sigma)}\right)}{\left(k_{\{\nu,\sigma\}}k^{\{\nu,\sigma\}}-m_{(\nu,\sigma)}{}^2c^2+i\varepsilon_{(\nu,\sigma)}\right)^2} & \frac{-\partial^\sigma\left(\overline{\psi}_{(\nu)}\tau_i\gamma^\nu\psi_B\overline{\psi}_G\tau_i\gamma^\mu\psi_{(\mu)}\right)}{\left(k_{\{\mu,\nu\}}k^{\mu}-m_{\{\mu,\nu\}}{}^2c^2+i\varepsilon_{\{\mu,\nu\}}\right)^2} & 0\end{pmatrix}, \quad (10.2)$$

where $\left(k_\mu k^\mu-m_{(\mu)}{}^2c^2+i\varepsilon_{(\mu)}\right)\left(k_\nu k^\nu-m_{(\nu)}{}^2c^2+i\varepsilon_{(\nu)}\right)\equiv\left(k_{\{\mu,\nu\}}k^\mu-m_{(\mu,\nu)}{}^2c^2+i\varepsilon_{(\mu,\nu)}\right)^2$ etc. is defined as a shorthand for the denominators simply to save space. Taking a second trace, $\mathrm{tr}\,\mathrm{tr}\, p^{\sigma\mu\nu}=0$.

If we look at any of the six off-diagonal entries in (10.2), for example, the term with a $\partial^\sigma\left(\overline{\psi}_{(\mu)}\tau_i\gamma^\mu\psi_G\overline{\psi}_B\tau_i\gamma^\nu\psi_{(\nu)}\right)$ numerator and a $\left(k_\mu k^\mu-m_{(\mu)}{}^2c^2+i\varepsilon_{(\mu)}\right)\left(k_\nu k^\nu-m_{(\nu)}{}^2c^2+i\varepsilon_{(\nu)}\right)$ denominator (second row, third column), we see something of a mismatch, with two fermions in the numerator and two massive vector boson propagators in the denominator. Fermions, of course, contain four degrees of freedom (spin up and down, particle and antiparticle), while massive vector bosons contain three degrees of freedom (two transverse, one longitudinal). So with both $\overline{\psi}_{(\mu)}$ and $\psi_{(\nu)}$, in this numerator there are two *fermions* totaling eight degrees of freedom, while with both $k_\mu k^\mu-m_{(\mu)}{}^2c^2+i\varepsilon_{(\mu)}$ and $k_\nu k^\nu-m_{(\nu)}{}^2c^2+i\varepsilon_{(\nu)}$ in the denominator there are two *massive vector boson* propagator denominators representing a total of six degrees of freedom.

Moreover, SU(3)QCD requires *massless* gluons, whereas the denominators in (10.2) are all for *massive* vector bosons. So this looks like approximate SU(3) flavor rather than exact SU(3) color symmetry. Moreover, (10.2) has vanishing trace. Moreover, knowing that all particle propagators have the general form $i\Sigma_{\mathrm{spins}}/\left(p^2-m^2+i\varepsilon\right)$ with spin sum Σ_{spins} being the completeness relation, we see that $\psi_G\overline{\psi}_B$ at the center of this numerator looks like a spin sum $\Sigma_s u\overline{u}=\not{p}+m$, but is not.

This is because ψ_G and ψ_B in $\psi_G\overline{\psi}_B$ are *not the same fermion but are two different fermions*. This final point indicates the way forward, because if we can turn ψ_G and ψ_B in $\psi_G\overline{\psi}_B$ into the same fermion we can use this as a fermion spin sum. Then, having a spin sum in the numerator and *two* massive boson propagators in the denominator, we can shuttle a degree of freedom from a boson to a fermion to simultaneously produce a fermion propagator and a massless boson propagator. This entails a form of spontaneous symmetry breaking which starts with (8.1) then breaks symmetry using (8.2), because the backbone of (8.2) does have the requisite same-fermion terms $\psi_{(\sigma)}\overline{\psi}_{(\sigma)}$, $\psi_{(\mu)}\overline{\psi}_{(\mu)}$ and $\psi_{(\nu)}\overline{\psi}_{(\nu)}$ needed to use the completeness relation.

11. Spontaneously breaking symmetry inside the Yang-Mills signal magnetic monopole

Keeping the fermion state definitions (9.1) exactly as is, we now examine the "backbone" of (8.2) formed by the three terms $\psi_{(\sigma)}\overline{\psi}_{(\sigma)}$, $\psi_{(\mu)}\overline{\psi}_{(\mu)}$ and $\psi_{(\nu)}\overline{\psi}_{(\nu)}$. Each wavefunction $\psi\left(x^\mu, p^\mu\right)=u\left(p^\mu\right)\exp\left(-ip_\sigma x^\sigma\right)$ has adjoint $\overline{\psi}=\overline{u}\exp\left(ip_\sigma x^\sigma\right)$, and because these are back-to-back with no γ^α between them, their p_σ are the same. But here, unlike with (10.1), because these are same-labelled with the same p_σ, we may not only write $\psi\overline{\psi}=u\overline{u}$, but given that $\Sigma_s u\overline{u}=\not{p}+m$ for the spin sum over fermion particle states, we may use $u\overline{u}$ as the basis for a spin sum which can lead to a fermion propagator. So, if we additionally take the sum Σ_s over particle spins, what we find in contrast to (10.1) is the now-diagonalized backbone:

$$\Sigma_s\left(\psi_{(\sigma)}\overline{\psi}_{(\sigma)}+\psi_{(\mu)}\overline{\psi}_{(\mu)}+\psi_{(\nu)}\overline{\psi}_{(\nu)}\right)$$
$$=\Sigma_s\begin{pmatrix}\psi_R\overline{\psi}_R & 0 & 0\\ 0 & \psi_G\overline{\psi}_G & 0\\ 0 & 0 & \psi_B\overline{\psi}_B\end{pmatrix}=\Sigma_s\begin{pmatrix}u_R\overline{u}_R & 0 & 0\\ 0 & u_G\overline{u}_G & 0\\ 0 & 0 & u_B\overline{u}_B\end{pmatrix}=\begin{pmatrix}\not{p}_R+m_R & 0 & 0\\ 0 & \not{p}_G+m_G & 0\\ 0 & 0 & \not{p}_B+m_B\end{pmatrix}. \quad (11.1)$$

Next, we use (11.1) to flesh out (8.2), as with (10.1) for (8.1) to obtain (10.2). In doing so we take the spin sums over all of the fermions inside of (8.2), which implies taking $\Sigma_s p^{\sigma\mu\nu}$ over the entire signal monopole system as well. Consequently, with this spin sum included and *c*=1:

$$2ic\varepsilon_0 \mathrm{tr}\,\Sigma_s p^{\sigma\mu\nu}/g^3=$$
$$\begin{pmatrix}\partial^\sigma\dfrac{\overline{\psi}_{(\sigma)}\tau_i\gamma^{[\mu}\left(\not{p}_R+m_R\right)\tau_i\gamma^{\nu]}\psi_{(\sigma)}}{\left(k_\sigma k^\sigma-m_{(\sigma)}{}^2+i\varepsilon_{(\sigma)}\right)^2} & 0 & 0\\ 0 & \partial^\mu\dfrac{\overline{\psi}_{(\mu)}\tau_i\gamma^{[\nu}\left(\not{p}_G+m_G\right)\tau_i\gamma^{\sigma]}\psi_{(\mu)}}{\left(k_\mu k^\mu-m_{(\mu)}{}^2+i\varepsilon_{(\mu)}\right)^2} & 0\\ 0 & 0 & \partial^\nu\dfrac{\overline{\psi}_{(\nu)}\tau_i\gamma^{[\sigma}\left(\not{p}_B+m_B\right)\tau_i\gamma^{\mu]}\psi_{(\nu)}}{\left(k_\nu k^\nu-m_{(\nu)}{}^2+i\varepsilon_{(\nu)}\right)^2}\end{pmatrix}. \quad (11.2)$$

This is the diagonalized counterpart of (10.2). And because of this, we may take another trace which, in contrast to $\mathrm{tr}\,\mathrm{tr}\left(p^{\sigma\mu\nu}\right)=0$ in (10.2), is non-vanishing, namely:

$$2ic\varepsilon_0 \mathrm{tr}\,\mathrm{tr}\,\Sigma_s p^{\sigma\mu\nu}/g^3=$$
$$\left(\partial^\sigma\frac{\overline{\psi}_{(\sigma)}\tau_i\gamma^{[\mu}\left(\not{p}_R+m_R\right)\tau_i\gamma^{\nu]}\psi_{(\sigma)}}{\left(k_\sigma k^\sigma-m_{(\sigma)}{}^2+i\varepsilon_{(\sigma)}\right)^2}+\partial^\mu\frac{\overline{\psi}_{(\mu)}\tau_i\gamma^{[\nu}\left(\not{p}_G+m_G\right)\tau_i\gamma^{\sigma]}\psi_{(\mu)}}{\left(k_\mu k^\mu-m_{(\mu)}{}^2+i\varepsilon_{(\mu)}\right)^2}+\partial^\nu\frac{\overline{\psi}_{(\nu)}\tau_i\gamma^{[\sigma}\left(\not{p}_B+m_B\right)\tau_i\gamma^{\mu]}\psi_{(\nu)}}{\left(k_\nu k^\nu-m_{(\nu)}{}^2+i\varepsilon_{(\nu)}\right)^2}\right). \quad (11.3)$$

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Next, we focus on the Yang-Mills aspect of the numerators irrespective of Dirac matrices, which we can do because γ^μ and τ_i with $\left[\gamma^\mu, \tau_i\right]=0$ are independent operators acting on independent parts of each Nx4 bra or ket in (9.1). We find the three structural combinations $\overline{\psi}_{(\sigma)}\tau_i\tau_i\psi_{(\sigma)}$, $\overline{\psi}_{(\mu)}\tau_i\tau_i\psi_{(\mu)}$ and $\overline{\psi}_{(\nu)}\tau_i\tau_i\psi_{(\nu)}$. So we use $\tau_i=\frac{1}{2}\lambda_i$ to calculate eight columns vectors $\tau_i\psi_{(\sigma)}$, eight $\tau_i\psi_{(\mu)}$, and eight $\tau_i\psi_{(\nu)}$. The adjoint $\overline{\psi}_{(\sigma)}\tau_i$, $\overline{\psi}_{(\mu)}\tau_i$ and $\overline{\psi}_{(\nu)}\tau_i$ are the Hermitian conjugates of these. The net result, easily confirmed, is*:

$$\overline{\psi}_{(\sigma)}\tau_i\tau_i\psi_{(\sigma)}=\tfrac{4}{3}\overline{\psi}_R\psi_R;\qquad \overline{\psi}_{(\mu)}\tau_i\tau_i\psi_{(\mu)}=\tfrac{4}{3}\overline{\psi}_G\psi_G;\qquad \overline{\psi}_{(\nu)}\tau_i\tau_i\psi_{(\nu)}=\tfrac{4}{3}\overline{\psi}_B\psi_B. \tag{11.4}$$

We then use (11.4) in (11.3) and multiply through by i to obtain:

$$-c\varepsilon_0 \operatorname{tr}\operatorname{tr}\Sigma_s p^{\sigma\mu\nu}=$$

$$i\tfrac{2}{3}g^3\left(\partial^\sigma \frac{\overline{\psi}_R\gamma^{[\mu}\left(p_R+m_R\right)\gamma^{\nu]}\psi_R}{\left(k_\sigma k^\sigma-m_{(\sigma)}{}^2+i\varepsilon_{(\sigma)}\right)^2}+\partial^\mu \frac{\overline{\psi}_G\gamma^{[\nu}\left(p_G+m_G\right)\gamma^{\sigma]}\psi_G}{\left(k_\mu k^\mu-m_{(\mu)}{}^2+i\varepsilon_{(\mu)}\right)^2}+\partial^\nu \frac{\overline{\psi}_B\gamma^{[\sigma}\left(p_B+m_B\right)\gamma^{\mu]}\psi_B}{\left(k_\nu k^\nu-m_{(\nu)}{}^2+i\varepsilon_{(\nu)}\right)^2}\right). \tag{11.5}$$

Above, the mismatch between numerators containing fermions and denominators with massive vector boson propagators becomes crystalized. But so too does the solution: In the numerators we have spin sums $p+m$ for fermions, mismatched in the denominators with a *pair* $\left(k^2-m^2+i\varepsilon\right)\left(k^2-m^2+i\varepsilon\right)$ of massive vector boson propagators. Indeed, the reason we respectively established the energy-momentum notation conventions p^μ, k^μ and q^μ for fermions, massive vector bosons and massless vector bosons at the start of Section 5, was so when we arrived at (11.5) this mismatch would be clear.

Now, with the i, let's work with the (11.5) term operated upon. e.g., by ∂^σ, ditto for the others. In the denominator we revert to indexes/labels showing the currents j^μ and j^ν which brought these fermions "to the dance" via (7.2). This $\left(k_\mu k^\mu-m_{(\mu)}{}^2+i\varepsilon_{(\mu)}\right)\left(k_\nu k^\nu-m_{(\nu)}{}^2+i\varepsilon_{(\nu)}\right)$ represents a total of six degrees of freedom, three for each of two massive vector bosons. Although the massive bosons were introduced by hand at (4.1), it is well-known how to give a renormalizable mass to these via $\mathfrak{L}=\partial^\sigma\phi^*\partial_\sigma\phi+g^2G^\sigma G_\sigma\phi^*\phi$ using the Higgs mechanism. We then break symmetry by releasing one degree of freedom from the $k_\mu k^\mu-m_{(\mu)}{}^2+i\varepsilon_{(\mu)}$ denominator and shuttling this over to be swallowed by the $k_\nu k^\nu-m_{(\nu)}{}^2+i\varepsilon_{(\nu)}$ denominator, or vice versa – it does not matter. Doing so, we *demote* $k^\mu\mapsto q^\mu$ into a *<u>massless</u> vector boson* such as a QCD gluon while consequently setting $m_{(\mu)}=0$. Simultaneously, we *promote* $k^\nu\mapsto p^\nu$ to the energy momentum of the fermion and $m_{(\nu)}\mapsto m_R$ and $\varepsilon_{(\nu)}\mapsto\varepsilon_R$ to the mass and ε of the R fermion. Then we rename denominator indexes back to those in ∂^σ. Thus:

$$i\frac{\overline{\psi}_R\gamma^{[\mu}\left(p_R+m_R\right)\gamma^{\nu]}\psi_R}{\left(k_\mu k^\mu-m_{(\mu)}{}^2+i\varepsilon_{(\mu)}\right)\left(k_\nu k^\nu-m_{(\nu)}{}^2+i\varepsilon_{(\nu)}\right)}\xrightarrow[\text{break symmetry}]{}\frac{1}{q_\sigma q^\sigma+i\varepsilon_{(\sigma)}}\frac{\overline{\psi}_R\gamma^{[\mu}i\left(p_R+m_R\right)\gamma^{\nu]}\psi_R}{p_{R\sigma}p_R{}^\sigma-m_R{}^2+i\varepsilon_R}. \tag{11.6}$$

Above, the original 3+3=6 degrees of freedom in the denominator are maintained, but redistributed to 2+4=6 degrees of freedom, two for the now-massless vector boson, and four for the red fermion. Accounting for all three additive terms in (11.5) which now contain the three R, G, B

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* For SU(N) in general, the factor $f=4/3$ shown for SU(3) in (11.4), is equal to ½ times the number of states in the adjoint representation over the number of states in the fundamental representation, that is $f=\frac{1}{2}\left(N^2-1\right)/N$. This is also equal to the magnitude of the principal Casimir operator for any SU(N), that is, $f\,\mathrm{Id}=\boldsymbol{\tau}^2=\Sigma_i\tau_i{}^2$.

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fermion states, we multiply by 3 so that 9+9=18 degrees of freedom at the outset in six massive vector boson propagators have become redistributed as 6+12=18 into three massless vector boson and three massive fermion propagators. If we suppose that the massive vector bosons on the left of (11.6) originally come from a scalar field via the Higgs mechanism, then (11.6) is a later step in a "cascade" wherein a degree of freedom is first passed from a scalar to a massless vector boson to make the latter massive, then is passed from the vector boson to a fermion whereby the vector boson reverts to being massless. Mindful that $(\not p+m)(\not p-m)=p_\sigma p^\sigma - m^2$, the upshot is that we now have a fermion propagator sitting in the middle of (11.6).

Finally, inserting (11.6) with necessary reindexing and relabeling into (11.5) we obtain:

$$-c\varepsilon_0 \mathrm{tr}\ \mathrm{tr}\ \Sigma_s p^{\sigma\mu\nu} = \tfrac{2}{3} g^3 \begin{pmatrix} \dfrac{1}{q_\sigma q^\sigma + i\varepsilon_{(\sigma)}} \partial^\sigma \dfrac{\overline{\psi}_R \gamma^{[\mu} i(\not p_R + m_R)\gamma^{\nu]}\psi_R}{p_{R\sigma} p_R{}^\sigma - m_R{}^2 + i\varepsilon_R} \\ + \dfrac{1}{q_\mu q^\mu + i\varepsilon_{(\mu)}} \partial^\mu \dfrac{\overline{\psi}_G \gamma^{[\nu} i(\not p_G + m_G)\gamma^{\sigma]}\psi_G}{p_{G\mu} p_G{}^\mu - m_G{}^2 + i\varepsilon_G} \\ + \dfrac{1}{q_\nu q^\nu + i\varepsilon_{(\nu)}} \partial^\nu \dfrac{\overline{\psi}_B \gamma^{[\sigma} i(\not p_B + m_B)\gamma^{\mu]}\psi_B}{p_{B\nu} p_B{}^\nu - m_B{}^2 + i\varepsilon_B} \end{pmatrix}. \tag{11.7}$$

This is now the Yang-Mills magnetic monopole with symmetry broken in two stages: First, by taking (8.1) where each fermion is "brought to the dance" with a current j^σ, j^μ or j^ν and turning it into (8.2) whereby the fermions now form part of an *indivisible monopole system*, no longer distinguishing fermions based on their initial "escort." In this stage, (10.2) is diagonalized and turned into (11.2). Second, by using (11.6) to transfer longitudinal vector boson degrees of freedom over to fermions thus rendering the remaining vector bosons massless and revealing complete fermion propagators. Now, with the vector bosons being massless, the SU(3) symmetry introduced at (9.1) becomes an *exact* symmetry like that of SU(3)QCD, rather than the approximate symmetry of SU(3) flavor, bringing us closer to these R, G, B being true QCD states.

12. Incorporating the massless vector boson propagators into the fermion normalizations

We can further simplify (11.7) by suitable normalization of the three fermion spinors. Often, a covariant normalization with an energy-dimensioned $N^2 = E + mc^2$ is employed for this. If we write this with $c=1$ as $1 = N_{R,G,B}{}^2 / (E+m)_{R,G,B}$ and place this with corresponding label in front of each of the three terms in (11.7), we end up with $N_R{}^2/(E+m)_R \left(q_\alpha q^\alpha + i\varepsilon_{(\alpha)}\right)$ in the top line, ditto for the others. Then, because the YM monopole is an indivisible system, we may now choose $N_R'^2/(E+m)_R \left(q_\alpha q^\alpha + i\varepsilon_{(\alpha)}\right) \equiv 1$ as a modified normalization in which $N_R'^2$ scales with the $q_\alpha q^\alpha + i\varepsilon_{(\alpha)}$ of a massless vector boson within the overall system. Doing this, (11.7) simplifies to:

$$-c\varepsilon_0 \mathrm{tr}\ \mathrm{tr}\ \Sigma_s p^{\sigma\mu\nu} = \tfrac{2}{3} g^3 \left(\partial^\sigma \frac{\overline{\psi}_R \gamma^{[\mu} i(\not p_R + m_R)\gamma^{\nu]}\psi_R}{p_{R\sigma} p_R{}^\sigma - m_R{}^2 + i\varepsilon_R} + \partial^\mu \frac{\overline{\psi}_G \gamma^{[\nu} i(\not p_G + m_G)\gamma^{\sigma]}\psi_G}{p_{G\mu} p_G{}^\mu - m_G{}^2 + i\varepsilon_G} + \partial^\nu \frac{\overline{\psi}_B \gamma^{[\sigma} i(\not p_B + m_B)\gamma^{\mu]}\psi_B}{p_{B\nu} p_B{}^\nu - m_B{}^2 + i\varepsilon_B} \right). \tag{12.1}$$

This becomes our final expression for the signal (non-perturbative) Yang-Mills magnetic monopole, which clearly embeds a propagator for each of its three fermions. Now we are ready to show why these monopoles have the same color and confinement properties as baryons, with interactions mediated by entities which have the same color properties as mesons.

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13. Yang-Mills magnetic monopoles have the color-neutral singlet wavefunction of baryons, and interact via objects with the color-neutral singlet wavefunction of mesons

We know that the RGB SU(3) symmetry in the monopole (12.1) is exact, because the vector bosons were made massless at (11.6). We know that in its unperturbed signal state, this monopole contains exactly three fermions. We know too, that monopole $p^{\sigma\mu\nu}$ is a third rank antisymmetric tensor. But, a key step in going from (8.1) to (8.2) in the first symmetry breaking stage, was to assign each fermion, not to the current which "brought it to the dance," but to the index of the partial derivative commonly operating on those fermions once they were "already at the dance." This led to the associations $\sigma \sim R$, $\mu \sim G$ and $\nu \sim B$, as very clearly seen in (12.1).

So, writing out the antisymmetry of the monopole indexes and relating these to their color associations once the fermions are all "at the dance," the wavefunction symmetry of the now-indivisible monopole may be schematically represented by:

$$p^{\sigma\mu\nu} \sim \sigma\mu\nu - \sigma\nu\mu + \mu\nu\sigma - \mu\sigma\nu + \nu\sigma\mu - \nu\mu\sigma \sim RGB - RBG + GBR - GRB + BRG - BGR \,. \quad (13.1)$$

This is precisely the antisymmetric color-neutral singlet wavefunction of a baryon, see eq. [2.70] of [16]. Indeed, one can argue that the antisymmetric indexes in $p^{\sigma\mu\nu}$ should have been a tip-off that YM magnetic monopoles would make good baryons. Though individual fermions and vector bosons inside the monopole carry color charges, *the entire monopole system is a color singlet*.

Next, we return to the differential forms relation $c\mu_0 \iiint p = -ig \oiint G^2 \neq 0$ of (3.3b) and again ask as we did at the end of Section 3: what are these $G^2 = \frac{1}{2}\left[G_\mu, G_\nu\right] dx^\mu dx^\mu$ entities which <u>do</u> net flow across a YM magnetic monopole surface? Lowering all free indexes in (12.1), then using $p = \frac{1}{3!} p_{\sigma\mu\nu} dx^\sigma dx^\mu dx^\nu$ to obtain the monopole 3-form, and using the antisymmetry among each infinitesimal element in $dx^\sigma dx^\mu dx^\nu$ along with index renaming as needed, and also using $c^2 \varepsilon_0 \mu_0 = 1$ while reconnecting this to (3.2b), we obtain:

$$\begin{aligned} &-c\varepsilon_0 \mathrm{tr}\ \mathrm{tr}\Sigma_s p = ic\varepsilon_0 \tfrac{1}{3!} \mathrm{tr}\ \mathrm{tr}\ \Sigma_s p_{\sigma\mu\nu} dx^\sigma dx^\mu dx^\nu = c^2 \varepsilon_0{}^2 d\, \mathrm{tr}\ \mathrm{tr}\ \Sigma_s g G^2 \\ &\partial_\sigma \left\{ \tfrac{2}{9}\tfrac{1}{2} g^3 \left(\frac{\overline{\psi}_R \gamma_{[\mu} i\left(p_R + m_R\right)\gamma_{\nu]}\psi_R}{p_{R\sigma} p_R{}^\sigma - m_R{}^2 + i\varepsilon_R} + \frac{\overline{\psi}_G \gamma_{[\mu} i\left(p_G + m_G\right)\gamma_{\nu]}\psi_G}{p_{G\mu} p_G{}^\mu - m_G{}^2 + i\varepsilon_G} + \frac{\overline{\psi}_B \gamma_{[\mu} i\left(p_B + m_B\right)\gamma_{\nu]}\psi_B}{p_{B\nu} p_B{}^\nu - m_B{}^2 + i\varepsilon_B} \right) dx^\mu dx^\nu \right\} dx^\sigma \end{aligned} . \quad (13.2)$$

We then take the triple integral of all sides and apply (1.6) via $\int_M dp = \int_{\partial M} p$, to find:

$$\begin{aligned} &-c\varepsilon_0 \iiint \mathrm{tr}\ \mathrm{tr}\ \Sigma_s\, p = ic^2 \varepsilon_0{}^2 \iiint d\left\{\mathrm{tr}\ \mathrm{tr}\ \Sigma_s g G^2\right\} = ic^2 \varepsilon_0{}^2 \oiint \mathrm{tr}\ \mathrm{tr}\ \Sigma_s\, g G^2 \\ &= \tfrac{2}{9} g^3 \oiint \tfrac{1}{2} \left(\frac{\overline{\psi}_R \gamma_{[\mu} i\left(p_R + m_R\right)\gamma_{\nu]}\psi_R}{p_{R\sigma} p_R{}^\sigma - m_R{}^2 + i\varepsilon_R} + \frac{\overline{\psi}_G \gamma_{[\mu} i\left(p_G + m_G\right)\gamma_{\nu]}\psi_G}{p_{G\mu} p_G{}^\mu - m_G{}^2 + i\varepsilon_G} + \frac{\overline{\psi}_B \gamma_{[\mu} i\left(p_B + m_B\right)\gamma_{\nu]}\psi_B}{p_{B\nu} p_B{}^\nu - m_B{}^2 + i\varepsilon_B} \right) dx^\mu dx^\nu \end{aligned} . \quad (13.3)$$

Above, the Gaussian integration has removed the ∂_σ operator from (13.2). Extracting the integrands from the surface integrals in (13.3) and using (2.2) written as $-\left(F - dG\right) = igG^2$, then using $G^2 = \frac{1}{2}\left[G_\mu, G_\nu\right] dx^\mu dx^\mu$, restoring all spacetime indexes and removing $dx^\mu dx^\nu$, we find:

$$\begin{aligned} &-c^2 \varepsilon_0{}^2 \mathrm{tr}\ \mathrm{tr}\ \Sigma_s \left(F_{\mu\nu} - \partial_\mu G_\nu + \partial_\nu G_\mu\right) = ic^2 \varepsilon_0{}^2 \mathrm{tr}\ \mathrm{tr}\ \Sigma_s\, g\left[G_\mu, G_\nu\right] \\ &= \tfrac{2}{9} g^3 \left(\frac{\overline{\psi}_R \gamma_{[\mu} i\left(p_R + m_R\right)\gamma_{\nu]}\psi_R}{p_{R\sigma} p_R{}^\sigma - m_R{}^2 + i\varepsilon_R} + \frac{\overline{\psi}_G \gamma_{[\mu} i\left(p_G + m_G\right)\gamma_{\nu]}\psi_G}{p_{G\mu} p_G{}^\mu - m_G{}^2 + i\varepsilon_G} + \frac{\overline{\psi}_B \gamma_{[\mu} i\left(p_B + m_B\right)\gamma_{\nu]}\psi_B}{p_{B\nu} p_B{}^\nu - m_B{}^2 + i\varepsilon_B} \right) \end{aligned} . \quad (13.4)$$

By inspection, (13.3) and (13.4) have the schematic color wavefunctions:

$$\iiint p \sim \oiint G^2 \sim \oiint \left(\overline{R}R + \overline{G}G + \overline{B}B\right) \neq 0; \qquad G^2 = \tfrac{1}{2}\left[G_\mu, G_\nu\right] dx^\mu dx^\mu \sim \overline{R}R + \overline{G}G + \overline{B}B \,. \quad (13.5)$$

This is precisely the required symmetric color-neutral singlet wavefunction for a meson. So in contrast to the U(1) magnetic monopoles of Maxwell for which there is no net magnetic field flux across any closed surface, there is a net flux of "chromo-magnetic" fields across the surface surrounding a Yang-Mills magnetic monopole, namely the G^2 first identified at (3.3b). We now see these have the required color wavefunction of mesons known to mediate baryon interactions.

Accordingly, we conclude that Yang-Mills magnetic monopoles have the antisymmetric color-neutral singlet wavefunction of baryons, and objects which net flow across their surfaces have the symmetric color-neutral singlet wavefunction of mesons. Together, these are the hadrons.

14. Act of confinement: dynamical hadronization from Maxwell's Yang-Mills equations

In his review of the MIT bag model in Section 18 of [6], Close reviews Gauss' theorem for electric charge – contained in $c\mu_0 \iiint *j = \oiint *F$ from (1.7a) – then "consider[s] the chromodynamics case which is analogous to" Gauss' theorem. He states: "if the demand that no quark current crosses the boundary is supplemented by the demand that colour gluons are also confined then Gauss' theorem implies that the system have zero colour charge." He continues that in the bag model, "the introduction of a pressure *B* that counterbalances the flow of colour flux automatically requires the system to be colour neutral. If colour symmetry is exact then the system must be a colour singlet." This is precisely true of (12.1): its color symmetry is exact because the symmetry breaking in (11.6) made its gauge bosons massless, and it is a color singlet.

Close then makes the critical points, emphasis added, that "*quark confinement arises as a result of colour confinement*," and that the bag model "imposition *ad hoc* of a boundary condition that confines the coloured gluons has, by Gauss, confined the coloured quarks." Importantly, he concludes that "*a dynamical origin for this boundary condition has not been presented*" by the bag model. Or it appears, by any other theory to date.

Here, (13.4) and (13.5) demonstrate "by Gauss" that the objects which net flow across the monopole surface are color-neutral. This means, conversely, that *objects which are not color neutral, i.e. which have a color charge, do not net flow across the surface but are confined.*[*] Inside these signal monopoles, each of the three fermions has a net color charge in the fundamental representation of an SU(3) gauge group which is exact because its vector bosons are massless, and each of the massless vector bosons has a net color charge in the adjoint representation of SU(3). We therefore conclude that these fermions and massless gauge bosons are confined. Consequently, we conclude that: the fermions in (12.1) are quarks; the now-massless gauge bosons are gluons; the signal monopole (12.1) is a baryon in a non-perturbative state with all "noise" filtered out; and the G^2 object (13.4) which net flows across the monopole-now-baryon surface is a meson.

Crucially, this is not an *ad hoc* result. It has a "*dynamical origin*" in the very fundamental physics of Maxwell's equations extended to the non-commuting gauge fields of Yang and Mills, coupled with Dirac's theory of fermions [12] and the requirement that each fermion in a system such as an atom or a nucleus or a nucleon or a baryon must occupy an exclusive quantum state. There is nothing new or unsettled in any of the individual elements which are combined to reach this dynamical result. What is new is simply understanding how these all combine together to produce the hadronic phenomenology of QCD, and how the rank-3 antisymmetric structure of a magnetic monopole is dynamically responsible for SU(3) – not SU(2) or SU(4) or anything else – being the gauge group underlying hadronic physics. So, we do not need to postulate $SU(3)_{QCD}$ as has been done ever since Gell-Mann [17] and Zweig [18] first discovered the quark model. The Yang-Mills magnetic monopoles dynamically make that postulate for us, all by themselves.

In cosmology, it is widely believed that "hadronization . . . occurred shortly after the Big Bang when the quark-gluon plasma cooled to the temperature below which free quarks and gluons cannot exist [19]." In view of all the above, we can now identify the symmetry breaking of Section 11 with hadronization of a free quark and gluon plasma believed to exist only at ultra-high GUT energies above ~10^{15} GeV, not far below the Planck scale $E_P = \sqrt{\hbar c^5 / G} \approx 1.220 \times 10^{19}$ GeV. Specifically,

[*] Because the confined objects are those with non-neutral net color, there is nothing in this result which prevents the electroweak photon, $W^{\pm}$ and Z from flowing across the monopole surface, because these are color-neutral.

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the signal monopole obtained in (10.2) *prior to symmetry breaking* has $\mathrm{tr}\,\mathrm{tr}\, p^{\sigma\mu\nu} = 0$ and is associated with energies above 10^{15} GeV where quarks are free and can mingle with leptons, and where baryons with confined quarks and gluons are not yet formed. The signal monopole obtained in (12.1) *after symmetry breaking* has $\mathrm{tr}\,\mathrm{tr}\, \Sigma_s p^{\sigma\mu\nu} \neq 0$ and is associated with lower energies where free quarks and gluons no longer exist but are confined in color-neutral hadrons. So, what we have colloquially referred to as the "monopole dance" is now seen to take place at some energy E_X a few orders of magnitude below the Planck energy. The pre-symmetry breaking (8.1) in which quarks are labelled with the spacetime indexes of their current density "escorts" shows the pre-hadronization baryon above E_X, which at (10.2) has $\mathrm{tr}\,\mathrm{tr}\, p^{\sigma\mu\nu} = 0$. The post-break (8.2) in which the monopole is made indivisible with quarks now labelled independently from their "escorts," shows the baryon below E_X once hadronization is complete, which at (12.1) now has $\mathrm{tr}\,\mathrm{tr}\, \Sigma_s p^{\sigma\mu\nu} \neq 0$ with confined color. The symmetry breaking from (8.1) to (8.2), which via (11.6) takes us from (10.2) to (12.1), then becomes synonymous with E_X-scale hadronization which is *dynamical*, not *ad hoc*. This is what Close refers to as the "act of confinement."

15. Filling the Yang-Mills Mass Gap

It is well-known that Maxwell's electric charge equation (1.1a) has no inverse, or, to be precise, that the inverse $\left(g^{\mu\nu}\partial_\sigma\partial^\sigma - \partial^\mu\partial^\nu\right)^{-1}$ of its operator on A_μ is infinite (singular). To deal with this, one of several approaches is required. A first option is to impose a gauge condition, often the covariant $\partial_\mu A^\mu = 0$. Then it is easy to obtain $\left(g^{\mu\nu}\partial_\sigma\partial^\sigma\right)^{-1}$ alone. Another is to introduce a Proca mass by hand, which is what we did at (4.1). But doing so means the theory is no longer renormalizable, so we must eventually find a way to remove this mass and introduce it some other way. Using $D \mapsto \partial$ to obtain a "signal" inverse, the inverse of the operator in (4.1) becomes the familiar finite (A.7). And we see from (A.7) that when $m = 0$ this inverse becomes infinite, which directly demonstrates why (1.1a) has no inverse. Using notations reviewed at the start of Section 5, this is not just because of $k_\nu k_\alpha / m^2$ in the numerator which can be removed when contracted with a source current because of the signal continuity equation $k_\nu j^\nu = 0$. More importantly it is because $k_\sigma k^\sigma - m^2$ in the denominator becomes zero when the vector boson is "on-shell," meaning that $k_\sigma k^\sigma - m^2 = 0$. And, when we remove the Proca mass so the boson is again massless, the denominator becomes $q_\sigma q^\sigma$ which on-shell is also $q_\sigma q^\sigma = 0$. These singularities are why we also need the $+i\varepsilon$ prescription to take a particle "off-shell," i.e., to render it "virtual."

In view of this, we sum the inverse (5.1) from the left with electric source density J^α and the right with J^ν, both from (2.3a). Recall, using $c^2\mu_0\varepsilon_0 = 1$, that $J^\sigma = j^\sigma - ic\varepsilon_0 g G_\tau F^{\tau\sigma}$ as reviewed following (3.1). As obtained in (B.2) the continuity equation $\left(p_\nu + gG_\nu\right)J^\nu = 0$ whereby the term with $\left(k_\nu k_\alpha + gk_\nu\left(G_\alpha\right)/c\right)/m^2c^2$ drops out. So, with $c = 1$ we obtain:

$$J^\alpha I_{\alpha\nu} J^\nu = J^\alpha \frac{{}_\vee -g_{\alpha\nu} + \dfrac{k_\nu k_\alpha + gk_\nu G_\alpha}{m^2}}{"k_\sigma k^\sigma - m^2 - g^2 G_\sigma G^\sigma + i\varepsilon"} J^\nu = -J_\sigma\left(k_\sigma k^\sigma - m^2 - g^2 G_\sigma G^\sigma + i\varepsilon\right)^{-1} J^\sigma, \quad (15.1)$$

Now, we see how to remove the Proca mass thus restoring renormalizability while maintaining a finite inverse: Because the term $g^2 G_\sigma G^\sigma$ arises naturally from the Yang-Mills gauge theory and is itself mass-dimensioned, and because this term has the same form as what is in $\mathfrak{L} = \partial^\sigma\phi^* \partial_\sigma\phi + g^2 G^\sigma G_\sigma \phi^*\phi$ whereby vector boson masses arise in a well-known way from spontaneous symmetry breaking, we simply use $g^2 G_\sigma G^\sigma$ to replace m^2 by removing the latter

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entirely. In other words, Yang-Mills gauge theory puts a mass-producing term right where it needs to be in the form it needs to have, so we can set $m=0$ above without adverse consequence. Likewise, neither do we need $+i\varepsilon$. So, removing these, and expanding $G_\sigma = \tau_i G_{i\sigma}$, we now have:

$$J^\alpha I_{\alpha\nu} J^\nu = -J_\sigma \left(k_\sigma k^\sigma - g^2 G_\sigma G^\sigma\right)^{-1} J^\sigma = -J_\sigma \left(k_\sigma k^\sigma - g^2 \tau_i \tau_j g_{\mu\nu} G_i{}^\mu G_j{}^\nu\right)^{-1} J^\sigma . \quad (15.2)$$

What is important now, is that $G_\sigma G^\sigma = \tau_i \tau_j g_{\mu\nu} G_i{}^\mu G_j{}^\nu$ is not an ordinary spacetime scalar, but rather, is an NxN matrix for any Yang-Mills gauge group SU(N), with adjoint structure established by the $\left(N^2-1\right)^2$ products $\tau_i\tau_j$. Also, some of the matrices $\tau_i\tau_j$ and thus $g_{\mu\nu}G^\mu G^\nu$ contain imaginary components, which can be seen even in the simplest case of SU(2) where $\tau_i = \frac{1}{2}\sigma_i$ and the Pauli identity $(\boldsymbol{\sigma}\cdot\mathbf{x})(\boldsymbol{\sigma}\cdot\mathbf{y}) = \mathrm{Id}\,\mathbf{x}\cdot\mathbf{y} + i\boldsymbol{\sigma}\cdot(\mathbf{x}\times\mathbf{y})$. And the $J^\sigma = j^\sigma - ic\varepsilon_0 g G_\tau F^{\tau\sigma}$ which are also NxN YM matrices will contain additional imaginary components. Moreover, nothing restricts (15.2) to SU(3). As reviewed in Section 9, this restriction is imposed by the magnetic monopoles. Here, we are dealing with Maxwell's Yang-Mills equation (3.1a) for electric source densities independent of magnetic monopoles, with the Proca mass of (4.1) now removed.

Because (15.2) is an NxN Yang-Mills matrix and a spacetime scalar, and additionally houses a finite-matrix (*f*) inverse which by definition is invertible, i.e. $\left(f^{-1}\right)^{-1} = f$, it has finite eigenvalues λ which are calculated in the usual way for a square matrix M via the determinant relation $0 = \left|M - \lambda\,\mathrm{Id}\right|$, where Id is an NxN unit matrix for any SU(N). These λ are obtained by:

$$0 = \left|J^\alpha I_{\alpha\nu} J^\nu - \lambda \mathrm{Id}\right| = \left|-J_\sigma \left(k_\tau k^\tau - g^2 G_\tau G^\tau\right)^{-1} J^\sigma - \lambda \mathrm{Id}\right| . \quad (15.3)$$

But (A.7) provides the basis for seeing what these eigenvalues look like in the non-perturbative limit with all recursion reviewed at (5.2) and (5.3) removed via $D_\sigma \mapsto \partial_\sigma$ so that $c\mu_0 J^\nu = D_\sigma F^{\sigma\nu}$ becomes $c\mu_0 j^\nu = \partial_\sigma F^{\sigma\nu}$ and J^σ becomes j^σ. In Section 5 notation with what is now a continuity equation $k_\nu j^\nu = 0$, these eigenvalues become:

$$\lambda = j^\alpha i_{\alpha\nu} j^\nu = j^\alpha \frac{-g_{\alpha\nu} + \dfrac{k_\nu k_\alpha}{m^2}}{k_\sigma k^\sigma - m^2 + i\varepsilon} j^\nu = \frac{-j_\sigma j^\sigma}{k_\sigma k^\sigma - m^2 + i\varepsilon} . \quad (15.4)$$

Finally, combining (15.3) and (15.4) and reintroducing the infinite recursive series notation of (5.3) with … wherever a substitution $G_\alpha\left(j^\nu, G_\alpha\right)$ or $F^{\mu\nu}\left(G_\alpha\left(j^\nu, G_\alpha\right)\right)$ is required, and noting the similarity of $J^\alpha I_{\alpha\nu} J^\nu - j^\alpha i_{\alpha\nu} j^\nu \mathrm{Id}$ to $-V = D_\sigma D^\sigma - \partial_\sigma \partial^\sigma$ discussed at the start of section 3 whereby the perturbation is the difference "noise = (signal + noise) – signal," we obtain:

$$\begin{aligned} 0 &= \left|J^\alpha I_{\alpha\nu} J^\nu - j^\alpha i_{\alpha\nu} j^\nu \mathrm{Id}\right| = \left|-J_\sigma \left(k_\tau k^\tau - g^2 (G_\tau \ldots)(G^\tau \ldots)\right)^{-1} J^\sigma - \frac{-j_\sigma j^\sigma}{k_\sigma k^\sigma - m^2 + i\varepsilon} \mathrm{Id}\right| \\ &= \left|-\left(j_\sigma - ic\varepsilon_0 g\left(G^\alpha F_{\alpha\sigma} \ldots\right)\right)\left(k_\tau k^\tau - g^2 (G_\tau \ldots)(G^\tau \ldots)\right)^{-1}\left(j^\sigma - ic\varepsilon_0 g\left(G_\beta F^{\beta\sigma} \ldots\right)\right) - \frac{-j_\sigma j^\sigma}{k_\sigma k^\sigma - m^2 + i\varepsilon} \mathrm{Id}\right| \end{aligned} . \quad (15.5)$$

This is the mass gap solution. Specifically, referring to page 6 of [2], for "any compact simple gauge group G," that is, for any SU(N), "a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap > 0 . . . namely there must be some constant $\Delta > 0$ such that every excitation of the vacuum has energy at least Δ . . ." "Excitations of the vacuum" is another phrase for non-zero perturbations $V_{\nu\sigma} = ig\left(G_\nu \partial_\sigma + \partial_\nu G_\sigma\right) + g^2 G_\nu G_\sigma \neq 0$ defined following (2.4), and these arise from the canonic promotion of $\partial_\sigma \mapsto D_\sigma$ reviewed in Section 2. And it is another phrase for what we have referred to here as the "noise" of Yang-Mills dynamics over Maxwell's U(1) electrodynamics. Indeed, $J^\alpha I_{\alpha\nu} J^\nu - j^\alpha i_{\alpha\nu} j^\nu \mathrm{Id}$ in the determinant (15.5), like *V*, is also a direct measure of "noise = (signal +

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noise) – signal." When these excitations of the vacuum $V_{\nu\sigma} \neq 0$, the inverse $G_\alpha\left(j^\nu, G_\alpha\left(j^\nu, G_\alpha\left(j^\nu, G_\alpha\left(j^\nu, G_\alpha(...)\right)\right)\right)\right)$ has the infinitely-recursive, highly nonlinear form of (5.3). But $J_\sigma\left(k_\tau k^\tau - g^2 G_\tau G^\tau\right)^{-1} J^\sigma$ is clearly invertible (again, $\left(f^{-1}\right)^{-1} = f$), so it will have a non-zero determinant and non-zero eigenvalues.

Further, $G_\sigma G^\sigma$ is a correctly-signed term for positive vector boson rest energy $mc^2 > 0$, has the same form as what appears in the symmetry-breaking $\mathfrak{L} = \partial^\sigma \phi^* \partial_\sigma \phi + g^2 G^\sigma G_\sigma \phi^* \phi$, and at (15.2) was used to remove the Proca mass to restore renormalizability while maintaining finite invertibility. Because $-J_\sigma\left(k_\tau k^\tau - g^2\left(G_\tau...\right)\left(G^\tau...\right)\right)^{-1} J^\sigma$ contains both real and imaginary numbers, these eigenvalues can be real, imaginary, or complex. So, when the eigenvalues (15.4) and then the $-m^2 + i\varepsilon$ in the eigenvalue denominator are computed using (15.5) for "*any* compact simple gauge group G," we will find that "every excitation of the vacuum has energy at least $\Delta" = mc^2 > 0$. Moreover, the imaginary parts of $J^\alpha I_{\alpha\nu} J^\nu$ in (15.5) can produce $+i\varepsilon > i0$ which corresponds physically to finite particle lifetimes. This is how Yang-Mills gauge theory reveals masses $m > 0$ and finite lifetimes via $\varepsilon > 0$ in the (15.4) denominator while maintaining renormalizability by removing Proca masses and $+i\varepsilon$, thereby filling the Yang-Mills mass gap.

The manifest recursion highlighted in (15.5) is also important for understanding how to carry out exact closed analytic calculations in Yang-Mills theory, as opposed to using numerical methods such as those of Lattice QCD. In this regard, Jaffe and Witten state on page 7 of [2] that "since the inception of quantum field theory, two central methods have emerged to show the existence of quantum fields on non-compact configuration space (such as Minkowski space). These known methods are (i) Find an exact solution in closed form; (ii) Solve a sequence of approximate problems, and establish convergence of these solutions to the desired limit." The foregoing (15.5) suggests a third method which is really a hybrid of (i) and (ii): find an exact *recursive kernel* in closed form, and then expand that kernel in successive iterations approaching the limit of infinite recursive nesting to identify the underlying infinite series.

Conclusion

For an entire century we have known experimentally about the existence of protons. For almost 90 years we have known about neutrons. But beyond knowing baryons contain three quarks with exact SU(3) chromodynamic symmetry, contain massless gluons in the adjoint representation of SU(3), have an antisymmetric color-neutral singlet wavefunction, and interact via symmetric color-neutral singlet mesons, we still cannot answer Rabi's simple query "who ordered that?," and we do not understand the dynamic basis of quark and gluon confinement and hadronization.

Here, we have shown that the magnetic monopoles of Yang-Mills gauge theory in their "signal" state contain three fermions in the fundamental representation of SU(3). Following symmetry breaking which moves a degree of freedom from the gauge bosons to the fermions, the gauge bosons become massless, SU(3) becomes an exact symmetry, and a propagator is established for each fermion. The monopoles then have the same antisymmetric color singlet wavefunction as a baryon, and the field quanta of the magnetic fields fluxing through their surface have the same symmetric color singlet wavefunction as a meson. Consequently, we can identify these fermions with colored quarks, the massless gauge bosons with gluons, the magnetic monopoles with baryons, the fluxing entities with mesons, and the symmetry breaking with hadronization, while establishing that the quarks and gluons remain confined following hadronization. The result is a quantum chromodynamic (QCD) theory of the hadrons. And using analytic tools developed along the way, we fill the Yang-Mills mass gap.

Finally, as previewed in the introduction, Rabi's question is answered: Protons, neutrons and other baryons were ordered by Maxwell, Gauss and Faraday together with Yang and Mills, and by

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Weyl who was the father of gauge theory, with the aid of Fermi-Dirac-Pauli via Dirac's quantum theory of the electron and the Exclusion Principle, and with seminal credit to Hamilton for pioneering non-commuting quaternions which more than a century later became the foundation of Yang-Mills gauge theory.

Appendix A: Calculation of the inverse for the Yang-Mills electric source equation

Because $\partial^\mu D^\nu = \partial^\mu \partial^\nu - ig\partial^\mu G^\nu$ is a non-symmetric tensor even in flat spacetime because in general $\partial^\mu G^\nu \neq \partial^\nu G^\mu$, it is important when calculating the inverse $I_{\alpha\nu}$ to make certain that the left- and right-side inverse calculations lead to the same $\delta^\mu{}_\alpha$ identity matrix, with $I_{\alpha\nu\mathrm{LEFT}} = I_{\alpha\nu\mathrm{RIGHT}}$, as shown at (4.4). Therefore, we shall carry out both a left- and a right-side calculation, then make certain that both of these inverses are one and the same. Based on (4.4), we expect the general form of the inverse to be:

$$I_{\alpha\nu\mathrm{LEFT}} = I_{\alpha\nu\mathrm{RIGHT}} = Ag_{\alpha\nu} + B\partial_\alpha D_\nu + C\partial_\nu D_\alpha , \tag{A.1}$$

where *A*, *B* and *C* are unknowns to be determined, and where we include both $\partial_\alpha D_\nu$ and $\partial_\nu D_\alpha$ given the non-symmetry of these terms. The left-placement of *A*, *B* and *C* in (A.1) is arbitrary *ab initio*, but once we do so, we must maintain consistent ordering thereafter. So, it would not be correct to write $I_{\alpha\nu\mathrm{RIGHT}} = g_{\alpha\nu}A + \partial_\alpha D_\nu B + \partial_\nu D_\alpha C$. It should also be noted from the term $\partial^\mu D^\nu G_\mu$ in (4.1) that the free index is in D^ν, while ∂^μ to the left of D^ν sums with G_μ. We will calculate in configuration space, then convert to momentum space in the usual way.

Using (A.1) in (4.4) as a left-side inverse and operating with the metric tensor produces:

$$\begin{aligned}\delta^\mu{}_\alpha &= \left(Ag_{\alpha\nu} + B\partial_\alpha D_\nu + C\partial_\nu D_\alpha\right)\left(g^{\mu\nu}\left(\partial_\sigma D^\sigma + m^2\right) - \partial^\mu D^\nu\right) \\ &= A\delta^\mu{}_\alpha\left(\partial_\sigma D^\sigma + m^2\right) - A\partial^\mu D_\alpha + B\partial_\alpha D^\mu\left(\partial_\sigma D^\sigma + m^2\right) - B\partial_\alpha D_\sigma \partial^\mu D^\sigma \\ &\quad + C\partial^\mu D_\alpha\left(\partial_\sigma D^\sigma + m^2\right) - C\partial_\sigma D_\alpha \partial^\mu D^\sigma\end{aligned} . \tag{A.2}$$

Matching up $\delta^\mu{}_\alpha$ with the term $A\delta^\mu{}_\alpha \partial_\sigma D^\sigma$ first reveals that $\delta^\mu{}_\alpha = A\delta^\mu{}_\alpha\left(\partial_\sigma D^\sigma + m^2\right)$, i.e., that:

$$A = \left(\partial_\sigma D^\sigma + m^2\right)^{-1} . \tag{A.3}$$

Because $D^\sigma = \partial^\sigma - igA^\sigma$ contains $A^\sigma = \tau_i A_i^\sigma$ which is an NxN square matrix for SU(N), we cannot simply write the above as $A = 1/\left(\partial_\sigma D^\sigma + m^2\right)$ which treats $\partial_\sigma D^\sigma + m^2$ as an ordinary denominator. Rather, this must itself be inverted *independently* of the spacetime inversion (4.4).

Substituting (A.3) into (A.2) then reducing now produces:

$$\begin{aligned}&\left(\partial_\sigma D^\sigma + m^2\right)^{-1}\partial^\mu D_\alpha \\ &= B\left(\partial_\alpha D^\mu\left(\partial_\sigma D^\sigma + m^2\right) - \partial_\alpha D_\sigma \partial^\mu D^\sigma\right) + C\left(\partial^\mu D_\alpha\left(\partial_\sigma D^\sigma + m^2\right) - \partial_\sigma D_\alpha \partial^\mu D^\sigma\right)\end{aligned} . \tag{A.4}$$

The left side above contains $\partial^\mu D_\alpha$ which matches to the same term inside $C\partial^\mu D_\alpha\left(\partial_\sigma D^\sigma + m^2\right)$. From this we conclude that the terms with *B* are not needed to calculate the inverse, i.e., that we can calculate the inverse with $B = 0$. This is a downstream consequence of the fact noted following (A.1) that in (4.1), the free index is in D^ν. Consequently, we further reduce (A.4) to:

$$C = \left(\partial_\sigma D^\sigma + m^2\right)^{-1}\left[\partial^\mu D_\alpha\left(\partial^\mu D_\alpha m^2 + \partial^\mu D_\alpha \partial_\sigma D^\sigma - \partial_\sigma D_\alpha \partial^\mu D^\sigma\right)^{-1}\right] . \tag{A.5}$$

Inserting (A.3) and $B = 0$ and (A.5) into (A.1) and reducing now produces the left-side inverse:

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$$I_{\alpha\nu\mathrm{LEFT}} = \left(\partial_\sigma D^\sigma + m^2\right)^{-1}\left\{g_{\alpha\nu} + \partial^\mu D_\alpha\left(\partial^\mu D_\alpha m^2 + \partial^\mu D_\alpha \partial_\sigma D^\sigma - \partial_\sigma D_\alpha \partial^\mu D^\sigma\right)^{-1}\partial_\nu D_\alpha\right\}$$
$$\equiv \frac{{}_\vee g_{\alpha\nu} + \dfrac{\partial^\mu D_{\alpha\vee}\partial_\nu D_\alpha}{"m^2\partial^\mu D_\alpha + \partial^\mu D_\alpha \partial_\sigma D^\sigma - \partial_\sigma D_\alpha \partial^\mu D^\sigma"}}{"\partial_\sigma D^\sigma + m^2"} \quad \text{(A.6a)}$$

In the bottom line above, simply to provide a compact visual comparison to the usual inverses for a massive vector boson, we have defined "quoted" denominators in which the inverses are represented as "denominators," but with the understanding that this is a shorthand for what is actually a matrix inverse. We also use subscripted $\vee$ to indicate where the "denominators" are placed when represented as inverses. In general, for a square matrix *M*, we shall use this shorthand to write $1/"M" \equiv M^{-1}$. Looking closely, note that $\partial^\mu D_\alpha$ appears in both the upper numerator and the upper "denominator," but cannot (yet) be cancelled using $\partial^\mu D_\alpha\left(\partial^\mu D_\alpha\right)^{-1} = \mathrm{Id}$ because of the $\partial_\sigma D_\alpha \partial^\mu D^\sigma$ term in the upper denominator in which D_α is commuted to the left of ∂^μ.

Next, we perform an identical calculation using (4.4), again using the general form (A.1), but now for $I_{\alpha\nu\mathrm{RIGHT}}$. The result for *A* is the same as in (A.3), and by matching up the $\partial^\mu D_\alpha$ terms as in (A.4) we again conclude that $B = 0$. So, we finally calculate *C* as in (A.5) and insert all the results into (A.1) to find that:

$$I_{\alpha\nu\mathrm{RIGHT}} = \left(\partial_\sigma D^\sigma + m^2\right)^{-1}\left\{g_{\alpha\nu} + \partial^\mu D_\alpha\left(\partial^\mu D_\alpha m^2 + \partial_\sigma D^\sigma \partial^\mu D_\alpha - \partial^\mu D^\sigma \partial_\sigma D_\alpha\right)^{-1}\partial_\nu D_\alpha\right\}$$
$$\equiv \frac{{}_\vee g_{\alpha\nu} + \dfrac{\partial^\mu D_{\alpha\vee}\partial_\nu D_\alpha}{"\partial^\mu D_\alpha m^2 + \partial_\sigma D^\sigma \partial^\mu D_\alpha - \partial^\mu D^\sigma \partial_\sigma D_\alpha"}}{"\partial_\sigma D^\sigma + m^2"} \quad \text{(A.6b)}$$

If we set $D \mapsto \partial$ throughout to turn the gauge-covariant derivatives into ordinary ones, then use $\left[\partial_\alpha, \partial_\beta\right] = 0$ in flat spacetime, and then convert from configuration into momentum space using the substitution $i\hbar\partial^\mu \mapsto p^\mu$ with $\hbar = 1$ while including $+i\varepsilon$, the quotes can come off the denominators, and we find that the non-perturbative $I_{\alpha\nu} \mapsto i_{\alpha\nu}$ inverse is:

$$i_{\alpha\nu} \equiv \frac{g_{\alpha\nu} + \dfrac{\partial_\nu\partial_\alpha}{m^2}}{\partial_\sigma\partial^\sigma + m^2 - i\varepsilon} = \frac{-g_{\alpha\nu} + \dfrac{p_\nu p_\alpha}{m^2}}{p_\sigma p^\sigma - m^2 + i\varepsilon}. \quad \text{(A.7)}$$

This will be recognized as the well-known inverse for a massive vector boson. Using the language of Jaffe and Witten in [2], this has no "excitations of the vacuum," and is used in the eigenvalues (15.4) for the mass gap solution (15.5).

Now, the requirement $M^{-1}M = MM^{-1} = \mathrm{Id}$ for any square matrix *M* tells us the inverse M^{-1} must be the same no matter the side from which it multiplies *M*, that is, $I_{\alpha\nu\mathrm{LEFT}} = I_{\alpha\nu\mathrm{RIGHT}}$. So, if we now set the two results in (A.6) to be equal, we find this will be so if and only if:

$$\partial^\mu D_\alpha \partial_\sigma D^\sigma - \partial_\sigma D_\alpha \partial^\mu D^\sigma = \partial_\sigma D^\sigma \partial^\mu D_\alpha - \partial^\mu D^\sigma \partial_\sigma D_\alpha \left(=0\right). \quad \text{(A.8)}$$

As it turns out, not only are both sides of (A.8) equal, but each is equal to zero which is why we included $\left(=0\right)$. This can be proved using the covariant commutator relation $\left[p_\mu, G_\nu\right] = -i\hbar\partial_\mu G_\nu$ for a field $G_\nu\left(t, \mathbf{x}\right)$ which is a function of space and time. The $\left[p_i, G_j\right] = -i\hbar\partial_i G_j$ space components originate in the Heisenberg commutator $\left[\hat{p}_x, \hat{x}\right] = -i\hbar$, while the time component is rooted in the Heisenberg-picture commutator $\left[\hat{H}, G_\nu\right] = -i\hbar\partial_0 G_\nu$ in view of the relation $\left\langle\overline{\psi}\right|\hat{H}\left|\psi\right\rangle = \left\langle\overline{\psi}\right|cp^0\left|\psi\right\rangle$, where $\hat{H}$ is a particle Hamiltonian and $cp^0 = E$ is the particle energy.

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We can see this in the following way: Substitute $D^\sigma = \partial^\sigma - igG^\sigma$ throughout (A.8). Then factor out one *g*, and use $\left[\partial_\alpha, \partial_\beta\right] = 0$ in flat spacetime to remove some terms, yielding:

$$\begin{aligned} & i\partial_\sigma G_\alpha \partial^\mu \partial^\sigma - i\partial^\mu G_\alpha \partial_\sigma \partial^\sigma + g\partial_\sigma G_\alpha \partial^\mu G^\sigma - g\partial^\mu G_\alpha \partial_\sigma G^\sigma \\ & = i\partial^\mu G^\sigma \partial_\sigma \partial_\alpha - i\partial_\sigma G^\sigma \partial^\mu \partial_\alpha + g\partial^\mu G^\sigma \partial_\sigma G_\alpha - g\partial_\sigma G^\sigma \partial^\mu G_\alpha \end{aligned} . \tag{A.9}$$

Next, convert the partial derivative which is just to the right of a gauge field in each of the eight terms, into momentum space using $i\hbar\partial^\mu \mapsto p^\mu$ with $\hbar = 1$. Then use $\left[p_\mu, G_\nu\right] = -i\partial_\mu G_\nu$ to commute these new p^μ to the left of the gauge fields. More terms cancel using $\left[\partial_\alpha, \partial_\beta\right] = 0$, so:

$$\begin{aligned} & \partial_\sigma p^\mu G_\alpha \partial^\sigma - \partial^\mu p_\sigma G_\alpha \partial^\sigma + ig\partial^\mu p_\sigma G_\alpha G^\sigma - ig\partial_\sigma p^\mu G_\alpha G^\sigma \\ & = \partial^\mu p_\sigma G^\sigma \partial_\alpha - \partial_\sigma p^\mu G^\sigma \partial_\alpha + ig\partial_\sigma p^\mu G^\sigma G_\alpha - ig\partial^\mu p_\sigma G^\sigma G_\alpha \end{aligned} . \tag{A.10}$$

Finally, revert to configuration space via $p^\mu \mapsto i\partial^\mu$, make one final use of $\left[\partial_\alpha, \partial_\beta\right] = 0$, then multiply through by –*i* and consolidate terms using $D_\mu = \partial_\mu - igG_\mu$ to obtain:

$$\begin{aligned} & \left(\partial_\sigma \partial^\mu - \partial^\mu \partial_\sigma\right) G_\alpha \left(\partial^\sigma - igG^\sigma\right) = \left[\partial_\sigma, \partial^\mu\right] G_\alpha D^\sigma = 0 \\ & = \left(\partial^\mu \partial_\sigma - \partial_\sigma \partial^\mu\right) G^\sigma \left(\partial_\alpha - igG_\alpha\right) = \left[\partial^\mu, \partial_\sigma\right] G^\sigma D_\alpha = 0 \end{aligned} . \tag{A.11}$$

The result is simply $0 = 0$ with both the left and right sides seen to equal zero.

This has two important consequences: First, it proves that *(A.8) is not some independent condition on the gauge fields*, but is simply a manifestation of the commutator relation $\left[p_\mu, G_\nu\right] = -i\hbar\partial_\mu G_\nu$ which covariantly combines both the canonical commutation relations in space and the Heisenberg equation of motion commutator in time. Second, because both sides of (A.11) are not only equal, but are each independently equal to zero, we prove that *each side of (A.8) is independently equal to zero* in view of the covariant commutator $\left[p_\mu, G_\nu\right] = -i\hbar\partial_\mu G_\nu$. A third logical consequence is that *unless* there is some other way to go from (A.8) from (A.11) *without* using $\left[p_\mu, G_\nu\right] = -i\hbar\partial_\mu G_\nu$ (which may be possible but is not apparent), then $\left[p_\mu, G_\nu\right] = -i\hbar\partial_\mu G_\nu$ is retro-proved by the linear algebra requirement that $M^{-1}M = MM^{-1} = \mathrm{Id}$ for a square matrix inverse, in the current context which includes the Heisenberg picture.

Because each side of (A.8) is equal to zero, we may set these same terms to zero in each of (A.6), which are then clearly equal to one another, $I_{\alpha\nu\,\mathrm{LEFT}} = I_{\alpha\nu\,\mathrm{RIGHT}}$. Then we can reduce each of (A.6) via $\partial^\mu D_\alpha \left(\partial^\mu D_\alpha\right)^{-1} = \mathrm{Id}$, then apply $D^\sigma = \partial^\sigma - igG^\sigma$, then apply $\partial_\sigma G^\sigma = igG_\sigma G^\sigma$ from (4.2) which, again, is a required condition for a massive vector boson in Yang-Mills gauge theory, then apply a final $i\partial^\mu \mapsto p^\mu$ conversion to momentum space and add the $+i\varepsilon$ prescription, to obtain:

$$\begin{aligned} I_{\alpha\nu} &= \left(\partial_\sigma \partial^\sigma + m^2 + g^2 G_\sigma G^\sigma + i\varepsilon\right)^{-1} \left(g_{\alpha\nu} + \frac{\partial_\nu \partial_\alpha - ig\partial_\nu G_\alpha}{m^2}\right) \\ &= \left(-p_\sigma p^\sigma + m^2 + g^2 G_\sigma G^\sigma + i\varepsilon\right)^{-1} \left(g_{\alpha\nu} - \frac{p_\nu p_\alpha + gp_\nu G_\alpha}{m^2}\right) = \frac{-g_{\alpha\nu} + \frac{p_\nu p_\alpha + gp_\nu G_\alpha}{m^2}}{"p_\sigma p^\sigma - m^2 - g^2 G_\sigma G^\sigma + i\varepsilon"} \end{aligned} . \tag{A.12}$$

It is easily seen that when $p_\nu G_\alpha = 0$ and $G_\sigma G^\sigma = 0$, this reduces to the standard inverse (A.7) for a massive vector boson. Conversely, this means the two terms $gp_\nu G_\alpha$ and $g^2 G_\sigma G^\sigma$ are what get added to the massive vector boson inverse by Yang-Mills gauge theory perturbations. And, as noted at (4.2) and used at (15.2), $g^2 G_\sigma G^\sigma$ is precisely the term in which boson masses are revealed via $\mathfrak{L} = \partial^\sigma \phi^* \partial_\sigma \phi + g^2 G^\sigma G_\sigma \phi^* \phi$ during the symmetry breaking of renormalizable gauge theory.

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Appendix B: The Yang-Mills continuity equation in terms of Dirac wavefunctions

To obtain a continuity equation in terms of Dirac wavefunctions, we start with Dirac's Yang-Mills canonic equation $i\gamma^\sigma D_\sigma \psi - m\psi = 0$. With the adjoint wavefunction $\overline{\psi} \equiv \psi^\dagger \gamma^0$ defined as usual, it is straightforward to obtain the adjoint equation $iD_\sigma{}^\dagger \overline{\psi}\gamma^\sigma + m\overline{\psi} = 0$. If we then sandwich each of these and add, because the igG_σ terms from D_σ and $D_\sigma{}^\dagger$ cancel out, we obtain:

$$\begin{aligned} 0 &= \overline{\psi}\gamma^\sigma D_\sigma \psi + D_\sigma{}^\dagger \overline{\psi}\gamma^\sigma \psi = \overline{\psi}\gamma^\sigma \left(\partial_\sigma - igG_\sigma\right)\psi + \left(\partial_\sigma + igG_\sigma\right)\overline{\psi}\gamma^\sigma \psi \\ &= \overline{\psi}\gamma^\sigma \partial_\sigma \psi + \partial_\sigma \overline{\psi}\gamma^\sigma \psi = \partial_\sigma\left(\overline{\psi}\gamma^\sigma \psi\right) = 0 \end{aligned} . \tag{B.1}$$

So even for Yang-Mills theory, *the continuity relation for Dirac wavefunctions only contains the ordinary derivative*.

Combining (B.1) with (2.4) this also means, matching up continuity zeros, that:

$$\begin{aligned} 0 &= c\mu_0 D_\nu J^\nu = D_\nu D_\sigma F^{\sigma\nu} = \partial_\nu \partial_\sigma F^{\sigma\nu} - V_{\nu\sigma} F^{\sigma\nu} = c\mu_0 \partial_\sigma\left(g\overline{\psi}\gamma^\sigma\psi\right) \\ &= c\mu_0\left(\partial_\nu - igG_\nu\right)J^\nu = -ic\mu_0\left(p_\nu + gG_\nu\right)J^\nu \end{aligned} , \tag{B.2}$$

which via $i\partial_\mu \mapsto p_\mu$ contains the continuity relation $\left(p_\nu + gG_\nu\right)J^\nu = 0$ mentioned after (2.4) and used in (15.1). Combining this with $c\mu_0 j^\nu = \partial_\sigma F^{\sigma\nu}$ from (3.1a) and using (2.4) we then obtain:

$$c\mu_0 \partial_\nu j^\nu = \partial_\nu \partial_\sigma F^{\sigma\nu} = c\mu_0 D_\nu J^\nu + V_{\nu\sigma} F^{\sigma\nu} = c\mu_0 \partial_\nu\left(g\overline{\psi}\gamma^\nu\psi\right) + V_{\nu\sigma}F^{\sigma\nu} = V_{\nu\sigma}F^{\sigma\nu}, \tag{B.3}$$

including the perturbation tensor $V_{\nu\sigma} = ig\left(G_\nu\partial_\sigma + \partial_\nu G_\sigma\right) + g^2 G_\nu G_\sigma$ defined at (2.4). When $V_{\nu\sigma} = 0$ this reduces following integration-sans-constant to the familiar $j^\nu = g\overline{\psi}\gamma^\nu\psi$.

Finally, because of the perturbations i.e. excitations of the vacuum of Yang-Mills theory, it is beneficial to define a four-vector κ^ν with dimensions of charge density, which is also an NxN matrix for SU(N), such that $j^\nu \equiv g\overline{\psi}\gamma^\nu\psi + \kappa^\nu$. Then, because (B.1) reveals $\partial_\nu\left(g\overline{\psi}\gamma^\nu\psi\right) = 0$ even in Yang-Mills theory, we can insert this definition into (B.3) to deduce that $c\mu_0\partial_\nu\kappa^\nu = V_{\nu\sigma}F^{\sigma\nu}$. Using $c^2\mu_0\varepsilon_0 = 1$, and mindful that $c\mu_0 J^\nu = c\mu_0 j^\nu - igG_\sigma F^{\sigma\nu} = D_\sigma F^{\sigma\nu}$, the net result is that:

$$j^\nu = g\overline{\psi}\gamma^\nu\psi + \kappa^\nu, \tag{B.4a}$$

$$\partial_\nu \kappa^\nu = c\varepsilon_0 V_{\nu\sigma}F^{\sigma\nu}, \tag{B.4b}$$

with the latter being a first-order differential equation for κ^ν. When the perturbation tensor $V_{\nu\sigma} = 0$ (B.3) reduces to $\partial_\nu j^\nu = \partial_\nu\left(g\overline{\psi}\gamma^\nu\psi\right) = 0$, so that with integration constants set to zero we recover the familiar $j^\nu = \overline{\psi}\gamma^\nu\psi$ with $\kappa^\nu = 0$. Likewise when $V_{\nu\sigma} = 0$ the continuity equation (B.3) becomes $\partial_\nu j^\nu = 0$, which in momentum space via $i\partial^\mu \mapsto p^\mu$ further means that $p_\nu j^\nu = 0$.

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