Article

On a contextual model refuting Bell's Theorem

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Abstract: Bell's theorem can be refuted by presenting a counterexample which predicts correctly the expectation values of QM. As Bell has only ruled out noncontextual models a contextual model with hidden variables would refute his theorem. Such a model is presented able to explain the spin measurement results with entangled photons or electrons. It is not ruled out by the Kochen-Specker Theorem. Consequences for the feasibility of quantum computers are discussed.

Keywords: Bell's Theorem; EPR paradox; hidden variables; contextual model; PACS: 03.65.Ta, 03.65.Ud

1 Introduction

'Bell's theorem states that the predictions of quantum theory (for measurements of spin on particles prepared in the singlet state) cannot be accounted for by any local theory.' See Goldstein et al. [1] who give an overview of the state of the discussion about Bell's theorem which was introduced in 1964 by the Irish physicist. In his paper "On the Einstein Podolsky Rosen Paradox" [2] he had developed the famous Bell inequality which any hidden variable theory describing entangled states has to obey in contrast to quantum mechanics (QM) which infringes that inequality. The inequality is a relation between expectation values of measurements taken at different settings of the instruments. Bell has stated his theorem in his own words [2]: "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant." In other words, nature is not local as it cannot be described with local realistic models. A general approach to logical Bell inequalities and their relation to contextual models is given by [3]. Bell's paper and many subsequent experiments [4] having proved the infringement of Bell's inequality by QM established the belief of many physicists that nature was nonlocal [5]. The theory of relativity would not be violated because no information transport over the quantum channel of entangled photons [6] is possible.

Bell's theorem can be refuted by presenting a counterexample which predicts correctly the expectation values of QM. Many authors have tried this. Some have developed models in which the influence of the measuring apparatus should cause the correlations [7]. Others blame various loopholes for measuring results that violate Bell's inequality [1]. These are all ruled out after the results of Delft physicists and others [8-10] have proved that QM correctly describes the measured correlations without any reference to external conditions. One counterexample was presented by Muchowski [11]. But the derivation is not completely convincing as it contains ambiguous values for measurement results. A better model is presented in the current manuscript with more compelling model assumptions. Khrennikov [12] states that 'Bell nonlocality is a wrong conclusion that the violation of Bell type inequalities implies the existence of mysterious instantaneous influences between distant physical systems'. De Zela [13] demonstrated the possibility of local contextual variables for the Bell theorem. The model he has presented "to exhibit Bell violations in a local-realistic context resorts to inner-product-type correlations, which are mathematically equivalent to Born's rule."

Looking thoroughly at what Bell has proved one sees that he only has ruled out a specific class of models namely those which are not contextual. Noncontextual models cannot describe QM measurement results as they infringe the Kochen Specker (KS) theorem. Noncontextuality is defined by KS as: 'If a QM system possesses a property (value of an observable), then it does so independently of any measurement context.' [14]. Contextual models do not infringe the KS theorem. The interdependence between contextuality and indistinguishability was discussed in detail also with respect to the KS theorem [15]. Indistinguishability can be considered not only between identical particles (bosons and fermions), but between information units themselves (i.e., qubits and their possible generalizations). 'This approach suggests information indistinguishability as means for nonlocal realistic entanglement, offering a relatively simple understanding for the limitation of realistic nonlocality in quantum theory' [16].

In this paper, a local realistic but contextual model is presented where the measurement results are predetermined but the polarization or the spin respectively is correlated with the setting of the instruments. That is a contextual approach verified by the fact that due to indistinguishability the polarization or the spin of entangled particles are generally not defined. The concept of indistinguishability of identical particles could be seen as one of the foundational principles of the quantum theory [17]. Quantum systems differ from classic particles, particularly by this effect. The current paper will help to understand the experimental results. Such an understanding can only be based on local effects. Although we refer to the singlet state as the basis of the investigation the model does not make use of the formalism of QM.

2 Materials and Methods

Theory only

3 A model describing the statistical behavior of entangled photons

3.1 Model overview

The model should reproduce the QM predictions of polarization measurements with entangled photons. Figure 1 shows how entangled photons are generated. Figure 2 shows the experimental arrangement with the coordinate system. With the system in singlet state the conditional probability $P_{B\beta|A\alpha}$ that photon 2 passes polarizer PB at β if photon 1 passes polarizer PA at α is after QM upon projecting the singlet state onto the polarizer directions

$$P_{B\beta|A\alpha} = \sin^2(\alpha - \beta). \quad (1)$$

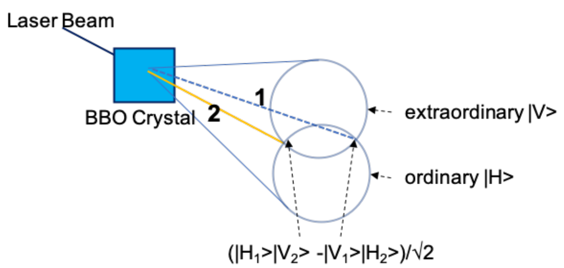

Figure 1: Entangled photons are generated, for example, by parametric fluorescence with a BBO crystal. The ordinary photon beam has the polarization 0° and the extraordinary photon beam comes with the polarization 90°. Each photon leaves the source in a cone of light. Both cone shells intersect in beam 1 and beam 2 which are thus each a mixture of horizontally and vertically polarized photons in equal share. However, due to indistinguishability their polarization is not defined and the photons are by superposition in singlet state
1/√2(|H1V2>-|V1H2>). The Hilbert space for the combined system is $\mathbf{H}_{12} = \boldsymbol{H}_1 \otimes \boldsymbol{H}_2$. Normalized base

vectors are |H1> and |V1> for system 1 at side 1 and |H2> and |V2> for system 2 at side 2. |H1> and |H2> correspond to the x-axis and |V1> and |V2> correspond to the y-axis.

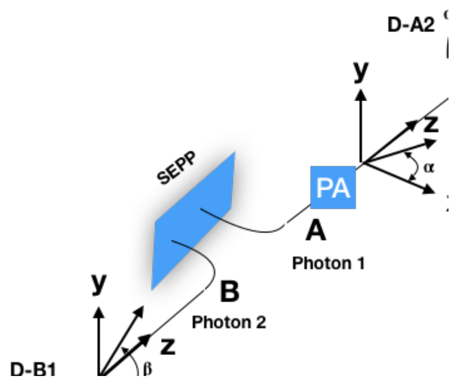

Figure 2: The SEPP (source of entangled photon pairs) emits photons in singlet state propagating in opposite directions towards adjustable polarizers PA and PB and detectors DA-1 and DA-2 on wing A and DB-1 and DB-2 on wing B. A coincidence measuring device not seen in the picture encounters matching events. The polarization angles are defined in the x-y-plane which is perpendicular to the propagation direction of the photons. The coordinate system is left-handed and the same for both wings with the x-axis in horizontal and the y-axis in vertical direction. The z-axis is in propagation direction of photon 1 and opposite to the propagation direction of photon 2.

3.2 Model assumptions

With polarization measurements, photons can choose one of two perpendicular polarizer exits. A hidden variable model has to describe which of these two possible exits a photon will take. Five model assumptions are introduced an overview of which is given below with a description afterwards in italic letters:

MA1 introduces the propensity state called p-state which represents the polarizer exit a photon will take.

MA2 introduces a statistical parameter λ which controls the p-state.

MA3 describes how photons from the singlet state are coupled together.

MA4 describes the polarization of a selection of photons from an entangled pair.

MA5 says that photons don't have a memory of their previous state after a measurement.

Model assumption MA1:

A propensity state (p-state) represents the polarizer exit a photon will take. A photon with p-state α will pass the polarizer exit α with certainty.

Note that the p-state only has a meaning with the specification of a polarizer position.

Model assumption MA2:

The p-state is controlled by a parameter λ which is equally distributed between 0 and +1. With the photon having polarization φ and $\delta = \alpha - \varphi$ we define an indicator function $A(\delta,\lambda)$ which indicates the p-state of the photon before a subsequent measurement. $A(\delta,\lambda)$ can have the values +1 and -1.

For $0 \leq \delta < \pi/2$:

$$A(\delta,\lambda) = +1 \quad \text{for} \quad 0 \leq \lambda \leq \cos^2(\delta), \tag{2}$$

meaning the photon is in p-state α given by the polarizer setting and

$$A(\delta,\lambda) = -1 \quad \cos^2(\delta) < \lambda \leq +1, \tag{3}$$

meaning the photon is in p-state $\alpha+\pi/2$ perpendicular to the polarizer setting.

Figure 3 shows the geometric relationships on which the model is based

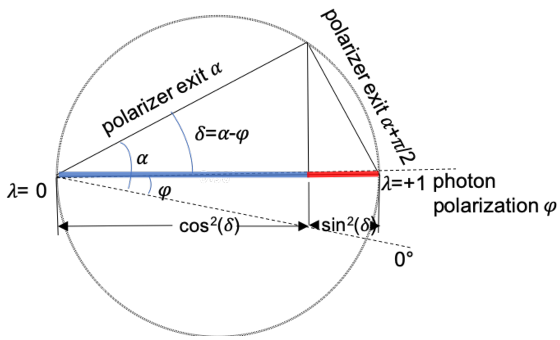

Figure 3: Geometrical derivation of a deterministic distribution of polarized photons onto polarizer outputs. The polarizer is set to the angle $\alpha/\alpha+90°$. The generated photon has a polarization φ. The difference is $\delta=\alpha-\varphi$. A parameter λ is evenly distributed over the generated photons in the value range $0 \leq \lambda \leq +1$. Photons with $\lambda \leq \cos^2(\delta)$ are assigned to the polarizer output α, while photons with $\lambda > \cos^2(\delta)$ take the output $\alpha+90°$.

For the case $\pi/2 \leq \delta < \pi$ we refer to the other exit of the polarizer. $\delta < 0$ is covered by changing the polarizer direction by 180°. Thus, $-\pi < \delta < -\pi/2$ is equivalent to $0 < \delta < \pi/2$ and $-\pi/2 < \delta < 0$ is equivalent to $\pi/2 < \delta < \pi$.

To account for the correlation between the entangled photons we introduce

Model assumption MA3:

Photons of an entangled pair share the same value of the parameter λ. The rules for the distribution of the generated photons onto the two output directions of the polarizer represented by equations (2) and (3) are also valid for the partner photon on wing B.

This reflects a property of the singlet state with the coordinate systems on both wings being of different handedness with respect to the propagation directions of the photons.

Model assumption MA4:

Selected photons from each wing of the singlet state with the selected p-state α have the polarization $\varphi=\alpha$. With a selection other than the initial context all information about the origin from the initial context is lost.

This means also that selected photons from each wing of the singlet state with p-state $\alpha+\pi/2$ have a polarization $\varphi = \alpha+\pi/2$. Photons with polarization α and $\alpha+\pi/2$ come in equal shares according to MA2. See also the remarks below equation (14). MA4 accounts for the fact that the polarization of photons from the singlet state is undefined due to indistinguishability but changed and redefined by entanglement. Thus, photons of a selection cannot be distinguished by their polarization. For a selection of the initial states 0° or 90° the polarization is not changed as it is already equal to the p-state of the selection.

MA4 is a contextual assumption as the polarization of a selection depends on the setting of a polarizer. However, it is a local realistic assumption as it assigns a real value to the physical entity polarization.

Model assumption MA5:

After leaving a polarizer exit λ is indeterminate and uniformly distributed.

MA5 stands for the fact that photons don't have a memory of their previous polarization after leaving a polarizer. The ensemble of photons covers the full range $0 \le \lambda \le +1$ after passing a polarizer and a photon has the polarization α after passing a polarizer with setting α.

3.3 *Predicting measurement results for single photons*

Using equation (2) the photon with polarization φ is found behind the output α of a polarizer with the probability

$$P_\delta = \int_0^{\cos^2(\delta)} d\lambda = \cos^2(\delta) . \quad (4)$$

For the case $\pi/2 \le \delta < \pi$ we refer to the other exit of the polarizer and have with $\vartheta^* = \delta - \pi/2$

$$P_\delta = \int_{\cos^2(\delta*)}^{1} d\lambda = 1-\cos^2(\vartheta^*) = \cos^2(\vartheta) \text{ as well.}$$

With $\delta = \alpha-\varphi$ we obtain the same P_δ for a photon in state $\cos(\varphi) * |H\rangle + \sin(\varphi) * |V\rangle$ from a projection onto

$\cos(\alpha) * \langle H| + \sin(\alpha) * \langle V|$ according to QM from Born's rule.

3.4 *Predicting measurement results for the initial context*

In this section we are going to show that a selection of photons by a polarizer set to α on wing A means for the thus selected perpendicular polarized peer photons they would hit a polarizer set to $\alpha+\pi/2$ on wing B e.g. they are in p-state $\alpha+\pi/2$.

Entangled photons are generated by a common source on wing A with the polarization $\varphi1 = 0°$ and on wing B with the polarization $\varphi2 = 90°$ or on wing A with the polarization $\varphi1 = 90°$ and on wing B with the polarization $\varphi2 = 0°$. The generated photons with polarization 0° and the photons with polarization 90° come in equal shares on either wing. This is the initial context. See figure 1.

First, we calculate measurement results for the pair of generated photon 1 with polarization 0° and generated photon 2 with polarization 90°. For instance, having a generated photon 1 with $\varphi1 = 0°$ and an assumed polarizer PA setting α we would get $\delta = \alpha - \varphi1 = \alpha$ and from equation (2)

$$A(\delta,\lambda) = +1 \quad \text{for} \quad 0 \le \lambda \le \cos^2(\delta), \quad (5)$$

meaning photon 1 is in p-state α.

From equation (3) we get

$A(\delta,\lambda) = -1$ for $\cos^2(\delta) < \lambda \leq +1$, (6)

meaning photon 1 is in p-state $\alpha+\pi/2$ perpendicular to the polarizer PA setting.

Defining an indicator function $B(\delta,\lambda)$ for measurement results on wing B we can apply model assumption MA3 for the correlation between the entangled photons on both wings. Here δ is again the angle between the polarizer setting and the polarization of the generated photon. Equations (5) and (6) do also apply adding 90° to all angles and exchanging $A(\delta,\lambda)$ with $B(\delta,\lambda)$.

Having thus a generated photon 2 with $\varphi 2 = 90°$ and an assumed polarizer PB setting $\alpha+\pi/2$ we would get $\delta = \alpha + \pi/2 - \varphi 2 = \alpha$ and from equation (2)

$B(\delta,\lambda) = +1$ for $0 \leq \lambda \leq \cos^2(\delta)$, (7)

meaning photon 2 is in p-state $\alpha+\pi/2$ given by the polarizer PB setting. From equation (3) we get

$B(\delta,\lambda) = -1$ for $\cos^2(\delta) < \lambda \leq +1$, (8)

meaning photon 2 is in p-state α perpendicular to the polarizer PB setting. Here p-state α and p-state $\alpha+\pi$ are equivalent.

As entanglement connects photons 1 on wing A with photons 2 on wing B by the same value of the parameter λ we obtain from equations (5) and (7) and (6) and (8) respectively that the p-states of peer photons are perpendicular to each other meaning if photon 1 is detected by PA at α its peer photon 2 is detected with certainty by PB at $\alpha+\pi/2$.

In the same way, we calculate measurement results for the pair of generated photons 1 with polarization 90° and generated photons 2 with polarization 0°.

Summarizing a brief rationale for the orthogonality of the p-states on both sides: If a photon with the parameter λ hits a polarizer exit on side A, which is rotated by the angle α relative to its polarization, the peer photon on side B also hits a polarizer exit, which is rotated by the angle α relative to its polarization as well. (the same local rules on both sides, the same value of δ and the same value of λ). So if on wing A the 0 ° polarized photon hits the polarizer exit α (p-state α), then on wing B the 90 ° polarized peer photon will surely hit a polarizer exit set to $\alpha + \pi/2$ (p-state $\alpha + \pi/2$), since the difference angle δ between polarizer position and polarization of the photon is equal to α on both sides.

With the p-states perpendicular to each other, the model predicts correctly measurement results with perpendicular polarizers on both wings. The reason for this is a common parameter λ and not a nonlocal action as we have seen.

So far we did not make use of the contextual assumption MA4 of the model. The derivation so far is local and noncontextual.

3.5 Predicting measurement results for an arbitrary context

We now calculate probabilities for arbitrary setting of the polarizers having polarizer PA set to α and polarizer PB set to β.

This means changing the selections of the photons. In the initial context 0°/90° the generated photons with 0° polarization and 90° polarization comprised the selection. Now the selection is changed. So is the polarization state of the photons which is defined by model assumption MA4.

If PA is set to α all selected photons 1 are in p-state α before selection. And the peer photons 2 belonging to the selected photons 1 are also selected thereby but in p-state $\alpha+\pi/2$ as we have seen above. With MA4 the polarization of the thus selected photons 2 is equal to the p-state $\alpha+\pi/2$. With the selected photons 1 in p-state α and peer photons 2 in polarization state $\alpha+\pi/2$ the selected peer photons 2 behave like a beam of single photons with polarization $\alpha+\pi/2$. The probability that this beam of photons 2 with polarization $\alpha+\pi/2$ passes PB at β can thus be obtained by equation (4) with $\delta = \beta-\alpha-\pi/2$ yielding

$$P_\delta = \int_0^{\cos^2(\delta)} d\lambda = \cos^2(\delta) = \cos^2(\beta-\alpha-\pi/2) = \sin^2(\beta-\alpha) \qquad (9)$$

where δ is the angle between the PB polarizer setting β and the polarization α+π/2 of the selected photon 2 which are peer to the selected photon 1 in p-state αin accordance with QM.

The expectation value for a joint measurement with photon 1 detected behind detector PA at α and peer photon 2 detected behind detector PB at β is

$$E(\alpha,\beta) = \Sigma(A*B*P_{A,\alpha}*P_{B,\beta|A,\alpha} \text{ for } A,B=+1,-1). \qquad (10)$$

where $P_{A,\alpha}$ is the unconditional probability to detect (A=1) or not detect (A= -1) photon 1 at α.

$P_{B,\beta|A,\alpha}$ is the conditional probability for photon 2 to have the outcome B (+1 for passing or -1 for not passing) at PB set to β if photon 1 had the outcome A (+1 for passing or -1 for not passing) at PA set to α. With P_δ from equation (9) we get in particular

$P_{+1,\beta|+1,\alpha} = P_\delta$ is the conditional probability to detect photon 2 at β if photon 1 was detected at α (B=+1, A=+1) and

$P_{-1,\beta|+1,\alpha} = 1 - P_\delta$ is the conditional probability to detect photon 2 not at β but at $\beta+\pi/2$ if photon 1 was detected at α. (B=-1, A=+1) and

$P_{+1,\beta|-1,\alpha} = 1 - P_\delta$ is the conditional probability to detect photon 2 at β if photon 1 was detected not at α but at $\alpha+\pi/2$.(B= +1,A= -1) and

$P_{-1,\beta|-1,\alpha} = P_\delta$ is the conditional probability to detect photon 2 not at β but at $\beta+\pi/2$ if photon 1 was detected not at α but at $\alpha+\pi/2$. (B=-1,A=-1).

Generated photons of polarization 0° and 90° contribute to the probability $P_{1,\alpha}$ to find a photon 1 at α according to equation (4) with fractions ½cos²(α) and ½cos²(α-π/2) respectively taking into account that the generated photons of 0° and 90° contribute in equal shares to the total stream of photons on either wing.

Thus, $P_{1,\alpha} = ½(\cos^2(\alpha) + \cos^2(\alpha-\pi/2)) = ½ = P_{-1,\alpha}$ (11)

With the above definitions we get from equations (9)-(11)

$E(\alpha,\beta)= ½(1* P_\delta -1*(1- P_\delta) -1*(1- P_\delta) +1* P_\delta) =$

$= P_\delta -(1- P_\delta) = \sin^2(\beta-\alpha) - \cos^2(\beta-\alpha) = -\cos (2(\beta-\alpha))$ (12)

in accordance with QM as well.

As the expectation value E(α,β) from equation (12) does exactly match the predictions of quantum physics it also violates Bell's inequality.

We have assumed that λ is not changed with the change of the polarization. So we have left to prove that λ is uniformly distributed in the interval $0 \le \lambda \le 1$ for the changed polarization. Equations (5) and (6) were derived assuming a generated photon 1 with φ1 =0° and polarizer PA setting α. Now we assume a generated photon 1 with φ_1 =90° and polarizer PA setting α+π/2. Then we would get δ = α +π/2 - φ_1= α and from equation (2)

$A(\delta,\lambda) = +1$ for $0 \le \lambda \le \cos^2(\delta)$, (13)

meaning photon 1 is in p-state α+π/2 given by the polarizer PA setting. From equation (3) we get

$A(\delta,\lambda) = -1$ for $\cos^2(\delta) < \lambda \le +1$, (14)

meaning photon 1 is in p-state α perpendicular to the polarizer PA setting. Note that p-state α and p-state α+π are equivalent.

Thus we see from equations (5) and (14) both generated photons with polarization 0° and 90° respectively contribute to the p-state α so that one half of photon 1 is in p-state α for $0 \le \lambda \le +1$ and from equations (6) and (13) we obtain the other half of photon 1 being in p-state $\alpha+\pi/2$ also for $0 \le \lambda \le +1$. In both cases λ is uniformly distributed in the interval $0 \le \lambda \le +1$ Thus, the proof is complete.

4 Extending the model to spin ½ particles

4.1 Model assumptions

The model does also apply to spin ½ particles by simply replacing every angle with its half in equations (2) and (3) and figure 3 as well and in all subsequent derivations yielding

Model assumption MA1a:

A propensity state (p-state) represents the instrument exit a particle will take. A particle with p-state α will pass the instrument (Stern-Gerlach apparatus) exit α with certainty.

Note that the p-state only has a meaning with the specification of an instrument position.

Model assumption MA2a:

The p-state is controlled by a parameter λ which is equally distributed between 0 and +1. With the particle having spin direction φ and $\delta = \alpha - \varphi$ we define an indicator function $A(\delta,\lambda)$ which indicates the p-state of the particle before a subsequent measurement. $A(\delta,\lambda)$ can have the values +1 and -1.

For $0<\delta<\pi$:

$$A(\delta,\lambda) = +1 \quad \text{for} \quad 0 \le \lambda \le \cos^2(\delta/2), \tag{15}$$

meaning the particle is in p-state α given by the instrument setting and

$$A(\delta,\lambda) = -1 \quad \text{for} \quad \cos^2(\delta/2) < \lambda \le +1, \tag{16}$$

meaning the particle is in p-state $\alpha+\pi$ opposite to the instrument setting.

Figure 4 shows the geometric relationships on which the model is based. For the case $\pi \le \delta < 2\pi$ we refer to the other exit of the instrument.

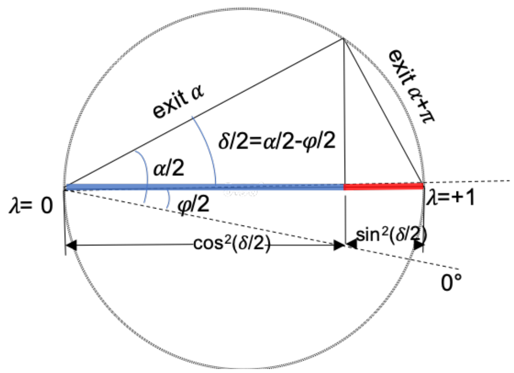

Figure 4: Geometrical derivation of a deterministic distribution of spin ½ particles onto instrument outputs. The instrument is set to the angle $\alpha/\alpha+180°$. The generated particle has a spin direction φ. The difference is $\delta=\alpha-\varphi$. A parameter λ is evenly distributed over the generated particles in the value range $0 \le \lambda \le +1$. Particles with $\lambda \le \cos^2(\delta)$ are assigned to the instrument output α, while particles with $\lambda > \cos^2(\delta)$ take the output $\alpha+180°$.

To account for the correlation between the entangled particles we introduce

Model assumption MA3a:

Particles of an entangled pair share the same value of the parameter λ. The rules for the distribution of the generated particles onto the two output directions of the instrument represented by equations (15) and (16) are also valid for the partner particle on wing B.

This reflects a property of the singlet state with the coordinate systems on both wings being of different handedness with respect to the propagation directions of the particles.

Model assumption MA4a:

Selected particles from each wing of the singlet state with the selected p-state α have the spin direction α. With a selection other than the initial context All information about the origin from the initial context is lost.

This means also that selected particles from each wing of the singlet state with p-state $\alpha+\pi$ have spin direction $\alpha+\pi$. Particles with spin direction α and $\alpha+\pi$ come in equal shares according to MA2a. MA4a accounts for the fact that the spin direction of particles from the singlet state is undefined due to indistinguishability but changed and redefined by entanglement. Thus, particles of a selection cannot be distinguished by their spin direction. For a selection of the initial states 0° or 180° the spin direction is not changed as it is already equal to the p-state of the selection. MA4a is a contextual assumption as the spin direction of the selection depends on the setting of an instrument. However, it is a local realistic assumption as it assigns a real value to the physical entity spin direction.

Model assumption MA5a:

After leaving an instrument exit λ is indeterminate and uniformly distributed.

MA5a stands for the fact that particles don't have a memory of their previous spin direction after leaving a instrument. The ensemble of particles covers the full range $0 \leq \lambda \leq +1$ after passing an instrument and a particle has the spin direction α after passing an instrument with setting α.

4.2 Predicting measurement results for single particles

Using equation (15) the particle with spin direction φ is found behind the output α of an instrument with the probability

$$P_\delta = \int_0^{\cos^2(\delta/2)} d\lambda = \cos^2(\delta/2) \qquad (17)$$

in accordance with QM.

For the case $\pi \leq \delta < 2\pi$ we refer to the other exit of the instrument and have with $\vartheta^* = \delta - \pi$

$$P_\delta = \int_{\cos^2(\delta*/2)}^{1} d\lambda = 1-\cos^2(\vartheta^*/2) = \cos^2(\vartheta/2) \text{ as well.}$$

4.3 Predicting measurement results for the initial context

In this section we are going to show that a selection of particles with spin direction α on wing A means for the thus selected peer particles they would certainly leave an instrument exit set to $\alpha+\pi$ on wing B e.g. they are in p-state $\alpha+\pi$.

Entangled particles are generated by a common source on wing A with the spin direction $\varphi_1 = 0°$ and on wing B with the spin direction $\varphi_2 = 180°$ or on wing A with the spin direction $\varphi_1 = 180°$ and on wing B with the spin direction $\varphi_2 = 0°$. This is the initial context.

First, we calculate measurement results for the pair of generated particle 1 with spin direction 0° and generated particle 2 with spin direction 180°.

For instance, having a generated particle 1 with $\varphi_1 = 0°$ and an assumed instrument PA setting α we would get $\delta = \alpha - \varphi_1 = \alpha$ and from equation (15)

$$A(\delta,\lambda) = +1 \quad \text{for} \quad 0 \leq \lambda \leq \cos^2(\delta/2), \tag{18}$$

meaning particle 1 is in p-state α.

From equation (16) we get

$$A(\delta,\lambda) = -1 \quad \text{for} \quad \cos^2(\delta/2) < \lambda \leq +1, \tag{19}$$

meaning particle 1 is in p-state $\alpha+\pi$ opposite to the instrument PA setting.

Defining an indicator function $B(\delta,\lambda)$ for measurement results on wing B we can apply model assumption MA3a for the correlation between the entangled particles on both wings. Here δ is again the angle between the instrument setting and the spin direction of the generated particle. Equations (18) and (19) do also apply adding 180° to all angles and exchanging $A(\delta,\lambda)$ with $B(\delta,\lambda)$.

Having thus a generated particle 2 with $\varphi_2 = 180°$ and an assumed instrument PB setting $\alpha+\pi$ we would get
$\delta = \alpha + \pi - \varphi_2 = \alpha$ and from equation (15)

$$B(\delta,\lambda) = +1 \quad \text{for} \quad 0 \leq \lambda \leq \cos^2(\delta/2), \tag{20}$$

meaning particle 2 is in p-state $\alpha+\pi$ given by the instrument PB setting. From equation (16) we get

$$B(\delta,\lambda) = -1 \quad \text{for} \quad \cos^2(\delta/2) < \lambda \leq +1, \tag{21}$$

meaning particle 2 is in p-state α opposite to the instrument PB setting. Here p-state α and p-state $\alpha+\pi$ are equivalent.

As entanglement connects particles 1 on wing A with particles 2 on wing B by the same value of the parameter λ we obtain from equations (18) and (20) and (19) and (21) respectively that the p-states of peer particles are opposite to each other meaning if particle 1 is detected by PA at α its peer particle 2 is detected with certainty by PB at $\alpha+\pi$.

In the same way, we calculate measurement results for the pair of generated particles 1 with spin direction 180° and generated particles 2 with spin direction 0°.

Summarizing a brief rationale for the counter directed p-states on both sides: If a particle with the parameter λ leaves an instrument exit on side A, which is rotated by the angle α relative to its spin direction, the peer particle on side B also leaves the instrument exit, which is rotated by the angle α relative to its spin direction as well. (the same local rules on both sides, the same value of δ, and the same value of λ) So if on side A the particle with spin direction 0° leaves the instrument exit α (p-state α), then on side B the particle with spin direction 180° will surely leave an instrument exit set to $\alpha+\pi$ (p-state $\alpha+\pi$), since the difference angle δ between instrument position and the spin direction of the particle is equal to α on both sides.

With the p-states opposite to each other, the model predicts correctly measurement results with opposite instrument setting on both wings. The reason for this is a common parameter λ and not a nonlocal action as we have seen. So far we did not make use of the contextual assumption MA4a of the model. The derivation so far is local and noncontextual.

4.4 Predicting measurement results for an arbitrary context

We now calculate probabilities for arbitrary setting of the instruments having instrument PA set to α and instrument PB set to β. This means changing the selections of the particles. In the initial context 0°/180° the generated particles with 0° spin direction and 180° spin direction comprised the selection. Now the selection is changed. So is the spin direction state of the particles which is defined by model assumption MA4a.

If PA is set to α all selected particles 1 are in p-state α before selection. And the peer particles 2 belonging to the selected particles 1 are also selected thereby but in p-state α+π as we have seen above. With MA4a the spin direction of the thus selected particles 2 is equal to the p-state α+π. With the selected particles 1 in p-state α and peer particles 2 having spin direction α+π the selected peer particles 2 behave like a beam of single particles with spin direction α+π. The probability that this beam of particles 2 with spin direction α+π passes PB at β can thus be obtained from equation (17) with δ = β-α-π yielding

$$P_\delta = \int_0^{\cos^2(\delta/2)} d\lambda = \cos^2(\delta/2) = \cos^2(½\,(\beta-\alpha)-\pi/2) = \sin^2(½\,(\beta-\alpha)) \qquad (22)$$

where δ is the angle between the PB setting β and the spin direction α+π of the selected particles 2 which are peer to the selected particles 1 in p-state α in accordance with QM.

The expectation value for a joint measurement with particle 1 detected behind detector PA at α and peer particle 2 detected behind detector PB at β is

$$E(\alpha,\beta) = \Sigma(A*B*P_{A,\alpha} *P_{B,\beta|A,\alpha} \quad \text{for } A,B=+1,-1). \qquad (23)$$

where $P_{A,\alpha}$ is the unconditional probability to detect (A=1) or not detect (A= -1) particle 1 at α.

$P_{B,\beta|A,\alpha}$ is the conditional probability for particle 2 to have the outcome B (+1 for passing or -1 for not passing) at instrument PB set to β if particle 1 had the outcome A (+1 for passing or -1 for not passing) at instrument PA set to α. With P_δ from equation (22) we get in particular

$P_{+1,\beta|+1,\alpha} = P_\delta$ is the conditional probability to detect particle 2 at β if particle 1 was detected at α (B=+1, A=+1) and

$P_{-1,\beta|+1,\alpha} = 1 - P_\delta$ is the conditional probability for particle 2 not to be detected at β but at $\beta+\pi$ if particle 1 was detected at α.
(B=-1, A=+1) and

$P_{+1,\beta|-1,\alpha} = 1 - P_\delta$ is the conditional probability for particle 2 to be detected at β if particle 1 was detected not at α but at $\alpha+\pi$.
(B= +1,A= -1) and

$P_{-1,\beta|-1,\alpha} = P_\delta$ is the conditional probability to detect particle 2 not at β but at $\beta+\pi$ if particle 1 was detected not at α but at $\alpha+\pi$. (B=-1,A=-1).

Generated particles of spin direction 0° and 180° contribute to the probability $P_{1,\alpha}$ to find a particle 1 at α according to equation (17) with fractions ½cos²(α/2) and ½cos²(α/2-π/2) respectively taking into account that the generated particles of 0° and 180° spin direction contribute in equal shares to the total stream of particles on either wing.

$$\text{Thus, } P_{1,\alpha} = ½(\cos^2(\alpha/2) + \cos^2(\alpha/2-\pi/2)) = ½ = P_{-1,\alpha} \qquad (24)$$

With the above definitions we get from equations (22-24)

$$E(\alpha,\beta) = ½(1* P_\delta -1*(1- P_\delta) -1*(1- P_\delta) +1* P_\delta) =$$

$$=P_\delta -(1- P_\delta) = \sin^2(½(\beta-\alpha)) -\cos^2(½(\beta-\alpha)) = -\cos(\beta-\alpha) \qquad (25)$$

in accordance with QM as well.

As the expectation value E(α,β) from equation (25) does exactly match the predictions of quantum physics it also violates Bell's inequality. The rest of the proof is done accordingly to that of photons in the preceding section.

5 Results, Discussion, and Conclusion

A local contextual model was presented which correctly meets the predictions of QM for polarization measurements with photons in singlet state as well as for spin measurements with spin 1/2 particles in singlet state thus refuting Bell's theorem.

The rule determining which instrument exit a particle will take is the same for both wings. Dependencies between the particles on either wing originate from the shared parameter λ and not from a nonlocal influence of particle 1 upon particle 2. With the same rules acting upon the generated particles on both wings, the measurement results are correlated without nonlocal effects. The model is also valid for single particles where the concept of a propensity state controlled by a parameter λ allows to predict measurement results for incompatible states simultaneously.

Measurement values at one wing are determined independently of the setting of the instrument at the other wing. Only the correlation between the measurements at both wings depends on the setting of the instruments. The latter is a contextual effect that comes into play as the spin/polarization of entangled particles is undefined. The model says that the spin/polarization of a selection of particles is changed by entanglement due to indistinguishability. However, this effect is local as we have seen. The purpose of the model was to show that a local contextual model is possible which correctly predicts expectation values in accordance with QM. The model's predictions therefore violate Bell's inequality thus refuting Bell's theorem.

After Bell's theorem was refuted we cannot conclude any more nature is nonlocal. This is no more a necessary consequence of QM infringing Bell's theorem. Experimental results with spin or polarization measurements can be explained without assuming non-local effects. If there is no basis for nonlocal effects anymore some conclusions can be drawn regardless of whether or not the model presented can replace QM in every detail. It is assumed that the observations on photons from an entangled pair are generally also valid for single photons.

Measured values are not generated upon the measurement, they already exist beforehand. Otherwise, a strong correlation between the outcomes of measurements at different sides would demand nonlocal effects. This supports Einstein's view of the meaning of the wave function as a description of an ensemble [18]. Thus, quantum mechanics does not violate the principle of causality, at least for spin measurements.

From the model point of view, we can also answer the question of what is the reason for the violation of Bell's inequality. It is not the fact that projective measurements do not commute. This is covered by model assumption MA2 and does not lead to a violation of Bell's inequality in the initial context. Rather, it is the indistinguishability of particles in the singlet state, described by model assumption MA4. Projective measurements are determined by the hidden variable λ. The observation that measurements in different spin directions do not commute is due to the fact (MA5) that there is no memory of the previous state after the measurement. The concept of superposition which implies the simultaneous existence of incompatible physical states is in question. If measured values exist beforehand mutually exclusive values cannot exist simultaneously. As a consequence the concept of a quantum computer is in question as it relies upon the assumption that a quantum system bears simultaneously information about two mutually exclusive outcomes [19]. As this assumption is no longer tenable the diversity of the solution of a quantum computer is considerably restricted.

Author Contribution

E.M is the only author and solely responsible for the entire manuscript

Competing interests

None

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