A further study of a scalar-tensor theory in Minkowski spacetime

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Abstract

In this work, we discuss a newly proposed scalar-tensor theory in Minkowski spacetime. The basic frame in Minkowski spacetime is obtained. Consider a charged particle in the electromagnetic field, we make an attempt to connect this theory with conventional quantum theory. Two specific models are chosen to calculate values of the time delay.

Keywords: scalar-tensor theory; Minkowski spacetime; scalar field

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I. INTRODUCTION

Gravity may be the most mysterious basic interaction, even if it is related to phenomena encountered in daily life, and it is easiest to think of without any advanced knowledge [1]. In 1915, Einstein completed the theory of General Relativity (GR) [2], which included gravity and any accelerated frame. It is worth noting that the theory completely consistent with the experimental results of the precession of Mercury's orbit, Lense–Thirring [3, 4] gravitomagnetic precession (1918), and the gravitational deflection of light by the Sun.

The success of GR made it a milestone in 20th century physics and one of the two pillars of modern physics: together with quantum field theory [5]. However, in the last thirty years several shortcomings came out in GR and people began to study whether GR is the only basic theory that can explain the gravitational interaction [1]. These problems basically come from cosmology and quantum field theory, and these shortcomings are related to observations and many theoretical aspects [6, 7]. These problems led to the emergence of various modified theories of gravity [8–21], which have made important progress in solving the problem of cosmological constant, inflation, or structure formation, etc. The most famous modified gravity theories are the scalar-tensor theory, $f(R)$ theory, $f(R,T)$ theory, Lovelock gravity, Einstein-Gauss-Bonnet gravity, etc [10, 14, 22–26]. Some of the most interesting theories are based on constructing of more general theory by abandon one or more of the several assumptions of GR. For example, the only degrees of freedom of the gravitational field are those of the metric, or simply choosing that the gravitational Lagrangian should be a linear function of scalar curvature [25].

As one of the most popular modified theories, the scalar-tensor theory has received widespread attention [23, 24]. As we know, GR is a geometrical theory or a metric theory, which is also called "tensor theory". Although usually we should set the degrees of freedom in the theory as few as possible, it does not rule out the introduction of additional scalar fields [24]. In the original scalar-tensor theory [27], the scalar field is related to a changing gravitational constant, thus the gravity can be adjusted. There can be many sources of scalar fields, it could be the dilaton from string theory [28, 29], the scalar field in a brane world [30], or comes the size of compactified internal space. This feature of scalar field theory is very attractive because it provides a lot of freedom for the birth of new theories. The variation principle played an important role in the development of modified gravity theories. Starting from the action, one can get the equation of motion of the field or the particle easily.

Observations show that the current expansion of the universe is accelerating. In order to explain this phenomenon, many dark energy models have been proposed [31–34]. There are many scalar field models in these models, such as quintessence, K-essence, tachyon, phantom and dilatonic models [31]. The scalar field plays an important role in these theories. For example, quintessence is described by an ordinary scalar field minimally coupled to gravity that lead to late-time inflation with particular potential [31]. Recently, a new theory involves scalar fields has been proposed [35]. In this theory, the action for the scalar fields has similar form with that of quintessence.

This paper is aimed at discuss the recently proposed theory [35] in Minkowski spacetime. We will try to connect this theory with conventional quantum theory in this space-time background.

II. A BRIEF REVIEW

Considering a spin-independent particle moving in an arbitrary background, we choose the action of fields as

$$I = I_0 + \int \sqrt{-g}\lambda \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta + \frac{1}{2} \left(\frac{mc}{\hbar} \right)^2 \mathcal{D} \zeta^2 \right] d^4x, \tag{1}$$

the Lagrangian describing the motion of the particle is

$$L = L_0 - \frac{1}{2} mc^2 \mathcal{D}. \tag{2}$$

Here, I_0 is the action of arbitrary theories we usually known, such as gauge theory [36–38] or various gravity theories. L_0 is the Lagrangian of the particle in arbitrary known theories and m is the proper mass of the particle. c is the speed of light and $\hbar$ is the reduced Planck constant, λ is a constant with the dimension of length. $\mathcal{D}(x)$ is a proper time field, which plays an important role in explaining the uncertainty of quantum theory. $\zeta(x)$ is a scalar field with its directly explanation as

$$\zeta^2 = \eta, \tag{3}$$

where η is the proper space density of the particle caused by restricting the space of its existence. It is reasonable to require the space density to satisfy the conservation law

$$\partial_\mu \left(\sqrt{-g} \eta u^\mu \right) = 0. \tag{4}$$

Similar to what we have done in conventional theories, we require that the space density of the existence be positive continuous, finite and single-valued at every point of space.

From above equations, one can get three basic equations:

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \zeta\right) + \left(\frac{mc}{\hbar}\right)^2 \sqrt{-g} \mathcal{D} \zeta = 0, \tag{5}$$

$$\partial_\mu \left(\sqrt{-g} \eta u^\mu\right) = 0, \tag{6}$$

$$g_{\mu\nu} u^\mu u^\nu + \mathcal{D} c^2 = -c^2. \tag{7}$$

They are very important in this theory.In above equations, $u_\mu = \frac{dx_\mu}{d\tau}$ is the four velocity and τ is the proper time. Eq. 5 shows that the relation between proper time field and the space density field. Eq. 6 is the conversation law of the proper space density and Eq. 7 is a space-time relation. We can calculate the specific form of $\mathcal{D}(x)$ from Eq. 3 and Eq. 5,

$$\mathcal{D} = -\left(\frac{\hbar}{mc}\right)^2 \frac{\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \sqrt{\eta}\right)}{\sqrt{-g}\sqrt{\eta}}. \tag{8}$$

III. BASIC FRAME OF THE THEORY IN MINKOWSKI SPACETIME

Starting from this section, we will discuss this theory in Minkowski spacetime. From Eq. 2, we can write the equation of motion for a spin-independent particle with mass m in Minkowski coordinates as

$$K_v - \frac{1}{2} mc^2 \partial_v \mathcal{D} = \frac{dp_v}{d\tau}, \tag{9}$$

with

$$\mathcal{D} = -\frac{\hbar^2}{m^2 c^2} \left(\partial_\sigma \partial_\sigma \ln \sqrt{\eta} + \partial_\sigma \ln \sqrt{\eta} \partial_\sigma \ln \sqrt{\eta}\right). \tag{10}$$

The space-time relation becomes

$$u_\mu u_\mu + \mathcal{D} c^2 = -c^2, \tag{11}$$

or

$$dx_\mu dx_\mu = -\left(1 + \mathcal{D}\right) c^2 d\tau^2, \tag{12}$$

and the conversation law of the proper space density can be rewritten as

$$\partial_\mu (\eta u_\mu) = 0. \tag{13}$$

Where $x_\mu = (x, y, z, ict)$, $p_v = m u_v$ is the four momentum, K_ν is the four force. Now we see there is an additional "force" related to the space density.

To make a clear explanation to the above formulae, Eq. 12 can be rewritten into a more familiar form,

$$dx_\mu dx_\mu = -c^2 d\bar{\tau}^2, \tag{14}$$

with

$$d\bar{\tau} = \sqrt{1+\mathcal{D}}d\tau, \tag{15}$$

where $\bar{\tau}$ is the proper time of the local coordinate which moves together with the particle and the space-time relation of the coordinate remains same to special relativity. However, the proper time of the particle is not equal to the proper time of the coordinate because there is a proper time field $\mathcal{D}(x)$, and the comprehension of the four velocity and the proper space density of the existence of the particle also need to be justified by this way. We would like to emphasize now that the new spacetime relation is very important, from which we know there also might be a time delay even in the case the velocity is zero.

Eqs. 9, 12 and 13 are the basic frame of the theory in Minkowski spacetime. The following discussion should go first with the energy. From Eqs. 9 and 11 we have

$$u_v K_v = 0. \tag{16}$$

Then by defining $\gamma = \frac{dt}{d\tau} = \frac{dt}{d\bar{\tau}}\frac{d\bar{\tau}}{d\tau}$, the mass-energy relation is defined as

$$\varepsilon = \frac{c}{i} p_4 = m\gamma c^2. \tag{17}$$

It is worth noting that this relation is similar in form to the relation given by Einstein [39], but has a wider meaning. Eq. 17 can be illustrated in the stationary state of the space density as

$$\frac{dm\gamma c^2}{dt} = -\frac{u_4 K_4}{\gamma^2} = \frac{u_i K_i}{\gamma^2} = \vec{v} \cdot \vec{F} \tag{18}$$

where $v_i = \frac{dx_i}{dt} = \frac{u_i}{\gamma}, F_i = \frac{K_i}{\gamma}, (i = 1, 2, 3)$. We can see the definition of energy is reasonable, since the increase of energy is equal to the work done by the force.

IV. ATTEMPT TO CONNECT QUANTUM THEORY

Let's continue our study with a charged particle moving in the electromagnetic field. In this case, force can be written as

$$K_v = qu_\mu \left(\partial_v A_\mu - \partial_\mu A_v\right), \tag{19}$$

where $A_\mu = (\vec{A}, i\frac{\varphi}{c})$ is the four electromagnetic potential. On the other hand, from Eqs. 13 and 9, we also have

$$K_v = u_\mu \left(\partial_\mu p_v - \partial_v p_\mu\right). \tag{20}$$

There could be many ways to solve the above equations. In this section, we use complex functions in our following discussion in order to reach the Klein-Gordon equation.

Define $u'_\mu = -\frac{\hbar}{m}\partial_\mu \ln\sqrt{\eta}$, $p'_\mu = mu'_\mu$, $U_\mu = u_\mu + iu'_\mu, P_\mu = p_\mu + ip'_\mu$, then Eqs. 13 and 11 can be composed as

$$\left[\frac{\hbar}{im}\partial_\mu + U_\mu\right] U_\mu = -c^2. \tag{21}$$

By considering Eqs. 19 and 20, we define $\pi_\mu = p_\mu + qA_\mu$, π_μ is the four canonical momentum, then we have $\nabla \times \vec{\pi} = 0$, also $\nabla \times \vec{\Pi} = 0$, for $\Pi_\mu = P_\mu + qA_\mu$. Now from Eq. 21 we have

$$\left(\Pi_\mu - qA_\mu\right)\left(\Pi_\mu - qA_\mu\right) + \frac{\hbar}{i}\partial_\mu\left(\Pi_\mu - qA_\mu\right) = -m^2c^2. \tag{22}$$

Because $\nabla \times \vec{\Pi} = 0$, we can define

$$\Pi_\mu = \frac{\hbar}{i\psi} \partial_\mu \psi, \tag{23}$$

where ψ is the wave function describing the state of the particle in the other way. It should be also continuous, finite and single-valued.

Then Eq. 22 becomes

$$\left[\frac{\hbar}{i}\partial_\mu - qA_\mu\right]\left[\frac{\hbar}{i}\partial_\mu - qA_\mu\right]\psi + m^2c^2\psi = 0. \tag{24}$$

Now we get Klein-Gordon equation [40, 41] in this theory, and its wave function gets its explanation automatically as $\psi = A \exp\left[\frac{i}{\hbar}\int^{x_\mu} \Pi_\mu\left(x'_\mu\right) dx'_\mu\right]$, $\psi^*\psi = \eta$, where A is a constant, and from Eq. 13 the equation of normalization could be $\int_\infty \gamma\psi^*\psi d^3x = 1$.

For stationary electromagnetic field, the total energy is

$$E = m\gamma c^2 + q\varphi = \frac{c\pi_4}{i}. \tag{25}$$

In the non-relativistic limit, Eq. 13 becomes

$$-\frac{\partial \eta}{\partial t} = \nabla \cdot (\eta \vec{v}). \tag{26}$$

Then we know

$$|p'_4| = \left|\frac{i\vec{v}\cdot\vec{p}'}{c} + \frac{\hbar}{2i}\nabla\cdot\frac{\vec{v}}{c}\right| \ll |\vec{p}'|. \tag{27}$$

Eq. 9 becomes

$$\vec{F} + \frac{1}{2m}\nabla\left(p'^2 - \hbar\nabla\cdot\vec{p}'\right) = \frac{d\vec{p}}{dt}. \tag{28}$$

It's worth noting that the additional "force" on the left side of Eq. 28 is radically different from conventional force, since it originates from restricting the space of the existence particle.

Defining $\Psi = e^{\frac{imc^2t}{\hbar}}\psi$, then Eq. 24 becomes

$$i\hbar\frac{\partial \Psi}{\partial t} = \frac{1}{2m}\left[\frac{\hbar}{i}\nabla - q\vec{A}\right]^2\Psi + q\varphi\Psi. \tag{29}$$

Schrödinger's non-relativistic equation [42] is obtained in this theory, now still we have $\Psi^*\Psi = \eta$. One can also see this theory meets with Born's statistical interpretation [43] in the non-relativistic limit.

Concerning the energy in the non-relativistic limit, we can now define $E' = E - mc^2$, then from Eq. 25 we have

$$E' = \frac{1}{2m}\left(p^2 - p'^2 + \hbar\nabla\cdot\vec{p}'\right) + q\varphi = \text{ real part of } \left[\frac{i\hbar}{\Psi}\frac{\partial\Psi}{\partial t}\right], \tag{30}$$

this is the expression of energy which meets with the hypothesis of Schrödinger.

It is worth mentioning that Eqs. 24 and 29 may be also used as a convenient way to calculate $\mathcal{D}(x)$, which plays a major role in this theory.

V. CLASSICAL APPROXIMATION

In this section, we use uncertainty relation in our discussion. As we usually know in the classical case, the mass of the object becomes much larger, and the volume of the object also becomes much larger so its position could be always being "measured" (by photons, etc.), then

$$|\mathcal{D}| = \frac{1}{m^2c^2}\left|-\hbar\partial_\sigma p'_\sigma + p'_\sigma p'_\sigma\right| \sim \frac{\hbar^2}{m^2c^2}\frac{1}{\Delta x^2}, \tag{31}$$

where $|\Delta x|$ is the measuring accuracy of position. It is not difficult to find that, quantum effect can not be ignored when the measurement accuracy is close to the Compton wavelength $\hbar/mc$.

As illustration, consider the following examples:

(1) For $\Delta x \sim 10^{-10}$ m and $m \sim 10^{-31}$ kg, Compton wavelength $\hbar/mc \sim 10^{-11}$ m. Quantum effect apparently can not be ignored, since $|\mathcal{D}| \sim 10^{-2}$ m. This means that we can not ignore quantum effect. The motion of the extranulear electron fits this situation, for example, the electron of hydrogen atom.

(2) For $\Delta x \sim 10^{-10}$ m and $m \sim 10^{-18}$ kg, Compton wavelength $\hbar/mc \sim 10^{-24}$ m $\ll 10^{-10}$ m. We can ignore the quantum effect safely, since $|\mathcal{D}| \sim 10^{-28} \ll 1$. In this case, classical theory is applicable, such as Brownian particles.

In the case of quantum effect can be ignored, Eq. 9 and 11 become

$$K_v = \frac{dp_v}{d\tau} \tag{32}$$

and

$$u_\mu u_\mu = -c^2. \tag{33}$$

VI. TWO EXAMPLES: THE CALCULATION OF TIME DELAY

As mentioned above, Eqs. 24 and 29 may be used as a convenient way to calculate $\mathcal{D}(x)$. In this part, we calculate $\mathcal{D}(x)$ and the time delay of one-dimensional harmonic oscillator and the ground state of the atom for example.

We first consider a one-dimensional harmonic oscillator problem in classical mechanics (see Appendix A for details). We have

$$\langle \gamma \rangle_{\mathrm{CMH}} = 1 + \frac{\hbar\omega}{2mc^2}\left(n + \frac{1}{2}\right), \tag{34}$$

which gives us the average value of the time delay. This is just a "reference time delay". The subscript "CMH" represents classical harmonic oscillator. In this part, we set $\hbar\omega/mc^2 = 10^{-6}$, then one can check that

$$\langle \gamma \rangle_{\mathrm{CMH}} = 1.00000025 \tag{35}$$

for $n = 0$.

The proper time field $\mathcal{D}_H$ corresponding to the wave-function of the hamonic osillator is

$$\mathcal{D}_H(\xi) = -\frac{\hbar\omega}{mc^2} H_n^{-1}(\xi)\, e^{\frac{\xi^2}{2}} \frac{d^2}{d\xi^2}\left(e^{-\frac{\xi^2}{2}} H_n(\xi)\right). \tag{36}$$

Eq. 14 shows that there is a time delay between $\bar{\tau}$ and τ. The time delay is apparently related to the spatial position, we can calculate the average value of the time delay

$$\langle\gamma\rangle_{\text{QMH}} = \int_{\xi_1}^{\xi_2} \sqrt{1+\mathcal{D}_H}\eta d\xi = \frac{1}{2^n\sqrt{\pi}n!}\int_{\xi_1}^{\xi_2} \sqrt{1+\mathcal{D}_H}e^{-\xi^2}H_n^2(\xi)\,d\xi, \tag{37}$$

$\sqrt{1+\mathcal{D}_H} \geq 0$ for any $\xi \in [\xi_1, \xi_2]$. The subscript "QMH" represents quantum harmonic oscillator. One can check that

$$\langle\gamma\rangle_{\text{QMH}} = 1.00000025 \tag{38}$$

for $n = 0$. We list some results in Table S1. To our limited knowledge, measuring the decay lifetime of radioactive minerals might be a way to verify these results.

Then, we consider the ground state wave function of the hydrogen atom. According to Bohr's quantification condition and the formula for the orbital radius of the hydrogen atom, it can be concluded that the velocity of the electron in the ground state of the hydrogen atom is $v = \alpha c = c/137$. $\alpha = e^2/\hbar c = \lambda_c/a_0$ is the fine-structure constant. If we consider the relativistic effect, we can obtain the reference time delay of a electron moving around the nucleus,

$$\gamma_{\text{CMA}} = \frac{1}{\sqrt{1+\left(\frac{1}{137}\right)^2}} \approx 1.000026641, \tag{39}$$

where the subscript "CMA" represents Bohr model.

On the other hand, the wave function given by the Schrödinger equation is

$$\Psi = \left(\frac{1}{\pi a_0^3}\right)^{\frac{1}{2}} e^{-\frac{r}{a_0}}, \tag{40}$$

and the corresponding proper time field $\mathcal{D}_A$ is

$$\mathcal{D}_A = \left(\frac{\lambda_c}{a_0}\right)^2 \left(\frac{2a_0}{r} - 1\right), \tag{41}$$

where $a_0 = \hbar^2/me^2$ is Bohr radius. The average value of the time delay is

$$\langle \gamma \rangle_{\text{QMA}} = \int_{\infty} \sqrt{1 + \mathcal{D}_A} \eta d^3x = \frac{1}{\pi a_0^3} \int_0^{\infty} \sqrt{1 + \mathcal{D}_A} e^{-\frac{2r}{a_0}} \cdot 4\pi r^2 dr \approx 1.000026638, \tag{42}$$

where the subscript "QMA" represents the atom in quantum theory.

It is not difficult to find that the calculation result of this theory is close to the result given by the Bohr model. Using the same technique, we can calculate the time delay and the reference time delay of the ground state electron of an atom with a nuclear charge of $Z > 1$. We list some of the results in Table S2. One thing we would like to point out is that a replacement of the electron by an other particle dose little affects to our results. To our limited knowledge, measuring the decay lifetime of exotic atoms might be one of the effective verification method. The above two calculations may provide directions for experiments.

VII. CONCLUSIONS

In this paper, we discussed the newly proposed scalar-tensor theory in Minkowski space-time. The basic frame of the theory in Minkowski spacetime is given. From this frame, we tried to connect with the conventional quantum theory through theoretical derivation and discuss some related issues. Harmonic oscillator and hydrogen atom were chosen as examples to calculate their time delay.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

COMPETING INTERESTS

Te authors declare no competing interests.

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Appendix A: Time delay: harmonic oscillator

Suppose a restoring force $f = -kx$ acts on a particle of mass m, where k is a positive constant. The motion of the particle is simple harmonic motion, that is

$$x = A\cos\omega t, \tag{A1}$$

$$v = -\omega A\sin\omega t, \tag{A2}$$

where $\omega = \sqrt{\frac{k}{m}}$ and A is the amplitude. The total energy of the particle is $E = \frac{1}{2}m\omega A^2$. In the case of non-relativity theory, the time of one cycle of the particle motion is represented by T. Now we consider the influence of relativity. For observers who are stationary relative to the coordinate system, the period of particle motion (T') becomes longer. One can obtain

$$\langle\gamma\rangle_{\mathrm{CMH}} = \frac{1}{T}\int_0^T \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}dt \approx \frac{1}{T}\int_0^T \left(1+\frac{1}{2}\frac{\omega^2 A^2\sin^2\omega t}{c^2}\right)dt = 1+\frac{E}{2mc^2}. \tag{A3}$$

According to the corresponding principle, quantum calculation must be consistent with classical calculation. Then we can get the average value of the time delay

$$\langle\gamma\rangle_{\mathrm{CMH}} = 1+\frac{\hbar\omega}{2mc^2}\left(n+\frac{1}{2}\right). \tag{A4}$$

This is just a “reference time delay”.

In conventional quantum mechanics (QM), the wave functions and energy levels of a one-dimensional harmonic oscillator can be solved by Schrödinger equation, namely

$$\Psi_n = N_n H_n\left(\xi\right) e^{-\frac{\xi^2}{2}} = N_n H_n\left(\frac{x}{x_0}\right) e^{-\frac{x^2}{2x_0^2}} \tag{A5}$$

and

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, \quad \left(n = 0, 1, 2, \cdots\right), \tag{A6}$$

where $H_n\left(\xi\right)$ is the Hermite polynomials and N_n is the normalization coefficient, their specific form can be written as

$$H_n\left(\xi\right) = \left(-1\right)^n e^{\xi^2}\left(\frac{d}{d\xi}\right)^n e^{-\xi^2} \tag{A7}$$

and

$$N_n = \left(x_0\sqrt{\pi}2^n n!\right)^{-\frac{1}{2}} = \left(\sqrt{\frac{\hbar\pi}{m\omega}}2^n n!\right)^{-\frac{1}{2}}. \tag{A8}$$

The proper time field $\mathcal{D}_H$ corresponding to the wave-function of the hamonic osillator is

$$\mathcal{D}_H\left(\xi\right) = -\frac{\hbar\omega}{mc^2} H_n^{-1}\left(\xi\right) e^{\frac{\xi^2}{2}} \frac{d^2}{d\xi^2}\left(e^{-\frac{\xi^2}{2}} H_n\left(\xi\right)\right). \tag{A9}$$

Appendix B: Time delay calculation results: ground state of atom

n \ $\frac{\hbar\omega}{mc^2}$	10^{-6}	10^{-3}
0	1.00000025	1.00025
1	1.00000075	1.00075
2	1.00000125	1.00125
3	1.00000175	1.00175
4	1.00000225	1.00225
5	1.00000275	1.00274
6	1.00000325	1.00324
7	1.00000375	1.00374
8	1.00000425	1.00424
9	1.00000475	1.00473
10	1.00000525	1.00523
11	1.00000575	1.00573
12	1.00000625	1.00622
13	1.00000675	1.00672
14	1.00000725	1.00721
15	1.00000775	1.00771
16	1.00000825	1.00820
17	1.00000875	1.00869
18	1.00000925	1.00919
19	1.00000975	1.00968
20	1.00001025	1.001017

n \ $\frac{\hbar\omega}{mc^2}$	10^{-6}	10^{-3}
0	1.00000025	1.00025
1	1.00000075	1.00075
2	1.00000125	1.00125
3	1.00000175	1.00175
4	1.00000225	1.00225
5	1.00000275	1.00275
6	1.00000325	1.00325
7	1.00000375	1.00375
8	1.00000425	1.00425
9	1.00000475	1.00475
10	1.00000525	1.00525
11	1.00000575	1.00575
12	1.00000625	1.00625
13	1.00000675	1.00675
14	1.00000725	1.00725
15	1.00000775	1.00775
16	1.00000825	1.00825
17	1.00000875	1.00875
18	1.00000925	1.00925
19	1.00000975	1.00975
20	1.00001025	1.001025

Table I. **Table of calculation results of the harmonic oscillator.** The table on the left is the calculation result of $\langle\gamma\rangle_{\mathrm{QMH}}$, and the table on the right is the calculation result of the "reference time delay" $\langle\gamma\rangle_{\mathrm{CMH}}$.

Z	$\langle\gamma\rangle_{\mathrm{QMA}}$	γ_{CMA}
1	1.000026638	1.000026641
2	1.000106530	1.000106576
3	1.000239615	1.000239843
4	1.000425787	1.000426507
5	1.000664905	1.000666658
6	1.000956790	1.000960410
7	1.001301227	1.001307905
8	1.001697969	1.001709312
9	1.002146741	1.002164823
10	1.002647234	1.002674660
11	1.003199119	1.003239070
12	1.003802037	1.003858328
13	1.004455611	1.004532738
14	1.005159441	1.005262629
15	1.005913109	1.006048361

Table II. **Table of calculation results of ground state electron of atom.** The first column is the number of nuclear charges.