

Article

3-D Kernel Density Imaging based on the Euler Deconvolution of Tensor Gravity Data

Shujin Cao^{1,2,3*}, Bo Yang¹, Guangyin Lu², Xiangyun Hu³, Yajing Mao¹, Shuanggui Hu^{2,4} and Ziqiang Zhu^{2,*}

1 School Resource & Environment and Safety Engineering, Hunan University of Science and Technology, Xiangtan 411201, China

Shujin.cao@hnust.edu.cn (S.C.); yangbo@mail.hnust.edu.cn (B.Y.); luguangyin@csu.edu.cn (G.L.); xyhu@cug.edu.cn (X.H.); mao-yajing@mail.hnust.edu.cn (Y.M.); hushuanggui808@csu.edu.cn (S.H.); zhuziqiang@csu.edu.cn (Q.Z.)

2 School of Geosciences and Info-Physics, Central South University, Changsha 410083, China

3 Institute of Geophysics & Geomatics, China University of Geosciences, Wuhan, 430074, China;

4 School of Mathematics and Statistics, Central South University, Changsha 410083, China

* Correspondence: shujin.cao@hnust.edu.cn, zhuziqiang@hnust.edu.cn

Abstract: Traditional discrimination techniques for Euler deconvolution use only the color spectrums of structural indexes, without considering the spatial distribution characteristics and inherent relationships among the Euler solutions to separate adjacent causative sources. In the present study, a new approach was developed for discriminating uncorrelated Euler solutions from coherent solutions based on the focusing levels indicated by the probability density distributions generated using multivariate kernel density estimations (KDE). A novel multiple coverage technique was proposed by using a series of different sized moving windows over gridded gravity data, which formed tight clusters of Euler solutions for different sized causative sources. The results of the probability density distributions were obtained using a 3-D KDE method for the Euler solution subsets {x, y, z} of synthetical models, and real data from a survey conducted in British Columbia (Canada) which had successfully established more credible and meaningful geological models when compared with three other subsets.

Keywords: full tensor gradient gravity; kernel density imaging; color spectrums; multiple-sources; Euler deconvolution; tight clusters

1. Introduction

The crucial features of gravitational and magnetic fields is that they enhance and maintain the edges of geologic bodies. With the development of gravity gradient measurement methods and their corresponding interpretation techniques, many research findings regarding persistent edges have been published in recent years which have utilized such potential data sources as edge recognition [1]; eigenvector analyses [2]; and focusing inversion techniques [3]. Full tensor gravity gradient (FTG) technology is a multi-component gravimetry technique for measuring the different components of gravity gradient tensor [4]. In recent years, full tensor measurements have been widely carried out, and large amounts of high-precision tensor gravity data have been obtained. It has been found that the three-dimensional inversion of full tensor gravity data can achieve good results. In addition,

full tensor gravity gradiometric surveys tend to have much higher bandwidths than traditional gravity surveys, and are able to deliver images with higher resolution [5]. However, FTG data impose higher requirements on their corresponding technical interpretations due to the fact that they often generate more complex anomalies for given sources [6].

Euler deconvolution processes based on Euler's homogeneity equation use gridded potential field data (for example, total fields and gradient fields) with series of square moving windows for the purpose of determining the source locations and depths of potential field anomalies for assumed structure indexes. Therefore, such methods are suitable for the analysis and interpretation of large-area potential field data [7, 8]. The aforementioned methods have strong adaptability and flexibility, as well as simple calculation processes. The convergence rates of the tensor Euler deconvolution of full tensor gravity data have been observed to be better than those of traditional Euler deconvolution [9].

A linear system of equations is generally composed for each moving window, and solved using singular value decomposition or an equivalent technique in order to obtain such Euler solutions as the coordinates x , y , z , and the structure index N . The regional B and the β (assume $\beta_x - \beta_z$ for a 3-D application) are constants which normally vanish, with the exception of the cases where $N = 0$ [10, 11]. However, in practice, traditional Euler deconvolution has been found to easily produce spurious solutions when a system of equations is over-determined. Therefore, it is often difficult to determine the best solutions and discriminate the spurious solutions within an entire Euler solution dataset for geological bodies. In order to overcome the aforementioned difficulties, several methods have been proposed based on various posterior estimation strategies for the purpose of filtering out spurious solutions [12]. In previous studies, Gerovska et al. (2003) [13] filtered spurious Euler solutions using the properties of differential similarity transformation, which was based on the error estimations of singular point coordinates and structure indices. In another related study, Yao et al. (2004) [14] proposed three new criteria: Horizontal gradient filtering criterion; distance constraint criterion; and convergence constraint criterion, which could be utilized to filter spurious Euler solutions for the interpretations of magnetic and gravitational anomalies.

Kernel density estimations are used to reject solutions that are sparsely located and which are not parts of dense clusters. These estimations provide solid foundations for density-based clustering algorithm construction. Ugalde and Morris (2010) [8] applied clustering and kernel density distribution techniques to Euler solutions in order to obtain coherent filtering solutions from uncorrelated solutions. However, a drawback was encountered in that the aforementioned algorithm typically over-relied on the number of manually determined abnormal sources. The clustering analyses of Euler solutions are used to classify Euler solution sets according to similarity, and then classify the sets with the greatest similarity into one category, which occupy local regions of featured spaces. The aggregation centers of each local area play representative roles and indicate the spatial information of the corresponding geological bodies. Beiki (2013) [15] confirmed that the dimensionality of magnetic anomalies is the ratio between the smallest and intermediate eigenvalues acquired. Then, the removal of spurious Euler solutions can be completed using truncated singular value decomposition for the quasi-2D magnetic sources. On the other hand, the interpretation processes of the Euler solutions regarding the locations and depths of sources tend to utilize color levels or different sized circles. However, such methods have been found to have difficulty discriminating among different focusing degrees, such as the dense distributions (for example, the centroid of geological bodies) and sparse distributions (for example, the gaps among geological bodies) of the Euler solutions.

In this research investigation, Euler solutions were obtained using tensor Euler deconvolution of FTG data, which avoided the calculation of the potential field derivatives for

coordinates x and y , and depth z , and did not predict the structure index N . Then, using a comparative analysis process, the probability density curves, images, and iso-surfaces were obtained using different dimensions of KDE for the combinations of the Euler solution datasets, and 1D and 2D KDE producing too many probability density distribution curves or images, making it difficult to analyze. Furthermore, illustration of the results of the probability density distributions, which were 5D data obtained by quaternion KDE for the entire Euler solution dataset, were also found to be difficult to achieve. Consequently, this study utilized 3D KDE to determine the focusing levels for finding the probability density peaks and removing the spurious Euler solutions. It was found that the proposed algorithm was able to successfully establish a greater amount of credible and meaningful geological bodies when compared with other discrimination techniques for Euler deconvolution.

2. Tensor Euler deconvolution

Full tensor gravity data is composed of the derivatives along the tangent vectors of gravity anomalies. The first derivative, second derivative, and the extension of a gravity field also satisfy the Euler equation. The tensor Euler deconvolution equation is composed of the traditional Euler Equation (1) [16] and two similar equations (2) and (3) [17,18].

$$x_0 \frac{\partial g_z}{\partial x} + y_0 \frac{\partial g_z}{\partial y} + z_0 \frac{\partial g_z}{\partial z} + N(B_z - g_z) = x \frac{\partial g_z}{\partial x} + y \frac{\partial g_z}{\partial y} + z \frac{\partial g_z}{\partial z} + \beta_z, \quad (1)$$

$$x_0 \frac{\partial g_y}{\partial x} + y_0 \frac{\partial g_y}{\partial y} + z_0 \frac{\partial g_y}{\partial z} + N(B_y - g_y) = x \frac{\partial g_y}{\partial x} + y \frac{\partial g_y}{\partial y} + z \frac{\partial g_y}{\partial z} + \beta_y, \quad (2)$$

$$x_0 \frac{\partial g_x}{\partial x} + y_0 \frac{\partial g_x}{\partial y} + z_0 \frac{\partial g_x}{\partial z} + N(B_x - g_x) = x \frac{\partial g_x}{\partial x} + y \frac{\partial g_x}{\partial y} + z \frac{\partial g_x}{\partial z} + \beta_x, \quad (3)$$

where B_a is the abnormal background of gravity; g_a is the gradient gravity; $g_{a\beta}$

represents the component of FTG, $\{\alpha, \beta \in x, y, z\}$; and B_a is very small. Therefore, B_a

can be removed from Eqs. (1) to (3), and N will not be predicted [19]. Next, square moving windows over gridded gravity data were used to determine the locations of the causative sources. Each of the windows led to a solution involving the location of the causative sources $\{x, y, z\}$ and structure index N using singular value decomposition or equivalent techniques.

As a result, Eqs. (1) to (3) were rewritten in matrix form as follows:

$$Am = b, \quad (4)$$

where $A = [A_1, \dots, A_n]$, $A_i = \left[\left(\frac{\partial g_a}{\partial x} \right)_i, \left(\frac{\partial g_a}{\partial y} \right)_i, \left(\frac{\partial g_a}{\partial z} \right)_i, (\beta_a)_i \right]$, $b = [b_1, \dots, b_n]$,

$b_i = \left[\left(x_0 \frac{\partial g_a}{\partial x} \right)_i + \left(y_0 \frac{\partial g_a}{\partial y} \right)_i + \left(z_0 \frac{\partial g_a}{\partial z} \right)_i \right]$; m represents the entirety of the Euler solution

datasets; and m_i is the i^{th} Euler solution denoting $[x_i, y_i, z_i, N_i]$.

3. Multivariate KDE of the Euler solution datasets

In the present study, the linear system (4) was still solved as essentially a Euler homogeneous equation. Therefore, the entirety of the Euler solution datasets still required filtering of the spurious Euler solutions, as well as the determination of the optimum Euler solutions. Fitzgerald et al. (2004) [19] discussed traditional discrimination techniques for Euler deconvolution, such as the rejection of non-physically meaningful Euler solutions (such as inadmissible structure indexes N falling outside the observed survey); ratios of estimated depth errors and depths greater than 50%; and the moderate singularity ratios of 2-D bodies. Due to the fact that the ratios of the estimated depth errors and depths had varied under the different geological conditions, it may be difficult to set a constant value for a particular survey area. Godtliebsen et al. (2002) [20] and Chaudhuri and Marron (1999) [21] proposed a novel technique for determining the significant features of low-dimensional data instances, such as local extrema, valleys, ridges, saddle points, and steep gradients. In another related study, Hernan and William (2010) [22] applied kernel density distribution techniques in order to reject sparse clusters classified by a fuzzy c-means clustering algorithm. However, it has been found that fuzzy clustering analysis methods have several drawbacks, such as pre-determined numbers of clusters, local optimization, and classification uncertainties. Therefore, in order to overcome the difficulties, this study introduced multivariate KDE for discriminating uncorrelated or spurious Euler solutions from coherent solutions located near anomalous geological sources prior to filtering any Euler solutions.

The Euler deconvolution tends to yield many similar or duplicate solutions, and tight clusters have been found to be good representations of causative sources. In other words, the focusing levels of the Euler solutions have the ability to discriminate the spurious solutions in an entire Euler solution dataset for geological bodies. At the present time, a new form of focusing energy level has been introduced which utilizes the probability density obtained by multivariate KDE methods [18].

This study observed that in the available statistical data, KDE was considered to be a non-parametric approach for calculating the probability density distributions of data instances. For example, when compared with parametric approaches, non-parametric approaches make full use of all the data points and provide several advantages, such as asymptotic unbiasedness, square consistency, and uniform convergence. At the present time, KDE is a very popular tool for analyzing low-dimensional data instances [23,24].

Therefore, by letting $\mathbf{X} = (X_1, \dots, X_i, \dots, X_n)$ provide a sample of independent and identically distributed random variables with a common density f , the kernel density estimator of $\hat{f}$ can be defined as follows:

$$\hat{f}(\mathbf{X}, h) = \frac{1}{n} \sum_{i=1}^n K_{d,h}(\mathbf{X}_i - \mathbf{X}), \quad (5)$$

where

$$K_{d,h}(\mathbf{X}) = \prod_{i=1}^d K_{h_i}(\mathbf{X}_i) = \prod_{i=1}^d K\left(\frac{\mathbf{X}_i}{h_i}\right) / h_i. \quad (6)$$

Then, for the purpose of convenience, where function K is the 1-D kernel function, $h > 0$, which is the bandwidth. In the formula, the multivariate kernel function $K_{d,h}$ is the inner product of the multiple 1-D kernel functions [24], and d indicates the dimension of the dataset $\mathbf{X}$.

Theoretically speaking, any function can be used as a kernel function. However, for the convenience and rationality of density function estimations, this study used the Gaussian Kernel as follows:

$$K(\mathbf{X}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\mathbf{X}^2}{2}}, \quad (7)$$

where

$$\sup |K(\mathbf{X})| < \infty, \int_{-\infty}^{\infty} K(\mathbf{X}) d\mathbf{X} = 1. \quad (8)$$

Therefore, in accordance with above definition, the extended form of the d-dimensional multivariate kernel density estimator $\hat{f}$ of f can be written as follows:

$$\hat{f}(\mathbf{X}, \mathbf{H}) = \frac{1}{n} \sum_{i=1}^n K_{\mathbf{H}}(\mathbf{X}_i - \mathbf{X}), \quad (9)$$

where $K_{\mathbf{H}}(\mathbf{X})$ indicates the multivariate kernel. This formulation varies somewhat from the aforementioned the definition (5), where $\mathbf{H} = h^2$, and $d \times d$ represents the bandwidth matrix.

Therefore, by taking the partial derivative of Eq. (10) with respect to f , the yield kernel density derivative estimator will be as follows:

$$\nabla \hat{f}(\mathbf{X}, \mathbf{H}) = n^{-1} \sum_{i=1}^n \nabla K_{\mathbf{H}}(\mathbf{X} - \mathbf{X}_i), \quad (10)$$

where ∇ is the column vector of the d partial first derivatives, and

$$\nabla K_{\mathbf{H}}(\mathbf{X}) = |\mathbf{H}|^{-1/2} \mathbf{H}^{-1/2} \nabla K(\mathbf{H}^{-1/2} \mathbf{X}), \quad (11)$$

This study used the multivariate KDE to analyze the various combinations, such as $\{x\}$, $\{y\}$, $\{z\}$, $\{N\}$, and $\{x, y, z\}$, of the Euler solution datasets for the purpose of avoiding the involvement of the above-mentioned discrimination techniques [25,26].

4. Multiple coverage technique for tensor Euler deconvolution

The parameters of this study’s designed model are detailed in Table 1. In the table, (x0, y0, z0) represent the centroid of the cubes. There were five cubic bodies with the same density contrast ($0.36 \times 10^3 \text{ kg/m}^3$) over the background. A rectangular survey grid was divided into $40,000 = 200 \times 200$ observation points, which were located on the Earth’s surface. The observational height was set as 25 m, and the sampling intervals were 100 m in the x - and y -directions. In addition, in order to form tight clusters of Euler solutions, the entire survey area was covered using a series of different sized moving windows.

Table 1. Parameters of the designed model

Model no.	x0	y0	z0	Side length
1	9750	9750	500	500
2	8500	8500	1000	1000
3	6000	6000	2000	2000
4	1000	1000	4000	4000
5	-7400	-7400	6400	8000

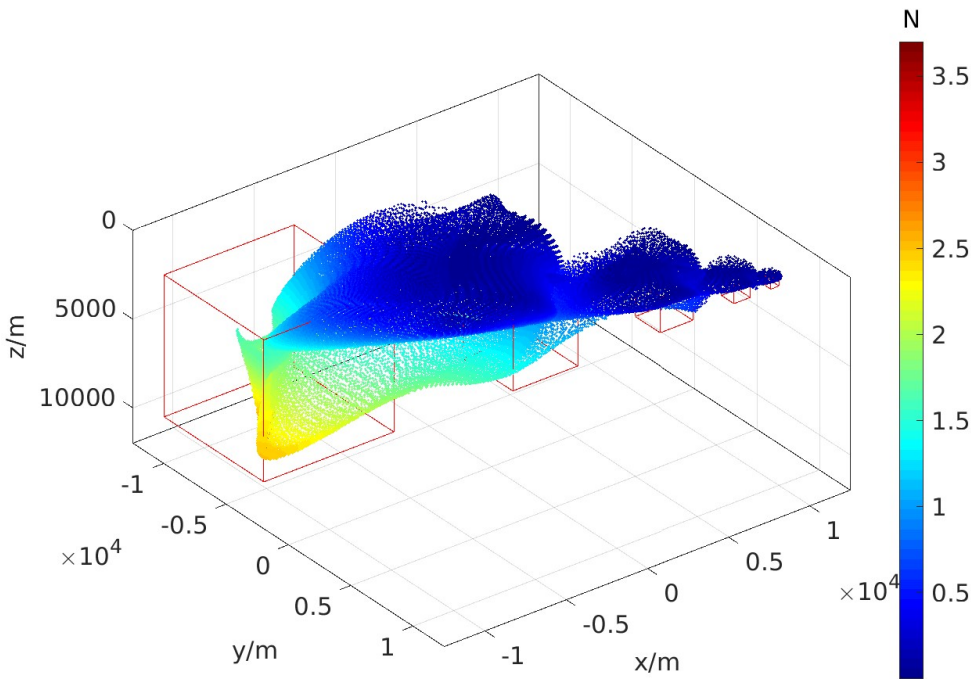


Fig. 1 Euler solutions

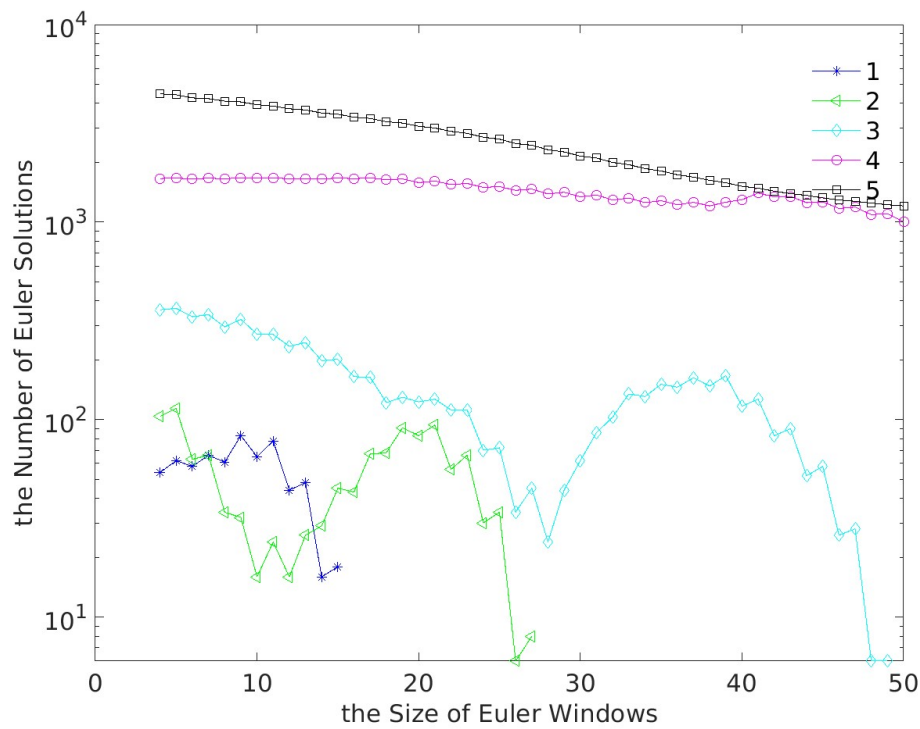


Fig. 2 Relationships between the sizes of the moving windows and the number of Euler solutions

Fig. 1 details the results of the tensor Euler deconvolution with a multiple coverage technique using different sized windows (ranging from 4 to 50) over gridded gravity data. It was found that when compared with the results of the tensor Euler deconvolution published by [18], the effects of the focusing levels of the Euler solutions were higher. In addition, there was a very tight cluster of Euler solutions observed in the vicinity of the No. 5 cube, which was related to the fact that the relatively large causative sources had made it difficult to accurately estimate using small-sized moving windows. Therefore, there were many spurious solutions observed with structural indices greater than two, especially for $z > 10,000$. The clusters for the other cubes were found to deviate from themselves and were disturbed by the No. 5 cube, in accordance with the equivalence principle.

The relationships between the sizes of the moving windows and the number of Euler solutions are detailed in Fig. 2. The Euler solutions were counted for each cube when the inside of the cube was 1.5 times larger than the designed cubes.

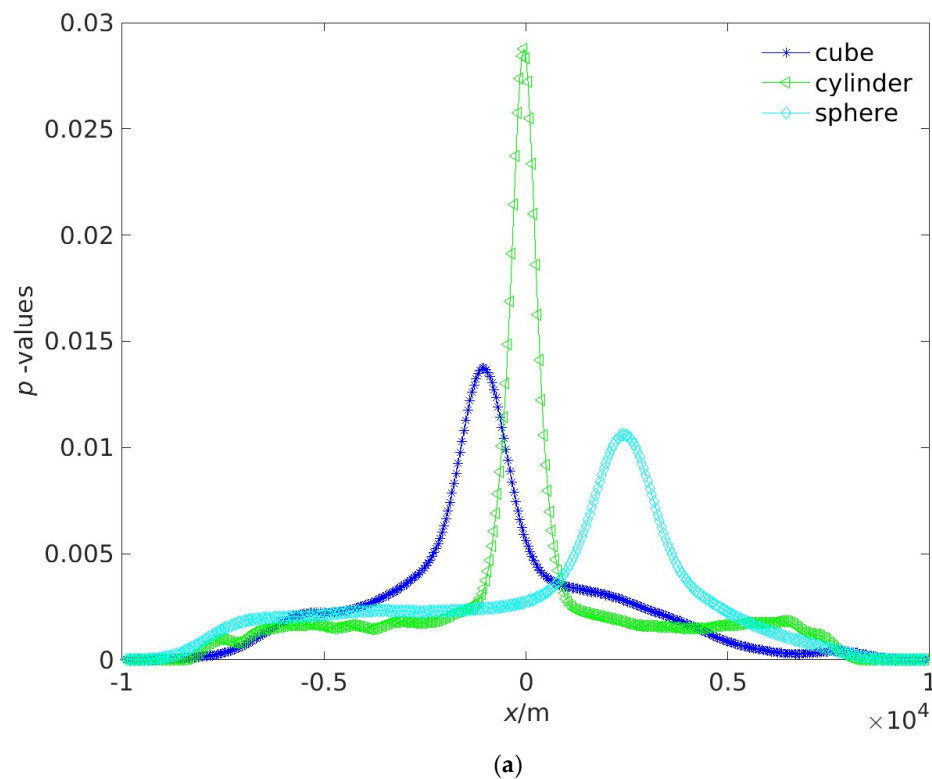
The relatively large causatives were not affected by the small-sized moving windows. However, this had easily led to the Euler solutions decreasing with the large structure indexes N . Curves 4 and 5 show that the number of Euler solutions of both the No. 4 and No. 5 had displayed similar trends toward the same level as the sizes of the moving windows increases. For example, Curve 1 indicated that when the size of the moving window was greater than 15, there was no solution for the smallest cube in the design model. This was essentially the same effect observed for the No. 2 cube in Curve 2. These two results were very unstable. As detailed in Fig. 2, Curve 3 of the No. 3 designed cube, which was an intermediate sized cube

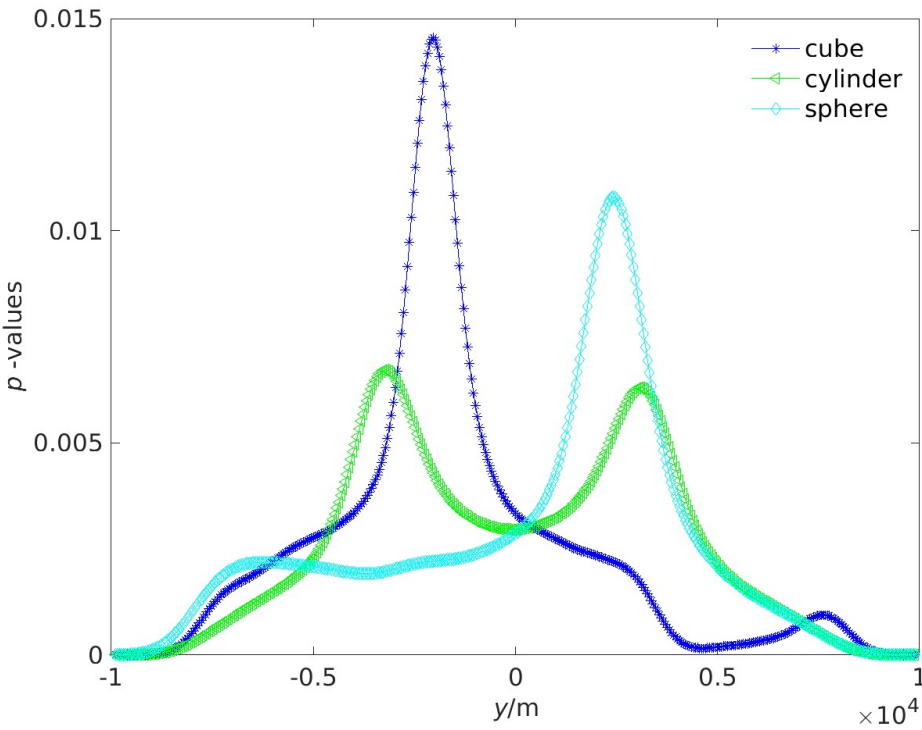
among all the cubes, had displayed relative stability. Therefore, by comprehensively considering the aforementioned curves, this study concluded that the selection of the sizes of the moving windows was difficult for the small causative bodies. However, the multiple coverage technique for Euler deconvolution had effectively formed tight clusters.

5. Model studies

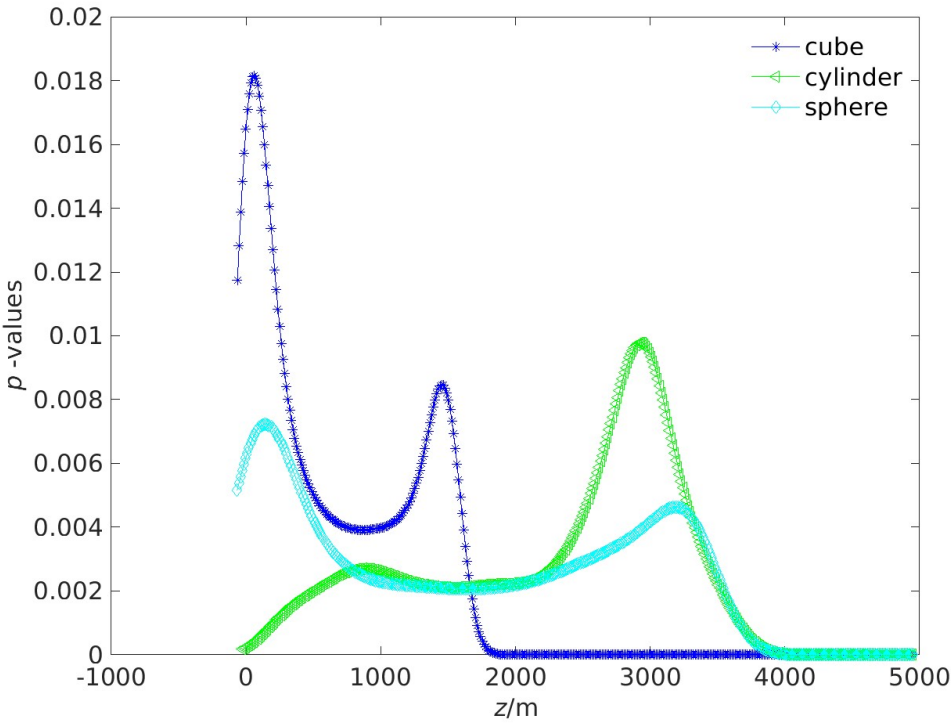
5.1 Verification of the validity of the KDE algorithm

On the premise of not rejecting any Euler deconvolution solutions, three simple models were designed in this study for the purpose of verifying the validity of the multivariate KDE algorithm. Analytical solutions were used to calculate the three simple models. The dimensions of the cubes were $1,000 \text{ m} \times 1,000 \text{ m} \times 1,000 \text{ m}$, with the centroid located at $(-1,000, -2,000, \text{ and } 1,500)$. The radii, lengths, and centroid of the horizontal cylinders were $1,000 \text{ m}$, $4,000 \text{ m}$, and $(0, 0, 2,500)$, respectively. The sphere radius was $1,000 \text{ m}$, with its centroid was located at $(2,500, 2,500, \text{ and } 3,500)$. There were three bodies with the same density contrast ($0.36 \times 10^3 \text{ kg/m}^3$) located over the background. The observation height was set as 25 m and the space-sampling interval was 100 m in both the x - and y -directions. The rectangular survey grid was divided into $40,000 = 200 \times 200$ observation points located on the Earth's surface. Then, the tensor Euler deconvolution algorithm and inverted FTG data contaminated by 3% noise were used for the x , y , z , and N with singular value decomposition.





(b)



(c)

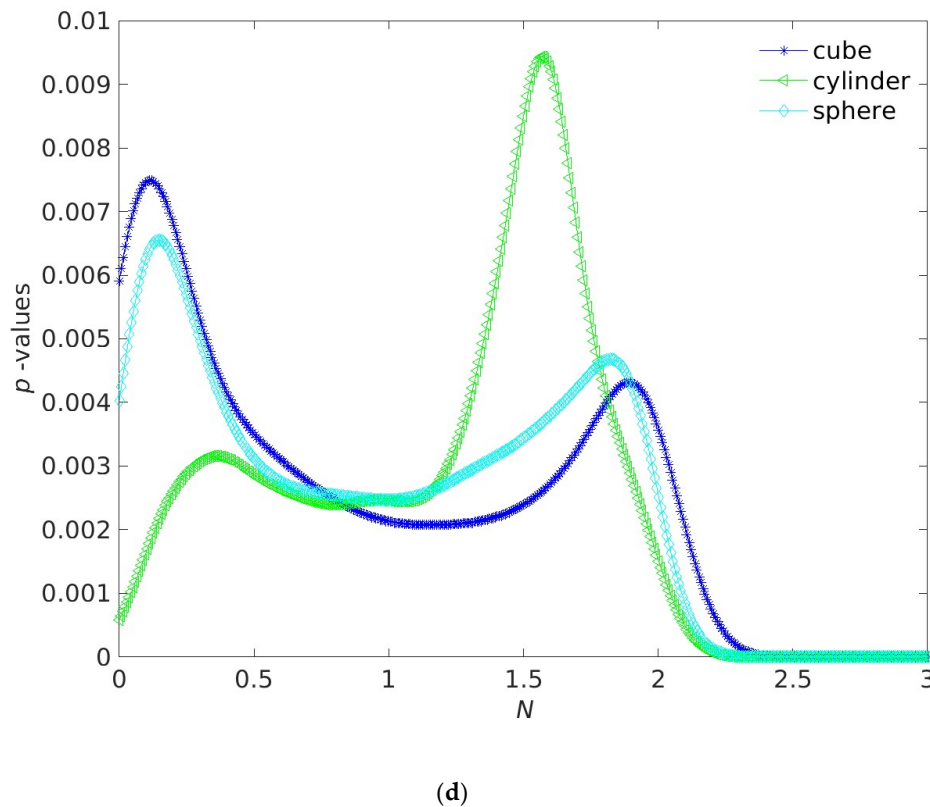


Fig. 3 Probability density curves obtained using 1-D KDE for: (a) Coordinate x; (b) Coordinate y; (c) Coordinate z; (d) Structure index N

The probability values of a 1-D subset (for example, x, y, z, and N) of the Euler solutions were obtained using a 1-D KDE method, as shown in Fig. 3. The kernel function in the KDE method was a Gaussian function, and its density function was a probability density estimation function. Further details can be referenced from Bowman and Azzalini (1997). Fig. 3a shows the results of the coordinate x of probability density peaks, which were determined to be equal to -1,000 m, 0 m, and 2,600 m, for the cube, cylinder, and sphere, respectively. Those three values were found to be in accordance with the designed models. Fig. 3b shows that there was a very good agreement between the coordinate y of the probability density peaks and this study's designed models of the cube and sphere. It was found that when compared with other methods, there were two equal peak values of the probability density at the top and bottom ends of the cylinder ($y = \pm 4,000\text{m}$). Fig. 3c details that there were two peaks in the probability density curve for each of the designed models. The first peak had corresponded to the shallow buried Euler solutions, which were considered to be spurious Euler solutions due to the instability of the Euler equation. The second peaks of three curves corresponded exactly to the theoretical values of the designed models. The same situation can also be observed in Fig. 3d.

The best locations and depths of the causative sources obtained from the optimum estimates had successfully verified the validity of the 1-D KDE algorithm. However, only single probability density peaks were found to correspond to the theoretical values. Therefore, in order to overcome these difficulties, this study used a 2-D KDE method to analyze the six subsets of the Euler solutions $\{x, y, z, N\}$. Due to the limitations in the length of this research report, only the results of the $\{x, y\}$ combination of the Euler solution datasets are presented, as shown in Fig. 4.

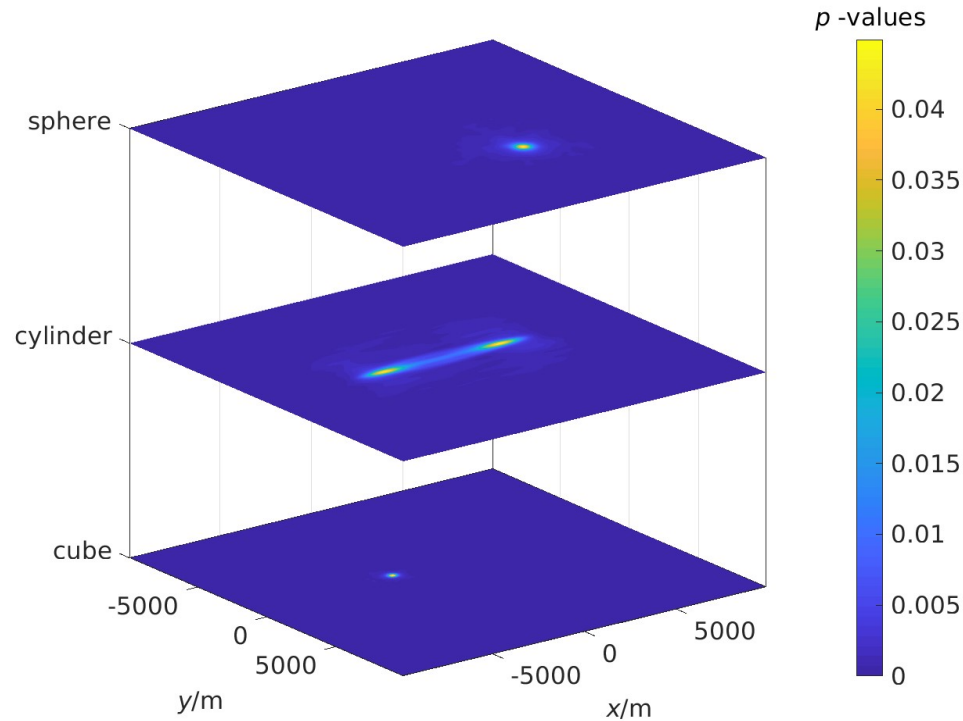
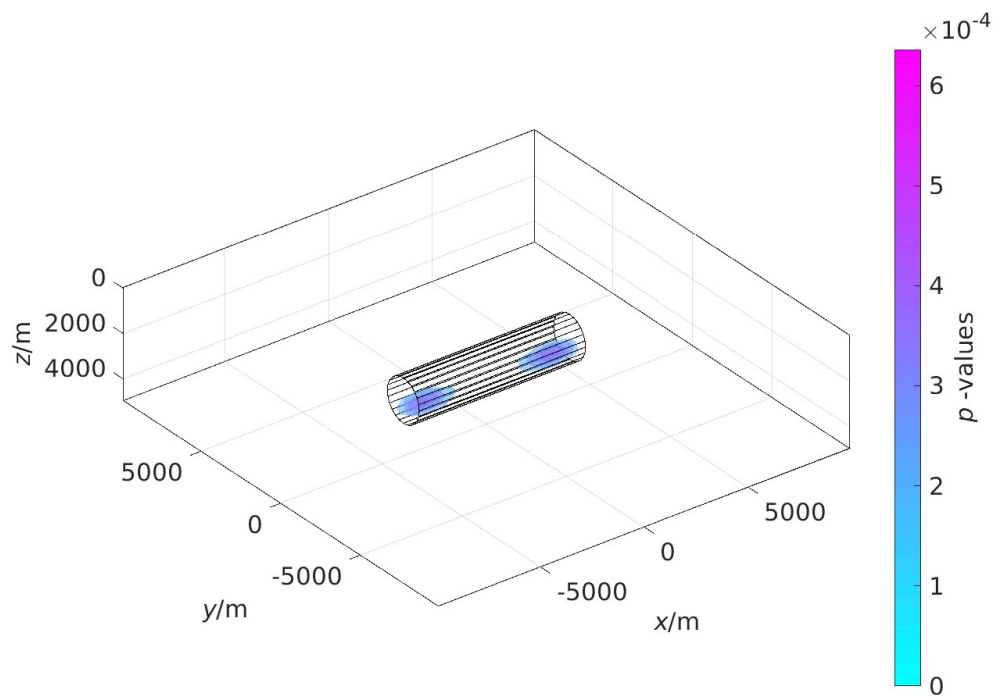


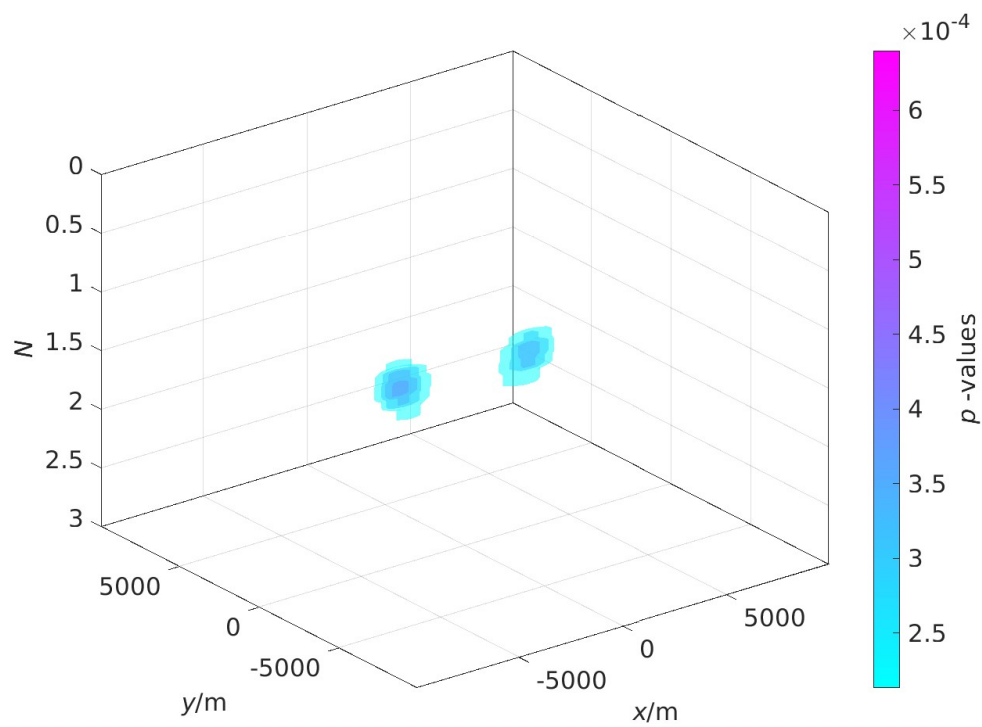
Fig. 4 Probability density images obtained using KDE for the $\{x, y\}$ Euler solution subsets

As shown in Fig. 4, the probability density of the cube and the sphere had only one peak, and there were two peaks in the probability density image corresponding to the ends of the cylinder. In regard to the other combinations, due to the limited length of this report, only the relevant results were attached. The probability density of the Euler solutions along two poles of the cylinder were characterized by a scattering distribution. In accordance with the equivalence principle, that type of phenomenon could also be produced by two adjacent cubes, indicating that ambiguities existed.

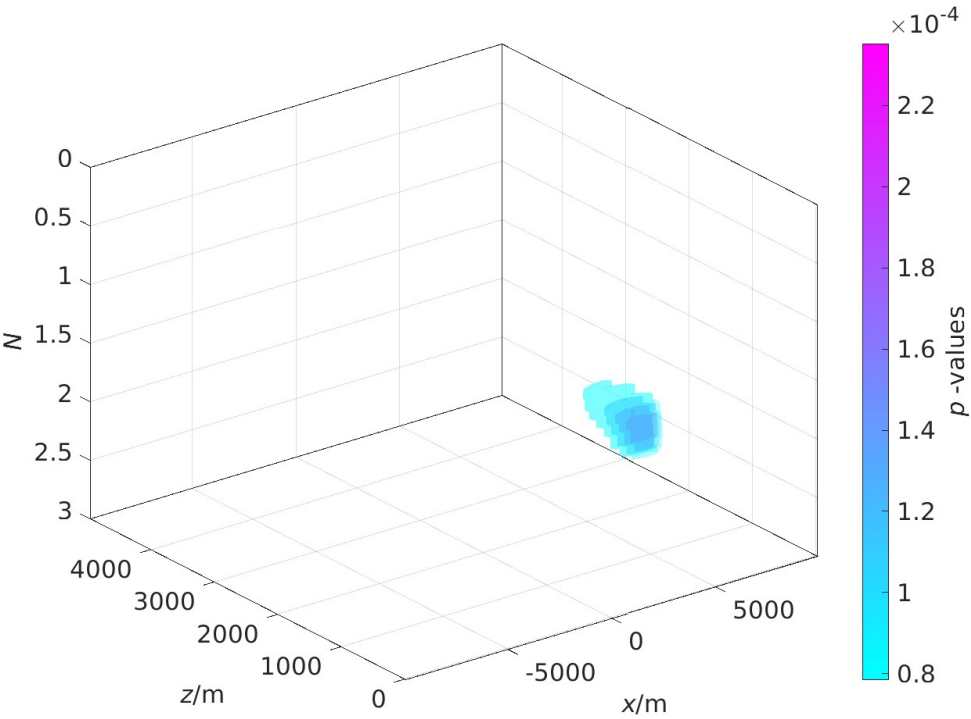
The locations and structural index were easily interpreted in this study using 2-D KDE. However, the 2-D KDE for the different combinations of Euler solution datasets had the drawback of producing too many results for the probability density distributions, making the data difficult to analyze. Furthermore, the visualization of the results using the 5-D data obtained by quaternion KDE for the entirety of the Euler solution datasets was found to be very challenging. Consequently, this study introduced 3-D KDE for the Euler solution subsets, such as $\{x, y, z\}$, $\{x, y, N\}$, $\{x, z, N\}$, and $\{y, z, N\}$. Then, there were a total of twelve results for the probability density distributions for the previously mentioned three designed models. In addition, for ease of presentation, this study only illustrated the results of the probability density distributions of the cylinder. For all of the experimental tests of the 3-D KDE in the remainder of this study, the significance level α was equal to 0.05 for three levels of the transparency color of the 3-D contours of the probability density data. The colors indicated the focusing levels of the clustered dataset of the Euler solutions. Further details are available from the research presented by [25,27].



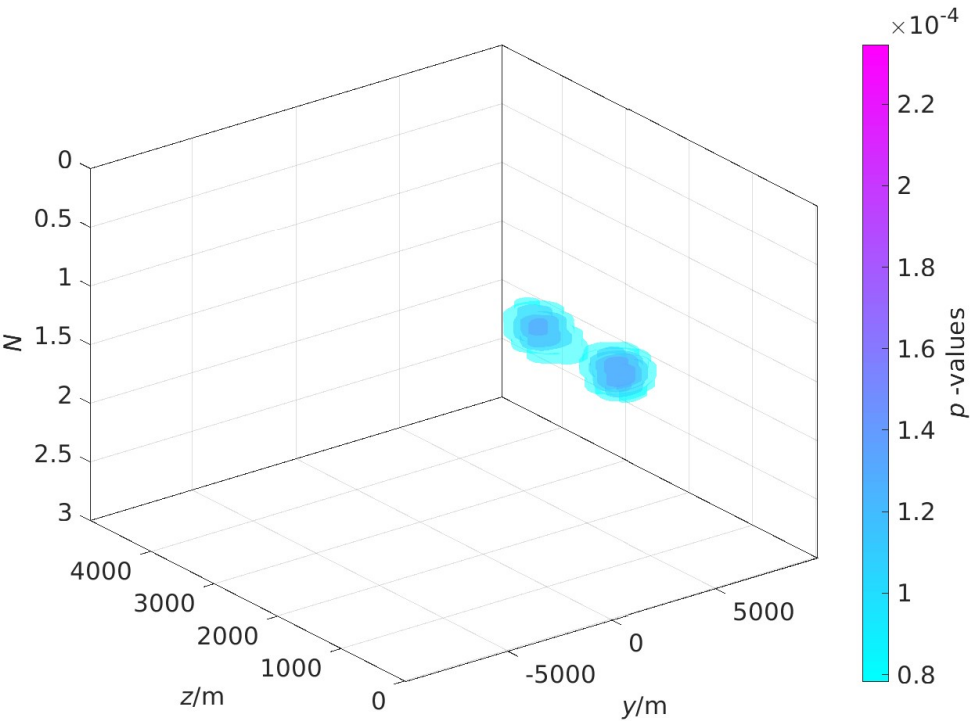
(a)



(b)



(c)



(d)

Fig. 5 Probability density iso-surfaces obtained using KDE for the 3-D data consisting of: (a) {x, y, z}; (b) {x, y, N}; (c) {x, z, N}; and (d) {y, z, N}

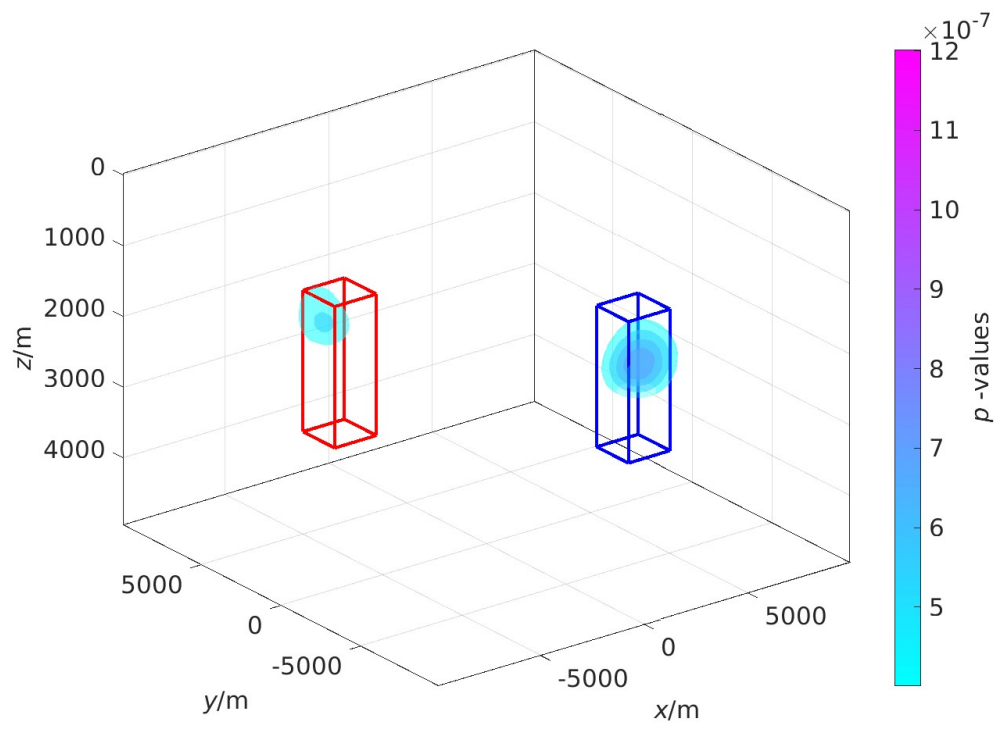
The bandwidth parameters of the 3-D KDE were observed to be more stable than those obtained with the 2-D KDEs, as shown in Table 2. Further details of the bandwidth parameters are available in the research presented by [25]. In this study’s experimental tests of the cylinder, the iso-surfaces for the probability density distributions obtained using 3-D KDE for all the subsets, including the y component, were found to have two cluster sources along the Y-axis direction, as detailed in Figs. 5a, 5b, and 5d. The above-mentioned three results provided mutual corroboration. Moreover, those results were found to be in accordance with Figs. 3 and 4. Therefore, by comprehensively considering the aforementioned figures, it was concluded that the final solution for the cylinder tests was a single causative body with a structural index between 1.3 and 2.4.

Table 2. Bandwidths for the 3-D KDE

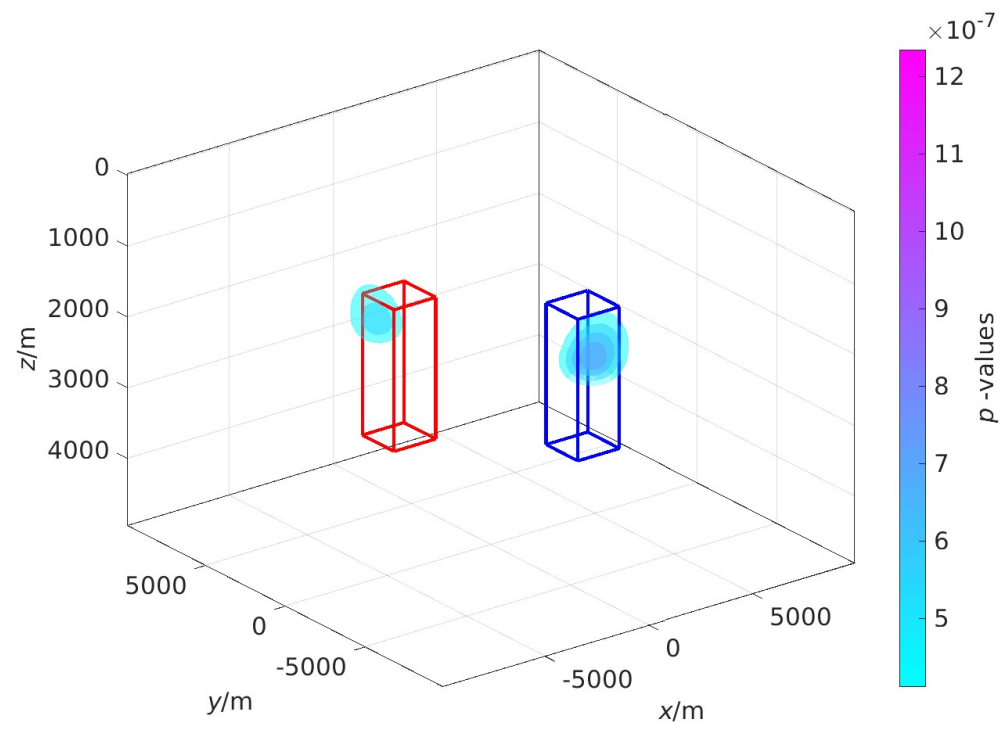
Subset	x	y	z	N
x, y, and z	163.13213	624.20116	97.51559	\
x, y, and N	163.13213	624.20116	\	0.07009
x, z, and N	163.13213	\	97.51559	0.07009
y, z, and N	\	624.20116	97.51559	0.07009

5.2 Influences of the adjacent causative bodies on the 3-D KDE for the Euler solution subset {x, y, z}

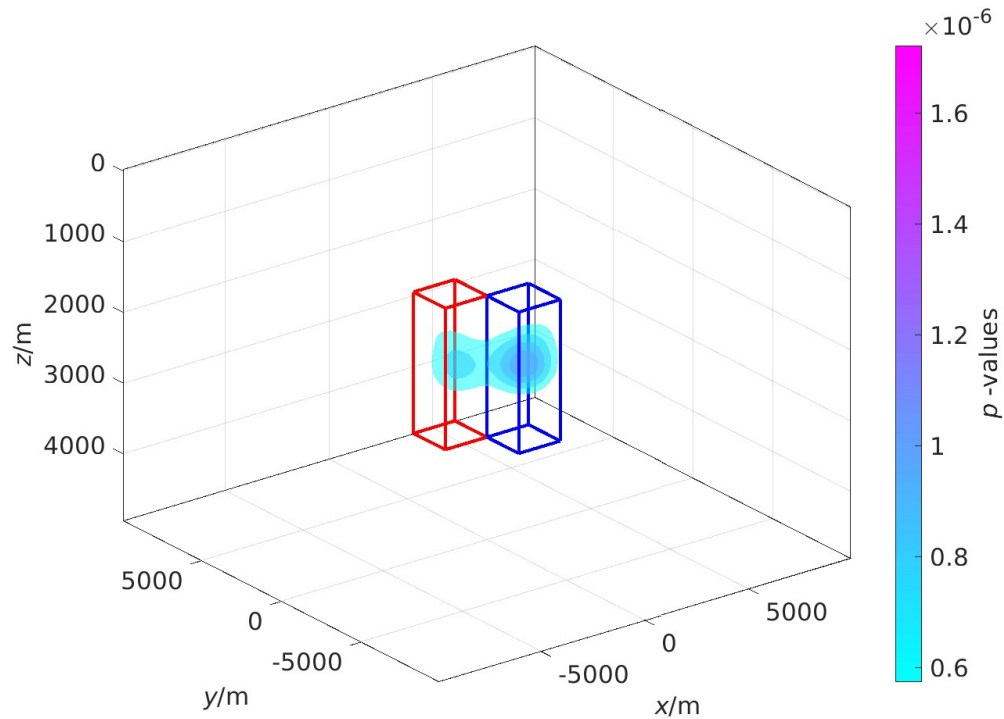
It should be pointed out that the results of the three subsets {x, y, N}, {x, z, N}, and {y, z, N} were easily confused with the adjacent causative bodies or similar causative bodies, as shown in Fig. 5. The interpretations of the Euler solution datasets using 3-D KDE relied mainly on the subset {x, y, z}, while making the assistance of the other subsets subsidiary. Therefore, this study chose to illustrate the results for subset {x, y, z} in the remainder of this research report. Fig. 6a details the forward modeling of the FTG for the designed models. There were cubic bodies with densities of $0.36 \times 103 \text{ kg/m}^3$ and $1.27 \times 103 \text{ kg/m}^3$ for the left and right, respectively, in which the centroids were (-4,000, 4,000, 2,500) and (-4,000, 4,000, 2,500) for the left and right, respectively. The size of each body was 2,000 m $\times$ 2,000 m $\times$ 2,000 m. The rectangular survey grid was divided into 40,000 = 200 $\times$ 200 observation points located on the Earth’s surface. The observation height was set as 25 m, and the sampling interval was 100 m in the x- and y-directions. Then, the two causative bodies were allowed to approach each other with L = 1,500 (moving step) in the x- and y- directions at the same time. In the current investigation, 3-D KDE was used to analyze the Euler solutions subset {x, y, z} for the FTG data contaminated by 3% noise.



(a)



(b)



(c)

Fig. 6 Probability density iso-surfaces obtained by the KDE for the 3-D data $\{x, y, z\}$ of: a) $L = 4,000$; b) $L = 2,500$; and c) $L = 1000$

Figs. 6a to 6c show that the two geological bodies with different gaps had been directly discriminated using 3-D KDE, and the depths and probability density distributions were in accordance with the designed model.

6. Case study completed in British Columbia (Canada)

The province of British Columbia (Canada) has been thoroughly explored for copper and gold porphyry deposits. However, the region is covered by a thick layer of sand and gravel left behind by glaciers. In the aforementioned case study, airborne gravity surveys were effectively used to identify potential kimberlite targets and assist geophysicists understand the geology of the British Columbia area. The survey blocks consisted of two parallelograms with two pairs of parallel sides. The first parallelogram measured approximately 386 km in length and 120 km in width. The second was 120 km in length and 60 km width and appended to the north, as illustrated in Fig. 7a [28].

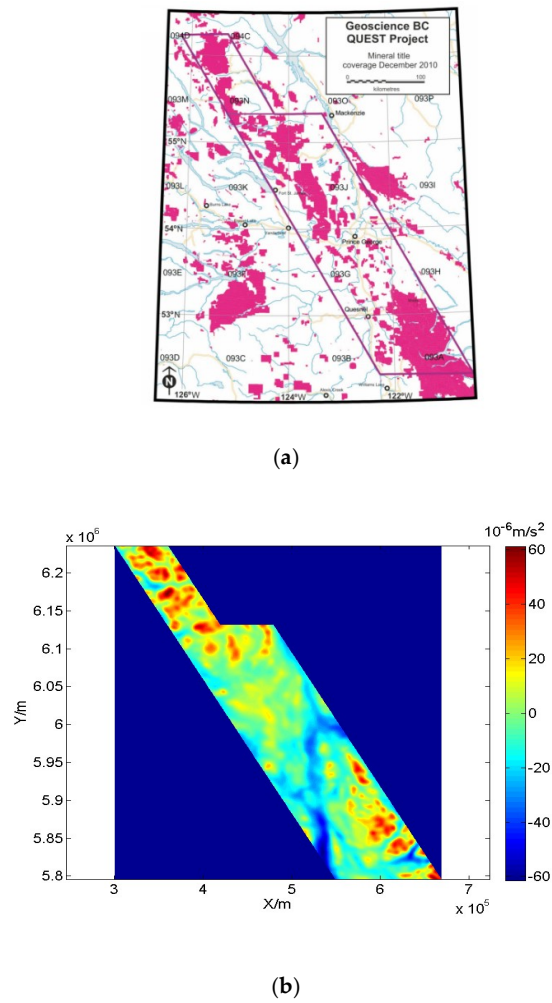
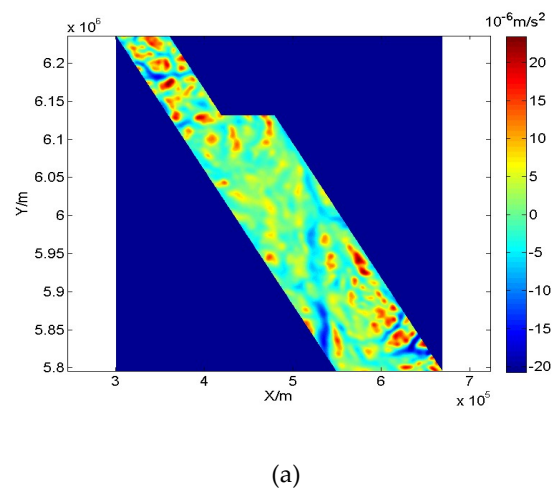
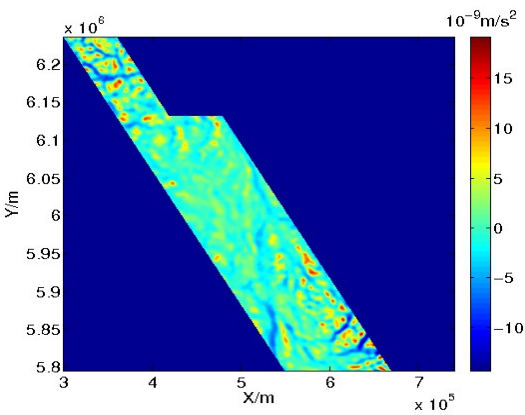


Fig. 7 (a) Map of the survey area [28]; (b) Gravity field

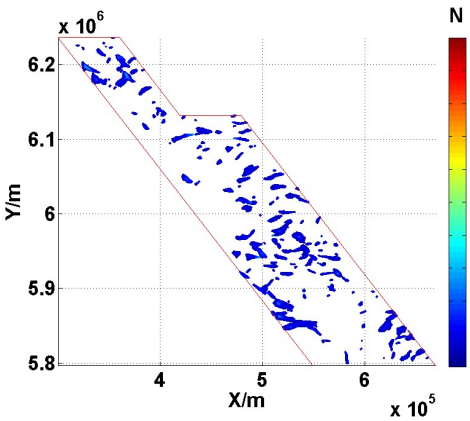




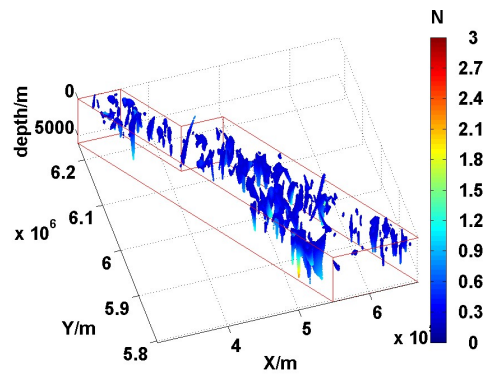
(b)

Fig. 8 (a) Local gravity anomaly; (b) Gzz component of the FTG

The measured heights ranged from 240 m to 2,850 m above the mean sea level, making it necessary to continue the irregular gravity anomaly upward to a horizontal plane. Fig. 8a details the Bouguer gravity anomaly. The gravity anomaly data were continued upward to 3,000 m and 6,000 m above the mean sea level in order to obtain a new gravity anomaly and the background field, respectively. Then, the background field from the new gravity anomaly was selected in order to obtain the local gravity anomaly. The FTG data were converted from the local gravity anomaly using a Fast Fourier Transform Method [29]. However, due to the space limitations of this report, only the Gzz component of the FTG was illustrated, as shown in Fig. 8b.



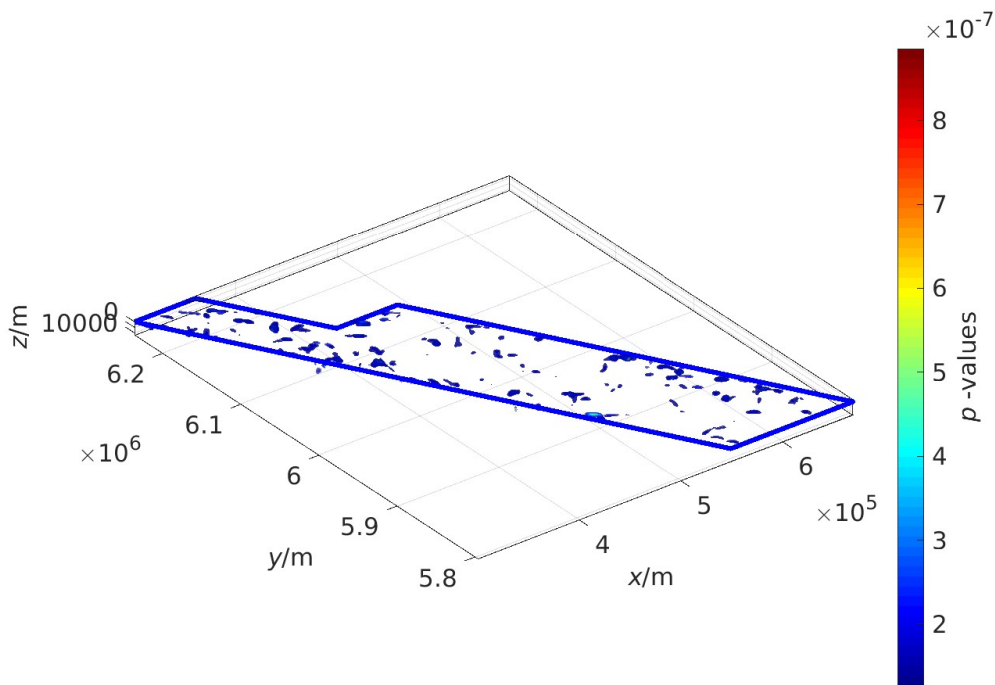
(a)



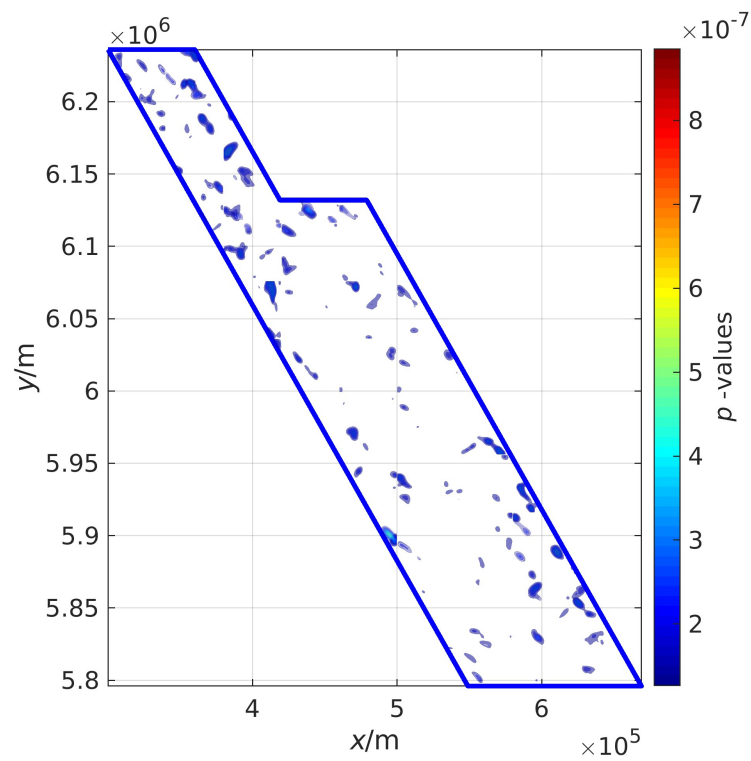
(b)

Fig. 9 (a) 2-D color spectrum of the Euler solutions; (b) 3-D color spectrum of the Euler solutions

In order to form tightly clustered datasets of the Euler solutions, the entire survey gravity map was covered with a series of different sized moving windows, which ranged in size from seven to 25. There were a total of 1,088,276 solutions generated using tensor Euler deconvolution for the FTG data before the solutions were filtered. The Euler solutions were illustrated with 2-D and 3-D color spectra, respectively, as shown in Fig. 9. It was only possible to roughly determine the number of causative sources, as shown in Fig. 9a. In particular, difficulties were encountered in discriminating the adjacent causative bodies (Fig. 9a), and also determining the depths of the causative bodies (Fig. 9b).



(a)



(b)

Fig. 10 Probability density iso-surfaces obtained using 3-D KDE for the{x, y, z}: **(a)** Perspective view; **(b)** Plan view

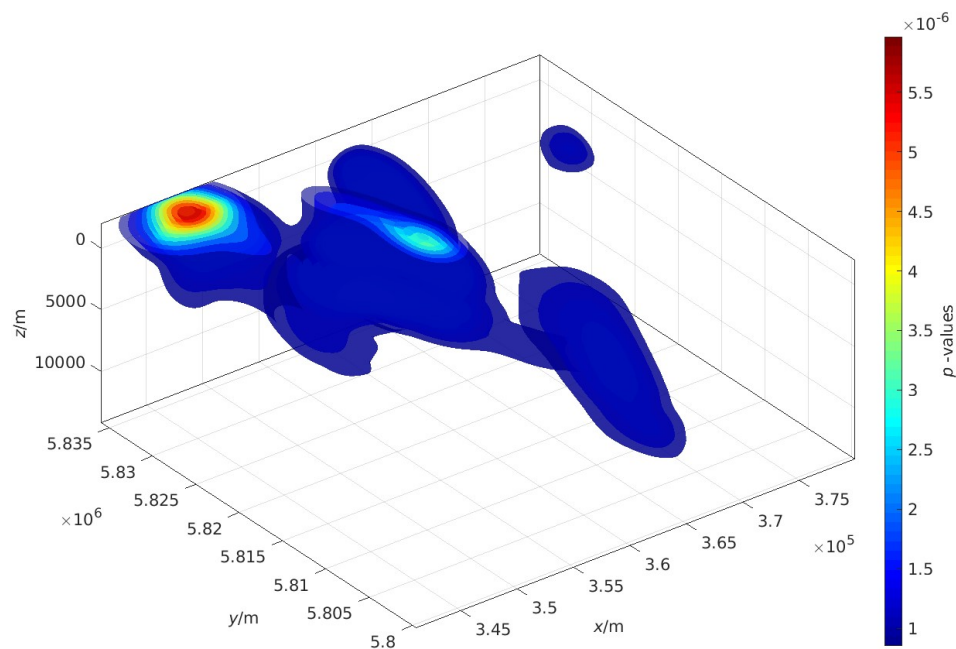


Fig. 11 Probability density iso-surfaces obtained using 3-D KDE for the $\{x, y, z\}$ in a specific part of the survey area

Fig. 10 details the probability density distributions obtained using the 3-D KDE for the Euler solutions subset $\{x, y, z\}$. It was found that when compared with the interpretations obtained using the color spectrum shown in Fig. 9, the proposed algorithm had successfully established more credible and meaningful geological models, as shown in Fig. 11. This study was able to easily determine the locations and the depths of the causative sources, as well as the weights among all the causative sources from the probability density distributions.

7. Conclusions

This study developed a new approach to discriminating uncorrelated Euler solutions from coherent solutions. The proposed method was based on the focusing levels indicated by probability densities generated using multivariate KDE. A novel multiple coverage technique applied using a series of different sized moving windows which forced the causative sources to form tighter clusters. A number of conclusions could be drawn by calculating one synthetic model. This study's conclusions were as follows:

- (1) The clusters generated by Euler deconvolution had easily deviated from each other when influenced by the equivalence principle.
- (2) It was observed that difficulties were encountered when choosing the most effective sizes for the moving windows for the smaller causative bodies.
- (3) The multiple coverage technique for the Euler deconvolution had effectively formed tighter clusters.

In the present study, the comparative analysis probability density curves and images were obtained using various dimensions of KDE for the different combinations of Euler solution datasets. The conclusions were as follows:

(1) Multiple probability density peaks existed for one geological body in the results obtained using 1-D and 2-D KDE. Therefore, it was difficult to determine which of the probability density peaks corresponded to the true values.

(2) For the 2-D KDE, it was found that there were too many results for the probability density distributions, which made it difficult to analyze the entirety of Euler solution datasets.

(3) The visualization of the results with the 5-D data obtained by 4-D KDE for the entirety of the Euler solution datasets was found to be difficult to achieve.

(4) In regard to the 3-D KDE, the results of the three subsets $\{x, y, N\}$, $\{x, z, N\}$, and $\{y, z, N\}$ were easily confused with the adjacent causative bodies or similar causative bodies. Therefore, the 3-D KDE for the Euler solutions subset $\{x, y, z\}$ of the synthetical model, and the real data from a survey conducted in British Columbia (Canada), were used to successfully establish more credible and meaningful geological models, when compared with the other three subsets.

The approach presented in this study was considered to have the potential to also be applied to gravity, magnetic, and full tensor gradient magnetic data.

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