

Theoretical analysis of three-dimensional seepage through the fracture with surrounding rock deformation

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Abstract: The formation of rock fractures in nature has a certain relationship with water seepage. In order to analyze the shape of rock fracture, we propose a rock deformation theoretical model in three-dimensional space considering with the condition of seepage water pressure, and establish the mass conservation equation, seepage equation, surrounding rock displacement equation, so as to deduce the rock deformation control equation under seepage pressure action. The numerical analysis of the nonlinear quadratic partial differential equation obtains the fracture deformation distribution in the example, and verifies with the calculated fracture morphology. And we further use this principle to analyze the fracture morphology in nature and prove the rationality of the theory.

Keywords: fracture; theoretical model; seepage pressure; rock deformation

1. Introduction

There are many fractures in nature, many of them are the products of geological structure, and some of them are results of groundwater seepage[1-3]. A lot of scholars thought the deformation of rock fracture groundwater seepage to be neglectible for the low seepage pressure. And the influence on seepage is very small. Because the deformation action under seepage is ignored, the seepage problem is only analyzed from the angle of liquid flow[4-8]. Many of these studies take into account the fluid itself or the roughness of surrounding rock and lack of analysis of deformation of surrounding rock. Some scholars also consider to homogenize the fissures, to average the seepage velocity or seepage aperture[9-12]. Although it can approximate to solve the description of rock fracture permeability, it can not reflect the deformation of different positions of the fractures seepage. There is a certain distance from the actual situation. In rock fracture there should be interaction between the liquid phase and the solid phase in the seepage space[13-15]. This is a typical fluid-solid coupling motion problem. Liquid phase permeation will be accompanied by different seepage pressure, sometimes this seepage pressure value is very large, which will cause deformation and displacement of surrounding rock mass[16]. For hard rock, this deformation and displacement may be small and the influence is not very large; but for some weak rocks, under the action of high seepage pressure, this deformation and displacement may be very large and the influence can not be ignored. Therefore, it is necessary to analyze the deformation and displacement of surrounding rock under the action of seepage pressure, so as to obtain its influence on permeability[17-20]. There are many numerical simulation analyses considering the deformation of permeable surrounding rock, but the experimental results are few with measurements of the surrounding rock deformations[21-27]. Therefore, many of the results of fluid mechanics research lack

the theoretical analysis of deformation factors of surrounding rock.

The objective of this paper is to establish a nonlinear quadratic partial differential equation of time and space considering the deformation of surrounding rock under the action of seepage pressure in three dimensional infinite space. Under the action of elastic deformation range of seepage pressure, the deformation and displacement of surrounding rock occur, and its form is a circular cone shape. It is compared with the existing calculation simulation, which shows the rationality of the theory. Natural circular cone shape fractures and caverns formations are analyzed based on the theory.

2. Theoretical model

For theoretical analysis of three-dimensional seepage flow through fracture under surrounding rock deformation the model is depicted as shown in fig.1. The fracture is cylinder. fracture radius is r_0 . Surrounding rock boundary can be infinite comparing with the dimension of the fracture. Because of the water injection process the water always fills the fracture space. The density of fluid is ρ . At the time of t_1 the fluid is filling the boundary of flat shape with the inlet speed of u . Outlet

speed is $u + \frac{\partial u}{\partial x}$. Inlet pressure is P and outlet pressure is $P + \frac{\partial P}{\partial x} dx$. At time of $t_1 + dt$ the seepage boundary changes for the rock displacement u and fracture diameter is r . The surrounding rock boundary is caused by the fluid pressure P . Rock deformation is completely elastic. The elastic modulus is E and Poisson's ratio is ν .

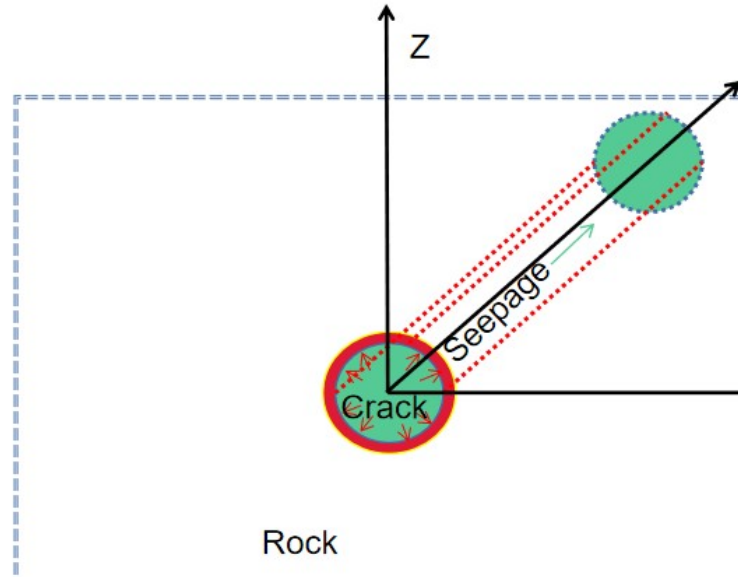


Fig.1 Theoretical model for fracture seepage in infinite space

3. Theoretical analysis

As shown in the fig.1 the inlet mass of fluid can be written as:

$$\rho u \pi r_0^2 dt \quad (1)$$

For outlet mass is

$$\rho \left(u + \frac{\partial u}{\partial x} dx \right) \pi r_0^2 dt \quad (2)$$

So mass change is

$$\rho \left(u + \frac{\partial u}{\partial x} dx \right) \pi r_0^2 dt - \rho u \pi r_0^2 dt = \rho \pi r_0^2 \frac{\partial u}{\partial x} dx dt \quad (3)$$

And for the incompressible water the change mass can also be

$$\rho \pi (r^2 - r_0^2) dx \quad (4)$$

Combining with eq.3 and eq.4, the formula can be

$$\frac{\partial u}{\partial x} = \frac{(r^2 - r_0^2)}{r_0^2} \frac{1}{dt} \quad (5)$$

Differential equation of Euler fluid can be written as:

$$\frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{\partial P}{\partial x} \quad (6)$$

According to elastic mechanics the rock deformation can be calculated as

$$\frac{dr}{dp} = \frac{1+\nu}{E} r_0 \quad (7)$$

The partial differential equation about the dependent variable r and the independent variable x, t can be obtained by the analysis of formula 5,6,7 as:

$$\frac{E}{\rho(1+\nu)r_0} \frac{\partial^2 r}{\partial x^2} = \frac{r+r_0}{r_0^2} \frac{\partial^2 r}{\partial t^2} + \frac{1}{r_0^2} \frac{\partial r}{\partial t} \frac{\partial r}{\partial t} \quad (8)$$

Eq.8 is the control equation of surrounding rock deformation under seepage pressure through three dimensional rock fracture. Eq.8 is a second order nonlinear partial differential equation which cannot be deduced for getting the analytic solution. But the approximate solution can be obtained by numerical analysis method.

4. The example and discussions

Because it is difficult to obtain the analytical solution in formula (8), we can use numerical solutions to analyze the specific problem. According to the spatial problem, the specific fracture seepage parameters are set as shown in table 1, and the pressure set at the water injection port is constant to 1 MPa at the position of $x=0$. It can be obtained by numerical simulation that the surrounding rock is deformed under the action of seepage pressure, and half of the fracture radius deformation distribution of seepage fracture is shown in fig .2. The red part is the deformation which is triangle and in three dimensional space the fracture shape is like truncated cone. That suggests in theory rock fractures figures are similar truncated cone patterns. At the inlet position the r is $5.005 \times 10^{-5} \text{m}$ and is about 10 percents of the r_0 . For the soft rock as the elastic modulus decreases, the r will be larger. So the seepage ability for fracture will be increased. As the seepage pressure increases, the radii of truncated cone cone changes to larger values. When the seepage pressure exceeds the strength, the surrounding rock will be damaged. The inlet position radius will be expanded larger than the outlet position. The seepage pressure can also be simulated along seepage fracture extension direction. For the constant inlet pressure the seepage velocity is

constant. The eq.8 can also be used to solve the changing pressure with the time. For the eq.8 is based on the elastic mechanics, the deformation can be linear superposition as the inlet seepage pressure increases. The seepage channel also changes after the displacement of the surrounding rock, so the permeability of the fissure also changes. In this example, the surrounding rock has undergone a radius change of about 10%, which indicates that its permeability is greatly increased. And with the increase of seepage pressure, its variable is further increased, which can not be ignored in the actual permeability calculation. The surrounding is clay and its elastic modulus is only 1-200 MPa. The surrounding displacement will reach the millimeter level, which is multiplied for the original fracture aperture, and the change of its permeability can not be ignored.

Table 1 Example parameters of seepage through the fracture

r_0 (mm)	l (mm)	ρ (g/mm ³)	E (Pa)	μ
0.5	1800	1	2.00×10^{10}	0.20

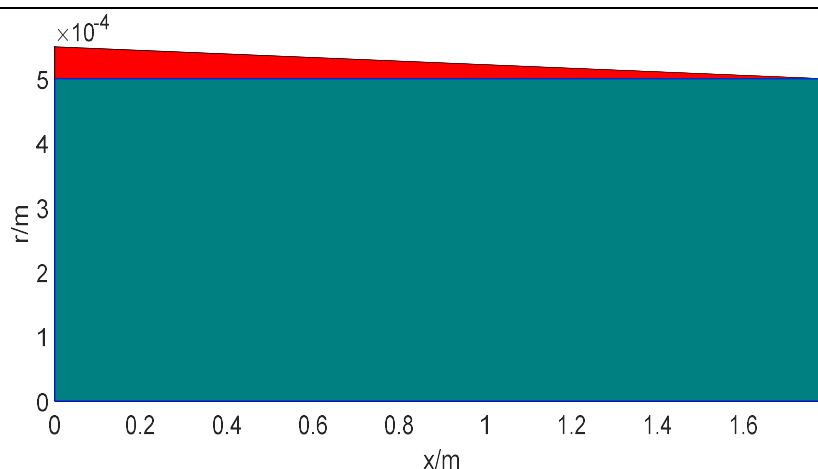


Fig.2 Half of the fracture radius deformation distribution of seepage fracture

Qiu Z. et al. (2019). has calculated the fracture interference effects on fracture geometry for wellbore strengthening[22]. The width of fracture along the fracture length direction has been analyzed as shown in fig.3. The width distribution has the similar shape as like the circle cone profile. That suggests the fracture under the seepage pressure has the cone shape pattern. Zhilong Lian, et al.(2008) studied the fracture formation and fracture morphology under the action of hydraulic fracturing[28]. The distribution morphology of fracture width along the fracture length direction is shown in fig.4. The fracture also shows a similar morphology with our theoretical

analysis results under the action of hydraulic support, which indicates that the fracture distribution of rock under the action of permeable water pressure has a similar cone geometry. Mohammadnejad(2009) depicted corresponding fracture opening profiles, showing that as the injection rate increases, both the length and the width of the fracture increase as well. And the fracture widths profiles were the similar with the cone shape.

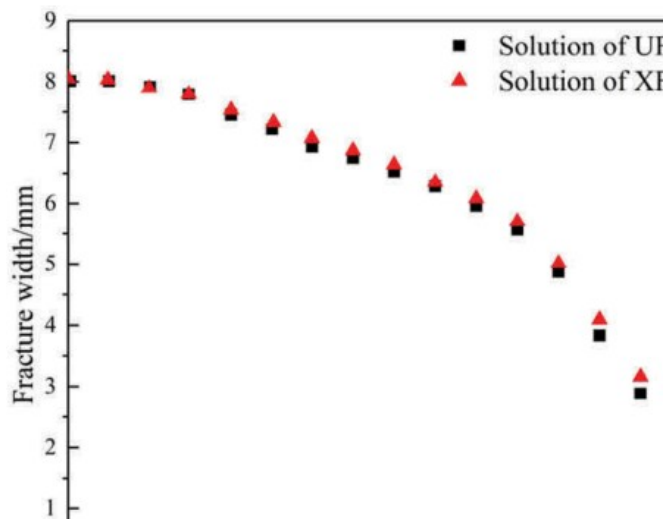


Fig.3 Fracture width along the fracture length direction(Qiu Z. et al., 2019)

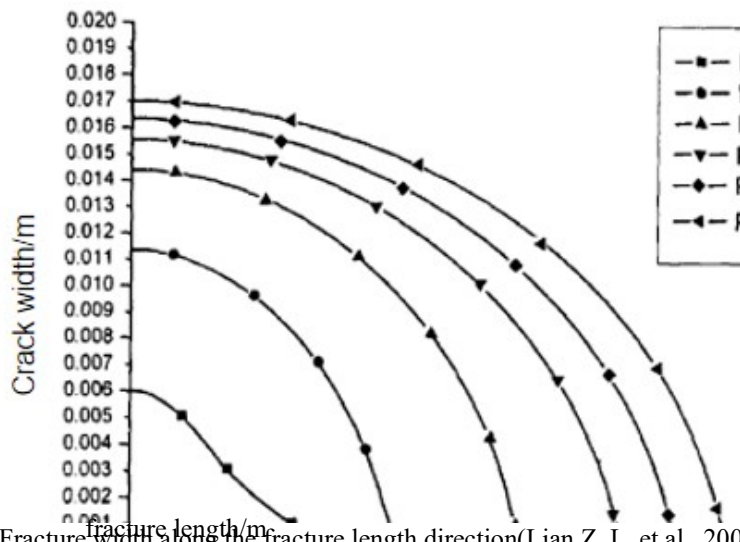


Fig.4 Fracture width along the fracture length direction(Lian Z. L. et al., 2008)

For the natural fractures or caverns are like truncated cone shape which radii are larger at the earth surface and smaller underground. Volcanic formation has a similar theoretical basis to the pressure formed by the rising flow of lava at the bottom. Based the theory the lava flow formed a volcanic form with large depth deformation and small surface deformation in the vertical direction.

At the inlet the surrounding rock bears the larger pressure than at the outlet. The displacement and damage are also more serious at the inlet than at the outlet. Under the action of pressure surrounding rock will occur elastic deformation, and the pressure is more general to break through the strength parameters of detachment. The above geological phenomena in nature indicate that larger pore sizes will be formed in the fluid injection position, such as the floor drain and volcanoes. The above manifestations show that the theory is reasonable and requires the introduction of plasticity and fracture theory to further analyze the formation processed of the above geological phenomena.

5. Conclusions

In this paper, the governing equations of the surrounding rock deformation under the condition of seepage flow are established, and the deformation form of the surrounding rock is analyzed by numerical calculation. It is concluded that the seepage fracture has the shape of a circular platform. In the existing fractures in nature, the seepage pressure has formed a lot of fracture forms similar to the circular cone, which can be established by this theory the elastic mechanical basis of the state, which provides supports for the development of plasticity and damage. We can study fractures based on this theory for soft rock deformation under high seepage pressure.

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Declaration of Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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