Measuring co-axial hole size of finite-size metallic disk based on a dual-constraint integration feature using multi-frequency eddy current testing

Ruochen Huang, Mingyang Lu*, Xiaohong He, Anthony Peyton, Wuliang Yin*

Abstract — This paper presents a new approach of eddy current methods for determining the size of the co-axial hole in the metallic circular disk. In recent decades, for the air-cored sensor probe, the impedance change due to the presence of an infinite metal plate can be calculated by the Dodd-Deeds model. However, in practical measurements, the sample cannot match with the condition required - ‘infinite’, thus the Dodd-Deeds model could not be applied to the disk with finite size and certainly not a co-axial hole in the center. In this paper, a dual-constraint analytical method is proposed. That is, the upper and lower limits of the integration are substituted with specific values instead of the original 0 and ∞. Besides, it is found that, once the outer radius of the disk is fixed (i.e. the lower limit of integration is fixed), the upper limit reduces linearly as the size of the coaxial hole increases. Both the FEM simulation and experiments have been carried out to validate this method. The radius of the hole can be estimated based on the dual-constraint integration feature.

Index Terms— Hole size measurement, finite-size metallic disk, eddy current testing, non-destructive testing.

I. INTRODUCTION

Non-destructive testing techniques have been applied in the fields of aerospace [1-3], rail transport [4-5], and pipeline testing [6-7]. Due to its advantage of high sensitivity and strong adaptability to the specimen, eddy current testing is widely used for the thickness measurement, liquid level measurement, and defect detection [8-12]. In the eddy current testing, an alternating current is injected into the excitation coil to generate an alternating magnetic field, then the eddy current is induced in the conductive samples. Thus, the receiving coil receives the signal contributed by both the magnetic field from the transmitter and that reflected from the sample.

With the rapid development of computational platforms, the electromagnetic problems can be calculated by numerical analysis algorithms, particularly the finite element method (FEM) [3, 13] and the boundary element method (BEM) [14]. FEM is suitable for samples with anisotropic materials but time-consuming. Ye et al designed an eddy current (EC) probe array with rotating exciting currents for the defect inspection. Besides, the numerical method plays an essential role to validate that the designed probe can detect defects in different layers [15-16].

In recent decades, the Dodd-Deeds analytical method [17-18] has been commonly used for evaluating various eddy current problems, which is more efficient than the numerical approach like FEM. Moulder et al proposed a reliable method to predict the conductivity of the sample and thickness of the substrate metal based on the Dodd-Deeds model [9]. Luloff et al examined Desjardins et al’s model of a non-coaxial sensor probe above two separated sample plates by utilizing the Dodd-Deeds model. It was found that both methods are matched with the experiments under the low frequency [19]. In [20], by combining the analytical solution and FEM simulations, the ferrite fraction can be inferred with the error typically less than 8%. Moreover, the measurement accuracy also suffers from the effect of the lift-off distance of the sensor. Yin et al proposed a lift-off immune method to extract the thickness profiles of the samples under different lift-off distance [21]. By incorporating the non-ideal behavior of the eddy current sensor, the model-based inversion method proposed in [22] proved its reliability in terms of lift-off elimination. Various lift-off compensation algorithms [23-25] have been proposed to eliminate the influence of the lift-off variation in order to predict the material properties more accurately. However, the fundamental formulation of mentioned approaches, i.e. the Dodd-Deeds formulation, cannot be applied for the tested sample with finite dimension, or even with a co-axial hole.

In this paper, to explore the analytical formulation for the finite-size disk sample with a co-axial hole, a dual-constraint integration feature has been found in the Dodd-Deeds model. By introducing two boundary limits, the inductance change due to the presence of the circular sample plate with a co-axial hole can be calculated. In the following sections, the mathematical formulation is presented and validated by the FEM simulation. Furthermore, measurements of the disk sample different sizes

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of the co-axial hole have been carried out. Results show that the size of the co-axial hole can be predicted accurately by utilising the proposed dual-constraint integration feature.

II. DUAL-CONSTRAINT INTEGRATION FEATURE

For the EC testing, the impedance change has been acted as a key indicator for the measurement of materials. The change of the impedance due to the sample plate with different kinds of material properties can be computed with various methods. For the air-cored EC sensor, the Dodd-Deeds method has provided the general analytical solution for the circular sensing coil located above the half-space sample plate. Since many test pieces cannot be treated as the infinite sample due to its finite planar size, our modified analytical solution by introducing an initial integration point \((\alpha r_s)\) has been proposed previously to tackle this problem [26].

From our previous research, as shown in Fig. 1, the vector potential generated by the excitation coil placed above the sample plate with finite dimensions can be derived as

\[
\mathbf{A}(r, z) = \frac{\mu_0 N}{2} \frac{1}{\alpha} \int_{D_0}^r |\Omega(\alpha)| P(\alpha) K(r, z, \alpha) \phi(\alpha) \, d\alpha
\]

where

\[
\phi(\alpha) = \frac{\alpha_1 + \mu_0 \alpha_1 (\alpha_1 - \mu_0) e^{-\alpha_2 D_0} - \alpha_1 + \mu_1 \alpha_1 (\alpha_1 - \mu_1) e^{-\alpha_2 D_0}}{\alpha_1 + \mu_0 \alpha_1 (\alpha_1 - \mu_0) e^{-\alpha_2 D_0} - \alpha_1 + \mu_1 \alpha_1 (\alpha_1 - \mu_1) e^{-\alpha_2 D_0}}
\]

\[
K(r, z, \alpha) = 2 - e^{-\alpha (z - l_e_2)} - e^{-\alpha (z - l_e_1)} + e^{-\alpha z_1} - e^{-\alpha z_2}
\]

\[
P(\alpha) = \int_{\alpha_{l_e_1}}^{\alpha_{l_e_2}} \tau J_1(\tau) \, d\tau
\]

Here, \(I\) denotes the excitation current, \(N\) denotes the turns of the excitation coil, \(I_1\) denotes the first-order Bessel function of the first kind, \(D_0\) denotes the sample thickness, \(\mu_0\) and \(\mu_1\) denote the vacuum magnetic permeability and the relative permeability of the sample plate; \(\sigma\) denotes the electrical conductivity of the sample plate, \(\omega\) denotes the excitation frequency, \(l_{e_1}\) and \(l_{e_2}\) denote the lower and upper height of the excitation coil, \(r_{e_1}\) and \(r_{e_2}\) denote the inner and outer radii of the excitation coil.

As can be seen from equation (2) and (3), \(\alpha_1\) is a frequency-dependent term, the frequency-phase feature of the inductance change due to the presence of the sample plate is mainly dominant by the phase term, \(\phi(\alpha)\), which includes sample thickness and conductivity. Besides, other parts are mainly related to the coil geometry and independent of the properties of the sample plate. According to the differential equation in [2], the magnetic vector potential \(\mathbf{A}\) induced in the conductor satisfies

\[
\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{1}{r} \frac{\partial \mathbf{A}}{\partial r} + \frac{\partial^2 \mathbf{A}}{\partial z^2} - \frac{A}{r^2} + \omega^2 \mu_1 \sigma_1 \mathbf{A} - j\omega \mu_1 \sigma_1 \mathbf{A} = 0
\]

\[
A = R(r)Z(z)
\]

Assume that \(Z(z)\) satisfies with the condition shown in equation (8),

\[
\frac{1}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} = \alpha^2 - \omega^2 \mu_1 \sigma_1 + j\omega \mu_1 \sigma_1
\]

Then combine equations (6) and (7) with equation (8),

\[
\frac{1}{R(r)} \frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r R(r)} \frac{\partial R(r)}{\partial r} + \alpha^2 - \frac{1}{r^2} = 0
\]

Since \(R(r)\) is a function which is related to the first-order Bessel function of the first kind, equation (9) can be expressed as

\[
(\alpha r)^2 + \frac{\alpha}{\phi(\alpha)} \left(2(I_0(\alpha r) - J_2(\alpha r)) - \alpha (3I_1(\alpha r) + J_3(\alpha r))\right) = 1
\]

As can be seen from equation (10), the product of \(\alpha\) and \(r\) is the solution of the equation, in consequence, the integration domain of \(\alpha\) is inversely proportional to the sample radius, then, the integration domain can be referred to the sample radius.

As shown in Fig. 2, for a metallic plate with finite radius, the lower limit is \(\alpha_{r_e}\) which corresponds to the position \(r = r_e\) while the upper limit is \(\infty\) which corresponds to the center of the plate \(r = 0\). Similarly, for the metallic plate with a hole in the center, \(r\) is in the range of \(r_{i_1}\) and \(r_{o_1}\), the integration domain should be from \(\alpha_{r_o}\) to \(\alpha_{r_i}\).

![Fig. 1. Illustration of the integrational path for the modified analytical solution](image-url)

![Fig. 2. EC sensor above the circular sample plate with a finite dimension (a) without the co-axial hole (b) with the co-axial hole](image-url)
Thus, as shown in Fig. 3, the magnetic vector potential for
the excitation coil above the plate with a co-axial hole in the center is

\[
\mathbf{A}(r, z) = \frac{\mu_0 N_e}{2} \int_{\alpha_{r_0}}^{\alpha_{r_1}} \int_{\alpha_{p_0}}^{\alpha_{p_1}} \frac{\rho (\alpha)}{\alpha^2} K(r, z, \alpha) \phi(\alpha) d\alpha
\]

Then the induced voltage on the receiver and the impedance change due to the sample plate are calculated as,

\[
V = j \omega N_p \int_S \mathbf{A} \cdot ds = \frac{j2\pi \omega N_p}{(l_{p1} - l_{p2})^2 (r_{p1} - r_{p2})^2} \int_{l_{p1}}^{l_{p2}} \int_{r_{p1}}^{r_{p2}} r \mathbf{A}(r, z) r dr dz
\]

\[
\Delta Z(\omega) = \Delta L = \frac{\Delta \mathbf{A}}{j \omega} \int_{\alpha_{r_0}}^{\alpha_{r_1}} \int_{\alpha_{p_0}}^{\alpha_{p_1}} \frac{P^2(\alpha)}{\alpha^6} e^{-\alpha(2l_1 + l_2 - l_1 + \gamma)}(1 - e^{-2\alpha(l_2 - l_1)}) \phi(\alpha) d\alpha
\]

where: x can be i or o (the inner or outer radius of the sample plate).

The inductance change due to the sample plate should be

\[
\Delta L(\omega) = \Delta Z = \frac{\Delta \mathbf{A}}{j \omega} \int_{\alpha_{r_0}}^{\alpha_{r_1}} \int_{\alpha_{p_0}}^{\alpha_{p_1}} \frac{P^2(\alpha)}{\alpha^6} e^{-\alpha(2l_1 + l_2 - l_1 + \gamma)}(1 - e^{-2\alpha(l_2 - l_1)}) \phi(\alpha) d\alpha
\]

where: \(N_p\) denotes the turns of the receiving coil, \(r_e\) denotes the average radius of the receiver, \(l_{p1}\) and \(l_{p2}\) denote the lower and upper height of the receiver, \(r_{p1}\) and \(r_{p2}\) denote the inner and outer radii of the receiver, \(l\) denotes the lift-off of the sensor probe, \(g\) denotes the gap between the transmitter and receiver, \(r_i\) denotes the radius of the co-axial hole and \(r_o\) denotes the radius of the sample plate.

III. NUMERICAL VERIFICATION

A. FEM modelling

In order to validate the proposed method, FEM simulations have been carried out in commercial software - Comsol. In the simulation, the exciter coil, the receiver coil, and the sample plate are co-axially deployed. The sensor parameter is listed in Table I. The inner and outer radii of the sensor coil were set to 28 and 28.25 mm respectively. The gap between the exciter coil and the receiver coil was set to 5 mm. The electrical conductivity and the relative magnetic permeability of the sample plate were kept to that of the copper - 59 MS/m and 1 respectively. The current with the amplitude of 1 A was injected into the exciting coil while the induced field can be detected by the receiver coil. Three cases have been simulated to test the feasibility of the method. The models of the sample plate with a thickness of 1 mm are shown in Fig. 4.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SENSOR PARAMETERS</th>
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<tbody>
<tr>
<td></td>
<td>The exciter coil</td>
</tr>
<tr>
<td></td>
<td>Inner radius (r_{e1})</td>
</tr>
<tr>
<td></td>
<td>28 mm</td>
</tr>
<tr>
<td></td>
<td>Inner radius (r_{p1})</td>
</tr>
<tr>
<td></td>
<td>28 mm</td>
</tr>
<tr>
<td>Number of turns</td>
<td>The exciter coil</td>
</tr>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Height of the exciter coil (l_{e1})</td>
<td>8 mm</td>
</tr>
<tr>
<td>Height of the receiver coil (l_{e2})</td>
<td>8 mm</td>
</tr>
<tr>
<td>The gap between two coils (g)</td>
<td>5 mm</td>
</tr>
<tr>
<td>Plate thickness (O_{p})</td>
<td>1 mm</td>
</tr>
</tbody>
</table>

As can be seen from Fig. 4, the sample plate with a radius of 50 mm is firstly simulated, then the second step is to simulate the large plate with a small co-axial hole inside. By combining these two features, the inductance change due to the presence of the finite-sized sample plate with a co-axial hole can be calculated by using the analytical method in (14).

B. Analyzing

1. The finite sample plate (r: 0-50mm)

Figure 5 depicts the real part and the imaginary part of the inductance change using the analytical and numerical FEM method. Compared with the results obtained from the simulation, the proposed method can demonstrate the inductance change well which is perfectly harmonious with the numerical FEM results.
As can be seen from Fig. 5(a), there is no zero-crossing point in the real part of the inductance change because of the non-magnetic metallic sample plate. Besides, under the low frequency, the real part of the inductance change is close to zero. Then, as the frequency increases, the inductance increases gradually until reaching its saturated value. It almost remains stable under the high frequency. Meanwhile, as shown in Fig. 5(b), the imaginary part of the inductance change is nearly zero under low and high frequencies. However, it can be observed that a peak value exists. From our previous studies, the corresponding frequency – termed as the peak frequency feature, is determined by the properties of the sample plate.

2). The infinite sample plate with a co-axial hole (r: 25 mm - inf)

The inductance due to the hole of a sample with an extremely large planar size is plotted in Fig.6. As shown in Fig. 6, by introducing the upper limit of the integration domain, the method holds for the coil placed above the infinitely large (half-space) sample plate with a hole. Besides, there is a good match of the inductance change between the proposed method and the numerical FEM simulation. It can be observed that the magnitude of the imaginary part reduces (compared to Fig. 5) with the peak frequency increased from approximately 200 Hz to 300 Hz.

3). The finite disk sample plate with a co-axial hole (r: 25 mm - 50 mm)

Figure 7 describes the change of the real part and imaginary part of the inductance due to the presence of the finite-size circular sample plate with a co-axial hole. Combining two boundary limits, the inductance change can be well described by the proposed analytical method. There is a slight discrepancy near the frequency 1 kHz. It is may because the discretization of the mesh domain in the modelling. Due to the skin depth effect, a much denser mesh is needed for the FEM modelling. Besides, the mesh domain of the free space region could affect the accuracy.

More combinations have been simulated to verify the method in Fig. 8. Moreover, it is found that the upper/lower limit of $\alpha$ ($\alpha_r$, $\alpha_i$) for the analytical formula is proportional to the reciprocal of the inner/outer radius of the sample ($r_i/r_o$). That is, $\alpha_r = \frac{3.518}{r_i}$ and $\alpha_i = \frac{3.518}{r_o}$, which fits the manipulated formulation in equation (13)).
**IV. HOLE SIZE MEASUREMENTS**

**A. Experimental setup**

Experimental studies have also been conducted and used for the radius measurement of the co-axial hole in the disk sample plate. Fig. 9 shows the entire setup of the experiment system. The probe was situated co-axially to the specimen with different...
radii of the holes. Parameters of the sensor are listed in Table I. The radii of the holes are 12.5, 15, 17.5, and 20 mm respectively. The material of the sample plate is copper and the thickness is 20 µm. The impedance analyser was running under the swept frequency mode. Hence, the inductance between the exciting coil and the receiver coil with and without the sample plate can be obtained.

B. Hole size measurements

Figure 10 demonstrates the measurement results of the inductance change of the EC sensor above the circular sample plate with various radii of co-axial holes. It can be seen that, as the radius of the co-axial hole increases, the magnitude of the imaginary inductance reduces with the increased peak frequency. The size of the co-axial hole can be predicted by utilising the peak frequency feature. It can be noticed that, as the peak frequency in Fig. 10 (20 µm case) is much larger than that in Fig. 8 (1 mm case), the eddy current for the peak-frequency in Fig. 10 is more restraint to the surface of the sample plate. Consequently, the eddy current density under the peak-frequency in Fig. 10 is larger than that in Fig. 8. Thus, the frequency feature becomes more sensitive to the diameter of the sample.

From the experimental data, the peak frequency for each sample plate can be obtained by finding the minimum value of the imaginary part of the inductance. Then we fit the measured peak frequency with the simulations by searching for the upper limit - $\alpha_r$,in (14) (e.g. sweeping the limit from 1000 to 100 in a certain increment, i.e. 1). Further, the relationship between the upper limit - $\alpha_r$, and the radius of the co-axial hole $r_i$ is used for the size prediction. Table II lists the actual and measured size by the air-cored EM sensor. The error between the actual and measured radius is within 5 %. The difference between the actual and predicted values is due to the approximation of the relationship in (13) and the error caused by the measured peak frequency of the imaginary inductance change.

V. CONCLUSIONS

In the paper, the analytical solution for the impedance of a sensor above the circular disk sample with a co-axial hole has been proposed, which has introduced a dual-constraint integration feature. The dual constraint of the integration has been found inversely proportional to the radii of disk samples and co-axial holes. This method was validated by both the numerical FEM modelling and experiments. By referring to the inversely proportional relationship between the upper constraint, the radius of the hole can be derived. The error for the radius measurement of the co-axial hole can be controlled within 5 %. The reconstruction error may be due to the approximated peak frequency from the measurements and the offset of the experimental system.

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<thead>
<tr>
<th>TABLE II</th>
<th>ACTUAL AND MEASURED RADIUS FOR THE COPPER PLATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Actual radius of hole (mm)</td>
</tr>
<tr>
<td>Copper</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

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