Revisiting Our Quantum World

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Abstract

Of course, now we know that quantum mechanics has been a fundamental structure of our world since our universe came into being. However, it has been only a century since the experimental and theoretical discovery of quantum mechanics and its extensions into many implications and applications. In particular, there are implications across many disciplines that most likely will affect education, health, security, etc. Examples are given of the need for starting education as early as possible in schools, the use of nano-robots to deliver drugs targeted to specific molecular sites, to developing new cryptographic systems to safeguard our privacy.

Keywords: education, neuroscience, nano-robots, financial markets, financial options, security, computation, optimization

1. Introduction

The history of computers is relatively new, less than 90 years old, within the lifetime of some readers of this book! Clearly, humans have been quite busy during this time, developing powerful computers to replace pen/pencil and paper.

An outline of the short (in time only) history of computers includes:

- The foundations of the mathematical foundation of computing is generally attributed to Turing in 1936 (Turing, 1937).

- The first computer is generally considered to be the ENIAC, completed in 1946.

- The first quantum algorithm that has had far-reaching implications is generally attributed to Shor in 1994 (Shor, 1994).

- The physical-mathematical foundations of quantum computing is generally attributed to Benioff in 1980 (Benioff, 1980).

- The initial history of quantum computers per se usually is generally attributed to Feynman in 1982 (R. Feynman, 1982).

- The first commercially available quantum computer is generally considered to be built by D-WAVE in 2017, building on a small 2-qubit quantum computer built circa 1997-1998.

The basic unit of a quantum computer is a qubit (quantum bit), the quantum analog of a classical-computer bit. A classical bit can be a state 0 or 1, whereas a qubit can be in a linear combination, called a superposition, of 0 and 1. For a classical computer with 2 bits, there are 4 possible states, but only one state can be realized at any given time. For a quantum computer with 2 qubits, there are 4 possible states, all of which can be realized simultaneously and “superposition” of all linear combinations are possible. There are multiple theories vying for testing, in some cases offering radically different interpretations of reality based on quantum mechanics, e.g., https://www.scientificamerican.com/article/this-twist-on-schroedingers-cat-paradox-has-major-implications-for-quantum-theory/.

When such a quantum state is subject to decoherence, e.g., by a classical measurement process, it becomes a classical state, wherein it becomes just one classical state. This decoherence is at the heart of why it is so difficult to build a quantum computer, as any classical perturbation causes quantum states to collapse into classical states. Decoherence still is referred to by many people as the “collapse” of the wave-function; however, even some of the issues reported here are not quite well explained using the term
collapse.

1.1. Factorization of large numbers using Shor’s algorithm
Most people who read about quantum computers associate this upcoming technology with new methods of computation that will be game-changers for security. That is because many articles have been written on how this new technology will change the way just about all major passwords are encrypted using factorization of large numbers.

1.2. Faster searches using Grover’s algorithm
The probability of a quantum state is given by the absolute square of the wave-function of that state. Because the wave-function lives in a complex space with both real and imaginary numbers, the absolute square multiplying the wave-function times its Hermitian transpose, changing the sign of the imaginary part, gives a real number, whereas the simple square will not unless the imaginary part is zero.

In 1996, Grover developed an algorithm to statistically search spaces of size \( N \) in time on the order of the square-root of \( N \), whereas a classical algorithm requires time statistically on the order of \( N \) (Grover, 1997). Basically, Grover’s algorithm uses the wave-function to perform the search, whereas classical algorithms must sample the probability of states.

1.2.1. Quantum key distribution
As expected, since quantum computers may crack most classical encryption that depends on factorization, there have been efforts to find methods of quantum encryption. One such method is quantum key distribution (QKD) (Fang et al., 2020), which recently has been used to transmit secure messages up to and back from a satellite.

1.3. Many quantum computers
There are many quantum computers now available from many companies/institutions. As of about a year ago, the author cited 17 (Ingber, 2018b).

D-WAVE (Canada)
DeepMind (Canada)
Facebook
Google
IBM
Intel
Microsoft
National Laboratory for Quantum Information Sciences (China)
Nippon Telegraph and Telephone
NOKIA Bell Labs
NSA
Post-Quantum
Rigetti
Russian Quantum Center
Toshiba
Quantum Circuits
Quantum Technologies (European Union)

As of the time of writing this chapter, there is an explosion of many more companies and institutions implementing quantum computers. For example, see https://eas.caltech.edu/engenious/16/chair_message and https://www.caltech.edu/about/news/preparations-begin-aws-center-quantum-computing-caltech discusses how Caltech is entering this domain using Amazon’s AWS platform.

The advances in the number of qubits available for computation have been expanding exponentially; see https://www.statista.com/chart/17896/quantum-computing-developments/ . In October 2019 Google announced a breakthrough in computational power; see https://www.weforum.org/agenda/2019/10/quantum-computers-next-frontier-classical-google-ibm-nasa-...

Although companies are exploring different machines, it is clear that the scales of thought of quantum computation are expanding rapidly. Quantum computing is being taken quite seriously even as it struggles to compete with classical computing on large scales.

1.4. Quantum mechanics is still a mystery

It must be appreciated that quantum mechanics (QM) is still a mystery to us all. There are unresolved issues about the basic assumptions, called axioms, on which QM rests. There is a steady stream of research examining these issues. For example, https://phys.org/news/2020-08-quantum-paradox-reveals-contradiction-widely.html reports on such aspects published as this chapter is being written (Bong et al., 2020). The issue arises when considering Wigner's observer observing the observer of a quantum experiment, a point raised by Wigner in 1961 (Wigner, 1961). It is well established that when a macroscopic body, like a large machine or a person, interacts with a quantum event, the wave-function describing that event suffers decoherence, and the event is pinned into a probabilistic event which is what is typically measured. That is, the measurement just measures one of the possible manifestations of the wave-function, not the wave-function itself. When consideration is made for an observer observing a sensitive quantum system making a quantum measurement on a different quantum system, this is where the analysis can fork into different possibilities as yet not well understood.

1.5. What’s at stake?

Why is this new quantum technology important, and how can it affect our lives? Some rationale will be given in this chapter. Keep in mind that just about all of this is still reasonably considered as being speculative, in terms of what new quantum technologies can actually deliver on large social scales. However, in terms of a financial options calculation, where the expected gain/loss over the duration of the option is a sum of products of expected payoffs/losses times probabilities of becoming mature technology, I believe that all readers will see that the the expected gain is quite large for humanity.

It may “hurt” to try to understand some quantum technologies — in the sense of getting emotionally disturbed by being presented with a lot of new information. To make this transition a bit easier, it will help to look at two example systems, one that has been with in the womb — our brain, and the other a vital part of our social interactions — money. Then, we also may get some understanding of another system, with the key word “blockchain” that many readers may have seen in the press, and how developments in the quantum world may deeply affect us in the near future.

The following sections will describe quantum influences in several disciplines. Becoming familiar with the quantum world requires education, which is important to acquire as early as possible in single-digits of age, to develop intuitions that will help us to learn more if we wish, and at least prepare us to make informed decisions affecting all our lives. Neuroscience will lead to new drugs. Quantum cryptology will lead to new secure methods of transactions, including money. Quantum blockchains will lead to new secure ledgers of information that affect bank transactions, government databases, and storing and retrieving your personal information. This all is much too important to leave in the hands of a few people, even if you feel you could trust them. Instead, we all must become vital participants in our futures.

2. Education

The quantum world is not “intuitive.” As testified by thousands of years before 1900 or so, no one truly imagined the nature of the quantum world. Even Democritus or Leucippus, his mentor, both circa 400-500 B.C., who are credited with postulating “atomism”, had no clue as the nature of the atomic world as we know it now, and we are still learning more about this world that of course has always been with us.

Indeed, it took experiments only possible in the 1900’s to understand the nature of quantum mechanics. For example, the famous double-slit experiment, sometimes reputed to be modeled on a similar
experiment that may have been performed by Young in 1801, by Davisson and Germer in 1927 (Davisson & Germer, 1927) showed that truly atomic particles possess a wave-particle duality. Quantum entities (like an electron) sometimes, e.g., in collisions behave as particles, and sometimes, e.g., in defraction like in the double-slit experiments they behave as waves. When they behave as waves, they can demonstrate interference patterns, which we now attribute to the wave-function describing their paths in space-time. When they behave as particles, it is the absolute square of this wave-function that describes the probability of their space-time location.

Quantum particles also exhibit the “uncertainty principle”, wherein variables that are mathematically “conjugate” within the experimentally verified Planck constant $\hbar = 1.0545718 \times 10^{-34}$ meters-squared kilogram/sec. For example, momentum and positions are conjugate variables, as are energy and time. There are limits to the precision/resolution that can be obtained by both members of conjugate variables, the product being limited to $\hbar$. This means that the resolution of momentum multiplied by the resolution of position (in the same system) can only be measured to within $\hbar$; at their minimum levels of resolution, an increase of resolution of one of them incurs a corresponding loss of resolution in the other.

The primary take-home lesson is that all this is not intuitive. We only know all this because of verified experiments and verified theories since circa 1900.

So, how can we expect children to enter the new world of quantum technologies that are rapidly being created, based solely on their interactions with the world devoid of knowledge of these quantum experiments and theories? The clear answer is that they can not and they will not learn any of this unless they are exposed to these experiments and theories. While the mathematics and physics theory may require some years of education to understand quantum mechanics, basic intuitions of this world can easily be imparted by exposing children to many experiments as early as (pre-)kindergarten, even before their formal education begins! For many people, without this early exposure to the quantum world, they will be clueless and helpless to make informed decisions, e.g., like voting on funding new technologies, selecting drugs based on these technologies, etc.

Therefore, we must start as early as possible, educating students and citizens about the quantum world in which they live.

There are several online sites that offer courses in quantum computing, e.g.,

https://www.edx.org/learn/quantum-computing

https://www.coursera.org/courses?query=quantum%20computing

and in quantum mechanics, e.g.,

https://www.edx.org/learn/quantum-physics-mechanics

https://www.coursera.org/courses?query=quantum%20physics

Most courses can be taken at no charge.

3. Neuroscience

It is likely possible to control brain function by targeting specific mechanisms at molecular levels, instead of using drugs that are widely systemic and have multiple contraindications.

As an example discussed in a recent paper (Ingber, 2018a), large-scale neuronal firings can be influences by regenerative molecular Ca$^{2+}$ generated at tripartite neuron-astrocyte quantum-scale sites. This suggests a mechanism by which nano-robots may control background synaptic activity that in turn control attention and memory, by delivering pharmaceutical agents. This interaction, if indeed found to be present in neocortex, would represent interactions over huge ranges, interactions between quantum molecular scales and highly synchronous neural firings among millions of neurons.

Of the $10^{11}$ cells in the human brain, about half are neurons, and the other half are glial cells. Astrocytes likely comprise the majority of glial cells. There are many papers which examine the roles of astrocytes on synaptic processes (Bellinger, 2005; Innocenti et al, 2000; Scemes & Giaume, 2006; Agulhon et al, 2008; Pereira & Furlan, 2009; Reyes & Parpura, 2009; Araque & Navarrete, 2010; Banaclocha et al, 2010; Volterra et al, 2014).
Regenerative intercellular calcium waves (ICWs) can travel over 100s of astrocytes, encompassing many neuronal synapses. These ICWs are documented in the control of synaptic activity (Ross, 2012). \([\text{Ca}^{2+}]\) (concentration of \(\text{Ca}^{2+}\)) affect increased release probabilities at synaptic sites, likely due to triggering release of gliotransmitters. (Free \(\text{Ca}^{2+}\) waves are considered here, not intracellular astrocyte calcium waves in situ which also increase neuronal firings.) During selective attention tasks, partially confirmed by fitting scalp electroencephalographic (EEG) recordings, free regenerative \(\text{Ca}^{2+}\) waves, arising from tripartite interactions, couple to the magnetic vector potential \(A\) produced by highly synchronous collective firings. \(A\) is derived from the neocortical dipole moment \(Q\), averaged over many neurons, in turn proportional to the current \(I\) which develops the electric potential on the scalp measured as EEG.

3.1. Reasonable Estimates

Estimates used for \(Q\) come from experimental data. When coherent activity among many macrocolumns associated with short-term memory (STM) is considered, \(|A|\) may be orders of magnitude larger. There is direct coherence between \(\text{Ca}^{2+}\) waves and the activity of \(A\). A simple classical calculation shows \(qA\), where \(q\) is the charge of a \(\text{Ca}^{2+}\) ion, from macroscopic EEG to be on the order of \(10^{-28}\) kg-m/s, while the momentum \(p\) of a \(\text{Ca}^{2+}\) ion is on the order of \(10^{-30}\) kg-m/s.

The recent XSEDE.org project developed by the author (Ingber, 2011; Ingber, 2012b; Ingber, Pappalepore & Stesiak, 2014; Ingber, 2015; Ingber, 2016b), investigates Top-Down influences of macroscopic patterns of neuronal firings, measured by scalp EEG during attentional memory tasks, on Bottom-Up free microscopic \(\text{Ca}^{2+}\) ions created tripartite interactions, which interact with background synaptic activity that in turn influence large synchronous firings of neurons.

During the development of this project, it was recognized that the proposed mechanism for this Top-Down Bottom-Up synergy could be influenced by nano-robots, to deliver pharmaceutical agents under conscious control of a person (Ingber, 2015).

3.2. EEG Influences on Ca Waves (EICW)

3.2.1. SMNI Model of Neocortical Interactions

Since 1980 a Statistical Mechanics of Neocortical Interactions (SMNI) model has been developed in a couple score of papers, scaling synaptic interactions to neuronal firings, to minicolumnar firings, to macrocolumnar firings, to regional firings — the scale measured by scalp EEG (Ingber, 1982; Ingber, 1983; Ingber, 1984; Ingber, 1994; Ingber, 1997; Ingber, 2012b). At the columnar level, SMNI has detailed properties of short-term memory (STM), e.g., capacity (auditory 7 ± 2 and visual 4 ± 2), duration, stability, primacy versus recency rule, and Hick’s law. Only recently have neuroscientists confirmed by some experiments (Salazar et al., 2012; Asher, 2012), that some STM are processed by highly synchronized patterns of neuronal firings.

At the regional scale SMNI has derived the EEG wave equation (Ingber & Nunez, 2010), and has been tested by fitting EEG Training data using Adaptive Simulated Annealing (ASA) (Ingber, 1993; Ingber, 2012a), testing on out-of-sample Testing sets, including centers sponsored by the National Institute on Alcohol Abuse and Alcoholism (NIAAA) (X.L. Zhang, Begleiter, Porjesz, Wang & Litke, 1995; X.L. Zhang, Begleiter & Porjesz, 1997; X.L. Zhang, Begleiter, Porjesz & Litke, 1997). This data set was used in earlier SMNI studies (Ingber, 1997; Ingber, 1998).

3.2.2. Neurology Considered

Astrocytes influence glutamate, the main excitatory excitatory neurotransmitter in neocortex, by taking in some glutamate released by presynaptic neurons and converting it into glutamine which can enter presynaptic neurons where it can be re-converted into glutamate. GABA is the main inhibitory neurotransmitter in neocortex, produced by inhibitory neurons by also utilizing glutamic acid (which when stripped of a hydrogen atom is glutamate) from astrocytes (Patel et al., 2001; Walls et al., 2015). \(\text{Ca}^{2+}\) waves arise from nonlinear cooperative regenerative processes from internal stores, involving several...
biochemical steps (Ross, 2012; Pitta et al, 2012; Goldberg et al, 2010).

3.2.3. Classical/Quantum Vector Potential influence on Ca\textsuperscript{2+} Momenta

Columnar firings develop electromagnetic fields described by a magnetic vector potential, referred to as the SMNI vector potential (SMNI-VP). Early included the “Smoking Gun” that implicates top-down interactions at molecular scales (Ingber, 2011; Ingber, 2012b). Some papers calculated the approach in a classical physics context (Ingber, 2012c). After these papers, detailed interactions were described of SMNI-VP firing states with Ca\textsuperscript{2+} waves, in both classical and quantum mechanics (Ingber, 2011; Ingber, 2012b), and quantum contexts (Ingber, Pappalepore & Stesiak, 2014; Ingber, 2015), using the “canonical momentum” \( \mathbf{p} + q\mathbf{A} \). The theoretical construct of the canonical momentum \( \Pi = \mathbf{p} + q\mathbf{A} \) is firmly entrenched in classical and quantum mechanics (R.P. Feynman, 1949; Schulman, 1981). See https://www.feynmanlectures.caltech.edu/II_15.html for a nice introduction.

Columnar firings are reasonably modeled as a wire/neuron with current \( I \) along a length \( z \) observed from a perpendicular distance \( r \) from a line of thickness \( r_0 \). The integral (sum of contributions) of \( I \) over \( r \) gives the magnetic vector potential \( \mathbf{A} \) (Jackson, 1962). This gives an insensitive log dependence on distance. The contribution to \( \mathbf{A} \) includes many minicolumnar lines of current from 100’s to 1000’s of macrocolumns, contributing to large synchronous bursts of EEG (Srinivasan et al, 2007). \( \mathbf{E} \) and \( \mathbf{B} \) are derivatives of \( \mathbf{A} \) with respect to \( r \), giving inverse-polynomial sensitivities. They do not possess this logarithmic insensitivity and do not linearly accumulate strength across macrocolumns.

This study is robust against much theoretical modeling as experimental data is used wherever possible.

The author used path-integrals to derive a closed-form solution of the propagation of the Ca\textsuperscript{2+} wave-packet composed of many Ca\textsuperscript{2+} ions (Ingber, 2018a). This wave function was used to calculate the probability distribution of this wave-packet, which was used to modify synaptic interactions in the SMNI model, tested by fitting this to EEG data. The calculation is quite sensitive to \( \hbar \) which is a test of the sensitivity to quantum interactions at the large scale of columnar firings of neurons.

3.2.4. Fitting SMNI Model Including EICW to EEG Data

The influence of \( \mathbf{A} \) on the background-noise of synaptic parameters SMNI used ASA to fit 28 parameters across a circuitry underlying 6 electrode sites using NIAAA data (Ingber, Pappalepore & Stesiak, 2014; Ingber, 2015).

The momenta of Ca\textsuperscript{2+} ions are influenced during EEG events like N100 and P300 potentials common in selective attention tasks (Srinivasan et al, 2007). Codes for SMNI fits to EEG data (Ingber, 1997; Ingber, 1998) were used as templates. The influence of Ca\textsuperscript{2+} waves is tested by parameterizing synaptic parameters to be dependent on Ca\textsuperscript{2+} wave activity.

To date, results of fits to EEG data only demonstrate that fits to an \( \mathbf{A} \) model are reasonably better than fits to the no-\( \mathbf{A} \) model.

3.3. Conscious Control of Nano-Robots Influencing Attention and Memory

Demonstration that nano-robots may be used in this context were calculated (Ingber, 2015), assuming Ca\textsuperscript{2+}-wave momentum-sensors acting as a piezoelectric devices (Alivisatos et al, 2013; Wang, 2012). At the beginning of a Ca\textsuperscript{2+} wave (100’s of ms), a change of momentum is on the order of \( 10^{-30} \) kg-m/s. A Ca\textsuperscript{2+} wave packet of 1000 ions with onset time of 1 ms produces a force on the order \( 10^{-24} \) N (1 N \( \equiv \) 1 Newton = 1 kg-m/s\(^2\)). Nano-robots drawn to this site could deposit chemicals/drugs that interact with the Ca\textsuperscript{2+} wave process. If the receptor area of the nanosystem were 1 nm\(^2\) (resolution of scanning confocal electron microscopy (SCEM)), this would require a pressure sensitivity of \( 10^{-6} \) Pa (1 Pa = 1 pascal = 1 N/m\(^2\)).

The nanosystem could be switched on/off at a regional/columnar level by sensitivity to local electric/magnetic fields.
3.4. Free Will
In addition to researching STM and multiple scales of neocortical interactions, there is interest in possible quantum influences on highly synchronous neuronal firings to understand possible connections to consciousness and “Free Will” (FW) (Ingber, 2016a; Ingber, 2016b).
If neuroscience ever establishes experimental evidence from quantum-level processes of tripartite synaptic interactions with large-scale synchronous neuronal firings, then FW may be established using the Conway-Kochen quantum no-clone “Free Will Theorem” (FWT) (Conway & Kochen, 2006; Conway & Kochen, 2009).
Thus, a Ca$^{2+}$ quantum wave-packet can generate a state proven to have not previously existed, since quantum states cannot be cloned.

3.5. Quantum Zeno Effects
The wave function of the Ca$^{2+}$ wave packet was calculated by the author, and it was demonstrated that despite multiple collisions during their regenerative processes over long durations of hundreds of ms, typical Ca$^{2+}$ waves support a Zeno or “bang-bang” effect which may promote long coherence times (Facchi, Lidar & Pascazio, 2004; Facchi & Pascazio, 2008; Wu et al, 2012; Giacosa & Pagliara, 2014; P. Zhang et al, 2014; Kozlowski et al, 2015; Patil et al, 2015; Muller et al, 2016).
Of course, the Zeno/“bang-bang” effect may exist only in special contexts, since decoherence among particles is known to be very fast, one of the fastest processes known (Preskill, 2015).
However, the constant collisions of Ca$^{2+}$ ions, causing some ions to enter or leave the wave packet, may perpetuate at least part of the wave, permitting the Zeno/“bang-bang” effect.

qPATHINT provides an opportunity to explore the coherence stability of the wave due to serial shocks of this process. qPATHINT was developed by the author and has been used to calculate the evolution of the quantum path-integral, an algorithm developed by Feynman in 1948 (R.P. Feynman, 1948) to propagate short-time probabilities over long time spans. PATHINT is the underlying classical code developed for classical systems (Wehner & Wolfer, 1983a; Wehner & Wolfer, 1983b; Wehner & Wolfer, 1987; Ingber, Fujio & Wehner, 1991; Ingber, Wehner et al, 1991), further developed by the author for arbitrary dimensions.
The path-integral presents a mathematically equivalent representation of both multivariate stochastic differential equations with multiplicative noise (nonlinear coefficients of Gaussian noise) and of multivariate partial differential equations, but also offering numerical and intuitive advantages like derivations of common concepts like force, momentum, etc (Langouche et al, 1979).

4. Finance
4.1. Quantum Money
Many countries are exploring “digital currencies”. E.g., see https://www.federalreserve.gov/newsevents/speech/brainard20200813a.htm. These include the use of blockchains for security, but controlled by central governments. This can be viewed as just the first step to considering new currencies, which of course will compete, if not dominate, with distributed blockchains like Bitcoin.
In the near future quantum computing will continue to be investigated for applications to financial products. To offer hedging as well as speculation, financial derivatives will developed on these products. Then, qPATHTREE and qPATHINT are poised to calculate financial derivatives in these quantum complex spaces. This presents contexts beyond using quantum computers to calculate financial derivatives, since the space of the dependent variables themselves may live in quantum worlds (Baaquie et al, 2002; Piotrowski et al, 2005; Accardi & Boukas, 2007; Meyer, 2009; Aaronson & Christiano, 2012; Jogenfors, 2016).
4.2. Financial Options On Quantum Money

To numerically calculate the path integral, especially for serial changes in time in the presence of random shocks — not approachable with standard Monte Carlo techniques — PATHINT and the quantum version qPATHINT were developed. The codes are written for arbitrary N dimensions, and have been used for several papers in both classical and quantum systems (Ingber, 2000; Ingber & Wilson, 2000; Ingber, 2016a; Ingber, 2017a; Ingber, 2017b).

Many systems propagate in the presence of continual “shocks” which include:
- future dividends
- changes in interest rates
- changes in asset distributions used in American options algorithms

which are included in these calculations.

5. Blockchains


The blockchain is a ledger that is validated, entry by entry, e.g., for any item, contract, or whatever. Still today, this validation via encryption pretty much hiding identities of people utilizing them is at the core of much strain in crypto-products like crypto-currency (such as Bitcoin, Ethereum, Ripple, etc.). Proof of Work was the original design and still has the largest scale via Bitcoin, but this requires lots of energy and time for the constant verification process; the potential drain on energy is even considered to be a threat to climate change!

While theoretically, anyone can participate in the validation process, in practice only a handful of large data banks, consuming towns and cities, effectively do most of the validation. This has given rise to some products like Ripple that do away with the democracy of validation by validators, and these are endorsed by many large institutions, e.g., many banks, who prefer this more efficient control over resources, dismay ing many ardent supporters of blockchains.

As in SMNI, the core of the quantum no-clone “Free Will Theorem” (FWT) theorem can have important applications in this domain. For example, quantum currency cannot be cloned. Such currencies are exceptional candidates for very efficient blockchains, e.g., since each “coin” has a unique identity (Meyer, 2009; Aaronson & Christiano, 2012; Bartkiewicz et al, 2016; Jogenfors, 2016).

As in SMNI, there are issues about the decoherence time of such “coins”.

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References

Note that in accordance with common practice, website URLs are not necessarily referenced at the end, but often appear within the body of text. E.g., see https://libanswers.snhu.edu/faq/8627 or https://www.scribbr.com/apa-examples/cite-a-website/ or https://libguides.css.edu/APA7thEd/Webpages.


M. Goldberg, M. De Pittà, V. Volman, H. Berry & E. Ben-Jacob (2010) Nonlinear Gap Junctions Enable Long-Distance Propagation of Pulsating Calcium Waves in Astrocyte Networks. Public Library of Science Computational Biology. 6(8), 1-14.


