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Article

The Fast Discrete Interaction Approximation Concept

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Abstract: Hasselmann and coauthors proposed the discrete interaction approximation (DIA) as the best tool replacing the nonlinear evolution term in a numerical wind-wave model. Much later, Polnikov and Farina radically improved the original DIA by means of location all the interacting four wave vectors, used in the DIA configuration, exactly at the nodes of the numerical frequency-angular grid. This provides nearly two-times enhancing the speed of numerical calculation for the nonlinear evolution term in a wind-wave model. For this reason, the proposed version of the DIA was called as the fast DIA (FDIA). In this paper we demonstrate all details of the FDIA concept for several frequency-angular numerical grids of high resolution, with the aim of active implementation the FDIA in modern versions of world-wide used wind-wave models.

Keywords: wind-wave modeling; nonlinear waves; kinetic integral; interacting waves; optimal configuration.

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1. Introduction

Nonlinear interactions between waves play a very important role in description of wind wave evolution governed by the equation[1]

$$\frac{D}{Dt}N(\mathbf{k},\mathbf{x},t) = IN(N_\mathbf{k}) + NL(N_\mathbf{k}) - DISS(N_\mathbf{k}),$$

where *D*/*Dt* is the total derivative, $N_\mathbf{k} \equiv N(\mathbf{k},\mathbf{x},t)$ is the wave-action spectrum in the wave vector **k**-space, at location **x**, and time *t*; *IN, NL, DISS* are the evolution terms responsible for the input, conservative nonlinear transfer among wave components, and dissipation of wave action, respectively. The nonlinear evolution term *NL* is described by the six-fold Hasselmann kinetic integral I_{NL} with a very complicated integrand [2]

$$\frac{\partial N(\mathbf{k}_4)}{\partial t} = I_{NL}(N) \equiv$$
$$\equiv 4\int M^2_{1,2,3,4}\{N(\mathbf{k}_1)N(\mathbf{k}_2)[N(\mathbf{k}_3)+N(\mathbf{k}_4)] - N(\mathbf{k}_3)N(\mathbf{k}_4)[N(\mathbf{k}_1)+N(\mathbf{k}_2)]\}\delta_{1+2-3-4}d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \quad (1)$$

where $M^2_{1,2,3,4} \equiv M^2(\mathbf{k}_0,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ is the second power of the matrix elements corresponding to the four-wave nonlinear interactions, $\delta_{1+2-3-4} \equiv \delta(\sigma_1+\sigma_2-\sigma_3-\sigma_4)\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)$ is the Dirac delta-functions responsible for the resonant feature of the four-wave interactions, and

$\sigma_i = \sigma(\mathbf{k}_i)$ is the radian frequency of the wave component with wave vector $\mathbf{k}_i$. Due to this complicity, the integral should be replaced by some theoretically justified approximation, to be used in a practical numerical wind-wave model. The best approximation was proposed by Hasselmann et al. [3}, based on replacing integral I_{NL} by the only configuration of four interacting waves, located at the vicinity of a singular subsurface. This subsurface in the six-fold **k**-space is defined by the resonance conditions

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4, \tag{2a}$$

$$\sigma_1 + \sigma_2 = \sigma_3 + \sigma_4. \tag{2b}$$

The wave vectors $\mathbf{k}_i$ are usually represented in the frequency-angular space, (σ, θ), where the wave frequencies σ_i are related to $\mathbf{k}_i$ by the dispersion relation, in the deep-water case having the kind

$$\sigma(\mathbf{k}_i) = \sigma_i = (gk_i)^{1/2}. \tag{3}$$

The proposed approximation is called as the discrete interaction approximation (DIA). Example of the four vectors configuration used in the original DIA [3] is schematically shown in Figure 1.

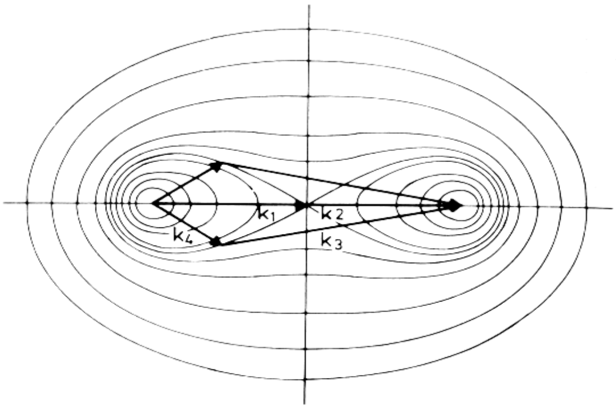

Figure 1. The configuration used in the original DIA.

2. Details of the discrete interaction approximation

The configuration used in the original DIA in the polar coordinates (σ, θ) has the following parameters (see Figure 1):

1) $\mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k}$, where wave vector $\mathbf{k} = (\sigma, \theta)$ is located at the node of the numerical grid $\{\sigma_i, \theta_j\}$;

2) $\mathbf{k}_3 = \mathbf{k}_+$, where $\mathbf{k}_+$ is represented by $\sigma_+ = \sigma(1+\lambda)$ and $\theta_+ = \theta + \Delta\theta_+$; (4a)

3) $\mathbf{k}_4 = \mathbf{k}_-$, where $\mathbf{k}_-$ is represented by $\sigma_- = \sigma(1-\lambda)$ and $\theta_- = \theta - \Delta\theta_-$;

4) In consistency with the resonant conditions (2) , the parameters of the configuration are

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$$\lambda = 0.25,\ \Delta\theta_+ = 11.5^o, \text{ and } \Delta\theta_- = 33.6^o. \tag{4b}$$

The nonlinear transfer at all the mentioned **k**-points takes the form [3]

$$NL(\mathbf{k}_-) = I(\mathbf{k},\mathbf{k}_+,\mathbf{k}_-),\quad NL(\mathbf{k}_+) = I(\mathbf{k},\mathbf{k}_+,\mathbf{k}_-),\quad NL(\mathbf{k}) = -2I(\mathbf{k},\mathbf{k}_+,\mathbf{k}_-), \tag{5}$$

where

$$I(\mathbf{k},\mathbf{k}_+,\mathbf{k}_-) = Cg^{-8}\sigma^{19}\left[N^2(\mathbf{k})\left(N(\mathbf{k}_+)+N(\mathbf{k}_-)\right) - 2N(\mathbf{k})N(\mathbf{k}_+)N(\mathbf{k}_-)\right]. \tag{6}$$

In Eq. (6), the fitting constant, C, has value C =3000 which is valid for the integration grid used by Hasselmann et al, [3].[1] The net nonlinear transfer at any fixed (σ,θ)-point of the numerical grid is found by running Eqs. (5, 6) through all the points of the frequency-angle integration grid $\{\sigma_i,\theta_j\}$.

The main advantage of this approximation is its evident simplicity and rather good accuracy for a certain initial spectrum [3]. For this reason, it is widely used in practical wind-wave modeling [1]. The third generation wind-wave numerical models, WAM [4] and WAVEWATCH(WW) [5], are the examples of successful implementation of the DIA. One technical shortages of the DIA routine is a presence of intermediate and cumbersome interpolation procedures induced by the mismatch of the spectral grid nodes and vectors $\mathbf{k}_+$, $\mathbf{k}_-$. This leads to the time-consuming about 50% CPU for the nonlinear evolution-term calculation during the numerical simulation of wave-field evolution.

The radical improvement of the DIA was done by Polnikov and Farina [6], based on locating all the interacting wave vectors of the DIA configuration exactly at the nodes of the frequency-angular grid used in both the kinetic integral and numerical model, $\{\sigma_i,\theta_j\}$. This provides nearly two-times enhancing the speed of numerical calculation for the *NL* term in a wind-wave model. For this reason, the proposed version of the DIA was called as the fast DIA (FDIA). Below, we demonstrate all details of the FDIA elements, based on several frequency-angular numerical grids of high resolution, with the aim of active implementation of the FDIA in modern wind-wave models.

3. The concept of the FDIA

In the original version of DIA [3], two of four interacting vectors (i.e., $\mathbf{k}_+$, $\mathbf{k}_-$) are not located at the nodes of integrating grid, what leads to necessity of the spectrum interpolation. For this reason, the speed of numerical wave forecast calculations is reduced remarkably. The main idea of the fast DIA (FDIA) is to use four-wave configurations which are adjusted to the integration grid for the kinetic integral. To specify this idea, first of all, one should introduce the principal parameters of the grid. Then, the features of configurations in FDIA could be described.

Integration grid for kinetic integral will be considered in the polar co-ordinates where each of interacting wave vector $\mathbf{k}_i$ is represented by the frequency-angular point (σ_i,θ_i). Usually, the integration grid is given by the formulas

$$\sigma_i = \sigma_0 \cdot q^{i-1} \qquad (1 \le i \le I), \tag{7a}$$

$$\theta_j = -\pi + (j-1)\cdot\Delta\theta \qquad (1 \le j \le J \text{ and } \Delta\theta = 2\pi / J). \tag{7b}$$

Thus, parameters of the grid are as follows:

[1] In principal, the value of C depends not only on the grid resolution parameters but on the kind of the source term of the model, as well.

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- the lowest frequency, σ_0;
- the frequency exponential increment, q;
- the maximum number of frequencies, *I*;
- the angle resolution in radians, $\Delta\theta$;
- and the maximum number of angles, *J*.

To our aims, the principal parameters are q and $\Delta\theta$, as far as they define the resolution of the grid. The numbers *I* and *J* should be rather great (several tens), but for the concept under consideration their explicit values *I* and *J* are not principal. Note only that the FDIA concept is valid for the rather fine grid (to save an accuracy) when

$$q \le 1.1 \quad \text{and} \quad \Delta\theta \le \pi/10 . \tag{8}$$

Everywhere below, the restriction (8) is supposed to be met. Initially, the FDIA was proposed in [6] for the resolution parameters

$$q = 1.05 \quad \text{and} \quad \Delta\theta = \pi/18 , \tag{9}$$

what is related to the "standard" integration grid introduced in [7] for the exact calculation of the kinetic integral.

In the FDIA, the basic configuration, the most close to the original DIA, is described by the following ratios [2]:

1) $\mathbf{k}_4 = \mathbf{k}$, (10a)

where the wave vector $\mathbf{k}$ is located at a current grid node and represented in the polar co-ordinates by the proper frequency σ and angle θ ;

2) $\mathbf{k}_3 = \mathbf{k}_+$ (10b)

where $\boldsymbol{k}_+$ is represented by $\sigma_3 = \sigma(1+\lambda_{34})$ and $\theta_3 = \theta + \Delta\theta_{34}$;

3) $\mathbf{k}_1 \approx \mathbf{k}_2 \approx (\mathbf{k}_4 + \mathbf{k}_3)/2 \equiv \mathbf{k}_a/2$ (10c)

where a specially introduced the reference wave vector $\mathbf{k}_a$ is directed along the angle $\theta_a = \theta + \Delta\theta_{a4}$, and its value is given by the formula (12) (see below).

Thus, we have two main (independent) parameters of configuration:

- the frequency increment, $1+\lambda_{34}$, defining the value of σ_3,
- and the proper angular difference, $\Delta\theta_{34} = \theta_3 - \theta_4$, defining the angle between vectors $\mathbf{k}_4$ and $\mathbf{k}_3$.

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[2] As far as $\mathbf{k}_4$ is conventionally taken as an external variable (see [7]), we have changed the order of interacting vector with respect to original DIA (compare (4a) and (10a,b,c)).

Varying these parameters, one can vary the configuration as a whole, including the values of dependent parameters θ_a and $\sigma_a = (\sigma_4 + \sigma_3)$.

The main differences between the configurations used in FDIA and in original DIA are as follows:

(a) all wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$, and $\mathbf{k}_4$ should be allocated at the nodes of the integration grid;

(b) vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ may be unequal, i.e. they may have some (but small) discrepancies in both values and directions;

(c) the resonance conditions (2) may be met rather approximately.

The main common feature of the configurations is that all of them are to be allocated in the vicinity of the "figure-of-eight" in the **k**-space (see, Figure 1). This requirement is expressed by the following ratios [7,8]

$$k_a = \sigma_a^2 / 2, \tag{11}$$

where $\mathbf{k}_a$ and σ_a in terms of independent variables: σ, σ_3, and $\Delta\theta_{34}$ have the kind

$$k_a = \left[\sigma_4^4 + \sigma_3^4 + 2\sigma^2\sigma_3^2 \, cos\,(\Delta\theta_{34})\right]^{1/2}, \tag{12}$$

and

$$\sigma_a = \sigma_4 + \sigma_3. \tag{13}$$

Eqs. (11)-(13) determine the value of angular difference $\Delta\theta_{34}$, for the given σ_4 and σ_3. After that, the expression for $\Delta\theta_{a4} = \theta_a - \theta_4$ is deduced from the resonant condition (2) and from the definition of $\mathbf{k}_a$ via the right part of (10c) by the following formula:

$$\Delta\theta_{a4} = \mathrm{arctg}\left[\frac{\sigma_3^2 \sin(\Delta\theta_{34})}{\sigma_3^2 \cos(\Delta\theta_{34}) + \sigma_4^2}\right]. \tag{14}$$

To fix the FDIA configuration, it needs to define several integer values corresponding to requirement (a) mentioned above (allocation of the vectors on the grid). According to (7a), this requirement can be expressed by the following equations:

$$\sigma_3 = \sigma_4 \cdot q^{m3}, \quad \sigma_1 = \sigma_4 \cdot q^{m1}, \quad \sigma_2 = \sigma_4 \cdot q^{m2}, \tag{15a}$$

and

$$\Delta\theta_{34} = n3 \cdot \Delta\theta, \qquad \Delta\theta_{a4} = na \cdot \Delta\theta \tag{15b}$$

Here $m1$, $m2$, $n3$, and na are the integer values to be found for any given integer $m3$. The first two are found from requirement (10c), and the latter two do from formulas

$$n3 = \text{Int}(\Delta\theta_{34} / \Delta\theta), \quad na = \text{Int}(\Delta\theta_{a4} / \Delta\theta) \tag{15c}$$

and previously determined $\Delta\theta_{34}$ and $\Delta\theta_{a4}$ (as it is described above). In (15c), the function Int(…) means the integer number which is nearest to the value of the argument.

Requirement (b) mentioned above (inequality of vectors $\mathbf{k}_1$ and $\mathbf{k}_2$) means that one can use the following choice for modulus parameters of the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$:

$$m1 = m2 \quad \text{or} \quad m1 = m2 \pm 1, \tag{16}$$

and the corresponding choice for the angle parameters of the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$:

$$n1 = n2 = na \quad \text{or} \quad n1 = n2 \pm 1 = na \pm 1 \tag{17}$$

where $n1$, $n2$ are the angular parameters of the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ corresponding to Eqs. (10c), (15b). Sign (±) means the permutation symmetry for vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ (see footnote below after formulas 22).

The choice of (± 1) means a possible inequality of vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. Eqs (15b) and (17) mean that for a certain configuration, given by values $m1$, $m2$, $m3$, and $n1$, $n2$, $n3$, the angle parameters of interacting vectors have the values

$$\theta_3 - \theta_4 = \pm n3 \cdot \Delta\theta; \quad \theta_1 - \theta_4 = \pm n1 \cdot \Delta\theta; \quad \theta_2 - \theta_4 = \pm n2 \cdot \Delta\theta \tag{18}$$

where sing (±) denotes a set of two mirror configurations (see [3]).

Taking into account the change of the interacting wave vectors order (mentioned above in footnote 2), the net expression for *NL*-term in the FDIA is given in the (σ, θ) - coordinates by the formulas

$$NL(\sigma_4, \theta_4) = I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma_4, \theta_4), \tag{19a}$$

$$NL(\sigma_3, \theta_3) = I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma_4, \theta_4), \tag{19b}$$

$$NL(\sigma_1, \theta_1) = -I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma_4, \theta_4), \tag{19c}$$

$$NL(\sigma_2, \theta_2) = -I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma_4, \theta_4), \tag{19d}$$

where

$$\begin{aligned} & I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma_4, \theta_4) = \\ & = C\sigma^{11}\left[S_1 S_2 (S_3 + (\sigma_3 / \sigma_4)^4 S_4) - S_3 S_4 ((\sigma_2 / \sigma_4)^4 S_1 + (\sigma_1 / \sigma_4)^4 S_2) \right], \end{aligned} \tag{20}$$

$S_i = S(\sigma_i, \theta_i)$, and C is the fitting constant depending of the grid parameters. The final value of 2D-function for *Nl*-term is found by the running calculations of (19)-(20) through the whole integration grid $\{\sigma_i,\ \theta_j\}$, similarly to the original DIA procedure.

After some numerical simulations for the grid (7a,b) with parameters q = 1.1, $\Delta\theta = \pi/12$ (typical for the WAM), the fitting constant C in (20) is found to be equal to 12000 [3]. The growth of C in our case is related to the exclusion of the abovementioned interpolation procedures.

Hereby, the algorithm of the FDIA configuration calculations is fully described. The certain set of configurations will be given in the next section. It needs only to add that effectiveness of the FDIA against DIA was numerously and successfully verified in comparison with the WAM [10-12] and the WW[13].

4. FDIA parameters for several certain configurations and grid resolutions

4.1. Parameters of configuration

On the example of typical WAM integration grid with parameters

$$q = 1.1 \quad \text{and} \quad \Delta\theta = \pi/12 \ (\text{or } \Delta\theta = 15^{o}), \qquad (21)$$

we shall demonstrate the choice of configuration parameters for FDIA: $m1$, $m2$, $m3$, and $n1$, $n2$, $n3$. For this aim, from Eqs. (11) - (13) and (15a), we calculate values $\Delta\theta_{34}$ and $\Delta\theta_{a4}$, varying independently the value of $m3$. Results of calculations are given in Table 1.

Table 1. Principle and auxiliary parameters for grid (21). The shaded line has parameters most closely corresponding to the original DIA configuration used in the WAM.

$m3$	$\Delta\theta_{34}$, deg.	$\Delta\theta_{a4}$, deg.	$x=\dfrac{\log(\omega_a/2)}{\log(q)}$	$m2$=Int(x+0.5)
3	23.8	15.2	1.61	2
4	32.3	22.2	2.19	2
5	41.5	30.3	2.79	3
6	51.6	39.8	3.42	3
7	62.7	50.9	4.07	4

From Table 1 it is seen that for the grid (21), the case with $m3$ = 5 (shaded) is the most close to the original DIA configuration for which angle $\Delta\theta_{34} \equiv \theta_{-} + \theta_{+} \cong 45^{o}$ (see (4.b)).

4.2. Parameters of some accuracy efficient FDIA configurations

In paper [6] it was derived some criterion for the accuracy efficiency of DIA configuration. Based on this criterion, we found several the most efficient FDIA configurations presented below. Taking into account the ratios (15a, b), one may construct the following efficient configurations.

4.2.1. For the original DIA configuration, the following FDIA parameters are the most efficient:

$$m3 = 5, \quad m2 = m1 = 3; \qquad (22a)$$

and

[3] This fitting coefficient C is tuned to the total source function of the wind wave model proposed in [9]..

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$$n3 = \text{Int}(\Delta\theta_{34} / \Delta\theta) = 3; \quad n2 = n1 = na = \text{Int}(\Delta\theta_{a4} / \Delta\theta) = 2. \tag{22b}$$

4.2.2. If we adopt existence of unequal values σ_1, σ_2, then parameters could be [4]

$$m3 = 5, \quad m2 = 2, \quad m1 = 3; \quad \text{and} \quad n3 = 3, \quad n2 = 3, \quad n1 = 2 \tag{23a}$$

or

$$m3 = 5, \quad m2 = 2, \quad m1 = 3; \quad \text{and} \quad n3 = 3, \quad n2 = 3, \quad n1 = 1. \tag{23b}$$

Pay attention that from Table 1 it is seen some others configurations which could be used for DIA. As it was shown in [14], some of them are more efficient that the original one given by (22a,b). The relative efficiency for these configurations was checked by means of the especial method constructed to this task and presented in [6].

4.2..3. For the multiple, three-configuration DIA (3C-DIA), in the fast version, we have found the following three constituents making an efficient construction (see [6, 14])

$$1)\ m3 = 4,\ m1 = 2,\ m2 = 3; \quad \text{and} \quad n3 = 2,\ n2 = 2,\ n1 = 1; \tag{24a}$$

$$2)\ m3 = 5,\ m1 = m2 = 3; \quad \text{and} \quad n3 = 3,\ n2 = n1 = 2; \tag{24b}$$

$$3)\ m3 = 7,\ m1 = m2 = 4; \quad \text{and} \quad n3 = 4,\ n2 = n1 = 3. \tag{24c}$$

4.2.4. For the grid parameters with *more fine angular resolution* as

$$q = 1.1 \quad \text{and} \quad \Delta\theta = \pi / 18 \ (\text{or } \Delta\theta = 10^{\circ}), \tag{25}$$

Table 1 has the same kind, whilst the proper configurations are as follows.

For the original DIA, the proper configuration in the FDIA version has parameters

$$m3 = 5, \quad m2 = m1 = 3; \text{ and} \quad n3 = 4, \quad n2 = n1 = 3. \tag{26}$$

The modified FDIA of the most better accuracy have configurations of the type

$$m3 = 5, \quad m2 = 2, \quad m1 = 3; \quad \text{and} \quad n3 = 4, \quad n2 = n1 = 3 \tag{27}$$

or

$$m3 = 5, \quad m2 = 2, \quad m1 = 3; \quad \text{and} \quad n3 = 4, \quad n2 = 4,\ n1 = 3, \tag{28}$$

and some others, corresponding to modifications (16), (17) for unequal $\mathbf{k}_1$ and $\mathbf{k}_2$.

4.2.5. For the multiple 3C-DIA[14], in the FDIA the most effective are the following joint three configurations

$$1)\ m3 = 4,\ m1 = m2 = 2; \quad \text{and} \quad n3 = 3,\ n2 = n1 = 2; \tag{29a}$$

$$2)\ m3 = 5,\ m1 = m2 = 3; \quad \text{and} \quad n3 = 4,\ n2 = n1 = 3; \tag{29b}$$

$$3)\ m3 = 7,\ m1 = m2 = 4; \quad \text{and} \quad n3 = 6,\ n2 = n1 = 5. \tag{29c}$$

5. FDIA parameters of configurations for very high resolution grid

5.1. Single conjurations. For applications which can be applied in the future, the following ***very high resolution grid*** is preferable

$$q = 1.05 \quad \text{and} \quad \Delta\theta = \pi / 18 \ (\text{or } \Delta\theta = 10^{\circ}). \tag{30}$$

Principal and auxiliary parameters for this grid are presented in Table 2.

Herewithin, in a case of the grid (30), the most efficient FDIA single configurations, which could be used in practice, are presented in Table 3 (for a proof of relative efficiency of these configurations among all other configurations, see [14]).

[4] Pay attention that the configuration has symmetry with respect to permutations $k_1 \leftrightarrow k_2$, that means permutation $(m2, n2) \leftrightarrow (m1, n1)$.

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Table 2. Principle and auxiliary parameters for grid (30). The shaded line has parameters most closely corresponding to the original DIA configuration used in the WAM.

$m3$	$\Delta\theta_{34}$, deg.	$\Delta\theta_{a4}$, deg.	$x = \frac{\log(\omega_a/2)}{\log(q)}$	$m2$=Int(x+0.5)
5	20.1	12.5	2.65	3
6	24.4	15.7	3.22	3
7	28.7	19.2	3.80	4
8	33.2	22.9	4.39	4
9	37.8	27.0	4.99	5
10	42.7	31.4	5.60	6
11	47.7	36.1	6.23	6
12	53.1	41.3	6.87	7
13	58.7	46.9	7.51	8
14	64.7	53.0	8.17	8
15	71.2	59.7	8.84	9

Table 3. Principle parameters for the set of the most efficient single configurations for grid (30)..

Index of configuration	$m3$ (general)	$m1$	$m2$	$n3$ (general)	na (general)	$n1$	$n2$
S1	8	4	5	3	2	2	2
S2	8	4	5	3	2	3	2
S3	9	5	5	4	3	3	3
S4*	9	4	5	4	3	3	2
S5	10	5	6	4	3	3	3
S6	10	6	6	4	3	3	3

Notes. 1. Index of configuration includes the symbol of the single configuration type, S, and the conventional number of configuration (for notations, see [14]).Supindex "*" means that the configuration has unequal $n1$ and $n2$

2. Configuration S6 is marked as the closest one to the original DIA configuration.

3. Parameters $m3$, $n3$, na are marked as "general" as far as $m3$ is independent parameter, and $n3$, na are directly defined by formulas (15a,b) and constant for a given $m3$.

5.2. Multiple constructions of single configurations.

Finally, we add that some multiple configurations (i.e., constructions of several single configurations[14]) which are more efficient than the simple configurations mentioned in Table 3. These constructions are presented in Table 4 and Table 5 given for auxiliary configurations.

Table 4. The set of the most efficient double-configuration constructions

Index of Construction	Composition of two simple configurations
M5	S1+ S8
M6	S1+ 0.7·S8*
M7	S1+ S10
M8	S1+ 0.7·S10

Note. The coefficient in front of configuration means the weight of a proper single configurations from Tables 3 and 5.

Table 5. Auxiliary simple configurations.

Index of configuration	$m3$ (general)	$m1$	$m2$	$n3$ (general)	na (general)	$n1$	$n2$
S8*	11	6	7	5	4	4	3
S10	12	7	7	5	4	4	4

Notes. For legend see notes for Table 3. Super-index "*" means that the configuration has unequal $n1$ and $n2$. These are the parameters of the auxiliary simple configurations used in Table 4.

5.3. Finally , it is worthwhile to mention one 3C-DIA construction parameters for the grid (30):

$$m3 = 8,\quad m1 = 4,\quad m2 = 5;\ n3 = 3,\quad n1 = n2 = na = 2;\qquad \text{(config. S1 from Tab. 2)} \tag{31a}$$

$$m3 = 10,\quad m1 = 5,\quad m2 = 6;\ n3 = 4,\quad n1 = n2 = na = 3;\qquad \text{(config. S5 from Tab. 2)} \tag{31b}$$

$$m3 = 12,\quad m1 = 7,\quad m2 = 7;\ n3 = 5,\quad n1 = n2 = na = 4;\qquad \text{(config. S10 from Tab. 5)} \tag{31c}$$

This construction is more effective than one in original DIA (see Table 6), but it is less effective than ones given in Table 4.

5.4. In {6] it was proposed some formulas for estimating the conventional efficiency of the DIA and FDIA aimed to their comparison. The values of conventional efficiency of the constructions and configuration mentioned above in this section are presented in Table 6 for completeness.

Table 6. Efficiency parameters for the constructions considered for the grid (30)

Index of construc-tion	S1	S2*	S3	S4*	S6 (original DIA)	M5	M6	M7	M8	3C-FDIA
Eff_1	5.26	6.07	4.98	5.82	4.3	-	-	-	-	-
Eff_2	-	-		-	-	6.57	6.43	6.43	6.39	4.4

Note. Values Eff_1 are applicable for simple configurations, whilst values Eff_2 do to two-configuration constructions [2].

6. Discussion

The DIA was proposed in 1985 [3], and for long time was unchanged for the reasons of complexity of the point. Some ideas of improving the DIA was declared in [15], but the radical step was made in [6], based on the own routine for the exact calculation of the kinetic integral [7]. This allows formulating the criterion of comparing an efficiency of different versions for DIA and its modifications. Finally, the idea of locating the interacting wave vectors at the nodes of the numerical grid was proposed and realized in [6]. It has happened that this modification provides not only an enhancing the speed of calculation of NL-term but the better accuracy, as well. The calculation speed is increased due to eliminating interpolation procedures in the original DIA, the better accuracy of FDIA is due to the better choice of the configuration.

This double positive effect is due the fact of rather crude efficiency of the original DIA, and better choice of the configuration [6]. The FDIA provides the increase of accuracy in 10%, and the speed of NL-term calculation is enhanced nearly twice. The tables of comparison the accuracy and time-consuming of FDIA and DIA are not given here to save the room of this paper. They are presented in the numerous early papers [6, 8, 11, 13].

Based on these results, the FDIA was implemented in the National Institute of Oceanography in India [12]. It is still left to spread this positive result to the new versions of the world-wide used models: WAM and WW. Present paper is namely aimed to prompt this implementation.

7. Conclusions

Details of the discrete interaction approximation (DIA) are presented and the concept of the fast DIA (FDIA) is comprehensively described.

Numerous versions of FDIA for different numerical grids are presented, including the single and multiple DIA configurations in a high resolution case.

The preference of the FDIA against the original DIA in accuracy and time-consuming are mentioned and explained. Some estimations of increased efficiency of the FDIA are shown.

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