Communication

New Results in 5D Theory and Some Problems of Astrophysics and Cosmology

Boris G. Aliyev [1]

Architecture dep., V.I. Surickov Moscow State Art Institute
Tovarishcheskij per. 30, Moscow, 109004, Russia, bgaliyev@mail.ru

Abstract: It is shown that the 5D Ricci identities give us a way to create a new viewpoint on the origin of the Maxwell equations, magnetic monopole problem, and also on some problems of the Astrophysics and Cosmology. Specifically, the application of the identities together with the monad and dyad methods makes it possible to introduce the new concept of the rest mass of the elementary particles. The latter leads to the new connections between the General Relativity and quantum field theories, as well as to a better understanding of the magnetic monopole problem and the origins of the Maxwell equations. The obtained results also provide a new insight into the mechanism of the accelerated expansion of the 4D Universe.

Keywords: monad and dyad methods; effective rest mass concept; scalar gravitational field; 5D geodetic equation; cylindrical symmetry condition; 5D Ricci identities; Maxwell equations; magnetic monopole; topological second-order transition in cosmology

1. Introduction

The article considers some old and new problems of modern physics, astrophysics, and cosmology in the framework of the 5D theory, using the well-developed and well-known monad and dyad methods [1] (pp. 184-207), [2,3] in General Relativity. Reviewing the papers of the different authors who have worked in the framework of the 5D theories, one can see that almost all of them [4-7] came to the 5D optics under the requirement that ratio e/m has to be constant. We demonstrate that relaxing this requirement leads to new, fundamental, unexpected, and very surprising consequences concerning the effective rest mass concept (ERM), the origins of the first and second pairs of the Maxwell equations, and the role of the 5D Ricci identities in the understanding of the some of these problems. Moreover, we hope that our findings can advance our current understanding of the magnetic monopole problem, as well as the other important problems of modern astrophysics and cosmology, including the ones of the dark matter and the dark energy.

2. The basic elements of the monad and dyad methods in 5D theory

Here I would like to represent briefly the basic information about the monad method in 5D theory with the chronometrical gauging of the monad vector and also about the dyad method with the mixed, chronometric and then, kinemetric gauging of the dyad vectors. These methods allow us to make the procedure of V_5-reducing: $V_5 \mapsto V_4 \mapsto V_3$ more correctly or, in other words, the orthogonal (4+1)-splitting and then (3+1+1)-splitting of the V_5 with the subsequent expressing of the 5D theory in terms of the 4D or 3D geometrical and physical values. In the framework of the monad method with the chronometrical gauging of the monad vectors we can represent the 5D metric G_{AB} as follows: $G_{AB} = g_{AB} - \lambda_A \cdot \lambda_B$ (here we have put the signature of the V_5 to be equal $(+----)$).

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[1] Communication address: Aliev, Boris; Altendorfer Str. 28, 09113, Chemnitz, Germany (Aliev, it is in a German manner and Aliyev, it is in an English manner).

The indexes $A,B,C,...=0,1,2,3,5.$ The 5D interval takes on the following form: $dI^2=ds^2-d\lambda^2$. Here $ds^2=g_{AB}\cdot dx^A\cdot dx^B$ and $d\lambda=\lambda_A\cdot dx^A$. The space-like monad vectors can be written as follows:

$$\lambda_A=\frac{G_{A5}}{\sqrt{-G_{55}}},\lambda^A=\frac{G_5^A}{\sqrt{-G_{55}}},\sqrt{-G_{55}}=\varphi\,.$$

The vector λ_A is tangential to the x^5 coordinate line and orthogonal to the space-time hypersurface V_4. Also g_{AB} is the metric tensor of V_4. Then, the orthonormal condition gives us: $\lambda^A\cdot\lambda_A=-1$; $\lambda^A\cdot g_{AB}=0;$ and, of course, we have the well-known relations: $G^{AB}\cdot G_{AB}=5$; $G^{AB}\cdot G_{BC}=\delta_C^A;$ $g^{AB}\cdot g_{AB}=4;$ $g^{AB}\cdot g_{BC}=\delta_C^A.$ Now we construct the monad differentiation operators: $\lambda^A\cdot\partial_A=\partial_\Lambda^+$ and $g_A^B\cdot\nabla_B=\overline{\nabla}_A^+$. Then, we can build the x^5 and V_4 projected directional normal and covariant derivatives. In the case of the dyad method the 5D metric and 5D interval take on the following form: $G_{AB}=\tau_A\cdot\tau_B-\lambda_A\cdot\lambda_B-h_{AB};$ $dI^2=d\tau^2-d\lambda^2-dl^2$; $d\tau=\tau_A\cdot dx^A;$ $dl^2=h_{AB}\cdot dx^A\cdot dx^B$. Now we can also write the orthonormal time-like (the case of the space-like vectors s λ_A was commented above) vectors of the dyad in kinemetric gauging as follows:

$$\tau^A=\frac{G^{0A}}{\sqrt{G^{00}}},\tau_A=\frac{G_A^0}{\sqrt{G^{00}}}\ ,\tau^A\cdot\tau_A=1,\lambda^A\cdot\tau_A=0,$$

and $\tau^A\cdot h_{AB}=\lambda^A\cdot h_{AB}=0.$ Note, that 3D metric of the V_3 has the following properties: $h^{AB}\cdot h_{AB}=3,h^{AB}\cdot h_{BC}=-\delta_C^A.$ Vector τ^A is the tangential one to the x^0 coordinate line. We can also build the same type of the differentiation operators: $\partial_\tau^+,\partial_\Lambda^+$ are the $\tau-$ and $\lambda-$ projected directional normal derivatives and $g_A^B\cdot\nabla_B=\overline{\nabla}_A^+-$ V_3 stand for the projected covariant derivative. We will mark these projected values with a tilde over the operators. See [1-3] for more details.

3. The new Rest Mass Concept and Some Cosmological Problems

Let us consider the geodetic equation in 5D theory, which has the standard form:

$$\frac{d^2x^A}{dI^2}=-P_{BC}^A\cdot\frac{dx^B}{dI}\cdot\frac{dx^C}{dI}\,. \qquad (1)$$

Here 5D interval $dI^2=G_{AB}\cdot dx^A\cdot dx^B$ and the indexes $A,B,C,...=0,1,2,3,5.$ G_{AB} is the 5D metric and P_{BC}^A stands for the 5D Christoffel symbols. Making here the (4+1)-splitting with the help of the monad method and eliminating the 5th coordinate, then imposing the cylindrical symmetry condition (CSC) along the 5th coordinate (it means that we have the Killing equations for the 5D metric G_{AB} along the tangent vector to the 5th coordinate $\xi_A=\varphi\cdot\lambda_A$: Lie derivative $L_\xi G_{AB}=\xi_{A;B}+\xi_{B;A}=0$) we may obtain from Eq. (1) the following system:

$$\frac{D^{+}(\varphi \cdot \hat{p})}{ds} = 0; \qquad (2)$$

$$\frac{D^{+}\overline{p}^{\alpha}}{ds} = \frac{Q_0}{c^2} \cdot \overline{u}^{\beta} \cdot F_{\beta \cdot}{}^{\alpha} + \partial^{\alpha +}\hat{m}_0. \qquad (3)$$

Here ,

$$\frac{D^{+}}{ds} = \overline{u}^{\alpha} \cdot \overline{\nabla}^{+}_{\alpha} - \hat{u} \cdot \partial^{+}_{\Lambda}.$$

The indexes $\alpha, \beta, \gamma, ... = 0,1,2,3.$

We have also used an integral of the Eq. (2) and inserted it to the Eq. (3). It should be also noted, that according to the monad method we have put here the ERM $\hat{m}_0$ of the Lorentz type in the following form (here m_0 is an initial rest mass of the 5D test particle and $\overline{p}^{\alpha} = \hat{m}_0 \cdot \overline{u}^{\alpha}$):

$$dI^2 = ds^2 - d\lambda^2, \hat{m}_0 = m_0 \cdot \hat{\beta}, \hat{\beta} = (1-\hat{u}^2)^{-1/2}, \hat{p} = \hat{m}_0 \cdot \hat{u}, \hat{u} = d\lambda / ds.$$

A use of the above-mentioned integral of Eq. (2) directly leads to the following exact expression for the ERM:

$$\hat{m}_0 = \sqrt{m_0^2 + \frac{Q_0^2}{4 \cdot k_0 \cdot \varphi^2}} = \sqrt{m_0^2 + \frac{n^2 \cdot \hat{m}_{Pl}^2}{\varphi^2}}, \qquad (4)$$

Here k_0 is the Newtonian gravitational constant, $Q_0 = n \cdot e$ (e is an electric charge of the electron), and also we have put here the 5D Plank mass $\hat{m}_{Pl} = e/(2 \cdot \sqrt{k_0})$. Expression (4) may be transformed to the more compact and convenient form by introducing the mass angle:

$$\chi_n = ar\sinh \frac{n \cdot \hat{m}_{Pl}}{m_0 \cdot \varphi} : \ \hat{m}_0 = m_0 \cdot \cosh \chi_n .$$

On the other side, we may represent this expression as a modulus of a complex quantity: $\hat{m}_0 = \sqrt{m_{0z} \cdot \overline{m}_{0z}}$, where $m_{0z} = m_0 + i \cdot n \cdot \hat{m}_{Pl} / \varphi$. The same may be written in the exponential form $m_{0z} = \hat{m}_0 \cdot e^{i \cdot \psi_n}$, where the phase $\psi_n = \arctan\{n \cdot \hat{m}_{Pl} /(m_0 \cdot \varphi)\}$. Here $\overline{m}_{0z}$ is a complex conjugate to the m_{0z} value. It motivates us to assume a possible connection with the quantum properties of the particles. In these terms, one may consider a photon as a complex null-particle $(0;0)$, a neutrino as a real particle $(m_\nu ;0),$ and a hypothetical tachyon as an imaginary particle $(0; m_\tau)$. It seems to be natural to extend out the equivalence principle of the

General Relativity on the scalar gravitational field (SGF) also, and because of it to put forward the hypothesis that every 5D particle must contain an electric charge. Then, the neutral 5D particles must be composite ones. In this sense, there are only two verily 4D particles: the photon and the neutrino. An additional argument to introduce the ERM is a fact that only in this case it is possible to do the further (3+1+1)-splitting in the 5D theory with an additional elimination of the time coordinate, using the dyad method [6]. Only in this case we obtain the well-known system of the geodetic equations, which is very familiar to the following system that we usually obtain in 4D theory, making the similar (3+1)-splitting of the 4D geodetic equation [8,9]:

$$\frac{D^+\hat{m}}{d\tau}=\hat{\bar{p}}^i\cdot(\bar{F}_i-\bar{v}^k\cdot\bar{D}_{ik})-\frac{Q_0}{c^2}\cdot\bar{v}^i\cdot\bar{E}_i+\frac{Q_0\cdot\sqrt{1-\bar{v}^2}}{2\cdot\sqrt{k_0}\cdot\varphi}\cdot\partial_\tau^+\hat{m}_0\,, \qquad (5a)$$

$$\frac{D^+\hat{\bar{p}}^i}{d\tau}=\frac{Q_0}{c^2}\cdot(\bar{E}^i-\bar{v}^k\cdot\bar{H}_{\cdot k}^{\ i})-\hat{m}\cdot\bar{F}^i+2\cdot\hat{\bar{p}}^k\cdot\bar{D}_k^i+\frac{Q_0\cdot\sqrt{1-\bar{v}^2}}{2\cdot\sqrt{k_0}\cdot\varphi}\cdot\partial^{+i}\hat{m}_0\ . \qquad (5b)$$

Here

$$\frac{D^+}{d\tau}=\partial_\tau^+ +\hat{v}\cdot\partial_\Lambda^+ +\bar{v}^i\cdot\bar{\nabla}_i^+\,.$$

The indexes $i,j,k,...=1,2,3.$.

In another case, it occurred that it is simply impossible to perform the (3+1)-splitting of the equation (3) because the equations (5a) and (5b) are interlaced so strongly that we have no chances to disconnect them. The last terms in equations (5a) and (5b) are the scalar 3-forces which are caused by the dependence of the ERM on the SGF. Here we have put $\hat{m}=\hat{m}_0/\sqrt{1-\bar{v}^2}$, $\bar{v}^i=d\bar{x}^i/d\tau$, $\hat{v}=d\lambda/d\tau$. Then, $\hat{\bar{p}}^i=\hat{m}\cdot\bar{v}^i$, $\hat{p}=\hat{m}\cdot\hat{v}=\hat{m}_0\cdot\hat{u}$, and $d\tau=c\cdot dt$. It should be noted also that the scalar 3-forces in equations (5a) and (5b) vanish for the zero rest mass particles, as it occurs in all of the scalar-tensor theories due to the factor $\sqrt{1-\bar{v}^2}$. The ERM $\hat{m}_0$ in the 5D theory appears just as the relativistic mass m in 4D theory:

$$dl\to ds\Rightarrow ds^2=d\tau^2-dl^2;m=m_0\cdot(1-v^2)^{-\frac{1}{2}}=m_0\cdot\beta;v=\frac{dl}{d\tau};d\tau=c\cdot dt$$

and it is easy to prove that the same result we obtain whenever we increase the dimension: $V_n\mapsto V_{n+1}$.

Thus, following the idea of P. Ehrenfest [10], one can see here that the increase in the dimension adds some new features to the physical nature of the particles.

The geodetic equation (3) we can easily bring to the following form:

$$\hat{m}_0\cdot\frac{D^+\bar{u}^\alpha}{ds}=f_L^\alpha+f_{BD}^\alpha. \qquad (6)$$

Here

$f_L^{\alpha} = \frac{Q_0}{c^2} \cdot \bar{u}^{\beta} \cdot F_{\beta \cdot}^{\alpha}$ is the 4D Lorenz force and $f_{BD}^{\alpha} = -\frac{Q_0^2 \cdot P^{\alpha\beta} \cdot \Phi_{\beta}}{4 \cdot k_0 \cdot \hat{m}_0 \cdot \varphi^2}$ is the 4D scalar one or, as we have proposed [9], the Brance-Dicke force. Here $P^{\alpha\beta} = g^{\alpha\beta} - \bar{u}^{\alpha} \cdot \bar{u}^{\beta}$ is a tensor of the orthogonal projection upon the 4D-velocity $\bar{u}^{\alpha}$ direction and $\Phi_{\beta} = \overline{\nabla}_{\beta}^{+} \ln \varphi$. It should be marked that the Lorentz force for big masses usually is equal to zero because it depends on the Q_0 linearly. But the Brance-Dicke force depends on the Q_0^2 and although it is very weak it can be very significant, being accumulated for the big masses (BME – the big masses effect). In addition, the scalar force is negative and has the repellent property. Thus, it may be one of the reasons of the Universe expansion. I believe also that the expression (4) for the ERM permits us to think that an electric charge e superposes simultaneously the role of the scalar charge, so it can explain to us why we could not find this scalar charge so far. Also, one can hope that an interplay between the SGF and an electric charge can explain how the dark matter may contribute to the total mass of the 4D Universe, which is immersed in some sense into the scalar ocean. It should be added also that the SGF contributes to the braking radiation force (BRF) [11] and permits us to generalize the concept of the one: $g^{\alpha} = g_E^{\alpha} + g_S^{\alpha} + g_{ES}^{\alpha}$. Here we should tell the total BRF may be represented as a sum of the electromagnetic, scalar and mixed parts, where the first one has the following form:

$$g_E^{\alpha} = \frac{2 \cdot e^3 \bar{u}^{\gamma}}{3 \cdot \hat{m}_0 \cdot c^3} \cdot \left(\bar{u}^{\beta} \cdot \overline{\nabla}_{\gamma}^{+} F_{\beta \cdot}^{\alpha} + \frac{e \cdot P^{\alpha\delta}}{\hat{m}_0 \cdot c^2} \cdot F_{\beta\delta} \cdot F_{\gamma \cdot}^{\beta} \right);$$

and the second one has the form as below:

$$g_S^{\alpha} = -\frac{e^4 \cdot P^{\alpha\beta} \cdot \bar{u}^{\gamma}}{6 \cdot c \cdot k_0 \cdot \varphi^2 \cdot \hat{m}_0^2} \cdot \left(\overline{\nabla}_{\gamma}^{+} \Phi_{\beta} - 2\Phi_{\beta} \cdot \Phi_{\gamma} + \frac{3 \cdot e^2 \cdot \Phi_{\beta} \cdot \Phi_{\gamma}}{4 \cdot k_0 \cdot \phi^2 \cdot \hat{m}_0^2} \right);$$

And at last the third one has the next form:

$$g_{ES}^{\alpha} = -\frac{e^5}{6 \cdot c^3 \cdot k_0 \cdot \varphi^2 \cdot \hat{m}_0^3} \cdot \left(g^{\beta\delta} \cdot P^{\alpha\gamma} - 3 \cdot g^{\alpha\gamma} \cdot \bar{u}^{\beta} \cdot \bar{u}^{\delta} \right) \cdot F_{\beta\gamma} \cdot \Phi_{\delta} .$$

See [9] for more details.

4. The 5D Ricci identities and some problems of the Astrophysics and Cosmology

We should remember some interesting and perspicacious ideas of the scientific classics about the connections between world geometry and physical interactions. Following one of the greatest mathematicians of the XIXth century - W.K. Clifford [12] – let us consider the well-known in Riemannian geometry Ricci identities in the case of the 5D theory:

$$R_{\cdot BCD}^{A} + R_{\cdot DBC}^{A} + R_{\cdot CDB}^{A} = 0 \quad . \qquad (7)$$

The (4+1)-splitting of the V_5 gives us the following relations between the 4D physical and geometrical values [9] starting with the 4D Ricci identities:

$$R^{\alpha}_{\cdot\beta\gamma\delta} + R^{\alpha}_{\cdot\delta\beta\gamma} + R^{\alpha}_{\cdot\gamma\delta\beta} = 0 \quad . \tag{8}$$

Then, we can obtain the following very important connection between the electromagnetic tensor and the curls of the SGF gradients. It gives us a very specific trigger of the changing electromagnetic process:

$$\sqrt{\overline{\kappa}_0} \cdot \partial^{+}_{\Lambda}\varphi \cdot F_{\alpha\beta} = \Phi_{\alpha;\beta} - \Phi_{\beta;\alpha} = -m_{\alpha\beta} \, . \tag{9}$$

Here $\overline{\kappa}_0 = 4 \cdot k_0 / c^4$ and the SGF curls $m_{\alpha\beta}$ we may consider as the quasi-particles of the magnetic monopole type. In some sense, these quasi-particles are similar to solitons [13,16]. Importantly, that imposition of the CSC gives rise to the vanishing of these scalar curls, since only in this case the necessary and sufficient condition for SGF to be laminar ($\Phi_{\alpha;\beta} = \Phi_{\beta;\alpha}$) may be satisfied.

At last, making use of Eq. (9), we obtain the first pair of the Maxwell equations with a nonzero r.h.s. of the magnetic monopole type, namely:

$$F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = \frac{2}{\sqrt{\overline{\kappa}_0} \cdot \partial^{+}_{\Lambda}\varphi} \cdot \begin{vmatrix} \Phi_{\alpha} & \nabla^{+}_{\alpha} & \Phi_{\alpha} \\ \Phi_{\beta} & \nabla^{+}_{\beta} & \Phi_{\beta} \\ \Phi_{\gamma} & \nabla^{+}_{\gamma} & \Phi_{\gamma} \end{vmatrix} . \tag{10}$$

Imposition of the CSC here also gives rise to the vanishing of the r.h.s of Eq. (10), which reduces this equation to the conventional form:

$$F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = 0 \, . \tag{11}$$

Thus, we can conclude that the first pair of the Maxwell equations is obliged to the Riemannian structure of the V_5 and has another origin than the second one. The r.h.s. R in Eq. (10) we can rewrite in a more convenient form:

$$R = \frac{2}{\sqrt{\overline{\kappa}_0} \cdot \partial^{+}_{\Lambda}\varphi} \cdot \left(\Phi_{\alpha} \cdot m_{\beta\gamma} - \Phi_{\beta} \cdot m_{\alpha\gamma} + \Phi_{\gamma} \cdot m_{\alpha\beta} \right). \tag{12}$$

This gives us the possibility to represent the second pair of the Maxwell equations, which we have obtained from the 5D variation principle [15], with the same monopole type of the r.h.s., if we believe that R is the linear combination of the magnetic monopoles currents. With this purpose we write down the mixed, x^5 - and V_4 - projected, 5D field equations with the energy-momentum tensor of the 5D dust as a r.h.s.:

$$\overline{\nabla}^{+}_{\nu}(\varphi^{3} \cdot F^{\mu\nu}) = -\frac{8 \cdot \pi}{c^{2}} \cdot \sqrt{k_0} \cdot \varphi \cdot Q^{\mu}_{5} \, . \tag{12}$$

Here

$$Q^{AB} = \mu_0 \cdot c \cdot \frac{dx^{A}}{dI} \cdot \frac{dx^{B}}{d\tau} \quad ,$$

where μ_0 is a matter density [15]. But before we have not imposed the CSC, we may easily bring the Eq. (12) to the following form (further we will avoid to mark the projected values with a tilde), casting temporarily aside the r.h.s. in Eq. (12) :

$$\nabla^{+}_{\nu} F^{\mu\nu} = 3 \cdot \Phi_{\nu} \cdot m^{\nu\mu} = -\frac{4 \cdot \pi}{c} \cdot j^{\mu}_{m} \ . \qquad (13)$$

Here we can interpret j^{μ}_{m} as a magnetic monopole current, where we can believe that curl $m_{\mu\nu}$ corresponds, for example, to the «north»-particle of the magnetic monopole type and $m_{\nu\mu}$ - to the «south» one, or vice versa. Let us call them «n-monopole» and «s-monopole». Thus, we believe that there exist the two types of the magnetic monopole «charges»: n and s ones.

Finally, we can establish that just after we have imposed the CSC, as a result of it, the SGF curls disappear and the Eq. (2) gives us an integral of the movement along the 5th coordinate connected with an electric charge of the 5D test particle, so thus the energy-momentum tensor of the 5D dust (the r.h.s. in Eq. (12)) easily transforms in the second pair of the Maxwell equations and gives the new r.h.s. of the conventional type as follows below [15]:

$$\nabla^{+}_{\nu} F^{\mu\nu} = -\frac{4 \cdot \pi}{c \cdot \varphi^{2}} \cdot j^{\mu}_{e} \, . \qquad (14)$$

Here $j^{\mu}_{e} = \rho_0 \cdot \bar{v}^{\mu}$ is the 4D electric current (ρ_0 is an electric charge density).

The analysis of these results leads us to the fundamental idea about the evolution of the 4D Universe. The process of the transition to the CSC in the 4D Universe seems to be very familiar with the second-order transition in liquid helium. Then, we may hypothesize that here we have some kind of a second-order phase transition in cosmology. Let us call it the topological one [16]. It is quite possible that this transition is connected with the cooling of the expanding Universe. Also, we can suppose that, as a result of this transition, we have some kind of the superfluidity state of the scalar matter and it may accelerate the expansion of the 4D Universe [16]. This acceleration, as well known, was discovered more than twenty years ago with the help of the observations after the type Ia supernovas explosions. Also, we may add that this transition and, then, further compactification of the 5th coordinate is possibly caused by the Casimir effect [17]. Besides it, one can assume that in modern times we cannot find the magnetic monopoles in the 4D Universe, maybe only a few of the relict ones [9,18]. Also one can hypothesize, basing on the expression (4), that the ERM of the particles possibly varies with the cosmological time, depending on the scalar φ, which is connected with the 5D metric G_{55}. In the work [19] the authors have considered the 5D metric of the Kasner type, which depends on the cosmological time.

5. Discussion

This article was written because for a long time the author has accumulated a lot of very interesting results in the 5D theory of the Kaluza type, which has been almost forgotten during the last fifty years. It has occurred that if one goes beyond the 5D optics, certain new and very non-trivial properties of the matter and 4D Universe may appear. Following this approach, the author has succeeded to generalize the rest mass concept and understanding more deeply the quantum nature of the matter. The implementation of the monad method the (4+1)-splitting of the 5D Ricci identities makes it possible to understand how the Riemannian structure of the World affects its physical properties. It permits one to approach closer to the understanding of the magnetic monopole problem and the origins of the Maxwell equations. The obtained results also provide new insight into the mechanism of the accelerated expansion of the 4D Universe. The author believes that the application of the developed approach extends far beyond the specific problem discussed in the present paper. Its generalization beyond the 5D case may be also worth-while.

6. Conclusions

Finally, the author would like to express the hope that these results may open a new page in the investigations of the 5D theory and will attract more attention of the physical community.

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References

1. Vladimirov, Yu.S. *Classical gravity theory*; Krasand: Moscow, Russia, 2018; pp. 184-207.
2. Aliyev, B.G. Motion equations in the 5D unified field theory. In *Abstracts of the IX-th Int. Conf. on Gen. Relat. Grav.*, Vol. 3, Jena, Germany (GDR), 1980, p. 679.
3. Aliyev, B.G. 5D theory of the scalar-tensor gravitation and electromagnetism in dyad form. In *Problems of the theory of gravitation and elementary particles*, Stanyuckovich K.P., Ed.; Atomizdat: Moscow, Russia, 1979; issue 10, pp. 141-149, (in Russian).
4. Fock, V.A. Über die invariante Form der Wellen- und der Bewegungsgleichungen für einen geladenen Massenpunkt. *Zeitschr. für Physik*, **1926**, *Bd 39*, S. 226-232.
5. Klein, O. Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschr. für Physik*, **1926**, *Bd 37*, S. 895.
6. De Broglie, L. L'Univers a sinq dimensions et la mecanique ondulatoire. *J. de Physique ,Serie VI* **1927**, *Vol. 8*, № 2, p. 65.
7. Rumer, Yu.B. *Investigations on 5D Optics*; State Pablishing House of the Technical and Theoretical Literature: Moscow, Russia, 1956, (in Russian).
8. Aliyev, B.G. The behavior of the charged particles in 5D gravity theory. In *The modern problems of the general relativity theory*, Physical Institute of the Byelorussian Academy of Sciences: Minsk, Byelorussian republic, 1979; pp. 154-160, (in Russian).
9. Aliyev, B.G. The effective rest mass concept and magnetic monopole problem in 5D Theory. In Gravitation, Astrophysics, and Cosmology, Proceedings of the ICGAC-12, Moscow, Russia, June 28-July 4, 2015; Melnikov, V. and Jong-Ping Hsu, Eds; World Scientific: Singapore, 2016, pp.321-326.
10. Ehrenfest, P. In what way does it become manifest in the fundamental laws of Physics that space has three dimensions. In *Proc. Amsterdam Ac.* **1917**, *v. 20*, p. 200.
11. Landau, L.D. and Lifshitz, E.M. *The classical theory of fields, v.2, 3th Ed.;*: Pergamon Press, Maxwell House: New York-London, USA-GB, 1971.
12. Clifford, W.K. *Mathematical Papers;* MacMillan: New York-London, 1968.
13. Gross, D., Perry M. Magnetic monopoles in Kaluza-Klein theories. *Nucl. Phys. B*, **1983**, *v. 226*, p.29.
14. Aliyev, B.G. The rest mass concept and some problems of Cosmology in 5D Theory. In *Abstracts of the RUSGRAV-16 (16th Rus. Int. Conf. on Gen. Relat. Grav.);*, BFU named I. Kant: Kaliningrad, Russia, 2017, p. 91.

15. Aliyev, B.G. On the energy-momentum tensor of the 5D dust. In *Abstracts of the reports of the Int. school-seminar "Multidimensional Gravity and Cosmology"*; RGA (Rus. Grav. Assoc.): Moscow, Russia, 1994, p. 1.
16. Aliyev, B.G. The solitons and the topological second-order transition in 5D Theory. In *Abstracts of the RUSGRAV-16 (16th Rus. Int. Conf. on Gen. Relat. Grav.),* BFU named I. Kant: Kaliningrad, Russia, 2017, p. 91.
17. Mostepanenko, V.M. and Trunov, N.N. *Casimir Effect and its Applications;* Oxford University Press: Oxford, GB, 1997; pp. 191-193.
18. Chodos, A., Detweiler, S. Where has the fifth dimension gone? *Phys.Rev. D* **1980**, *v. 21*, p.2167.
19. Aliyev, B.G. Where has the magnetic monopole gone? In *Abstracts of the ICGAC-12, Moscow, Russia, June 28-July 4, 2015;* PFUR: Moscow, Russia, 2015, p. 110.