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Communication

The solution of the cosmological constant problem: the cosmological constant exponential decrease in the super-early Universe

Ol'ga Babourova[1] and Boris Frolov[2*]

[1] Moscow Automobile and Road Construction State Technical University (MADI), Department 'Physics', Leningradsky pr., 64, Moscow 125319, Russia; ovbaburova@madi.ru
[2] Moscow Pedagogical State University (MPGU), Institute of Physics, Technology and Information Systems, M. Pirogovskaya ul., 29, Moscow 119992, Russia; bn.frolov@mpgu.su
[*]Correspondence: frolovbn@mail.ru; Tel.: +7-903-267-7107

Abstract: The stage of a super-early scale-invariant Universe is considered on the basis of the Poincaré–Weyl gauge theory of gravity in a Cartan–Weyl space-time. An approximate solution has been found that demonstrates an inflationary behavior of the scale factor and, at the same time, a sharp exponential decrease in the effective cosmological constant from a huge value at the beginning of the Big Bang to an extremely small (but not zero) value in the modern era, which solves the well-known "cosmological constant problem".

Keywords: Poincaré–Weyl gauge theory, super-early Universe, effective cosmological constant

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1. Introduction

In [1] the gravitational field as the field of curvature and torsion was introduced as the gauge field of the localized Poincaré group. This led to the construction of the Einstein-Cartan theory of gravity [2], based on the Lagrangian in the form of a scalar of curvature of the Riemann-Cartan space.

The next step towards the construction of a Poincaré-gauge theory of gravity was made in [3–5], where Lagrangians quadratic in the curvature tensor were introduced, and the theorem on the sources of a gauge field was formulated and proved in general form, as well as in [6,7], where in addition Lagrangians quadratic in the torsion tensor where also introduced (for the Poincaré-gauge theory of gravity, see [8–12]).

Then in [13–17], the Poincaré–gauge local symmetry were supplemented by transformations of the Weyl symmetry subgroup – stretching and contraction (dilatations) of space-time. In [15–17], the gauge theory of the Poincaré–Weyl group was developed. In this theory, tetrad coefficients (which cannot be gauge fields, since they are tensor components), as well as connection, are some functions of real translational and rotational gauge fields.

It was shown that at this theory the space-time acquires the properties of a Cartan–Weyl space with a curvature 2-form $\mathcal{R}^a{}_b$ and a torsion 2-form $\mathcal{T}^a$, as well as a nonmetricity 1-form $\mathcal{Q}_{ab}$ with the Weyl condition, $\mathcal{Q}_{ab} = (1/4)\, g_{ab}\, \mathcal{Q}$. In this case, in addition to the metric tensor, a scalar field β appears, which by its properties coincides with the scalar field introduced earlier by Dirac [18].

In [19–21], it was hypothesized that the group of symmetry of space-time at the super-early stage of the evolution of the Universe – the epoch of the Big Bang and subsequent inflation – was not the Poincaré group, but the Poincaré–Weyl group. At this stage, the rest masses of elementary particles had not yet appeared, all interactions were carried out by massless quanta, and therefore these interactions had the property of scale invariance.

The proposed hypothesis is based on the assumption made by Harrison and Zel'dovich about the approximate scale invariance of the early stage of the evolution of the Universe, which underlies the calculation of the initial part of the spectrum of primary fluctuations of matter density in the early Universe (Harrison-Zel'dovich Plateau, see [22]) and have been confirmed by results of the COBE experiment for the study of the anisotropy of the brightness of the background radiation.

2. Lagrangian density and field equations

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When terms with squares of curvature have been omitted, then the Lagrangian density 4-form of the Poincaré–Weyl gauge theory of gravity has the form [20–22],

$$\begin{aligned}\mathcal{L} = 2f_0\beta^2\Big[(1/2)\mathcal{R}^a{}_b\wedge\eta_a{}^b - \beta^2\Lambda_0\eta + \rho_1\mathcal{T}^a\wedge *\mathcal{T}_a + \rho_2(\mathcal{T}^a\wedge\theta_b)\wedge *(\mathcal{T}^b\wedge\theta_a)\\ + \rho_3(\mathcal{T}^a\wedge\theta_a)\wedge *(\mathcal{T}^b\wedge\theta_b) + \xi\mathcal{Q}\wedge *\mathcal{Q} + \zeta\mathcal{Q}\wedge\theta^a\wedge *\mathcal{T}_a + l_1\beta^{-2}\mathrm{d}\beta\wedge *\mathrm{d}\beta\\ + l_2\beta^{-1}\mathrm{d}\beta\wedge\theta^\alpha\wedge *\mathcal{T}_\alpha + l_3\beta^{-1}\mathrm{d}\beta\wedge *\mathcal{Q}\Big] + \beta^4\Lambda^{ab}\wedge(\mathcal{Q}_{ab} - (1/4)g_{ab}\mathcal{Q}),\end{aligned} \tag{1}$$

where $*$ is the Hodge dualization operation, Λ^{ab} are Lagrange multipliers, η is a volume 4-form with components η_{abcd}, and $\eta_{ab} = (1/2!)\theta^c\wedge\theta^d\eta_{abcd}$.

According to E. Gliner [24,25], the cosmological constant Λ in the Einstein equations is interpreted as the energy of the physical vacuum. The term $\beta^2\Lambda_0$ in (1) describes the effective cosmological constant [20–22] (dark energy, the energy of the physical vacuum), depending on the Weyl-Dirac scalar field β (Λ_0 is the theory parameter providing the correct rate of inflation). The equality $\beta_0^2\Lambda_0 = 10^{120}\Lambda$ (see [26]) should be fulfilled, where Λ is the modern value of the cosmological constant, and β_0 is the value of the Weyl-Dirac scalar field at $t=0$.

The derivation of the variational field equations is based on the use of the commutation lemma on variation and dualization operations of external forms in the Cartan–Weyl space, proved in [20,27] and then modified taking into account the presence of a scalar Weyl–Dirac field. As a result of the variation procedure, we have obtained three field equations: Γ-equation, θ-equation, and β-equation, which one can see in Appendix.

3. Cosmological model for the super-early Universe

Let us consider the homogeneous, isotropic and spatially flat Universe with the Friedman–Robertson–Walker (FRW) metrics,

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{2}$$

and let us solve for this metrics the Poincaré–Weyl gravity field equations (A1)-(A3) for the scale invariant super-early stage of the Universe evolution.

It is known from previous works [20–22,29–31] that for this metrics the 2-form of torsion $\mathcal{T}_a = (1/3)\mathcal{T}\wedge\theta_a$ is completely determined by the 1-form of its trace $\mathcal{T} = *(\theta_c\wedge *\mathcal{T}^a)$. It is also known that in this case the anti-symmetric part of the Γ-equation (A1) is equivalent to the equations,

$$2(\rho_1 - 2\rho_2 - 1)\mathcal{T} + 3(1/4+\zeta)\mathcal{Q} = (6-3l_2)(\mathrm{d}\beta/\beta), \tag{3}$$

$$2(\rho_1 - 2\rho_2 + 4\zeta)\mathcal{T} + (16\xi + 3\zeta)\mathcal{Q} = (-3l_2 - 8l_3)(\mathrm{d}\beta/\beta). \tag{4}$$

As a result of the equations (3), (4), the 1-forms of the torsion trace $\mathcal{T}$ and the nonmetricity trace $\mathcal{Q}$ are equal to

$$\mathcal{T} = s(\mathrm{d}\beta/\beta), \qquad \mathcal{Q} = q(\mathrm{d}\beta/\beta). \tag{5}$$

Here the numbers s, q are determined by the coupling constants of the Lagrangian density (1). For these numbers we accept the condition,

$$\frac{q}{8} - \frac{s}{3} = 1, \tag{6}$$

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which was also accepted when finding spherically and axially symmetric solutions of the of Poincaré–Weyl theory of gravity [29–31].

As a result we obtain that the only non-null expressions of the θ-equations (A2) are

$$3H^2+6HU+U^2\left(k^2+3\right)=\Lambda_0\beta^2, \tag{7}$$

$$2H_t+2U_t+4HU+3H^2-U^2\left(k^2-1\right)=\Lambda_0\beta^2, \tag{8}$$

where we have introduced notations,

$$k^2=l_1+(1/2)sl_2+(1/2)ql_3\ , \tag{9}$$

$$H=a_t/a,\quad a_t=da/dt,\quad U=\beta_t/\beta,\quad \beta_t=d\beta/dt, \tag{10}$$

and the only non-null expression of the β-equations (A3) is

$$3H_t+6H^2+\left(U_t+U^2\ +3HU\right)\left(k^2+3\right)=2\Lambda_0\beta^2. \tag{11}$$

4. Cosmological solution for the super-early Universe

Now we solve the system of equations obtained in the previous section. Let us subtract the equation (7) from the equation (8). As a result we obtain,

$$H_t+U_t-HU-U^2\left(k^2+1\right)=0. \tag{12}$$

Then after subtracting the double equation (8) from the equation (11) and taking into account the equation (12), we obtain the equation

$$k^2\left(U_t+3HU+2U^2\right)=0,\qquad k^2\neq 0. \tag{13}$$

When $k^2\neq 0$, we obtain for two unknown functions $a(t)$ and $\beta(t)$ three equations (7), (12) and (13). Thus, the system of equations is over-determined.

Let us estimate the value of k^2. Based on the calculation of the flyby anomaly (upon receipt of the spherically symmetric solution [29,30]), the following estimate was obtained,

$$k^2\approx 10^{-20}\div 10^{-22}\,. \tag{14}$$

Therefore, in equations (7) and (12), we may neglect the terms with the coefficient k^2. Then the equation (7) will be reduced to the equation

$$H+U=\left(\lambda/3\right)\beta,\quad \lambda=\sqrt{3\Lambda_0}\,. \tag{15}$$

Here from the two possible signs, we take the “+” sign.

In this case the equation (12) with regard to (14) turns out to be a consequence of the equation (15). The system of equations reduces to the two equations (13) and (15) for two unknown functions $a(t)$ and $\beta(t)$, and the system of equations is becoming well defined.

It is easy to understand that in case $k^2=0$ the system of equations (7), (12) and (13) turns out to be under-determined, because in this case there is only one equation (15) for two unknown functions.

Equation (13) with regard to (15) is reduced to the equation,

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$$U_t + \lambda\beta U - U^2 = 0. \tag{16}$$

which is equivalent to the equation

$$\beta\beta_{tt} + \lambda\beta_t\beta^2 - \beta_t^2 = 0. \tag{17}$$

which is integrated by substitution $\beta_t = \beta^2 z(\beta)$. The first integral of this equation is

$$U = \beta_t/\beta = -\lambda\beta\ln(C\beta). \tag{18}$$

Here C is an integration constant.

We obtain the second equation of the system after substituting this solution into equation (15),

$$H = a_t/a = \lambda\beta\left(\ln(C\beta) + 1/3\right). \tag{19}$$

A boundary condition is

$$C\beta \to 1 \quad \text{when} \quad t \to \infty. \tag{20}$$

The approach to the limit value $C\beta \to 1$ for large t occurs exponentially, $\sim e^{\frac{-\lambda t}{C}}$.

Then we impose a requirement that the value of the effective cosmological constant $\beta^2(t)\Lambda_0$ coincides with the modern value of the cosmological constant:

$$\beta^2(t_U)\Lambda_0 = \Lambda, \qquad t_U = 13.\ 8 \text{ billion years}, \tag{21}$$

where t_U is the Universe lifetime. This requirement determines the value of the constant C.

The result of numerical integration of the system of equations (18) and (19) with the boundary condition (20) is presented graphically in [32], where can be seen for small and large times t the inflationary behavior of the scale factor $a(t)$, as well as for small times t a sharp exponential decrease in the effective cosmological constant $\beta^2(t)\Lambda_0$ from a huge value at the beginning of the Big Bang to an extremely small (but not zero) value in the modern epoch, which coincides with its observed value.

5. Conclusions

We proposed (following Harrison and Zel'dovich idea) that the Universe at the super-early stage had the property of scale invariance. In this case the symmetry group of the space-time is the Poincaré–Weyl group, and space-time is the Cartan–Weyl space with curvature and torsion 2-forms, as well as nonmetricity 1-form with the Weyl condition. And also, an addition to the metric tensor, a Weyl–Dirac scalar field β appears, which determines the dark energy amount.

The theory constructed is applicable to the super-early stage of a homogeneous Universe with the FRW metrics with a scale factor $a(t)$. We obtain two equations, (18) and (19), and using the numerical integration, we obtain the sharp exponential decrease of the effective cosmological constant $\beta^2\Lambda_0$ at small t and the exponential increase of the scale factor (Figure 1 and Figure 2) [32]. In the proposed theory, the inflation problem acquires new features: inflation becomes scale-invariant and the Weyl-Dirac scalar field becomes the infraton.

The result obtained can be considered as a solution to one of the main contradictions of the theory of the Universe evolution, which is called the "problem of the cosmological constant" [26,33–35]. The essence of this problem is a huge (about 120 order) difference between the value of the physical vacuum energy, described by the cosmological constant, in the initial stage of the Universe evolution (determined on the basis of quantum field theory) and its value, determined on the basis of modern observational data.

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The hypothesis on the sharply decrease of the effective cosmological constant as a consequence of the fields dynamics in the super-early Universe was expressed in [19–22]. The main thing of the solutions obtained is that these solutions demonstrates sharply decrease in the effective cosmological constant at small t and approach the modern value of the cosmological constant (but not zero) at large values of t.

The problem of the cosmological constant is one of the important problems of modern fundamental physics [35]. The solution of this problem will allow to reconcile the theory of the evolution of the Universe with modern physical concepts.

Appendix

Γ-equation:

$$\begin{aligned} & f_0\Big[-(1/4)\mathcal{Q}\wedge\eta_a{}^b+(1/2)\mathcal{T}_c\wedge\eta_a{}^{bc}+(1/2)\eta_{ac}\wedge\mathcal{Q}^{bc}+\mathrm{d}\ln\beta\wedge\eta_a{}^b+ \\ & +2\rho_1\theta^b\wedge*\mathcal{T}_a+2\rho_2\theta^b\wedge\theta_c\wedge*(\mathcal{T}^c\wedge\theta_a)+2\rho_3\theta^b\wedge\theta_a\wedge*(\mathcal{T}^c\wedge\theta_c)+ \\ & +4\xi\delta_a^b[2,\ 3,\ 4]\mathcal{Q}+\zeta(2\delta_a^b\theta^c\wedge*\mathcal{T}_c+\ \theta^b\wedge*(\mathcal{Q}\wedge\theta_a))+ \\ & +l_2\theta^b\wedge*(\mathrm{d}\ln\beta\wedge\theta_a)\ +\ l_3 2\delta_a^b*\mathrm{d}\ln\beta\Big]-\beta^2\Lambda_a{}^b=0\,. \end{aligned} \tag{A1}$$

θ-equation:

$$\begin{aligned} & \frac{1}{2}\mathcal{R}^b{}_c\wedge\eta_b{}^c{}_a\ -\ \beta^2\Lambda_0\eta_a \\ & +\rho_1\big[2\mathrm{D}*\mathcal{T}_a+\mathcal{T}_b\wedge*(\mathcal{T}^b\wedge\theta_a)+*(*\mathcal{T}^b\wedge\theta_a)\wedge*\mathcal{T}_b+4\mathrm{d}\ln\beta\wedge*\mathcal{T}_a\big] \\ & +\rho_2\big[2\mathrm{D}(\theta_b\wedge*(\mathcal{T}^b\wedge\theta_a))+2\mathcal{T}^b\wedge*(\theta_b\wedge\mathcal{T}_a)-\ *(\mathcal{T}^b\wedge\theta_c\wedge\theta_a)(\mathcal{T}^c\wedge\theta_b) \\ & -*(*(\mathcal{T}^c\wedge\theta_d)\wedge\theta_a)\wedge*(\mathcal{T}^d\wedge\theta_c)+\ 4\mathrm{d}\ln\beta\wedge\theta_b\wedge*(\mathcal{T}^b\wedge\theta_a)\big] \\ & +\rho_3\big[2\mathrm{D}(\theta_a\wedge*(\mathcal{T}^b\wedge\theta_b))+2\mathcal{T}_a\wedge*(\mathcal{T}^b\wedge\theta_b)-*(\mathcal{T}^b\wedge\theta_b\wedge\theta_a)(\mathcal{T}^c\wedge\theta_c) \\ & -*(*(\mathcal{T}^b\wedge\theta_b)\wedge\theta_a)\wedge*(\mathcal{T}^c\wedge\theta_c)+\ 4\mathrm{d}\ln\beta\wedge\theta_a\wedge*(\mathcal{T}^b\wedge\theta_b)\big] \\ & +\ \xi\big[-\mathcal{Q}\wedge*(\mathcal{Q}\wedge\theta_a)-*(*\mathcal{Q}\wedge\theta_a)*\mathcal{Q}\big] \\ & +\zeta[\mathrm{D}*(\mathcal{Q}\wedge\theta_a)-\mathcal{Q}\wedge*\mathcal{T}_a+\mathcal{Q}\wedge\theta^b\wedge*(\mathcal{T}_b\wedge\theta_a) \\ & +*(*\mathcal{T}_b\wedge\theta_a)\wedge*(\mathcal{Q}\wedge\theta^b)+2\mathrm{d}\ln\beta\wedge*(\mathcal{Q}\wedge\theta_a)] \\ & +l_1[-\mathrm{d}\ln\beta\wedge*(\mathrm{d}\ln\beta\wedge\theta_a)-*(*\mathrm{d}\ln\beta\wedge\theta_a)\wedge*\mathrm{d}\ln\beta)] \\ & +l_2[\mathrm{D}*(\mathrm{d}\ln\beta\wedge\theta_a)+\mathrm{d}\ln\beta\wedge\theta^b\wedge*(\mathcal{T}_b\wedge\theta_a)-\mathrm{d}\ln\beta\wedge*\mathcal{T}_a \\ & +*(*\mathcal{T}_b\wedge\theta_a)\wedge*(\mathrm{d}\ln\beta\wedge\theta^b)\ +\ 2\mathrm{d}\ln\beta\wedge*(\mathrm{d}\ln\beta\wedge\theta_a)] \\ & +\ l_3[-\mathrm{d}\ln\beta\wedge*(\mathcal{Q}\wedge\theta_a)\ -\ *(*\mathcal{Q}\wedge\theta_a)*\mathrm{d}\ln\beta]=0\,. \end{aligned} \tag{A2}$$

β-equation:

$$\begin{aligned} & \mathcal{R}^a{}_b\wedge\eta_a{}^b-4\beta^2\Lambda_0\eta\ +\rho_1 2\mathcal{T}^a\wedge*\mathcal{T}_a+\rho_2 2(\mathcal{T}^a\wedge\theta_b)\wedge*(\mathcal{T}^b\wedge\theta_a) \\ & +\ \rho_3 2(\mathcal{T}^a\wedge\theta_a)\wedge*(\mathcal{T}^b\wedge\theta_b)+\xi 2\mathcal{Q}\wedge*\mathcal{Q}+\zeta 2\mathcal{Q}\wedge\theta^a\wedge*\mathcal{T}_a \\ & +\ l_1(-2\mathrm{d}*\mathrm{d}\ln\beta-2\mathrm{d}\ln\beta\wedge\mathrm{d}\ln\beta)\ +\ l_2(-\mathrm{d}(\theta^a\wedge*\mathcal{T}_a))\ +l_3(-\mathrm{d}*\mathcal{Q})\ =\ 0\,. \end{aligned} \tag{A3}$$

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