ENTROPY GENERATION FOR MHD HEAT AND MASS TRANSFER OVER A NON-ISOTHERMAL STRETCHING SHEET WITH VARIABLE VISCOSITY

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This work probes the combined effects of magnetic field and viscous dissipation on heat field and examine the second law analysis (entropy generation) in an electrically conducting fluid under the effect of wall mass transfer over continuous stretched non-isothermal surface with variable viscosity. The viscosity of the fluid is assumed to be an inverse linear function of temperature. The governing equation for the problem are changed to dimensionless ordinary differential equations by using similarity transformation and solved numerically by using Rung Kutta and Shooting technique. Velocity, concentration and temperature distribution are obtained and used to compute the entropy generation and the Bejan number in the flow field. The effect of variable viscosity, Schmidt number, Hartman and Reynolds number on the velocity, concentration, temperature, entropy generation and Bejan number are studied and discussed.

Key words: Entropy generation, heat and mass transfer, Stretching sheet, variable viscosity

Introduction

Fluid flow over a stretching sheet is important in many practical applications such as extrusion of plastic sheets, paper production, glass blowing, metal spinning, polymers in metal spring processes, the continuous casting of metals, drawing plastic films and spinning of fibers, all involve some aspects of flow over a stretching sheet or cylindrical fiber (Paullet and Weidman [1]). The quality of the final product depends on the rate of heat transfer at the stretching surface. Literature survey shows that interest in the flows over a stretched surface has grown during the past decades. The problem of stretching surface with constant surface temperature was analyzed by Crane [2]. Later, the stretching sheet flow has been studied by several researchers to examine the sole effects of rotation, velocity and thermal slip conditions, heat and mass transfer, chemical reaction, MHD, suction/injection, different non-Newtonian fluids or possible combinations effects ([3-8]). Elbashbeshy and Basziz [9] studied the effect of variable viscosity and internal heat generation on heat transfer over a continuous moving surface. Salem [10] Studied the problem of flow and heat transfer of an electrically conducting viscoelastic fluid having a temperature-dependent viscosity over a continuously stretching sheet. Salem [11] has further studied the problem of steady laminar free-convection boundary-layer flow along a vertical wedge with the effect of temperature-dependent viscosity immersed in electrically fluid-saturated porous medium in the presence of internal heat generation or absorption. Entropy generation is associated with thermodynamic irreversibility, which is common in all types of heat transfer processes. Different mechanisms are responsible for the generation of entropy such as transfer across finite temperature gradient, magnetic effect, viscous dissipation effects, etc. Sahin [12] introduced the second law analysis to a viscous fluid in circular duct with isothermal boundary layer conditions. Also, Sahin [13] presented the effect of variable viscosity on the entropy generation rate through a duct subjected to constant heat flux. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by Saouli and Aiboud-Saouil [14]. Makinde [15-18] studied the entropy generation analysis for variable viscosity channel flow with non-uniform wall temperature , also Thermodynamic second law analysis for a gravity driven variable viscosity liquid film along an inclined heated plate with convective cooling and studied Second law analysis for variable viscosity hydromagnetic boundary layer flow with thermal radiation and Newtonian heating. Naseem and Khan [19] examined boundary layer flow past a stretching plate with suction, heat and mass transfer and with variable conductivity. Cortell [20] also found the flow and heat transfer of a fluid through porous medium over a stretching surface with internal heat generation. Combined effects of magnetic field and partial slip on obliquely striking rheological fluid over a stretching surface have been investigated by Nadeem et al. [21]. Akbar et al. [22] have studied the numerical analysis of magnetic field effects on Eyring-Powell fluid flow towards a stretching sheet. Heat transfer and entropy generation analysis of non-Newtonoan fluid flow through vertical microchannel with convective boundary condition has been investigated by Madhu et al [23]. Recently, second law analysis of unsteady MHD viscous flow over a horizontal stretching sheet heated non-uniformly in the presence of ohmic heating has been studied by Qasim [24]. Here, we examine the effects of temperature dependent fluid viscosity in an electrically fluid on the flow, thermal and entropy generation features over a linear stretching sheet in the presence of a constant transfer magnetic field with blowing at the sheet. We derive velocity, concentration and temperature distribution and use them to compute the entropy generation and the Bejan number in the flow field. We also study and examine the effect of variable viscosity, Hartman and Reynolds number on velocity, temperature and concentration.

Formulation of the problem

A steady laminar, incompressible electrically conducting, viscoelastic fluid flow caused by porous stretching plate. In the presence of magnetic field, the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected, the x-axis is taken in the direction of main flow along the plate and y-axis is normal to the plate, the boundary-layer equation for the problem can be written as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})-\frac{\sigma B_0^2}{\rho_\infty}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho_\infty c_p}(\frac{\partial u}{\partial y})^2+\frac{\sigma B_0^2}{\rho_\infty c_p}u^2+\frac{Q}{\rho_\infty c_p}(T-T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions are given by

$$u=cx\ ,\quad v=-v_w\ ,\quad T=T_w(x)=T_\infty+A(\frac{x}{L})^2,\quad C=C_w+B(\frac{x}{L})^2,\quad \text{at}\ \ y=0$$

$$u\to 0\ ,\quad T\to T_\infty,\quad C\to C_\infty \qquad\qquad \text{as}\ \ y\to\infty \tag{5}$$

where ρ_∞ and c_p are the density and specific heat at constant pressure, Q is the volumetric heat generation or absorption, σ is the electric conductivity, B_0 is the magnetic induction , K is the thermal conductivity, T is the temperature ,C is concentration of the fluid, D is the molecular diffusivity, T_w, C_w are the variable wall temperature and concentration, l is a characteristic length, c is constant and v_w represents suction velocity across the stretching sheet, the viscosity is considered to be of the form:

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\delta(T-T_\infty)]\ \text{or}\ \frac{1}{\mu}=a(T-T_r),\ a=\delta/\mu_\infty\ ,\ T_r=T_\infty-1/\delta \tag{6}$$

where μ_∞ and T_∞ are the fluid free stream dynamic viscosity and fluid free stream temperature; a and T_r are constants and their values depend on the reference state and thermal property of the fluid, i.e. δ. In general, $a>0$ for fluids such as liquids and $a<0$ for gases.

The governing equations (1)-(4) can be expressed in a simpler form by introducing the following similarity transformation:

$$\eta = \sqrt{\frac{c}{\nu_\infty}}\,y, \quad \psi = \sqrt{c\nu_\infty}\,x f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \varphi = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

Substituting eq.(7) into eqs(1)-(3) produces the following ordinary differential equations

$$f''' - \left(\frac{\theta}{\theta_r} - 1\right)(f'^2 - ff'' + Mf') - \frac{1}{\theta - \theta_r}\theta' f'' = 0 \tag{8}$$

$$\theta'' - \Pr\left(2\ f'\theta - f\ \theta' - \alpha\theta - EcMf'^2 + Ec\frac{\theta}{\theta - \theta_r} f''^2\right) = 0 \tag{9}$$

$$\varphi'' + Sc\ f\ \varphi' - 2Scf'\varphi = 0 \tag{10}$$

where the prime denote the differentiation with respect to similarity variable η. Boundary conditions are:

$$f(\eta) = F_w, \quad f'(\eta) = 1, \quad \theta(\eta) = 1 \quad \varphi'(\eta) = -1 \ \text{at} \quad \eta = 0 \tag{11}$$

$$f'(\eta) \to 1, \ \theta(\eta) \to 0, \ \varphi(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{12}$$

where $\theta_r = -\frac{1}{\delta(T_w - T_\infty)}$ is the variable viscosity parameter, $M = \frac{\sigma B_0^2}{\rho_\infty c}$ is the magnetic field, $\Pr = (1 - \frac{\theta}{\theta_r})^{-1}\Pr_\infty$ is the Prandtl number with $\Pr_\infty = \frac{\mu_\infty c_p}{k}$ being the ambient Prandtl number, $\alpha = \frac{Q}{\rho_\infty c C_p}$ is the heat source or sink parameter, $Ec = \frac{c^2 L^2}{C_p A}$ is the Eckert number, $Sc = \frac{D}{\nu_\infty}$ is the Schmidt number+ and $F_w = -\frac{v_w}{\sqrt{\nu_\infty c}}$ is the dimensionless wall mass transfer coefficient.

Entropy generation analysis:

The local volumetric rate of entropy generation in the presence of magnetic field is given by:

$$S_G = \frac{k}{T_\infty^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{T_\infty}u^2 + \frac{D}{C_\infty}\left[\left(\frac{\partial C}{\partial x}\right)^2 + \left(\frac{\partial C}{\partial y}\right)^2\right] + \frac{D}{T_\infty}\left[\frac{\partial T}{\partial x}\frac{\partial C}{\partial x} + \frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right] \tag{13}$$

The first term on the right hand side of Eq.(13) is the entropy generation due to the heat transfer across a finite temperature difference, the second term is the local entropy generation due to viscous dissipation, the third term is the local entropy generation due to the effect of the magnetic field and the final terms are due to the Lorentz force. The entropy generation number is

$$N_s = \frac{L^2 T_\infty^2 S_G}{k\,\Delta T^2} \tag{14}$$

Where k is the thermal conductivity and L is the characteristic length scale. Using the similarity variables defined in Eq.(7), we obtain the entropy generation number as

$$N_s = N_H + N_F + N_J + N_M$$

where N_H, N_F, N_J *and* N_M are respectively the dimensionless local entropy generation rate due to heat transfer, fluid friction, joule heating, and concentration defined as

$$N_H = \frac{4}{X^2}\theta^2(\eta) + Re_L\theta'^2(\eta),\ \ N_F = Re_L\frac{Br}{\Omega}\frac{\theta_r}{\theta_r - \theta}f''^2(\eta),\ N_J = Ha^2\frac{Br}{\Omega}f'^2(\eta),$$

$$N_M = \frac{4}{X^2}\lambda_1\varphi^2 + Re_L\lambda_2\varphi'^2 + \lambda_3[\frac{4}{X^2}\theta\varphi + Re_L\theta'\varphi'] \tag{15}$$

where Re_L, Br, Ω and Ha are respectively, the Renolds number based on the characteristic length, Brinkman number, the dimensionless temperature difference and the Hartman number. These parameters are given by

$$Re_L = \frac{Lu_L}{\nu_\infty},\ Br = \frac{\mu_\infty u_w^2}{k\,\Delta T},\ \Omega = \frac{\Delta T}{T_\infty},\ Ha = B_0 L\sqrt{\frac{\sigma}{\mu_\infty}},$$

Dimensionless terms denoted $\lambda_i (1 \le i \le 3)$, and called irreversibility distribution ratios, are given by

$$\lambda_1 = \frac{DT_\infty}{KC_\infty}(\frac{\Delta C}{\Delta T})^2, \qquad \lambda_2 = \frac{DT_\infty^2}{KC_\infty}(\frac{\Delta C}{\Delta T})^2, \qquad \lambda_3 = \frac{DT_\infty}{K}(\frac{\Delta C}{\Delta T}) \tag{16}$$

Also, it is necessary to define the local Bejan number that can be calculated as the ratio of entropy generation due to heat transfer N_H to the total entropy generation N_s i.e.

$$Be = \frac{N_H}{Ns} \tag{17}$$

Findings and discussion:

We solve numerically the system of coupled non-linear ordinary differential equations (8)-(10) togethers with boundary conditions(11)- (12) by using the fourth order Rung-kutta methods along with the shooting technique. We obtain numerical results to examine the behavior of velocity, temperature and concentration profiles along with entropy generation rate and Bejan number for a linear stretching sheet for sundry values of the fluid viscosity parameter θ_r, the magnetic field parameter M, the Ekeart number E_c, the heat source or sink parameter α, the Schmidt number S_c and the dimensionless wall mass transfer F_w.

Velocity, temperature and concentration distributions:

Figures 1-3 are graphical representation of dimensionless velocity, temperature and concentration profiles for different values of magnetic field parameter M in the absent and presence of temperature dependent viscosity θ_r throughout the boundary layer. It is found that as M increases, the fluid velocity decreases; this is due to presence of transfer magnetic fields which causes the emergency of drag force opposing the motion of the field and as a result it retards the flow velocity. This is accompanied with slight increase in the fluid temperature and concentration within the boundary layer. In addition, the velocity in the case of variable viscosity (plotted as dotted lines) is higher than that constant viscosity (plotted as solid lines) for all values of magnetic field parameter M and reverse trend is seen for temperature and concentration profiles. The rise in concentration and temperature profiles may be attributed to resistance offered by Lorentz force.

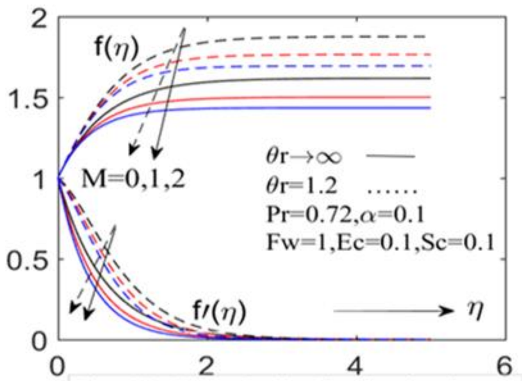

Fig. (1). The velocity distribution for different values of M and θr

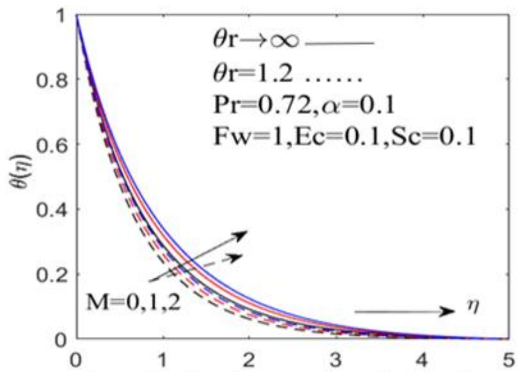

Fig. (2). The temperature distribution for different values of M and θr .

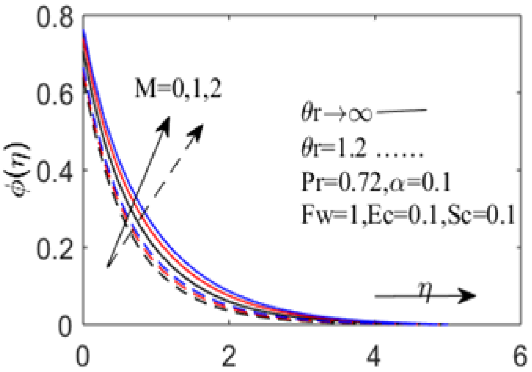

Fig. (3). The concentration distribution for different values of Mand θr.

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Figures.(4-9) display the influence of the Eckert number Ec and heat generation parameter α on the velocity, temperature and concentration profiles in the absence and presence of θ_r. The velocity and concentration are almost not affected with increase of E_c *and* α in the absence of the variable viscosity θ_r inside the boundary layer. From figures 4 and 6 one sees that the viscous dissipation and heat generation has negligible effect on the velocity and concentration in the case of constant viscosity since the viscous dissipation and heat generation are associated basically with energy equation. However, in the presence of variable viscosity, the momentum and energy equations are coupled, therefore, changes in values of viscous dissipation and heat generation causes change in the velocity profiles which are plotted as dotted lines. In the presence and absence of variable viscosity, the effect of viscous dissipation and heat generation increase temperature inside the thermal boundary layer. Physically, when the frication on plate increases due to fluid viscosity, more heat is generated and as a result the fluid temperature increases.

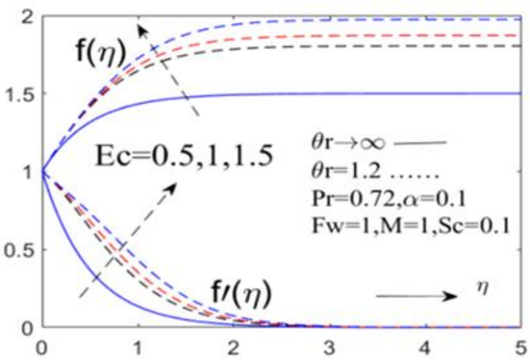


Fig. (4). The velocity distribution for different values of Ec and θr.

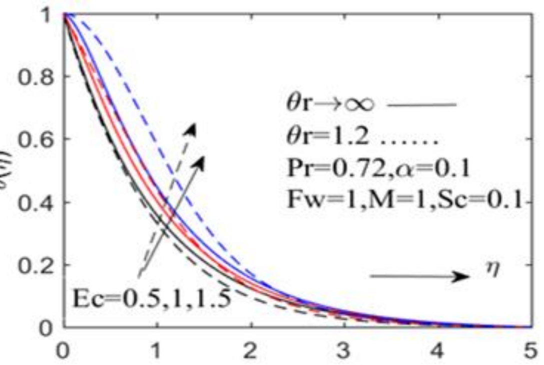


Fig. (5). The temperature distribution for different values of Ec and θr.

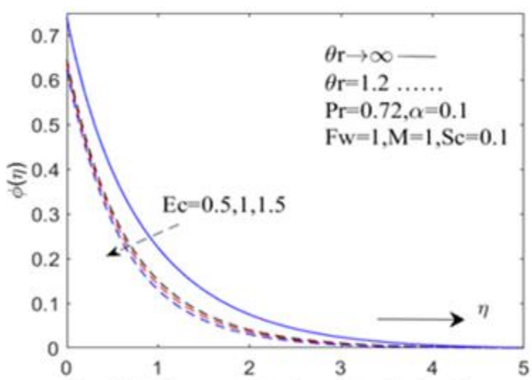


Fig. (6). The concentration distrubution for different values of Ec and θr.

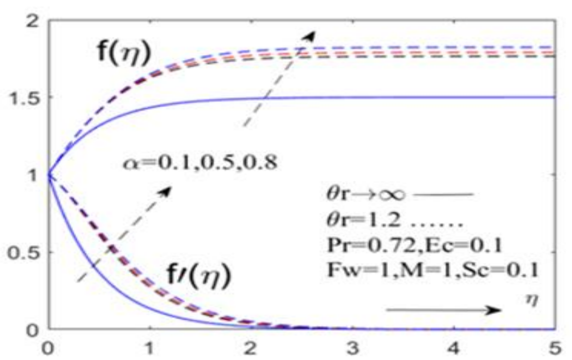


Fig. (7). The velocity distribution for different values of α and θr.

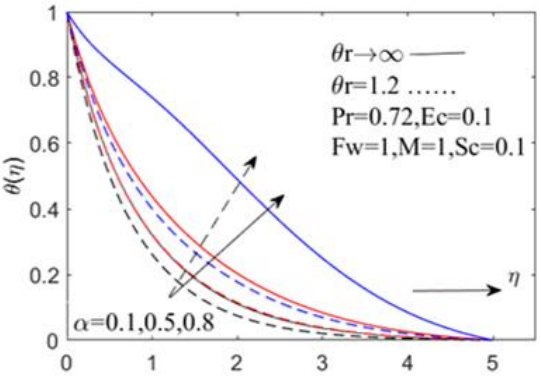


Fig. (8). The temperature distribution for different values of α and θr .

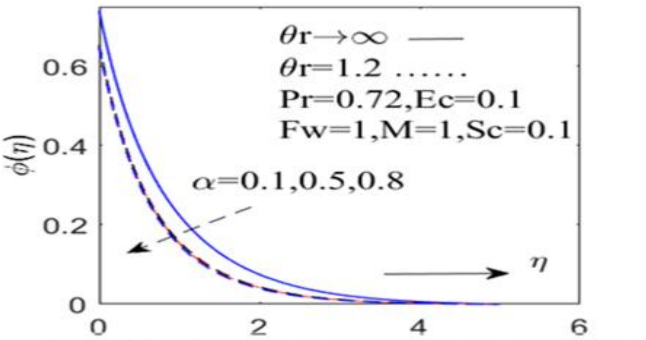


Fig. (9). The concentration distribution for different values of α and θr .

Figures 10-12 show the effect of Schmidt number Sc on the velocity, temperature and concentration in the absence and presence of variable viscosity θ_r . It is seen in Figures 10 and 12 that the variation of Schmidt number does not t have much effect on velocity and temperature profiles. However, as it is seen in Figure 11, the effect of increasing the values Sc is to decrease concentration distribution inside the flow region. Physically, the increase of Sc means decrease of molecular diffusivity. Hence, the concentration of species is higher for small values of Sc and lower for large values of Sc. Also it is observed that the concentration in the case of variable viscosity is lower than that of uniform viscosity for all values of Schmidit number Sc.

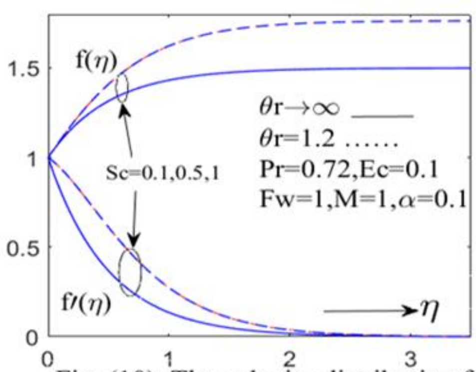


Fig. (10). The velocity distribution for different values of Sc and θr.

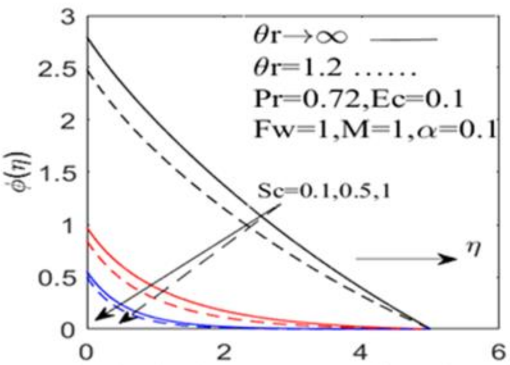


Fig. (11). the concentration distrebution for different values of Sc and θr .

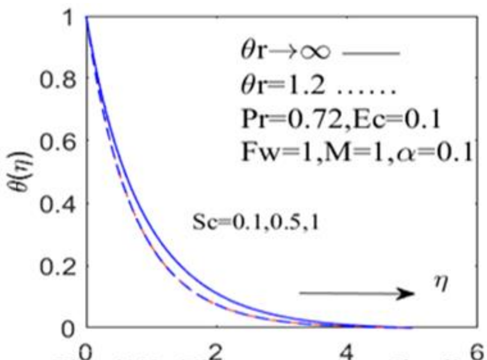


Fig. (12). The tempreture distribution for different values of Sc and θr .

In Figures 13, 14 and 15, the dimensionless velocity, temperature and concentration profiles are plotted for different values of suction parameter F_w in the absence and presence of variable viscosity parameter θ_r throughout the boundary layer. For M=1, $\alpha = 0.1$, Ec=0.1 and Pr=0.72, we observe that both profiles of horizontal velocity, temperature and concentration decrease with the increase of suction parameter. The same observation is made by Kandasamy [25] which is " the presence of wall suction decreases the velocity boundary layer thicknesses but decreases the thermal and solute boundary layer thickness, i.e. thin out the thermal and solute boundary layers". In addition, the velocities in the case of variable viscosity are higher than that of constant viscosity for all values of suction parameter and reverse trend is seen for temperature and concentration inside the boundary layer.

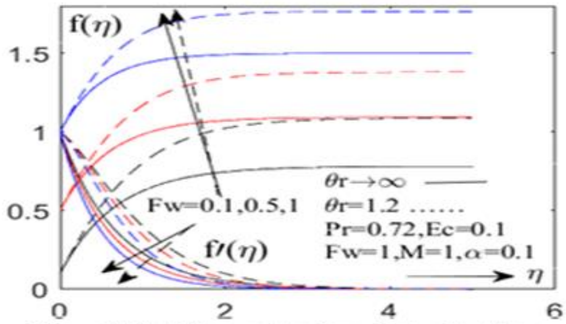


Fig. (13). The velocity distrubution for different values of Fw and θr .

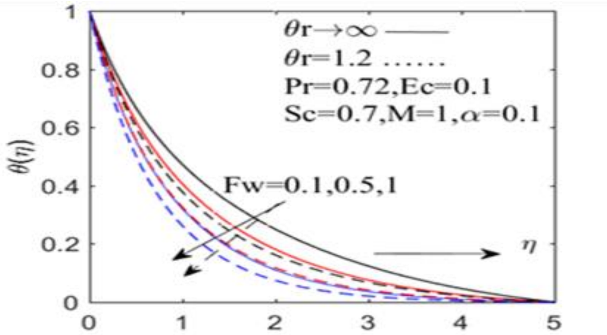


Fig. (14). The temperature distribution for different values of Fw and θr .

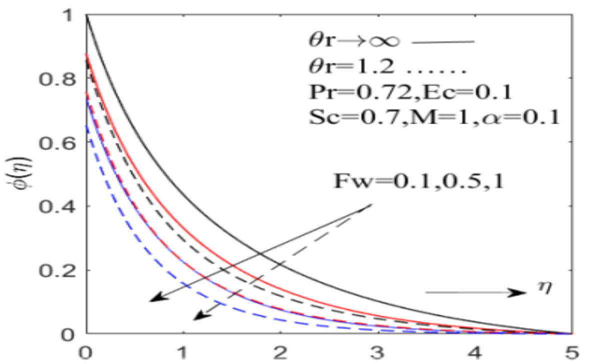


Fig. (15). The concentration distrebution for different values of Fw and θr .

Entropy generation rate:

Figures 16-21 show entropy generation number profiles $Ns(\eta)$ for different values of magnetic field parameter M, viscous dissipation parameter Ec, heat generation parameter α, suction parameter F_w, Schmidt number Sc, Hartmann number Ha and group parameter $Br\Omega^{-1}$ in the presence and absence of variable viscosity parameter θ_r .As it is observed in Figures 16,17 and 18, the entropy generation decreases across the boundary layer with increase of M, Ec and α, while the reverse trend is observed outside the boundary layer. However an increase in suction parameter F_w ,generates the opposite effect to magnetic field parameter M as shown in Figure 19. According to Figure 20, the increase of Schmidt number Sc could highly diminish the entropy generation number profiles throughout the boundary layer. In the presence of temperature-dependent viscosity, the effect of Schmidt number is to decrease the entropy generation number profiles throughout the boundary layer more than that the case of fluid with uniform viscosity for lower and higher values of Sc, i.e. variable viscosity with large values of Schmidt number causes a decrease in the entropy generation throughout the boundary layer. The effect of Hartmann number Ha causes the entropy generation number to slightly increase throughout the boundary layer, as it is observed in Figure 21.

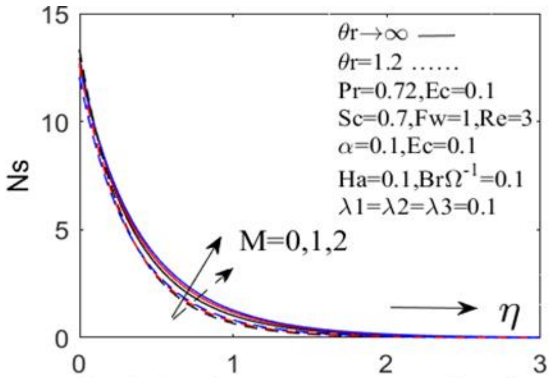


Fig. (16). The Entropy generation for different values of M and θr .

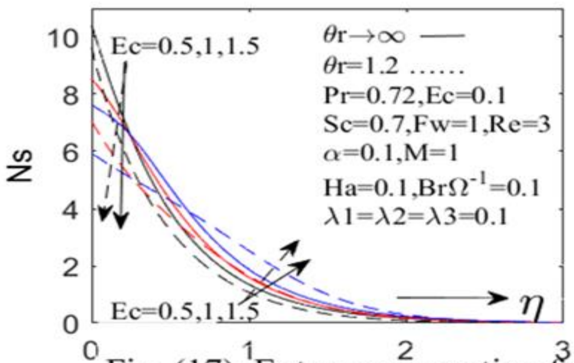


Fig. (17). Entropy generation for different values of Ec and θr .

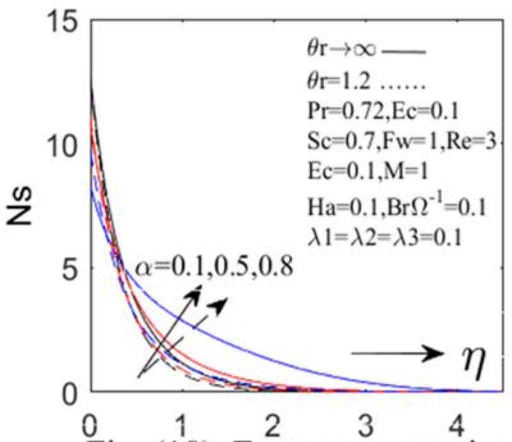


Fig. (18). Entropy generation for different values of α and θr .

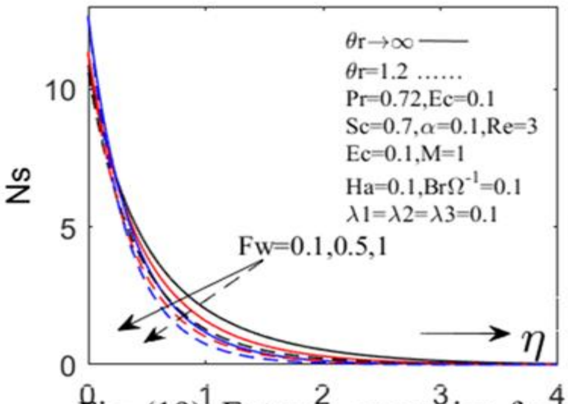


Fig. (19). Entropy generation for different values of Fw and θr .

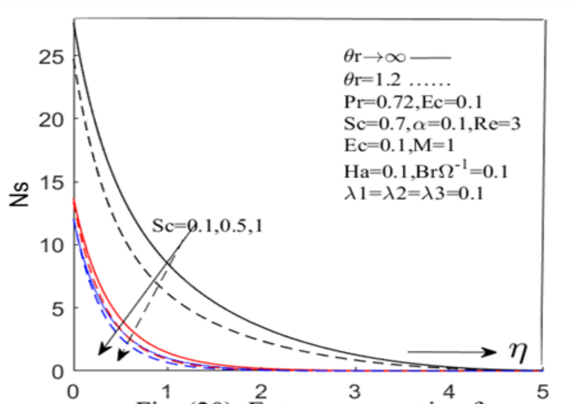


Fig. (20). Entropy generation for different values of Sc and θr .

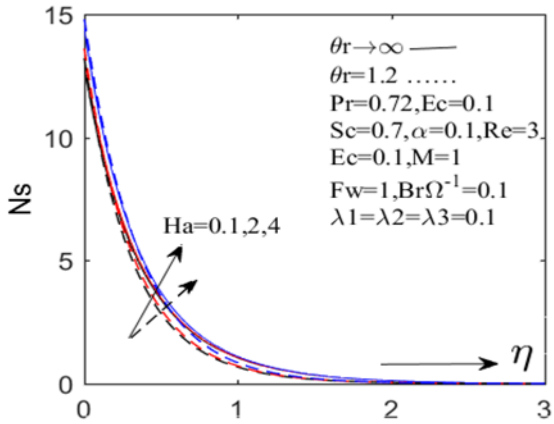


Fig. (21). Entropy generation for different values of Ha and θr.

Bejan number:

Figure 22 shows how the Bejan number profiles $Be(\eta)$ vary with the Hartman number. The Bejan number profiles increase due to increase in the Hartman number within the boundary layer in the absence as well as in the presence of temperature dependent fluid viscosity. In addition, the Bejan number of the fluid with constant viscosity is greater than that for the fluid with variable viscosity for all values of Hartman number. The effect of Brinkman group $Br\Omega^{-1}$ on Bejan number for three different values of Hartman number, namely, Ha=0, 0.5 and 1.5, is presented in Figure 23 in the absence and presence of variable viscosity parameter. The Bejan number increases due to an increase in the group parameter $Br\Omega^{-1}$ for $Ha > 0$ within the boundary layer. This increase in Bejan number is much at large values of Ha.

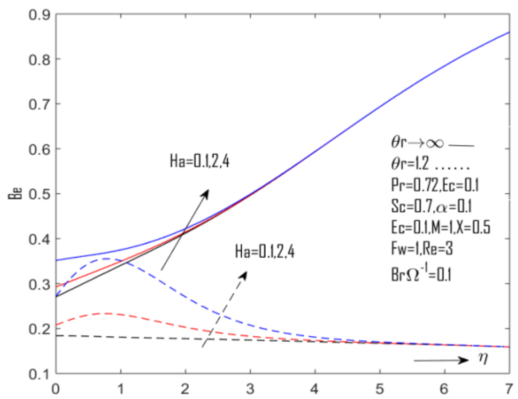
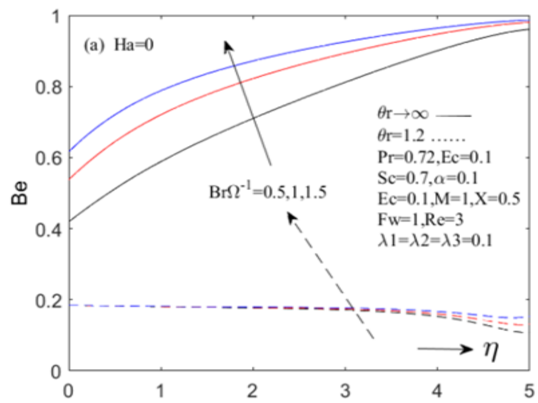
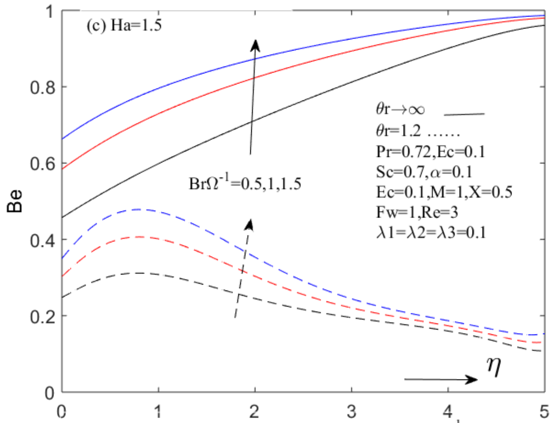


Fig. (22). The Bejan number for different values of Ha and θr.





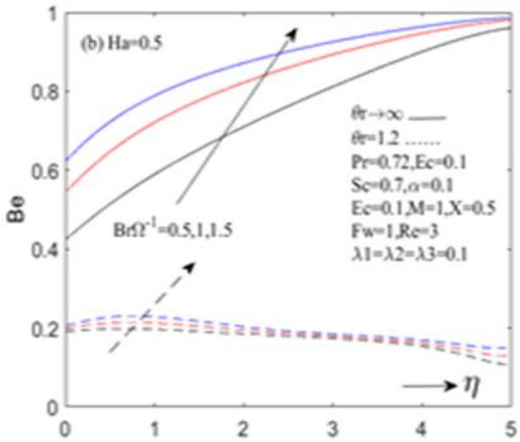


Fig. (23). (a) Bejan number for different values of $Br\Omega^{-1}$ and Ha=0. (b) Bejan number for different values of $Br\Omega^{-1}$ and Ha=0.5.(c) Bejan number for different values of$Br\Omega^{-1}$ and Ha=1.5.

Conclusion

Entropy analysis for the steady two dimensional laminar flow, heat and mass transfer of an incompressible fluid over a non-isothermal permeable stretching sheet in the presence magnetic field, variable viscosity, and heat generation is examined. The governing boundary layer equations are transferred using suitable similarity transformations three nonlinear coupled ordinary differential equations, which are then solved by using Rung-Kutta method with shooting technique. The effect of variable physical parameters on the velocity, temperature, concentration, entropy generation number, and Bejan number are analyzed. The results indicate that, increasing the magnetic field parameter tends to decrease the velocity profile but increases the temperature and concentration profiles. In addition, when the temperature dependent fluid viscosity is included, a considerable rise in the velocity and considerable reduction in the temperature and concentration profiles throughout the boundary layer are observed. Also it has been noticed that the increasing of Schmidt number Sc corresponds to lower concentration field $\varphi(\eta)$ for both constant and variable viscosity. The entropy generation inside the boundary layer slightly decreases with increase of magnetic field, Eckert number and heat generation but the opposite behavior is noticed outside the boundary layer. Moreover, by increasing the Schmidt number, the entropy generation is found to be smaller for the flow of variable fluid viscosity than that for the flow of constant fluid viscosity. The present study assures that the Schmidt number and temperature dependent fluid viscosity parameter may be taken as the dominant variables for entropy generation since their variations could considerably alter the entropy generation inside the boundary layer.

Nomenclature

Be	Local Began number
B_0	Magnetic induction
C_f	Skin-friction coefficient
Ns	entropy generation number
f	Nonsimilarity function
k	Therma conductivity
c_p	specific heat at constant pressure
M	Magnetic field parameter
Nu	Nusselt number
Pr	Prandtl number
q_w	wall heat flux
x, y	Dimensionless coordinates
Re_l	Local Reynolds number
T	Temperature
u, υ	Dimensionless velocities
U_w	Constant velocity
x, y	Dimensionless coordinates

Greek symbols

ν	Kinematics coefficient of viscosity
ρ	Fluid density
Ψ	Stream function
θ	Dimensionless temperature
σ_0	fluid electrical conductivity
κ	Porous medium permeability
ω	coordinate of the wavy surface
∇	Laplacian operator

Subscripts

∞	Conditions far away from the surface
'	differentiation with respect to η
w	wall surface

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