

Galactic symmetry

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Abstract

Differences between the quantum mechanical and relativistic concepts of time are explained by applying the strong equivalence principle to the electron in an atomic clock. The resulting clock model calls for the microscopic equations of motion of the electron to be formulated in Minkowski space, and for the photon to be described relativistically as a four-dimensional localization of energy. The resulting Lagrangian formulation of quantum mechanics is completely analogous to the more familiar nonrelativistic Hamiltonian model based on the Schrödinger equation. It accounts for the 720 degree rotation of a wave function as the absorption of one 360 degree electromagnetic wave cycle and the emission of another, yielding two wave cycles to correspond with one clock cycle. Because the properties of energy are universal the equations are able to be extended to include galaxies in spite of vast differences in lifetime. They show that symmetry exists between the electromagnetic fields of atoms and the gravitational fields of galaxies due to the presence in both of radial and transverse fields. The description of galaxy structure is fundamentally distinct because it is based on the conjugate variables energy and time.

Key Words: time; clocks; quantum mechanics; relativity theory; conjugate variables; non-inertial; space-time linearity; energy

1.0 Introduction

We do not have a theory of quantum gravity that successfully describes both quantum mechanics, governing the microscopic properties of matter; and relativity theory, its large scale behavior. It is hypothesized here that the greatest impediment to integrating these two foundational frameworks of theoretical physics is the question of time. Time is registered by clocks, devices that read out a one-dimensional sequence of increasing numbers. Currently there is no way to distinguish between clocks, a mechanism, and time, a concept. They are treated operationally as one and the same thing. On the one hand clock mechanisms are quantum oscillators consisting of electrons that transition between orbitals while registering time as a series of ticks. On the other hand, time is relativistic in nature and varies continuously according to a clock's motion and gravitational potential. In order to calculate the passage of time on the earth's surface corrections are applied to satellite time based on factors such as velocity and altitude relative to the earth's surface that are unrelated to clock function. The clocks in GPS systems are corrected for relative velocity and gravitational potential, but without knowing from a quantum mechanical perspective why it is necessary.

In other words, clocks register ticks absolutely in rigid succession, but time is perceived relativistically as a continuous variable. Moreover, corrections made to the orbiting clocks due to velocity and gravitational potential are applied simultaneously. If clocks are conceived of as operationally singular mechanisms that indicate time with a string of ticks caused by a transitioning electron, then it is difficult to comprehend how the electron, transitioning within the atom in a regular way, is able to process inputs from two physically distinct external sources, velocity and gravitational potential. Clocks are described quantum mechanically, but time is relativistic; and it seems contradictory to attempt to explain relativistic corrections with nonrelativistic theory. To resolve differences between the quantum mechanical and relativistic interpretations of time fundamental change will be necessary.

2.0 Non-inertial frames

Time is not an observable in quantum mechanics. Observables are measured by bringing a measuring device into contact with the physical system and different values of the observable are obtained depending on the state of the system. In quantum mechanics time measurements are performed without making contact with the observed physical system, a clock. So the time "measured" by clocks is not a quantum mechanical observable. It is just a numerical parameter, one of four coordinates, the same as in classical physics, and clocks are treated as simple measuring devices of absolute time. If time could actually be treated the same as spatial parameters, then a simple transformation of clock coordinates would be sufficient to make the quantum and relativity theories compatible. However, relativistic time varies *continuously* with respect to velocity and gravitational potential, and this contrasts sharply with quantum mechanical clocks which measure time in absolute increments of discrete amount. To understand why a fundamental difference between quantum mechanical time and relativistic time exists we must look at the origins of quantum theory.

Soon after the Bohr model of the atom was proposed in 1913 the assumption was made that the effects of gravitational field on an atom during the emission and absorption of radiation were so small that they could be neglected. At the time there were very few objections and no hard evidence to the contrary. The uncertainty principle was introduced upon further development of the quantum mechanical formalism providing "proof" and it contributed to the popular belief that a nonrelativistic formulation of quantum mechanics for atomic structure is both sufficient and complete. This caused the dynamic, classically inspired geometry of relativity theory to be set aside in favor of Hilbert space, an abstract complex linear vector space that is rigid.

Because quantum theory is limited to applications in inertial systems there was no immediate need for it to be extended to non-inertial geometries. However, in recent years new and totally unexpected experimental techniques have been developed that can link 10,000 atoms in a lattice that functions as a single quantum oscillator and atomic clock [1]. This allows error due to thermal radiation to be essentially eliminated from properties of atomic structure and extremely precise measurements of time to be performed. When quantum

oscillators of this type are used as clocks they are able to detect differences in the earth's gravitational potential as small as one millimeter [2]. If a different time coordinate can be associated with each point in space it means that the quantum mechanical concept of absolute time is no longer a viable concept. Time has a different value at each point in space. Because the tiniest of gravitational potentials cannot be ignored *all physical laws should be formulated in non-inertial frames*.

Nonrelativistic quantum mechanics is formulated in inertial frames; that is, frames in uniform relative motion and in the absence of gravitational potentials. To be applied to events in non-inertial frames it must first be formulated relativistically. Dirac noted that possibility in his paper on quantum electrodynamics [3], "The theory is non-relativistic only on account of the time being counted throughout as a c-number [classically], instead of being treated symmetrically with the space coordinates." In other words, for the time variable to be relativistically correct it must be treated the same as the space coordinates with both discrete and continuous properties. A fully relativistic theory of quantum mechanics formulated with space and time coordinates that are symmetrical must hold in all frames, also accelerated frames.

3.0 Gravitational perturbations

3.1 Deviation from the standard model

According to the standard model gravitons mediate the discrete exchanges of gravitational field energy at the quantum level. However, an experimental procedure to detect gravitons that could be performed in practice has never been developed. Because of their low cross section during interactions with matter it has been questioned whether the detection of gravitons is even possible [4]. An alternate method is to use the continuum space of general relativity theory and study the behavior of physical systems at the quantum level by perturbing them with a barely detectable gravitational field. A light beam that alters direction in propagation space by close proximity to a massive body, is one example. When stars are perfectly aligned the resulting gravitational lens can create the image of an "Einstein cross". The deviation of light from rectilinear motion is observed to occur continuously without perceivable corrections of a discrete nature.

A second example is the perturbation that occurs when slight differences in gravitational potential cause an atomic clock to change its rate. The changes in rate occur continuously in response to continuous changes of velocity and gravitational potential, a result that is in complete agreement with relativity theory, but is not compatible with the foundations of quantum theory which requires changes to be discrete. In order to describe the slight changes in clock rate caused by gravitational perturbation in a way that is consistent with relativity theory an interpretation of quantum mechanics for non-inertial frames is required, one that includes time in both discrete and continuous forms. In other words, we need a model of the atom that explains how the discrete ticks of a clock caused by an oscillating electron are able to change in response to continuous perturbations due to velocity and gravitational potential.

3.2 Classical space-time geometry

In general relativity theory space-time geometry curves in response to the influence of matter. If it is curved significantly it would indicate that the background space-time is non-Euclidean. However, deviations by light rays from an orthogonal background are not measured in curved spacetime. The curvature at any point is determined by measuring a tiny angle of deviation with respect to Euclidean space, where Euclidean space is defined as an orthogonally configured system of rods and clocks in empty space. Although determinations of distance are approximations as a result, they are routinely performed in the astronomical sciences without making adjustments for the possibility that space-time is curved. Despite its usefulness in performing accurate measurements of the curvature of light rays, Euclidean space-time is often dismissed in practice by citing the discrete nature of quantum mechanics for events that occur at the microscopic level. Thus in theories of quantum gravity the curved space-time of general relativity becomes a quantum field at the microscopic level, and like all quantum fields it has a granular structure in the same way that photons form the structure of an electromagnetic field [5]. Although it is possible to cite the discrete nature of photons as evidence of a quantum field as in string theory and loop quantum gravity, it is also possible to use the linearity of light to construct a *classical* field out of photons that is linear and orthogonal. Classically defined, orthogonal fields that use photons as “measuring rods” are routinely employed in the astronomical sciences to determine the distance between points in space.

Red shifts are used to determine the distance of luminous objects, their relative age, and the rotational velocity of galaxies. The time delay of a laser beam was used to determine our distance from the moon. If even an infinitesimal non-linearity were present as, for example, tiny differences in the homogeneity of space, the speed of light, or the properties of intervening matter; the billions of years in transit time of starlight would amplify them making useless attempts to assemble and compare data obtained by different collection methods or with respect to distinct wavelengths. Although the linearity of electromagnetic radiation during propagation is used to perform astronomical measurements at accuracies far beyond our ability to quantify them, it is not used at all at the microscopic level. The methods of quantum mechanics were derived from first principles to be stochastic and non-linear. Methods of renormalization ignore linearity in their interpretation of the behavior of quantum systems as a sum over histories or path lengths, and the same is true for entanglement phenomena which dismiss linearity altogether for wave function collapse. Theories concerning the quantum properties of matter arrive at extremely different conclusions about the linearity of space than do astronomically determined observations even though both are based upon the same experimentally determined properties of light. In the next section we shall show how it is possible to extend the highly successful, routinely practiced astronomical methods to the microscopic level in order to measure the distance between electron shells.

3.3 The gravitational perturbation of clocks

It is not possible to use a single clock to compare measurements in non-inertial systems of variable gravitational potential. To compare physical systems with differing gravitational potentials we use two separate clocks, one as standard and another as a system variable. Let a laboratory clock T_L that is fixed in space serve as standard and let a second movable clock T_S be introduced as a system variable. The system clock T_S may be subjected to different gravitational potentials and then compared with the laboratory clock T_L to determine how the clock rate changes. Clocks placed in locations of higher gravitational potential speed up with respect to a stationary clock, while clocks in uniform relative motion slow down. Thus, the state of the system T_S is determined by its velocity and its gravitational potential, both of which are observed and measured in continuous increments. Measurements of the time coordinate with respect to T_L are performed without making physical contact with the system, a normal procedure for time measurements in both relativity theory and classical theory.

The same property of light that allows astronomers to measure the distance of objects at the edge of the universe (spatial linearity) and compare the period of oscillation of red shifts (temporal linearity) can now be used to extrapolate to the very small dimensions of an atomic clock to analyze space-time at the microscopic level. The linearity of the radiation allows the coordinate differences of oscillating electrons in atomic clocks to be easily measured with respect to a flat space-time. Consider a well-known test for relativity theory using cesium beam atomic clocks based on the Cs_{133} isotope. The clocks are flown around the world first in an eastward and then in a westward direction, and later compared to a laboratory clock fixed on the earth's surface [6]. The experiment demonstrates a slowing of clocks Δt_k that is greatest in the eastward direction of flight due to the earth's rotation and a speeding up of clocks Δt_g at higher altitudes due to an increased gravitational potential. They are relativistic corrections to the time of the system clock T_S in the airplane which cannot be interpreted by a nonrelativistic model of clocks because they are located in a noninertial system of coordinates. When Δt_k and Δt_g are summed at the end of the flight they will equal the time on the atomic clock T_L in the laboratory.

$$T_L = T_S - \Delta t_g + \Delta t_k \quad 1)$$

Because the clock rate is locally determined both on the airplane and at the laboratory the frequency of the Cs atoms, 9,192,631,770 Hz, does not change during the approximately one week duration, $\Delta T \approx 6 \times 10^5$ sec, of the flights. However, during that time period T_S speeds up relative to T_L an amount equal to approximately 2.7×10^{-7} seconds due to the combined influence of clock velocity and gravitational potential. The corrections are due to the relative velocity and altitude of the airplane, so *they occur simultaneously*.

The linearity of time allows the classical interval ΔT , or time of flight of the airplane, to be subdivided into a series of identically constituted clock cycles of period τ that sum linearly.

$$\Delta T = \tau_m + \tau_{m-1} + \dots + \tau_{n-1} + \tau_n \quad 2)$$

where τ_m is the first clock cycle and τ_n is the last clock cycle. The clock periods τ correspond to single ticks of the clock, complete cycles of a transitioning electron between excited and ground states. Each of the cycles includes an infinitesimal correction with respect to the laboratory clock and their sum gives the total correction factor in 1). The correction for *each clock cycle* is calculated by dividing the total correction by the approximate number of oscillations during the measurement period ΔT . The linear correction factor that must be applied to the clock period τ due to gravitational field is,

$$\frac{\Delta \tau_g}{(\Delta T)(v_{cs})} \leq 2.7 \times 10^{-23} \text{ sec}$$

This corresponds to an uncertainty in the electron's path length for each period of oscillation of $ct \leq 8.1 \times 10^{-15} \text{ m}$. It is a distance that is much smaller than the indeterminacy of an electron relative to the nucleus due to the wavelength, $\lambda = 3.26 \times 10^{-2} \text{ m}$. The measurement of time is not governed by the uncertainty principle because clock ticks are quantum non-demolition measurements; that is, the uncertainty of the ticks does not increase from its measured value as the system evolves [7]. The measurement disturbs the system in a predictable way. It is a determination of the maximum thickness of the electron shell as opposed to a determination of the electron's position on the shell. That level of spatial precision is necessary to enable periodic electron transitions to reproduce the ticks of a clock with the observed precision. In other words, if the surfaces of the atom and its excited states were not completely uniform electron transitions would not be of uniform duration and clock ticks would be irregular.

3.4 Fractional shifts in wavelength

An improved measurement to "test the interplay of quantum mechanics and general relativity on the millimeter scale" was made between optical atomic clocks separated by a difference in height of 0.33 m [8]. After 40,000 seconds of data the authors found that the difference in gravitational potential of the clocks had caused a "fractional frequency shift" of 4.1×10^{-17} cycles/second. This corresponds to a fractional shift in wavelength $\Delta\lambda$.

$$\Delta\lambda = \frac{\Delta f}{f} \times \lambda = (4.1 \times 10^{-17})(2.6 \times 10^{-2} \text{ m}) = 1.06 \times 10^{-18} \text{ m}$$

The fractional shift in wavelength provides an estimate of the "perceived" thickness of an electron shell that is over two magnitudes greater precision than the measurement described in the previous paragraph.

The most recent and also most accurate clock experiments use a single crystal of 100,000 strontium atoms and ultraviolet light to differentiate between the gravitational potential of the crystal's upper and lower surfaces, a distance of one millimeter [2]. The fractional frequency instability given for that experiment, 7.6×10^{-21} , leads to the calculation of a fractional shift in wavelength $\Delta\lambda$, indicating the "tolerance" of electron shells during measurements of time.

$$\Delta\lambda = \frac{\Delta f}{f} \lambda = (7.6 \times 10^{-21})(6.98 \times 10^{-7} \text{ m}) = 5.3 \times 10^{-27} \text{ m}$$

Therefore each cycle of an atomic clock's "pendulum", an oscillating electron, is carried out between surfaces of indefinite thickness $\Delta x \leq 5.3 \times 10^{-27}$ m. The authors conclude, "These results suggest that there are no fundamental limitations to inter-clock comparisons reaching frequency uncertainties at the 10^{-21} level, offering new opportunities for tests of fundamental physics." Apparently the measurement of gravitational potential using atomic clocks is only limited by our ability to differentiate between clocks spatially; that is, by the granularity of matter itself. Thus the interrelationship of gravity and clock time at the quantum level involves all of the fundamental properties of matter in a single experiment.

It is possible to measure increased clock rate due to a gravitational potential of as little as one millimeter, and experimentalists have hypothesized that there is *no minimum distance*, and that clock rate changes are continuous with respect to changes in gravitational potential. To express the idea formally we describe clock function in terms of the Hamiltonian, $H=T+V$, where T refers to the electron's ground state energy and V to the energy required to raise it to a higher energy level. However, clock experiments show that if we increase the potential of the atomic clock by placing it in a higher gravitational potential it will cause a corresponding increase in the Hamiltonian δ that may be expressed as follows,

$$H=T+(V+\delta) \quad 3)$$

where δ may be made arbitrarily small because there are no "fundamental limitations" to changes in clock rate in response to continuous changes of the gravitational field. In non-relativistic quantum mechanics the Hamiltonian operator corresponds to discrete values of a system's energy spectrum, the eigenvalues, and continuous change does not occur.

In order to explain clock rate change in response to increases in gravitational potential of arbitrarily small amount fundamental change is necessary in the way we perceive energy exchange at the quantum level. The increases in potential cause increased clock energy, as manifested not by changes in electron transition energy, an invariant, but by an increased number of ticks of the system clock relative to the laboratory clock. The additional ticks increase in number relative to the length of time of the experiment and they are due to an exchange of energy between the gravitational and quantum systems. The exchanges of energy occur continuously, thereby contradicting nonrelativistic theory which proposes that exchanges of gravitational field energy are mediated discretely by gravitons. In order to account for continuous exchanges of energy relativistic equations of motion are required, a topic that will be taken up in the next section.

4.0 Lagrangian quantum mechanics

4.1 Differential equations of motion

The oscillating electron of an atomic clock transitions back and forth between discrete electron shells, but it moves continuously on its way from one shell to the other, and as it is moving it is being accelerated by the gravitational field. Due to the extremely short distance the electron travels, the tiny duration of its path, and infinitesimal changes produced by the gravitational potential; the curvature of space-time may be neglected. Therefore within the

infinitesimally small space-time region of the atom the equivalence principle is not just an approximation it is exactly true and the motion of the electron is subject to the laws of special relativity.

We are now prepared to answer the question posed in the introduction: How is it possible for the electron in an atomic clock to transition within the atom in a regular way while processing inputs from two physically distinct external sources, velocity and gravitational potential? The electron is subject to parameters acting in two distinct space-times. Most of its energy is determined by the discrete spacing between electron shells; but another infinitesimal part, due to the equivalence principle, is transmitted continuously by gravitational field acceleration. To derive *local* equations of motion for the electron that include both we use Minkowski space and the concept of invariant spacetime intervals [9] "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." Neither space nor time is assigned independent reality within the atom, rather a union of the two is preserved in the form of a spacetime interval.

The rate of an atomic clock does not speed up or slow down due to changes in the distance between electron shells. The distance between electron shells and the locally determined number of ticks per second of an atomic clock are invariant properties of matter because atomic structure is an invariant. Thus the electron in an atomic clock is subject to the invariant requirements of atomic structure due to the discrete nature of electron shells, yet at the same time it moves *continuously* in space-time during transitions between electron shells. For a complete determination of the equations of motion of the electron both discrete and continuous coordinates will be necessary.

From equations 1), 2), and 3) and the experimentally confirmed linearity of light we conclude that the electron of the atomic clock T_s oscillates at frequency ν_{Cs} with relativistic correction to *each cycle* $\Delta t(\dot{x})$ due to angular velocity relative to the center of the earth and correction to *each cycle* due to an acceleration of coordinates $\Delta t(\ddot{x})$ between ticks of the clock. An electron in free space accelerates under the influence of a gravitational field due to its mass by undergoing a change in velocity; however, an atomic electron cannot accelerate in the same way as a free electron because *the spacing between electron shells is an invariant*. To describe the influence of gravitational fields on the electron of an atom we use Minkowski space, hold the spatial coordinates constant, and let time vary. The use of Minkowski space has the advantage that the spacetime distance of an electron transition is invariant, so that all local frames of reference will agree on the total distance in spacetime between the ticks of a clock. The invariant properties of Minkowski space make it indispensable to a description of atomic structure. It allows the microscopic equations of motion of a transitioning electron to be assigned a differential equation for the variation of time with respect to space,

$$t(x) = \tau(x) + \Delta t(\dot{x}) - \Delta t(\ddot{x}) \quad (4)$$

where $\tau(x)$ is the invariant clock period of the atom, $\Delta t(\dot{x})$ represents a continuously applied correction due to kinematical time dilation, and $\Delta t(\ddot{x})$ represents a continuously applied speeding up of time due to increases in gravitational potential. Ordinarily the equation of motion of a particle describes its change in position with respect to time. However, the equation of motion of an electron in Minkowski space is given by *changes in time with respect to space*. Therefore experiments with atomic clocks in a gravitational field that we described in sections 3.3 and 3.4 suggest that time is not an absolute property of matter, rather its most fundamental expression is to be found in the relativistic function of clocks in a non-inertial frame.

4.2 Integral of the motion

Hamilton's principle states that the differential equations of motion for a physical system can be reformulated as an equivalent integral equation. It means that we can describe the dynamics of the system 4) as a variational problem based on the Lagrangian, which gives a complete accounting of the system's energy. In the case of the atomic clock the electron takes a path in configuration space with variation of the action (energy times time) and fixed boundary conditions (the electron shells) at the beginning and the end of the path. The integral equation describing the electron's path is an action functional $S[q(t)]$, something that takes a function as its input (the Lagrangian) and returns a scalar (Planck's constant), where \mathbf{q} (position) is expressed in generalized coordinates that are bounded in space by the electron shells. The Lagrangian of the electron's total energy includes the clock period τ , the velocities $\dot{q}(t)$ and coordinates $q(t)$ as inputs and returns a scalar.

$$S[q(t)] = \int_0^t L(\dot{q}(t), q(t), t) dt = \tau \quad 5)$$

We interpret 5) in flat space-time as the evolution $\mathbf{q}(t)$ of a quantum system during a *complete cycle*. The generalized coordinates describe the electron's path in a configuration space consisting of three coordinates that define the origin, or nucleus, and three coordinates that define the manifold on which the electron is constrained to move (the electron shells). In other words, the atomic clock/quantum oscillator is described in configuration space as having six parameters, three for the nucleus and three for the electron shells, a total of six degrees of freedom. Solving for the action we obtain an equation describing the cycles of a quantum oscillator, which are the individual ticks of an ideal clock.

It is now clear why nonrelativistic quantum mechanics does not correctly describe clock behavior. The wave function is defined continuously in three spatial dimensions, but to describe the continuous changes in clock rate due to velocity and acceleration two systems of coordinates are required, discrete to describe an electron's acceleration between electron shells due to gravitational potential and continuous to describe its relative velocity. By treating time symmetrically with the space coordinates in four dimensions a relativistic theory describing atomic structure is obtained that accurately describes the behavior of a quantum oscillator. The endpoints of oscillations are conceived of as events on the electron

shell designating the ticks of a clock. The action $S[x(t)]$ describes a complete cycle of the atomic clock by taking into account the invariant clock period τ , the acceleration of the electron $\Delta t(\ddot{x})$ due to gravitational potential, and a kinematic component $\Delta t(\dot{x})$ due to its continuous motion. Thus, atomic structure requires two complete four-dimensional space-times as a result of the equivalence principle; discrete coordinates to specify electron shells and continuous coordinates to describe an electron's continuous motion during transition.

By treating time symmetrically with the space coordinates to describe quantization a relativistic theory of atomic structure is obtained that is superior to nonrelativistic theory because it accurately reflects influences of velocity and acceleration on the time coordinate. It is a classically derived model that describes the space-time of atomic structure discretely in Minkowski space, while the intervening space-time in which atoms reside and move is continuous and curved. Despite vast amounts of experimental evidence demonstrating that energy quanta are discrete it does not follow logically that space-time is also discrete as it is portrayed in string theory and loop quantum gravity. They are not equivalent concepts. Energy resides in space-time, is contained by it, and is subordinate to it. This is expressed formally by 5) which describes quantization as a localization of energy within the space-time manifold of an atom's configuration space, an electron shell. It distinguishes the Minkowski space of the atom from the curved space of gravitational fields that atoms reside in.

4.3 The emission and absorption of energy

The properties of energy absorption and emission for a bounded electromagnetic system are of particular interest in our discussion for we wish to compare them with the long term time evolution of the absorption and emission of fields in a bounded gravitational system. However, it is not possible to make a direct comparison of gravitational fields to electromagnetic fields. Not only are they of vastly different strengths, but gravitational potentials do not share differences in polarity that are characteristic of electric charge. Despite glaring differences in their gross outward physical appearance, there are similarities that exist on a more subtle level. The kinetic flow of mass whether linear or rotational creates a transversely directed gravitational field that is analogous to the transverse magnetic field caused by current flow. The induced gravitomagnetic field, or force, is directed perpendicular to the mass flow and has been used to explain properties of relativistic jets emanating from the cores of quasars and other active galactic nuclei [10]. It is postulated that due to mass-energy equivalence transversely directed forces occur in response to mass and energy flows of all types and that they may be compared to electromagnetic flows.

In order to compare the gravitational and electromagnetic energy flows we require an interpretation of quantum mechanics that is more fundamental than the nonrelativistic formulation which treats energy as an observable. There is no better explanation for why a relativistic model is necessary than Dirac's [11].

"There is an alternative formulation for classical dynamics, provided by the Lagrangian. This requires one to work in terms of coordinates and velocities instead of

coordinates and momenta. The two formulations are, of course closely related, but there are reasons for believing that the Lagrangian one is the more fundamental.

In the first place the Lagrangian method allows one to collect together all the equations and express them as the stationary property of a certain action function. (This action function is just the time-integral of the Lagrangian.) There is no corresponding action principle in terms of the coordinates and momenta of the Hamiltonian theory. Secondly the Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant; while the Hamiltonian method is essentially non-relativistic in form, since it marks out a particular time variable as the canonical conjugate of the Hamiltonian function.

For these reasons it would seem desirable to take up the question of what corresponds in the quantum theory to the Lagrangian method of the classical theory."

In Dirac's first study, "The Lagrangian and the Action Principle", he theorizes, "We ought to consider the classical Lagrangian not as a function of the coordinates and velocities but rather as a function of the coordinates at time t and the coordinates at time $t+dt$ ". Rather than specify photon emission as an event that occurs at a particular point in time as in nonrelativistic theory, Dirac is seeking compatibility with relativity theory by calculating change in action over a space-time interval between *two* points in time. The results of the study proved to be accurate for they gave Feynman the idea to pursue a path integral formulation of quantum mechanics, for which he obtained the Nobel prize.

In the next section Dirac develops an idea complementary to the particle model with "An Application to Field Dynamics". "We may treat the problem of a vibrating medium in the classical theory by Lagrangian methods which form a natural generalization of those for particles. We choose as our coordinates suitable field quantities or potentials." To obtain a field equation that describes energy emission "by Lagrangian methods" we begin from a classical vantage point by using a Lagrangian density given by the fields and its first derivatives $L(\phi_i, \phi_{i,\mu})$. This allows for a complete accounting of the energy interactions that occur during an electron transition, where ϕ_i is the current density described *radially* and $\phi_{i,\mu}$ is the electromagnetic field strength described *transversely*. The transformation from a classical to a quantum viewpoint is realized by localizing the fields over a "region of space-time". Continuing with Dirac's analysis, "We introduce at each point of space-time a Lagrangian density, which must be a function of the coordinates and their first derivatives with respect to x, y, z , and t , corresponding to the Lagrangian in particle theory being a function of coordinates and velocities. The integral of the Lagrangian density over any (four-dimensional) region of space-time must then be stationary for all small variations of the coordinates inside the region, provided the coordinates on the boundary remain invariant."

Dirac's line of reasoning for fields may be applied to our description of the particle model 5) by integrating over a four-dimensional region of space-time with respect to invariant field boundaries coincident with the electron shells. The field model can then be

made to coincide with the time evolution of an electron transition. We introduce an action integral by considering an atomic oscillator, or equivalently an atomic clock, immersed in a radiation field with an outer electron that occupies either of two allowable energy states. Emission initiates from the excited state $R_2 = (x_2, y_2, z_2)$ at time t_2 and it finalizes at the ground state $R_1 = (x_1, y_1, z_1)$ at time t_1 . Each of the energy states R_2 and R_1 determines a locus of points where the fields vanish and therefore they represent invariant field boundaries. Continuing with Dirac's arguments, we require the integral of the Lagrangian density over the region of space-time between the excited and ground states to be "stationary for all small variations of the coordinates inside the region." Changes in action with respect to the electron shells are evaluated by integrating the Lagrangian density four-dimensionally thereby yielding a relativistic formulation of emission that is invariant, the same for all observers.

$$S[\phi_i(t)] = \int_{R_2}^{R_1} \int_{t_2}^{t_1} L(\phi_i, \phi_{i,\mu}) d^3x dt = h \quad (6)$$

The end points of the electron's path are located on equipotentials, space-like surfaces, and the action minimum is not equal to zero as in classical theory, but to Planck's constant h . The action $S[\phi_i(t)]$ is a functional, a function of the values of coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The boundaries of the fields are uniquely fixed four-dimensionally by the volume d^3x and the time interval t_2-t_1 . The field model given by 6) describes one cycle of fields, or photon emission, as opposed to the particle model 5) which describes one full clock cycle, the excitation and decay of an electron. Two field cycles, excitation and decay, equal one clock cycle. Evaluating 6), we obtain $E\tau=h$.

The absorption of energy by an atom may occur discretely as observed in the photoelectric effect; however, it is more often continuous as in the case of optical phenomena when it is due to superposed transverse fields. The continuous absorption of energy is not amenable to description by a Lagrangian density as employed in equation 6) because the fields are not bounded. Instead we use the particle model and describe the electron during a continuous excitation. The electron initiates its motion at time t_1 on the equipotential surface R_1 along a path r and when it arrives at R_2 it adopts the circular path $2\pi r$ of an orbital thereby assuming an angular momentum.

$$S[r(t)] = \int_{R_1}^{R_2} L \int_{t_1(r)}^{t_2(2\pi r)} dt = h \quad (7)$$

If the initial time t_1 of the excitation is arbitrarily set at zero, then the action integral for a path of unit length is evaluated as follows:

$$\Delta E 2\pi \tau = h \quad (8)$$

and simplifying, we have $E \tau = \hbar$. We use this expression rather than the more familiar $E = h\nu$ because frequency is a time-averaged quantity, and should be avoided when describing the discrete quantum mechanical nature of single emission. The results from 6) and 7) differ because we are treating them as two distinct physical processes. In 6) we treat energy as a localization of superposed continuous fields during absorption. In 7) the fields have been consolidated and represent the emission an independent entity, the photon.

In contrast to the Bohr semiclassical model of the atom this derivation of energy absorption assigns the correct value of zero for the ground state orbital angular momentum. Comparing them further we see that emission and absorption are not symmetric processes since it is impossible to obtain the field distribution leading to absorption from that which resulted due to emission. Combined they represent a fully relativistic model that explains quantization as a continuous four-dimensional flow of energy followed by its localization in space as a concentration of field, the photon.

4.4 Comparison with non-relativistic quantum mechanics

The overriding problem confronting quantum theory in a description of a radiating quantum system is how to describe two physical processes, absorption and emission, with a single equation. According to the Schrödinger wave equation they occur as a single process that evolves symmetrically in time. The wave function ψ used to describe an electron oscillating between two energy states performs two complete rotations, or a total of 720 degrees, before returning to its original state. If the electron is represented mathematically by a vector in Hilbert space $|\psi\rangle$ then one rotation of 2π results in a negative value $-|\psi\rangle$ and a second rotation of 2π brings the electron back to its original state $|\psi\rangle$. No physical interpretation is possible.

As noted in previous discussions, nonrelativistic interpretations of the time evolution of a quantum system based on the Hamiltonian model are inaccurate. The Lagrangian model, which describes quantization in real time as a four-dimensional absorption of energy by the electron in 7), followed by a four-dimensional localization of fields in 6) and release of a photon, is more fundamental. We interpret the first rotation of the wave function not as a rotation in *abstract space*, but as the change in phase of electromagnetic fields from 0 to 2π in *real space* during absorption. Thus one “rotation” of the wave function is interpreted as one full cycle of an electromagnetic wave and an increase in the electron’s energy from the ground state to an excited state. The second rotation occurs as the electron returns to the ground state and is interpreted as a localization of electromagnetic field energy and emission of a photon. The dual wave-particle nature of the photon is thereby realized as a physical transformation.

The external appearances of a radiating atomic system, the frequency and intensity of its spectral lines, are observables described by the matrix mechanical formulation, where each matrix gives a representation of both absorption and emission processes. Hermitian matrices are used so that when they are evaluated the upper elements, the absorptions, are multiplied with lower elements, the emissions, to obtain real values, the eigenvalues. Thus a

single expression describes two physical occurrences and from 8) we conclude that noncommutation is the result of an asymmetric exchange of momentum, $p\lambda = \hbar$. It occurs because an incoming exchange of momentum with a quantum system carries the opposite sign as compared to an outgoing exchange. We conclude that in the nonrelativistic description of quantum systems material bodies have external properties described by matrices and internal properties described by the wave function.

4.5 Relativistic absorption of energy

In the relativistic equation 6) the excitation energy is quantized by means of a localization of fields. It means that a physical separation exists between the matter and energy of an excited state due to field boundaries. An electron does not receive, or absorb energy when it is excited, rather excitation causes field boundaries to be erected that localize energy within the atom and create a “bound” photon. Localized electromagnetic fields within an atom have been observed experimentally and they are referred to as “stopped light”. The storage and retrieval of light are achieved for up to one minute in a rare earth element by converting light coherence in free space to atomic coherence in an excited state and back again [12]. The experiments provide experimental proof of the continuity of space between energy states. Discrete field boundaries are established that localize energy, while the transverse fields contained within them vary independently and are continuous in time. Because the fields are isolated from the observer by field boundaries the phenomenon of stopped light is being developed as a form of storage for quantum computing. Therefore these experiments confirm the accuracy of 6), that the absorption of electromagnetic field energy is a linear superposition of field that occurs continuously over a time period equal to the wave period and transforms spontaneously into a field source, the photon.

4.6 Universal properties of energy

In classical theory we treat energy as a simple physical variable or property of matter and the same unit of measurement, joules, is used to measure all of its forms. Since only quantitative comparisons are possible in the measurement of joules the universal properties of energy are under appreciated or ignored. This is understandable in the case of classical interactions since systems are not precisely defined. Nevertheless, we see the same practice in quantum mechanics where energy is described as an observable in the Schrödinger time independent wave equation. Because each of the myriad forms of energy is conceived of independently, we cannot compare them with each other in a meaningful way. This is true despite the fact that universal properties of energy do exist. The conservation of energy has never been known to fail for either quantum or classical systems whether microscopic or macroscopic. Another well-known characteristic of energy, equipartition, is valid for all classical systems. As pointed out earlier experimental techniques have been developed that can link 10,000 atoms in a lattice that functions as a single quantum oscillator and atomic clock [1]. Because the lattice is isolated from the environment it is hypothesized that characteristics of electromagnetic field energy can be compared with characteristics of gravitational field energy for isolated systems such as galaxies. The emission and absorption

of radiation described in 6) and 7) is an example of a quantum system converting electromagnetic field energy into matter by means of a four-dimensional transformation, or flow. Similarly gravitational systems in the form of galaxies are centers of energy emission and absorption naturally isolated from the environment and it should therefore be possible to compare the way energy is transformed with that of a quantum system despite their vast physical differences. *It is thereby postulated that the properties of energy, when described four-dimensionally with time, are independent of the material system which supports them.*

5.0 Gravitational field energy

5.1 Field transformation

We have completed our analysis of time, which we observe in the Minkowski space of clocks as discrete quantum mechanical ticks with continuously applied corrections due to velocity and gravitational potential. Because our model is relativistically correct it treats time evolution as an action in Minkowski space rather than as a “propagator” operating on states in Hilbert space. Diffuse forms of energy are localized four-dimensionally and then materialize as independent entities, or photons. Thus the conversion of electromagnetic fields into matter is described by means of a four-dimensional localization in Minkowski space. We wish to extend our interpretation of physical processes to study the nature of gravitational interactions as a localization of fields in the curved space-time of general relativity. This will allow energy-matter conversion processes to be compared relative to the same physical basis without causing confusion due to the wide separation of magnitudes, differences in polarity, and other observable differences that exist for electromagnetic and gravitational processes. The equipartition theorem is already routinely used in astronomical studies to determine the conditions for star formation from a molecular cloud and to estimate star temperature [13]. This and the experimentally confirmed principle of energy conservation leads to the assumption that the laws of energy in nature are valid universally for all isolated systems. For these reasons we expect energy-mass conversion processes for gravitational and electromagnetic systems to be similar.

Currently attempts to combine the effects of electromagnetism and gravity are of two forms; unification of the field equations, as in classical unified field theories; and unification of forces, as part of the standard model in theories of quantum gravity. The theories' claims to legitimacy are based on attempts to derive equations that can predict the behavior of test particles with respect to the conjugate variables position and momentum. The reason these types of unification have failed to produce the desired result is due to the simple fact that the fields are already unified in the form of electrons and other particles. The electromagnetic and gravitational fields coexist harmoniously within these particles and are superposed structurally without influencing each other. If the field sources can exist in close proximity, bound together in a single particle, without noticeable effect we should not expect to detect a relationship of the fields in the far less intense setting of empty space. In fact no experiment has detected influence between the fields no matter how high the intensity. Everything we

know about the fields indicates that in spite of being unified by particle structure, they are manifested and experienced independently. To attempt to unify fields by only looking at their external properties ignores this common origin. Instead we must seek a solution by taking the opposite viewpoint and asking, Why do fields that have the same physical origin interact according to completely distinct laws? To be sure a successful field theory must account for the many complexities of fields, but more importantly it must explain how this complexity can arise from simple structures. By looking at similarities in the way that gravitational and electromagnetic field energy transform into matter we compare the fields using the conjugate variables, energy and time, at a fundamental level.

In sections 4.1, 4.2, and 4.3 we conceived of quantization as the absorption of electromagnetic field energy by a linear superposition of field potentials that occurs continuously over a time period equal to the wave period and transforms by means of a four-dimensional localization of fields into the photon. The energy flow described by equations 6) and 7) may be expressed classically as a time-averaged, transverse radiation field given by the vector potential alone.

$$B=\nabla\times A$$

The linearly superposed field potentials transform into a field source, the photon, with distinct field geometry. We are motivated to adopt the same method, as the linear superposition of potentials, for describing the time evolution of gravitational fields in free space and their transformation into matter by hypothesizing that the laws of energy apply in an equivalent way.

5.2 General relativistic clocks

We wish to describe gravitational field energy similarly to the way electromagnetic field energy is described, as the time evolution of fields that begin in free space as linear potentials. Time evolution has different meanings as it is used in quantum mechanics and general relativity theory. The time variable is given by the ticks of a clock in the former by using atomic clocks and the linearity of light as measurement standards, while the latter gives the proper time independently of clocks. Clocks are undefined in general relativity theory because Einstein believed that “measuring rods and clocks would have to be represented as solutions of the basic equations” [14]. Although he was unable to extend the equations to incorporate clocks or measuring rods we can see how differences between theory and practice originate by looking at the Einstein equation defining the local curvature of space-time.

$$G_{\mu\nu}=\kappa T_{\mu\nu} \quad 9)$$

Time and clocks are treated in a fundamentally different way in this equation. Time is continuous on the left side in the form of proper time, with clocks and measuring rods present as test bodies. If clocks, or matter of any type is present in significant amounts it must be placed on the right side due to contributions of mass and energy. Thus, continuous time exists in free space whether or not clocks are present to measure it. Equation 9) lacks symmetry because there can be ideal clocks on either side, but real clocks are only allowable on the right.

5.3 Transformation of gravitational field energy

The four-dimensional evolution of gravitational fields may be demonstrated by constructing a model for a gravitating body that has slowly changing mass-energy density. We begin with a uniform distribution of hydrogen atoms, a simple form of clock, in free space distant from each other. Both proper time and clock time are determined in Minkowski spacetime and the gravitational field intensity of the system at infinity is found by simply summing particle masses. Now let the attractive force of their mutual gravitational field cause them to slowly coalesce into a spherical body such that particle momenta remain small. Due to the equivalence of mass and energy gravitational field intensity is determined by summing particle masses and binding energy, the energy required to remove particles to infinity. The period of clocks slows and Minkowski space-time is replaced by a Riemannian metric that is described by the Einstein tensor $G_{\mu\nu}$. An attempt to use space-time curvature to describe material structure would be unacceptable because it would place the continuous time of curvature on a more fundamental basis than the ticks of a clock, a quantum mechanical phenomenon. A background space that is continuous cannot be used to describe discrete phenomena that take place within the background space.

Accumulating atoms will cause an increasing gravitational pressure that generates heat and the emission of black body radiation, early signs of star formation. Eventually the Coulomb repulsion of the atoms is overcome by gravitational attraction in the form of a pressure gradient, protons are attracted to each other due to the strong force, and fusion initiates spontaneously in the star's core. The left side of the Einstein equation 9) interprets the star formation process as a continuous change in metric from Minkowski space to a Riemannian manifold with constant proper time. On the other hand, on the right side an accumulation of matter causes a slowing of clocks, a decreasing gravitational potential, and a corresponding dilation of proper time. Due to symmetry requirements, laws of a *decreasing* potential are governed by the same laws as for an *increasing* potential. Thus an increasing potential causes a speeding up of time and a decreasing potential causes a slowing down of time.

Laws describing the interaction of gravity and quantum mechanics were derived in 4.1 based on first principles; that is, on the invariance of atomic structure, the equivalence principle, and the variability of time. Due to symmetry requirements it is hypothesized that the equivalence principle holds identically for all gravitational field intensities, so that the equation of motion 4) that applies to matter in an increasing gravitational potential also applies in a decreasing potential. Therefore we describe equations of motion for matter in Minkowski space with proper time as a variable and acceleration and velocity as parameters. For a decreasing potential caused by the accumulation of matter there is an increasing dilation of proper time. After a time period of several billion years matter accumulates in a sufficient amount to slow proper time to zero within a gravitational field boundary, the event horizon, and a black hole forms. The event horizon is a boundary condition with proper time

that slows to zero enclosing a volume of matter in Minkowski space, whose properties are analogous to those of the electron shell.

5.4 Gravitational emission equation

In equation 6) we described the localization of electromagnetic field energy and photon creation four-dimensionally, using space-like field boundaries in the form of electron shells and the discrete times of transition to localize fields consisting of both radial and transverse fields. The localization of gravitational field energy by means of a superposition of potentials is also characterized by the formation of a space-like field boundary, the horizon, with discrete time $\Delta t=0$. In addition to the symmetry that exists in the above mentioned mass-energy conversion processes other symmetries are apparent when comparing field properties. The relativistic superposition of electromagnetic potentials is described by a Lagrangian density composed of radial and transversely oriented force fields with distinct physical origins. Similarly the superposition of gravitational potentials leads to a radially oriented baryonic force field observed in the galactic bulge and a transversely oriented force field that is manifested by the radial acceleration of matter contained in the disc. Symmetry arguments based on a comparison of gravitational and electromagnetic potentials support a theory that includes superposed forces of independent physical origin; a localized, radially oriented potential due to baryonic matter and a diffuse, transversely oriented $1/r$ potential which creates an acceleration equal to one angstrom/sec². Therefore a theory is favored that can explain galactic structure by means of a continuous field law in a way that is more closely aligned to the general relativistic concept of space-time than to a model based on dark matter or a modification of Newtonian laws.

We can show a formal relationship between the energy flow of photon creation, as expressed by the action integral 6), and black hole formation if it is assumed that the black hole represents a field source described by a Lagrangian density $L(\Phi_i, \Phi_\mu)$, where Φ_i represents radial fields due to baryonic matter and Φ_μ represents transverse fields generated by a different form of matter, localized within the same black hole. It differs from a MOND type theory of acceleration which postulates a modified Newtonian law with $1/r^2$ acceleration within the galactic bulge that gradually transforms at large distances to a $1/r$ acceleration [15]. The concept of a “two component” source for black holes gives a different interpretation of the Tully-Fisher relation, which states that multiplying the luminosity of a galaxy by the period of rotation yields a constant [16]. The units of the constant, energy times time, are of action and may be compared due to symmetry arguments to the quantization of electromagnetic fields in 6); that is, to the equation $E\tau=h$. It is hypothesized therefore that because the laws of energy apply universally in nature there must be an equation,

$$E\tau=H \quad 10)$$

describing the transformation of gravitational field energy that is equivalent to the quantization of electromagnetic field, where H is the gravitational equivalent of Planck's constant. It means that in a spiral galaxy we expect the two sources of field Φ_i and Φ_μ to be present within the black hole with the same intensity, just as the electric and magnetic fields

of a photon have the same intensity. The relation 10) also correctly predicts that observations of increased galactic emission energy, or energy flow, are an indication of a higher rotational velocity and a lower period of rotation, as observed.

It is a short step from postulating that the Tully-Fisher correlation represents solutions of the emission equation 10) to obtaining the equation. Symmetry arguments based on equation 6) suggest that the action integral of a galaxy's energy flow is a function of the values of the space-like coordinates on an event horizon R which are in turn a function of the continuous space-time variables of the matter within the surface, where integration is performed over all space.

$$S[\Phi_i(t)] = \int_{R(t=0)}^{\infty} \int_{t=0}^{\tau} L(\Phi_i, \Phi_\mu) d^3x dt = E\tau \quad (11)$$

The action integral $S[\Phi_i(t)]$ is a functional relating the energy within the surface of a black hole $L(\Phi_i, \Phi_\mu)$ to the fields external to it. It describes a transformation of material energy to field energy. The existence of an emission equation governing galaxy energy flow suggests that galaxy structure is the manifestation of a field emanating from a supermassive black hole, and it consists of a radial component Φ_i due to baryonic mass causing gravitational attraction and a transverse component Φ_μ of as yet undetermined physical origin within the event horizon causing radial acceleration. Because the integration is performed over all space it governs the dynamics of galaxy clusters as well. This hypothesis is supported by the observation that every galaxy has a black hole at its center and radial acceleration is directly proportional to its luminosity [17]. The forces are mapped on a flat background space in a reference system whose origin resides at the center of the black hole. The equations 6) and 11) both describe the emission of energy of a material system, atom and black hole, by including physically independent radial and transverse components. The fact that the emission equations are mathematically equivalent demonstrates a symmetry that is absent from theories that postulate the existence of unobserved forms of "dark" matter.

6. Conclusion

A fundamental difficulty that is encountered when attempting to unify quantum mechanics and general relativity theory is whether to adopt the discrete, Euclidean background space of quantum mechanics or the continuous, curved spacetime of general relativity theory. The question was resolved in sections 4.1, 4.2, and 4.3 by analyzing the interaction of barely detectable gravitational fields with an atomic clock. In 4.1 the electron's equation of motion is derived in Minkowski space by holding spatial parameters invariant and letting time vary with respect to gravitational acceleration and velocity. In 4.2 we applied Hamilton's principle to obtain an integral equation 5) demonstrating the need for continuous coordinates at the microscopic level to describe electron shells and in order to apply the equivalence principle to the electron during its transition. Finally in 4.3 we showed that the transformation of energy from Minkowski space into free space is not possible with a simple

mathematical transformation of coordinates; for the two spaces are separated by a field boundary and a *physical transformation* is required. When a photon is released into free space the field boundaries of the atom (the electron shells) represent a physical point of “no return” in much the same way as the field boundary of a black hole, its horizon, prevents the return of matter to free space.

The interdependent nature of the conjugate variables is evident when comparing the way the laws of gravity and electromagnetism are applied. We use the conjugate variables position and momentum of test particles to map *local* configurations of the electric, magnetic, and gravitational fields. The energy and time variables, on the other hand, are used to describe *nonlocal, globally determined* coordinates in space; as manifested by the space-like, equipotential surfaces of electron shells and the event horizons of black holes; and they also describe the time evolution of atomic clocks that are accurate to within 1 sec over the lifetime of the universe and the formation of galactic structure over time periods of the same length. The conjugate variables energy and time are an effective means for describing cyclic processes because they are based on the universal laws of energy conservation and equipartition. When expressed four-dimensionally by means of an action integral they give a relativistically correct description of the localization of electromagnetic and gravitational energy during absorption and emission processes with an experimentally proven accuracy based on atomic clocks that is far beyond our ability to measure. Descriptions of the natural laws that seek to derive equations of motion using the conjugate variables position and momentum are more restrictive due to the uncertainty principle and less able to predict the long term evolution of physical processes.

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