doi:10.20944/preprints202008.0550.v1

On the Jupiter's influence on the measurement of the gravitational constant

Zhi Cheng

gzchengzhi@hotmail.com

Abstract: This article analyzes in detail the influence of Jupiter on the measurement of the gravitational constant. Including a specific estimate of the change in the gravitational constant caused by the change of the earth's orbital position every half month. The influence of Jupiter's perihelion and aphelion on the gravitational constant is also estimated. On this basis, this article analyzes the data with specific time records such as BIPM-01, BIPM-14, JILA-10, UCI-14, HUST-09. The analysis results show that after considering the influence of Jupiter, the experimental results have been significantly improved.

Keywords: gravitational constant; measurement of gravitational constant; Jupiter

1 Introduction

The measurement of the gravitational constant is very difficult. Nevertheless, with the development of various related technologies since 2000, the measurement accuracy of the gravitational constant has been significantly improved. Therefore, some systematic errors that could not be considered in the past can also be taken into account, thereby effectively improving data consistency.

Nevertheless, the measurement accuracy of the gravitational constant is relatively low compared to other physical constants[1]. This has also led to the emergence of some theories about the gravitational constant that may change over time. The most famous of these is Dirac's theory of large numbers[2]. According to Dirac's theory of large numbers, as the universe continues to expand, the gravitational constant should also change. However, no definite evidence has been found in this regard. In addition, some authors have discovered that the gravitational constant measured in the past thirty years has a certain degree of oscillation[3,4].

As the planets in the entire solar system move around the sun, it will inevitably cause changes in the distribution of the gravitational field in the solar system.

Most of the mass of the solar system is concentrated in the sun. But even though Jupiter is only one-thousandth of the mass of the sun, it is enough to affect gravitational activity on the earth. One of the effects is the measurement experiment of the gravitational constant on the earth.

With the assumption that space-time compression will increase the gravitational constant, we can

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 August 2020 doi:10.20944/preprints202008.0550.v1

specifically calculate the influence of Jupiter's position on the gravitational constant measured on the earth. This article will give a more detailed estimation method.

2 Theoretical model and estimation

Let us now consider the relationship between the degree of compression of spacetime caused by the change in position between the sun and Jupiter. Figure 1 shows the position between the sun and Jupiter.

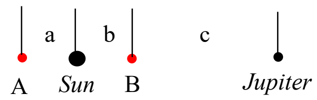

Figure 1. The position of Sun and Jupiter

In Figure 1, the small circle *Jupiter* represents planet Jupiter, and the large circle *Sun* represents the star sun. A and B respectively mark two positions on the earth. It can be seen from the figure that position B is located near the center of mass of the sun and Jupiter. Since the gravitational forces of Jupiter and the sun can cancel each other, the curvature of this spacetime is not as good as that of point A. However, due to the squeezing of the masses of Jupiter and the Sun, the spatial density of point B is greater than that of point A.

The larger the spatial density of point B, it means that the spatial measurement scale of this location is shorter. Although the spacetime of point A is more severely curved, the spatial density is not as large as that of point B. Therefore, the space measurement scale at point A is longer. Consider here that the distance between A and B is large enough from the sun, so that the time is basically flat. Believe that we only need to consider the density of space, and ignore the impact of time changes.

Then we consider the Schwarzschild radius corresponding to a mass. Consider that in Figure 1, points A and B are two different positions of the earth's orbit. The mass of the earth is m_e.

In this way, we can express the Schwarzschild radius of the earth as

$$r_{earth} = \frac{2GM}{c^2}$$

We can notice that in the flat spacetime observation system far away from the position of the sun and Jupiter, the observed Schwarzschild radius of the earth should not change, otherwise the solution of the Schwarzschild radius cannot be derived from the Einstein field equation. The problem now is that if you use the position of the earth at point B as the frame of reference, you will actually find that the Schwarzschild radius of the earth becomes longer. This is because the spatial scale used to measure the Earth's Schwarzschild radius has shrunk. Due to the conservation of energy, the mass of the earth cannot be changed regardless of whether it is at position A or at position B in Figure 1. According to the requirements of relativity, the speed of light is a constant. Therefore, an increase in Schwarzschild's radius means an increase in the gravitational constant *G*. For specific mathematical analysis, please refer to Appendix 1 of this article

This explains why the gravitational constant measured at position B is slightly larger than the gravitational constant measured at position A.

Let's make a simple estimate below.

Let the distance between point A and Sun be *a*; the distance between Sun and B as *b*; and the distance between B and Jupiter as *c*.

Suppose the mass of the earth is m_e. For the earth at point B, the sum of the gravitational potential energy of the sun and Jupiter is:

$$V_B = -Gm_e\left(\frac{M}{b} + \frac{m}{c}\right)$$

In the same way, for the earth at point A, the sum of the gravitational potential energy of the sun and Jupiter is

$$V_A = -Gm_e\left[\frac{M}{a} + \frac{m}{a+b+c}\right]$$

Here, **we assume that the magnitude of the spatial measurement scale change is proportional to the gravitational potential energy**, then

$$\frac{r_{earth}}{r'_{earth}} = \frac{V_A}{V_B} = \frac{\frac{M}{a} + \frac{m}{a+b+c}}{\frac{M}{b} + \frac{m}{c}} = \frac{G}{G'}$$

For convenience, suppose the earth moves around the sun in a circular orbit, then *a*=*b*

such

$$G' = \frac{1 + \frac{m}{M}\frac{a}{c}}{1 + \frac{m}{M}\frac{a}{2a+c}} G$$

Then substitute various parameters into the above formula, where

$$M = 2 \times 10^{30} kg$$

$$m = 2 \times 10^{27} kg$$

$$a = b = 1.5 \times 10^{11} m$$

$$a + c = 7.8 \times 10^{11} m$$

It can be calculated

$$G' = \frac{1 + \frac{1}{1000}\frac{1.5}{6.3}}{1 + \frac{1}{1000}\frac{1.5}{9.3}} G = 1.000077G = (1 + 7.7 \times 10^{-5})G$$

It can be seen that the biggest system uncertainty of the gravitational constant measured on the earth due to the different orbital position of Jupiter is about 77ppm. This uncertainty range is larger than the current most accurate measurement accuracy of the gravitational constant [3]. Therefore, the variation range of the gravitational constant can be roughly determined as

$$(6.674184 \pm 0.00051) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

3 Detailed calculation of Jupiter's influence on the gravitational constant

3.1 Jupiter and Earth are in circular orbits

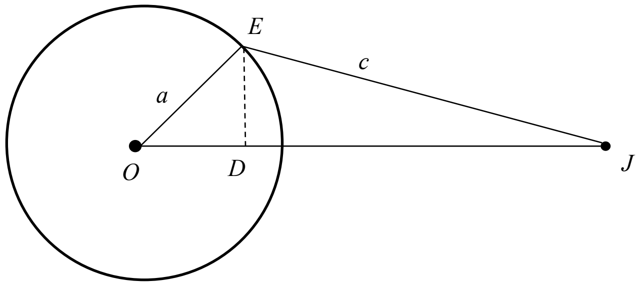

Figure 2. The positions of the earth, Jupiter and the sun

In Figure 2, *O* is the position of the sun, *E* is the position of the earth, and *J* is the position of Jupiter. Since Jupiter has an orbital period of 11.8 years, Jupiter and the sun can be regarded as stationary. For the convenience of calculation, cross point *E* to make a vertical line *ED* perpendicular to *OJ*.

OE is the orbital radius of Jupiter to the Sun. The orbit of Jupiter is approximately regarded as a circle here.

$$\angle EOD = \theta = \frac{x}{6}\pi$$

Where *x* is the time when the earth leaves Jupiter's opposition position, the unit is "month"

And the distance from Jupiter to Earth is

$$JE = c$$

Then

$$OD = acos\theta$$

$$ED = asin\theta$$

$$JD = OJ - OD$$

Therefore

$$c = \sqrt{a^2sin^2\theta + (OJ - OD)^2}$$

Then calculate the total potential energy of the Sun and Jupiter at point *E*

$$V_E = -G_0 m_e\left(\frac{M}{a} + \frac{m}{c}\right)$$

Where G_0 is the gravitational constant, m_e is the mass of the earth, *M* is the mass of the sun, and *m* is the mass of Jupiter.

Suppose Jupiter's opposition to the sun is the base point and its potential energy is V_0, then

$$V_0 = -G_0 m_e\left(\frac{M}{a} + \frac{m}{OJ - a}\right)$$

Note that the above potential energy is the result of observation in the distant flat space-time reference frame. Therefore, the obtained gravitational constant is set as G_0. However, the gravitational constant can be cancelled out.

$$G' = \frac{1 + \frac{m}{M}\frac{a}{OJ - a}}{1 + \frac{m}{M}\frac{a}{c}}G$$

Among the above parameters

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 August 2020 doi:10.20944/preprints202008.0550.v1

$$M = 2 \times 10^{30} kg$$

$$m = 2 \times 10^{27} kg$$

$$a = 1.5 \times 10^{11} m$$

$$OJ = 7.8 \times 10^{11} m$$

In this way, the change in the gravitational constant caused by the deviation from Jupiter’s opposition point every half month can be calculated, namely

$$G' = (1 + \alpha)G$$

Where G is the time of Jupiter's opposition to the sun, the value of the gravitational constant measured at the position of the earth. G' is the value of the gravitational constant measured at other locations.

The results are shown in Table 1

Table 1 The error of the gravitational constant caused by the time deviation from Jupiter opposition

Time (month)	1+α	Errors ($\times 10^{-11} m^3 kg^{-1} s^{-2}$)
0.5	1.000002	1.57215E-05
1	1.000009	5.92566E-05
1.5	1.000018	0.00012163
2	1.000029	0.000192522
2.5	1.000039	0.000263405
3	1.000049	0.000328628
3.5	1.000058	0.000385083
4	1.000065	0.000431393
4.5	1.000070	0.000467166
5	1.000074	0.000492474
5.5	1.000076	0.000507533
6	1.000077	0.000512529

3.2 The influence of Jupiter's perihelion and aphelion on the gravitational constant

Since the Earth’s eccentricity is relatively low compared to Jupiter, the Earth’s orbit can be regarded as a circle. Let us consider the influence of Jupiter's own perihelion and aphelion on the gravitational constant. The distance of Jupiter's perihelion is $7.41 \times 10^{11} m$, and the aphelion is $8.14 \times 10^{11} m$

By formula

$$G' = \frac{1 + \frac{m}{M}\frac{a}{d}}{1 + \frac{m}{M}\frac{a}{2a+d}} G$$

The distance between the earth and the Sun is a; when Jupiter opposes the sun, the distance between the earth and Jupiter is d.

If Jupiter is at the aphelion position, the difference in the gravitational constant measured during Jupiter's opposition is

$$G' = (1 + 7 \times 10^{-5})G$$

If Jupiter is at the perihelion, then

$$G' = (1 + 8.5 \times 10^{-5})G$$

In the above formula, G represents the gravitational constant measured on the surface of the earth when the earth is on the other side of Jupiter and the sun.

It can be seen that Jupiter's perihelion and aphelion have little effect on the gravitational constant measured on the earth. The impact is approximately $\pm 7ppm$.

However, when calculating, if you consider whether Jupiter is at the perihelion or the aphelion position, the calculation results should be improved. At that time, this improvement should be related to the accuracy of the instrument used. For example, theoretical calculations show that the measurement interval deviates from Jupiter's perihelion or aphelion by about 6 years, and the position of Jupiter's perihelion or aphelion has a greater influence on the result. However, if the time span is so large, factors such as the aging of equipment and materials may cause greater system errors.

4 Analysis of some time-recorded measurement values of gravitational constants

Since Cavendish measured the gravitational constant, humans have measured the gravitational constant many times. However, considering that the relative error of the measurement of the gravitational constant before 2000 is close to 150ppm. Such a relative error exceeds the estimation results of this article and my previous days, so these data are not suitable for analyzing the data of Jupiter's orbit on the earth's gravity measurement. For example, for the measurement result of TR&D-96 [2], the accuracy is too low, about 6.6730 (90). It can be seen that the gravitational constant measured before 2000 has no obvious correlation with the specific month. Therefore, below this accuracy, the changes in the positions of Jupiter and the Earth have little effect on the results.

Since 2000, the accuracy of the measurement of the gravitational constant has been improved to a certain extent. By 2018, the accuracy of the measurement of the gravitational constant has been improved to 12ppm [3]. Such a relative error exceeds the influence of Jupiter on the measurement of the gravitational constant. Therefore, some common changing laws should be found from these data and considered as systematic errors.

In addition, at present, the systematic error caused by different measuring devices is indeed relatively large. For example, the measurement accuracy of BIPM-01[4] and BIPM-14[5,6] are very high, but their results are very different from the measurement results of the HUST series, and it is difficult to explain the influence of Jupiter. Therefore, it can be determined that the difference of the measuring device will cause a large systematic error. Therefore, to compare the data and understand the influence of Jupiter on the measurement of the gravitational constant on the earth, the same device should be mainly used.

4.1 BIPM-01 and BIPM-14

The first measurement by Quinn et al. was from October 1 to October 30, 2000 [4]. The result of the measurement is $6.67559 \times 10^{-11} m^3 kg^{-1} s^{-2}$. In 2000, Jupiter's opposition time was November 28, 2000, and there was a **one-month** difference between the two. It shows that the earth and Jupiter are basically on the same side.

The second measurement was from August 7th to September 7th, 2007 [5,6]. The result of the measurement is $6.67545 \times 10^{-11} m^3 kg^{-1} s^{-2}$. Jupiter will oppose the sun in 2007 on June 5, 2007. There is a difference of **two months** between the two, and the earth has already begun to leave Jupiter. The measured result is slightly smaller.

According to Table 1, the first measurement is 0.000009*G* smaller than Jupiter's opposition point. Now add 0.000009*G* to the first measurement result to get

$$6.67565 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

The second measurement is 0.000029*G* smaller than Jupiter's opposition. Now add 0.000029*G* to the result of the second measurement, we can get:

$$6.67564 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

It can be seen that after considering the influence of Jupiter, the results of the two experiments have been significantly improved.

 doi:10.20944/preprints202008.0550.v1

4.2 JILA-10

The experiment of JILA-10 was mainly completed from May to June 2004 [7], and their measurement results are $(6.67234 \pm 0.00014) \times 10^{-11} m^3 kg^{-1} s^{-2}$. Considering that Jupiter's opposition in 2004 is March 4th. Therefore, during the measurement process from May to June, the earth is gradually moving away from Jupiter, so the measured value will gradually decrease. In the paper by Parks et al., Figure 2 shows the series of data measured during this time [7]. It can be seen that the value of the gravitational constant measured in June has dropped significantly.

Although there is no specific data, it can be roughly seen from the figure that the average difference between the data in May and the data measured in June is about $0.00001G$.

4.3 UCI-14

The measurement time of UCI-14 is 9-11/2000, 12/2000, 3-5/2002, 3-5/2006[8]. Among them, the measurement in 2004 was discarded due to too much noise signal. Newman et al. used three types of fibres. Among them, 9-11/2000 used the first fibre, 12/2000, 3-5/2002 used the second fibre, and 3-5/2006 used the third fibre.

Although the second fibre was used in 12/2000, the number of experiments was only more than one hundred, so the second fibre was mainly used in 3-5/2002.

In 2000, Jupiter opposed the sun on November 28. Therefore, the 9-11/2000 experiment differs from Jupiter's opposition time by about **2 months**. The 12/2000 experiment coincided with Jupiter's opposition to the sun. The opposition of Jupiter in 2002 was January 1st. Thus, the time difference between the 3-5/2002 experiment and Jupiter's opposition is about **4 months**. Jupiter's opposition in 2006 was May 4th. It can be seen that in 3-5/2006, the Earth was closest to Jupiter, a difference of about **half a month**. At this time, the maximum gravitational constant value should be measured. Although the earth was very close to Jupiter at the time of 12/2000, considering that the data measured in this month was relatively small and averaged by the data of 3-5/2002, the measurement result of fibre 2 was mainly affected by the measurement data of 3-5/2002. Considering that the earth began to move away from Jupiter in 3-5/2002, the value of the gravitational constant measured during this period should be relatively small.

Through the above analysis, we can conclude that the order of the experimental measurement results is:

$$\text{Fibre 3} > \text{Fibre 1} > \text{Fibre 2}$$

The actual measurement result is

Fibre 1: $6.67435(10) \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Fibre 2: $6.67408(15) \times 10^{-11} m^3 kg^{-1} s^{-2}$

Fibre 3: $6.67455(13) \times 10^{-11} m^3 kg^{-1} s^{-2}$

It can be seen from the results of UCI-14 that the first result in 2000 was greater than that in 2002, and 2006 was closer to Jupiter, and the result was the largest.

Therefore, according to Table 1, adding the data of Fibre1 to the effect of **two months**, that is, the difference of 0.000029G, you can get

$$6.67454(10) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

Add the data of Fibre2 to the impact of **four months**, that is, the difference of 0.000065G, you can get

$$6.67451(15) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

Adding the Fibre3 data to the impact of **half a month**, a difference of about 0.000002G, you can get

$$6.67457(13) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

It can be seen that the three sets of data have been significantly improved.

4.4 HUST-09

The first experiment of HUST-09 was from March 21, 2007 to May 20, 2007, and from April 19, 2008 to May 10, 2008 [9, 10]. The time of Jupiter's opposition in 2007 and 2008 are June 5, 2007 and July 9, 2008.

The second experiment was from August 25, 2008 to September 28, 2008, and from October 8, 2008 to November 16, 2008.

It can be seen that the first experiment is closer to the time of Jupiter opposition. The time interval is about **2 months**. The second experiment was a little farther away from Jupiter's opposition, about **two and a half months** on average. Therefore, theoretically, the result of the first experiment measurement will be larger than the result of the second experiment.

The reality is that the gravitational constant measured by Luo's team in the first experiment is:

$$(6.67352 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

The gravitational constant measured in the second experiment is:

$$(6.67346 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

According to the calculation in Table 1, if the data of the first experiment plus the influence of $0.000029G$ Jupiter, the result of the first experiment is

$$(6.67371 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

The second experiment took **two and a half months** longer than Jupiter's opposition to the sun. According to Table 1, it can be seen that the result of the second experiment is $0.000039G$ less than the gravitational constant of Jupiter's opposition point. So, the second experiment data plus the Jupiter influence of $0.000039G$, the second data will become:

$$(6.67372 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

This is exactly the same as the result after the first experimental correction. The experimental data has improved significantly.

5 Conclusion

For the measurement data of the gravitational constant before 2000, it is difficult to consider the influence of Jupiter on the measurement results of the gravitational constant on the earth due to insufficient accuracy. But after 2000, due to the improvement of various technologies, the accuracy of the measurement of the gravitational constant has also been improved to a certain extent. The current best accuracy has reached 12ppm. This has exceeded the estimated influence of Jupiter on the earth's gravitational constant. The estimated result of this paper is that if we consider the two extreme cases of Jupiter's opposition and the earth's farthest distance from Jupiter, the relative systematic error of measuring the gravitational constant on the earth will reach 77ppm. Based on this consideration, this paper retrieves the experiment data of the measurement of the gravitational constant since 2000, and specifically analyzed the results with more detailed time records. The analysis results show that if Jupiter's influence on the measurement of the gravitational constant is taken into account, the accuracy of the revised experimental data will be significantly improved.

References

[1] Rothleitner, C. & Schlamminger, S. Measurements of the Newtonian constant of gravitation, G. Rev. Sci. Instrum. 88, 111101 (2017).

[2] CANUTO, V., ADAMS, P. & TSIANG, E. Crystal structure and Dirac's Large Numbers Hypothesis. Nature 261, 438 (1976).

[3] J. D. Anderson, G. Schubert, V. Trimble and M. R. Feldman. "Measurements of Newton's gravitational constant and the length of day." EPL 110 (2015) 10002,

[4] Schlamminger, Stephan, Jens H. Gundlach, and R. D. Newman. "Recent measurements of the gravitational constant as a function of time." Physical Review D 91.12 (2015).

[5] Karagioz, O. & Izmailov, V. Measurement of the gravitational constant with a torsion balance. Meas. Tech. 39, 979–987 (1996).

 doi:10.20944/preprints202008.0550.v1

[6] Li, Q., Xue, C., Liu, J. *et al.* Measurements of the gravitational constant using two independent methods. *Nature* **560,** 582–588 (2018).
[7] Quinn, T. J., Speake, C. C., Richman, S. J., Davis, R. S. & Picard, A. A new determination of G using two methods. Phys. Rev. Lett. 87, 111101 (2001).
[8] Quinn, T. J., Parks, H. V., Speake, C. C. & Davis, R. S. Improved determination of G using two methods. Phys. Rev. Lett. 111, 101102 (2013).
[9] Quinn, T., Speake, C., Parks, H. & Davis, R. The BIPM measurements of the Newtonian constant of gravitation, G. Phil. Trans. R. Soc. A 372, 20140032 (2014).
[10] Parks, H. V. & Faller, J. E. Simple pendulum determination of the gravitational constant. Phys. Rev. Lett. 105, 110801 (2010).
[11] Newman, R., Bantel, M., Berg, E. & Cross, W. A measurement of G with a cryogenic torsion pendulum. Phil. Trans. R. Soc. A 372, 20140025 (2014).
[12] Luo, J. et al. Determination of the Newtonian gravitational constant G with time-of-swing method. Phys. Rev. Lett. 102, 240801 (2009).
[13] Tu, L. C. et al. New determination of the gravitational constant G with time-of-swing method. Phys. Rev. D 82, 022001 (2010).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 August 2020 doi:10.20944/preprints202008.0550.v1

Appendix 1: The Mathematical Principle of the Influence of the Change of Space-time Structure on the Gravitational Constant

1 Introduction

Figure 1(a) shows a space-time coordinate, reflecting the situation where space-time is compressed from left to right. The scale on the axis indicates the length of the space unit.

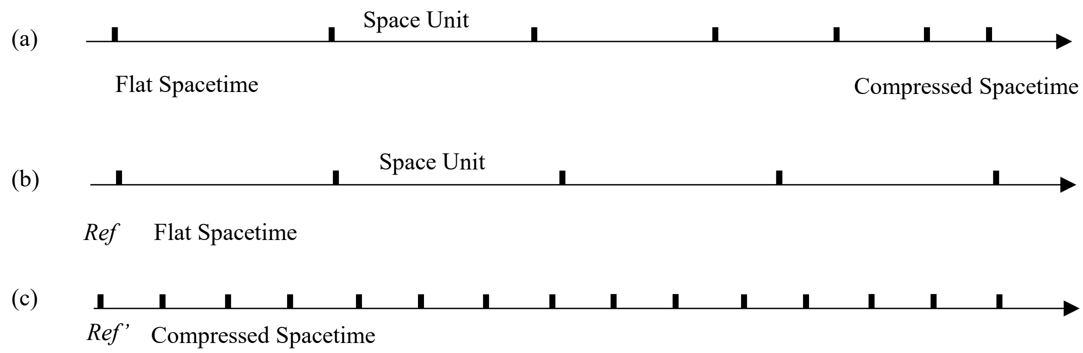

Figure 1. Flat and compressed spacetime

As can be seen from the top coordinate (a) in Figure 1, because space-time is compressed, it means that different spatial scales are displayed on the space-time coordinates. In the compressed space-time range, the space unit is relatively short, while in a relatively flat space-time, the space unit is relatively long. However, it is very inconvenient to deal with the problem of a coordinate system whose space unit length is changing. Therefore, (b) and (c) in Figure 1 introduce two coordinate systems with uniform scales, *Ref* and *Ref'*, respectively.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 August 2020 doi:10.20944/preprints202008.0550.v1

2 Changes in the length of different spacetime reference systems

Definition 1: Space-time reference system refers to a coordinate system with uniform scale with a specific space unit length as the unit of measurement, here referred to as the reference system.

In Figure 1(a), if we use the relatively flat space unit length on the left as a scale to mark the entire system, then this space-time can be called a flat space-time reference system, which is represented by *Ref* in Figure 1(b). It can be seen that the scale of the *Ref* reference system is uniform, but the scale interval is longer. And if the more severely compressed space unit length on the right is used as a scale to mark the entire system, then this space-time can be called a compressed space-time reference system. It is represented by *Ref'* in Figure 1(c). It can be seen that the scale of the *Ref'* spacetime reference system is relatively small.

Definition 2: The space unit length *I* represents the basic unit used to measure a length in a certain spacetime reference system.

Definition 3: Intrinsic length refers to the length measured in the space-time reference system whose space unit length is 1. The intrinsic space length can be expressed as a multiple of the space unit length.

Definition 4: The intrinsic length is expressed in bold font. Letters in unbold font indicate the length measured in a certain spacetime reference system. The length measured in a spacetime reference system multiplied by the space unit length of the corresponding reference system is equal to the intrinsic length.

which is

$$\boldsymbol{a} = aI$$

Where $\boldsymbol{a}$ is the intrinsic length, I is the length of the space unit of the reference system, and a is the length measured in the reference system.

With the above definitions of some terms, let's determine the axiom system.

Axiom 1: Space-time is an elastic substance that can be squeezed by mass

This is basically consistent with the assumption of general relativity. In general relativity, the space-time around the mass will be curved due to the existence of mass. And this bending of time and space also means that time and space are compressed.

Axiom 2: The compressed space-time has a shorter space unit length. A completely flat space-time without any mass and energy has an infinite space unit length.

In Figure 1, for a relatively flat space-time, the space unit length is limited. But compared to those compressed space-time, the space unit length is much larger. But if there is a completely flat space-time at infinity, it means that there is no mass or energy in it, so this space-time has no physical meaning. For a space-time without any physical meaning, there is no spacetime unit length. Conversely, if a space-time has a finite spacetime unit length, it means that the space-time can be measured and has physical meaning.

Therefore, the term "flat space-time" is used in this article to mean space-time with a very small amount of mass and energy. And completely flat space-time means space-time without mass and energy at infinity.

According to the above axioms, we can further reason and obtain a series of meaningful theorems.

Theorem 1: If the same length result is measured in different spacetime reference systems, the intrinsic length corresponding to the measured length of the compressed spacetime reference system is shorter.

As shown in Figure 1, if the measured result in any reference frame is a, then the length is observed in the compressed space-time reference frame *Ref'*, and the length can be expressed as:

$$\boldsymbol{a}' = aI'$$

Now switch to the flat space-time reference system *Ref*. Since the unit length of the *Ref* reference system is I, therefore

$$\boldsymbol{a} = aI$$

Since

$$I > I'$$

Then

$$\boldsymbol{a} > \boldsymbol{a}'$$

It shows that although the measurement results of the two reference systems are the same, their intrinsic lengths are different. The intrinsic length of the flat space-time reference frame is longer.

Theorem 2: With the same intrinsic length, the result measured in the compressed space-time reference system is longer than that of the flat space-time reference system

Prove:

In *Ref*

$$\boldsymbol{a} = aI$$

In *Ref'*

$$\boldsymbol{a} = a'I'$$

Since

$$I > I'$$

Then

$$a' > a$$

Corollary 1: The Schwarzschild radius measured by the compressed space-time reference frame is longer

Consider the Schwarzschild radius solved by Einstein's field equation as an intrinsic length $\boldsymbol{r_s}$.

If we observe $\boldsymbol{r_s}$ in a relatively flat spacetime reference frame *Ref*

We can get

$$\boldsymbol{r_s} = r_s I$$

Now we switch to the compressed space-time reference system *Ref'* to observe $\boldsymbol{r_s}$.

We can get

$$\boldsymbol{r_s} = r_s' I'$$

Since the unit length of the *Ref* reference system is larger, it means

$$I > I'$$

Then

$$r_s' > r_s$$

 doi:10.20944/preprints202008.0550.v1

3 Mass in different spacetime reference frames

Theorem 3: According to the requirements of conservation of energy, the rest masses observed in any space-time reference frame are equal.

Prove:

According to the relativistic mass-energy formula

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

Although the mass is located in a non-inertial frame of reference (such as a compressed space-time frame of reference), it is always possible to use an appropriate method, such as changing the inertial frame of reference so that p=0

therefore:

$$E = \sqrt{m_0^2 c^4} = m_0 c^2$$

It means that the static mass of the object will not change regardless of whether it is observed from the *Ref* or *Ref'* reference system.

4 The relationship between space unit length and gravitational potential

Conjecture: The length of space unit is inversely proportional to the gravitational potential

Regarding the relationship between space unit length and gravity, if space-time is regarded as an elastic substance, the squeezing of space-time by mass will compress the surrounding space-time. This space-time compression effect will cause a change in the spacetime unit length. According to the knowledge of elasticity, the displacement of space-time can be solved, and then the specific situation of space-time compression can be calculated, and then the relationship between space unit length and gravity can be calculated from this.

But such calculations will face some serious problems. For example, we don't know if space-time is an elastic substance, what is its elastic modulus? In addition, space-time is four-dimensional, and solving with three-dimensional elasticity knowledge will also face the problem of incorrect results caused by deformation in the time dimension.

Here we understand the relationship between gravity and space unit length from Axiom 2 proposed in this article.

According to Axiom 2, the space unit length of a completely flat space-time at infinity is infinite. Therefore, the relationship between unit length and gravity can be expressed in many forms, including: reciprocal relationship, inverse square relationship, multiple inverse square relationship, exponential relationship, etc.

The simplest one is the reciprocal relationship, namely:

$$g = \frac{a}{r}$$

Here g is used to represent a physical quantity related to gravity. It can be force, field strength, gravitational potential, etc. In addition, considering that the magnitude of gravity is usually proportional to mass, the above formula can also be expressed as:

$$g = \frac{bM}{r}$$

If we let

$$b = -G$$

Then the above formula becomes

$$g = -\frac{GM}{r}$$

This is actually the formula for calculating gravitational potential.

If the above relationship adopts the reciprocal relationship of multiple powers, it can be expressed as:

$$g = -\frac{GM}{r^{\alpha}}$$

If expressed in exponential form, then

$$g = -GMe^{-r}$$

This article adopts the reciprocal relationship, that is, the gravitational potential is used to express the magnitude of gravity.

 doi:10.20944/preprints202008.0550.v1

In this way, the relationship between space unit length and distance is proportional, and inversely proportional to gravitational potential. which is:

$$I = kr = -\frac{kGM}{V}$$

5 Gravitational constants in different spacetime reference frames

Theorem 4: The measured value of the gravitational constant of the compressed space-time reference frame is larger

Proof: For an object with a mass of *m*, the Schwarzschild solution can be obtained by using the Einstein field equation. This solution can be regarded as the intrinsic length of the Schwarzschild radius, namely

$$\boldsymbol{r_s} = \frac{\boldsymbol{G}m}{c^2}$$

According to Theorem 2, the Schwarzschild radius measured in a flat space-time reference frame is expressed as

$$r_s = \frac{Gm}{c^2}$$

The Schwarzschild radius measured in the compressed space-time reference frame is expressed as

$$r_s{}' = \frac{G'm}{c^2}$$

According to Theorem 3, no matter which space-time reference frame is in, the static mass of the object will not change. And c is a constant, so the only thing that can be changed on the right side of the equation is the gravitational constant. Therefore, the gravitational constant of the compressed space-time reference frame is represented by G'.

considering

$$r_s' > r_s$$

Then we can get

$$G' > G$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 August 2020 doi:10.20944/preprints202008.0550.v1

Theorem 5: The gravitational constant in different spacetime reference frames is proportional to the gravitational potential at that place

prove

$$\frac{G'}{G} = \frac{r_s'}{r_s} = \frac{\boldsymbol{r_s} I}{\boldsymbol{r_s} I'} = \frac{V'}{V}$$