

Article

Classical Mechanics as a Fundamental Law of Quantum Mechanics

Yehuda Roth ¹ *

¹ Oranim College; yudroth@gmail.com

Abstract: In our previous paper, we showed that the so-called quantum entanglement also exists in classical mechanics. The inability to measure this classical entanglement was rationalized with the definition of a classical observer which collapses all entanglement into distinguishable states. It was shown that evidence for this primary coherence is Newton's third law. However, in reformulating a "classical entanglement theory" we assumed the existence of Newton's second law as an operator form where a force operator was introduced through a Hilbert space of force states. In this paper, we derive all related physical quantities and laws from basic quantum principles. We not only define a force operator but also derive the classical mechanic's laws and prove the necessity of entanglement to obtain Newton's third law.

Keywords: post collapse entanglement; commutators pictures; force operator; labeling set

I. Introduction

Quantum resource theories, quantum entanglement in particular [3], are considered to be beyond classical understanding, offering a highly versatile and powerful framework for studying different phenomena in quantum physics [4]. In a previous paper [5] it was shown that classical particles do entangle but with no observable evidence because all measurements a classical observer can conduct, breaks this coherence to provide the well-known distinguishable states. This study proposes that the main evidence for that primary entanglement is Newton's third law.

The concept of quantum entanglement started with the famous E.P.R Gedanken experiment [6] that was initially presented to demonstrate the incompleteness of quantum theory. Later, by D. Bohm's interpretation [7], the EPR state was introduced as two $1/2$ -spins particles that are entangled to form a zero-spin entity. The key point is that no matter their distance apart, the particles still belong to the same entity-the zero-spin particle. This characteristic is the source for an almost ideal encryption theory: If a code is carried by a pair of entangled particles, a local observer that observes only a single particle cannot either encrypt or duplicate the code because of the lack of information after interacting with the particle (for example an implication, see ref. [8]). Following ref. [5] for classical entanglement, if in some space, prior to the classical observer measurement, classical particles were entangled, it would be interesting to be able to determine the identifying feature that defines them as one. This paper does not answer that question but motivates us to dig deeper into the entangled side of classical mechanics with the hope that this complementary feature will be revealed in the future.

Introducing entanglement in classical mechanics requires the exploration of the relation between classical and quantum theories. In this paper, this variation is introduced through the concept of particle-labeling. In classical mechanics, labeling relates to the distinguishability and locality of classical particles. In our approach, however, labeling implies a simple quantum technique where a spanning labeling set is the basis to which all operators are projected to. Other approaches concerning locality are reviewed as follows:

Non-locality and entanglement were shown to be related topics [9]. In recent years, non-local theories of

37 gravity and strings [10–23] have been presented. The attempts have been made through other theories
 38 to describe non-locality with quark confinement and anomalies [24,25]. Many theories introduce
 39 non-locality through the quantization of a class of relativistic non-local field equations, characterized
 40 by a non-local kinetic term in the Lagrangian [26]. El-Nabulsi [27] analyzes the implications of the
 41 quantum acceleratum operator, which is one of the main consequences of quantum non-local-in-time
 42 kinetic energy. Correspondingly, a non-local Heisenberg uncertainty is introduced.
 43 As mentioned earlier, an attempt to find a relation between quantum and classical mechanics was
 44 performed by introducing entanglement into the local classical mechanics [5]. However, the locality
 45 was not introduced implicitly. Instead, it was assumed that the localized particles can be labeled,
 46 implying that in quantum terminology they are represented with a labeling basis of states. The main
 47 idea was that even classical particles can be entangled but the local nature of the classical observer that
 48 labels the particles, breaks this coherence to provide the local classical mechanics laws. Although the
 49 quantum terminology was implemented in the paper of ref [5], the main formalism was fundamental
 50 in the sense that operators such as force, were introduced fundamentally through a series of projecting
 51 operators with no quantum justification. In addition, Newton's second law was assumed to be fulfilled
 52 in its operator form. In this paper, we reformulate classical mechanics beginning with pure quantum
 53 principles. We then prove that the consistency of our theory must be accompanied with classical
 54 entanglement.

55 II. Materials and Methods

56 This is a pure mathematical work. We implemented techniques of Hilbert and mostly Folk
 57 spaces to introduce classical mechanics in a quantum-like terminology. No experiment or computer
 58 proceeding were implemented.

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61 III. Introductory Section: Commutators Pictures and Representations

A trivial identity is

$$[\mathcal{H}, \vec{\mathcal{P}}] = -[\vec{\mathcal{P}}, \mathcal{H}] \quad (1)$$

where in the Heisenberg picture we have

$$[\mathcal{H}, \vec{\mathcal{P}}_{\langle E \rangle}] = i\hbar \frac{d\vec{\mathcal{P}}_{\langle E \rangle}}{dt} - i\hbar \frac{d}{dt} \vec{\mathcal{P}} \quad (2)$$

with,

$$\begin{aligned} \vec{\mathcal{P}}_{\langle E \rangle} &= \mathcal{U}_{\langle E \rangle} \vec{\mathcal{P}} \mathcal{U}_{\langle E \rangle}^\dagger \\ \mathcal{U}_{\langle E \rangle} &= \exp\left[-i\frac{\mathcal{H}t}{\hbar}\right]. \end{aligned} \quad (3)$$

We implement a conventional momentum operator, that is, $\vec{\mathcal{P}} \neq \vec{\mathcal{P}}(t)$ which causes the last term of eq. 2 to be canceled. This yield:

$$[\mathcal{H}, \vec{\mathcal{P}}_{\langle E \rangle}] = i\hbar \frac{d\vec{\mathcal{P}}_{\langle E \rangle}}{dt} \quad (4)$$

Using the space representation $\vec{\mathcal{P}} = -i\hbar \vec{\nabla}$ it can also be shown that,

$$[\vec{\mathcal{P}}, \mathcal{H}_{\langle p \rangle}] = -i\hbar \vec{\nabla} \mathcal{H}_{\langle p \rangle} \quad (5)$$

with,

$$\begin{aligned}\mathcal{H}_{\langle p \rangle} &= \mathcal{U}_{\langle p \rangle} \mathcal{H} \mathcal{U}_{\langle p \rangle}^\dagger \\ \mathcal{U}_{\langle p \rangle} &= \exp \left[i \frac{\vec{p} \cdot \vec{r}}{\hbar} \right].\end{aligned}\quad (6)$$

62 where we assumed that $\mathcal{H} \neq \mathcal{H}(t)$.

63 Thus, if we substitute eqs. 4 and 5 into eq. 1, we obtain the classical law $\vec{\nabla} \mathcal{H} = -\dot{\vec{p}}$ as an output
64 of a trivial mathematical identity. Clearly, this is wrong as both operators are different in picture and
65 representation. For example, the momentum operator in eq. 4 is presented in the energy form while
66 in eq. 5 it is shown in the conventional form. Nevertheless, this paper will show that despite their
67 different pictures, if we project both commutators of eqs. 4 and 5 onto a common representation, both
68 sides of eq. 1 will be equatable, again, to the classical laws of mechanics. In our paper we implement
69 the labeling basis of states as the common representation. In addition, we show that the phases that
70 are accompanied by the exponential operators $\mathcal{U}_{\langle E \rangle}$ and $\mathcal{U}_{\langle p \rangle}$ generate the system dynamics.

71 III.1. A Classical Terminology of a Post Collapse Entanglement

72 An observer terminology describes the observable representation. By selecting a basis of states,
73 the observer defines the concepts to be used in describing its measurement results[28]. Because the
74 operators $\vec{P}_{\langle E \rangle}$ and $\mathcal{H}_{\langle p \rangle}$ in eqs. 4 and 5 are represented differently, it make no sense to equate them.
75 Mathematically, this means that either variable can appear simultaneously at different sides of the same
76 equation provided that they possess the same dimensions, or that physical operators must appear with
77 the same representation. One method of resolving this scenario is by projecting operators to the same
78 representation. In this paper we suggest that this common representation becomes the labeling basis
79 of states that distinguishes between particles [5], provided that each particle is associated solely with
80 a well-defined momentum, and we can consider a diagonal momentum operator under the labeling
81 basis of states. On the other hand, forces analysis is different. Consider a two bodies problem: In
82 expressing mechanical relations between forces and momentum, each particle is associated with its
83 individual momentum. However, in dealing with forces with the desire to solve the equations of
84 motion, the two bodies which share forces causes an integration between both sets of equations. In a
85 way similar to the encryption implication mentioned previously, we can say that both particles are
86 essential to reveal all the system information. However, this is merely a of mathematical statement:
87 "If you need to solve the equations of motion, you have to consider both particles and not only one".
88 Experimentally, at any moment, forces can be detected on each isolated particle. This is because these
89 particles are already in a collapsed state, having adjusted to describe each isolated particle.
90 To define mutual forces, we consider that under the labeling basis, the force operator is non-diagonal.
91 Thus, the off-diagonal terms will correlate with different labeled states. It will be shown that this
92 non-diagonal representation gives rise to the pre-collapsed entangled states.

93 IV. Force Operator $\vec{F}_{\langle p \rangle}$ expressed in an off-Diagonal Momentum States

94 In ref. [5], we introduce the force state $|\vec{F}\rangle$ as an eigenstate of the force operator where the operator
95 was defined through a general formalism of projecting operators. In this section we implement eq. 5 to
96 obtain an explicit expression for the force operator.

We first use the identities:

$$\begin{aligned}\mathcal{U}_{\langle p \rangle} &= \sum_{\vec{p}} \exp \left[i \frac{\vec{p} \cdot \vec{r}}{\hbar} \right] |p\rangle \langle p|, \\ \mathcal{U}_{\langle p \rangle}^\dagger &= \sum_{\vec{p}} |p\rangle \langle p| \exp \left[-i \frac{\vec{p} \cdot \vec{r}}{\hbar} \right]\end{aligned}\quad (7)$$

to obtain:

$$\mathcal{H}_{\langle p \rangle} = \sum_{\vec{p}, \vec{p}'} \exp \left[-i \frac{(\vec{p} - \vec{p}') \cdot \vec{r}}{\hbar} \right] \langle \vec{p} | \mathcal{H} | \vec{p}' \rangle | \vec{p} \rangle \langle \vec{p}' | \quad (8)$$

Using the momentum operator, which is expressed through projecting operators,

$$\vec{\mathcal{P}} = \sum_{\vec{p}} \vec{p} |\vec{p}\rangle \langle \vec{p}|, \quad (9)$$

we find that,

$$\begin{aligned} \mathcal{P}\mathcal{H}_{(p)} &= \sum_{\vec{p}, \vec{p}', \vec{p}''} \exp \left[-i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right] \vec{p}'' \langle \vec{p}'' | \vec{p} \rangle \langle \vec{p}| \mathcal{H} | \vec{p}' \rangle |\vec{p}''\rangle \langle \vec{p}'| \underbrace{\langle \vec{p}'' | \vec{p} \rangle = \delta_{\vec{p}, \vec{p}''}}_{\Rightarrow} \\ \Rightarrow \mathcal{P}\mathcal{H}_{(p)} &= \sum_{\vec{p}, \vec{p}'} \vec{p} \exp \left[i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right] \langle \vec{p}| \mathcal{H} | \vec{p}' \rangle |\vec{p}\rangle \langle \vec{p}'|. \end{aligned} \quad (10)$$

Similarly, it can be shown that

$$\mathcal{H}_{(p)}\mathcal{P} = \sum_{\vec{p}, \vec{p}'} \vec{p}' \exp \left[i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right] \langle \vec{p}| \mathcal{H} | \vec{p}' \rangle |\vec{p}\rangle \langle \vec{p}'|. \quad (11)$$

yielding,

$$\left[\vec{\mathcal{P}}, \mathcal{H}_{(p)} \right] = \sum_{\vec{p}, \vec{p}'} (\vec{p} - \vec{p}') \exp \left[i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right] \langle \vec{p}| \mathcal{H} | \vec{p}' \rangle |\vec{p}\rangle \langle \vec{p}'|. \quad (12)$$

Consider a Hamiltonian with the form $\mathcal{H} = \frac{p^2}{2m} + \mathbf{V}$. Note that the factor $\vec{p} - \vec{p}'$ eliminates the diagonal term of the kinetic energy which gives,

$$\left[\vec{\mathcal{P}}, \mathcal{H}_{(p)} \right] = \sum_{\vec{p}, \vec{p}'} (\vec{p} - \vec{p}') \exp \left[i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right] \langle \vec{p}| \mathbf{V} | \vec{p}' \rangle |\vec{p}\rangle \langle \vec{p}'| \quad (13)$$

Defining the *potential energy variable* (its meaning, not as an operator)

$$V_{p,p'}(\vec{r}) = \exp \left[i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right] \langle \vec{p}| \mathbf{V} | \vec{p}' \rangle \quad (14)$$

we obtain,

$$\left[\vec{\mathcal{P}}, \mathcal{H}_{(p)} \right] = -i\hbar \vec{\nabla} \sum_{\vec{p}, \vec{p}'} V_{p,p'}(\vec{r}) |\vec{p}\rangle \langle \vec{p}'|. \quad (15)$$

Using eq. 5 and by identifying $\left[\vec{\mathcal{P}}, \mathcal{H}_{(p)} \right]$ in the classical equation $\vec{F} = -\nabla H$, we obtain the force operator,

$$\vec{\mathcal{F}}_{(p)}(\vec{r}) = -\vec{\nabla} \sum_{\vec{p}, \vec{p}'} V_{p,p'}(\vec{r}) |\vec{p}\rangle \langle \vec{p}'| \quad (16)$$

or,

$$\left[\vec{\mathcal{P}}, \mathcal{H}_{(p)} \right] = i\hbar \vec{\mathcal{F}}_{(p)}(\vec{r}) \quad (17)$$

97 We notice the following:

- 98 • $\vec{\mathcal{F}}_{(p)}$ is an off-diagonal operator. This means that when applied to a momentum state it changes
- 99 from \vec{p}' to \vec{p} as a force operator.
- 100 • The gradient operation that describes a physical law, is generated by the phases $\exp \left[i \frac{(\vec{p} - \vec{p}') \cdot r}{\hbar} \right]$.
- 101 • The time variable t is missing in eq. 17. It appears later when we present momentum in the
- 102 energy representation as described in section VI. Further discussion is provided in section VII.

103 Note that if $\langle \vec{p}| \mathbf{V} | \vec{p}' \rangle = \langle \vec{p}'| \mathbf{V} | \vec{p} \rangle$ and is real, the force operator is Hermitian; meaning that in

104 the diagonal representation the orthogonal force-eigenstates with real eigenvalues are well defined

105 quantities.

106 V. Force Operator $\vec{\mathcal{F}}_{\langle F \rangle}$ expressed in the Diagonal Force States

The purpose of this section is to show that the force operator can be represented with the standard tools of quantum mechanics. That is, as a series of projecting operators accompanied with eigenvalues. Assuming a Hermitian $\vec{\mathcal{F}}$, its eigenstates $|\vec{F}\rangle$ generate a complete orthogonal basis of states that can be represented through the identity operator:

$$\mathcal{I}_{\langle F \rangle} = \sum_{\vec{F}} |\vec{F}\rangle \langle \vec{F}| \quad (18)$$

Thus, using the relation

$$\vec{\mathcal{F}}_{\langle F \rangle} = \mathcal{I}_{\langle F \rangle} \vec{\mathcal{F}}_{\langle p \rangle} \mathcal{I}_{\langle F \rangle}^\dagger, \quad (19)$$

we obtain,

$$\vec{\mathcal{F}}_{\langle F \rangle} = - \sum_{\vec{p}, \vec{p}'} \sum_{\vec{F}, \vec{F}'} \left(\vec{\nabla} V_{p, p'}(\vec{r}) \right) \langle \vec{F}' | \vec{p} \rangle \langle \vec{p}' | \vec{F} \rangle |\vec{F}'\rangle \langle \vec{F}| \quad (20)$$

As the force operator is diagonal in this representation, we obtain the relations that identify the state $|\vec{F}\rangle$:

$$\begin{aligned} \sum_{\vec{p}, \vec{p}'} \left(\vec{\nabla} V_{p, p'}(\vec{r}) \right) \langle \vec{F}' | \vec{p} \rangle \langle \vec{p}' | \vec{F} \rangle &= \vec{F} \delta_{\vec{F}, \vec{F}'} \\ \vec{\mathcal{F}} &= - \vec{\nabla} \sum_{\vec{p}, \vec{p}'} V_{p, p'}(\vec{r}). \end{aligned} \quad (21)$$

Eqs. 21 enables us to write the force operator as a series of projecting operators:

$$\vec{\mathcal{F}}_{\langle F \rangle} = \sum_{\vec{F}} \vec{F} |\vec{F}\rangle \langle \vec{F}|$$

Using the definition $[\mathcal{P}, \mathcal{H}_{\langle p \rangle}]_{\langle F \rangle} \stackrel{\text{def}}{=} \mathcal{I}_{\langle F \rangle} [\mathcal{P}, \mathcal{H}_{\langle p \rangle}] \mathcal{I}_{\langle F \rangle}^\dagger$ we obtain:

$$[\mathcal{P}, \mathcal{H}_{\langle p \rangle}]_{\langle F \rangle} = i\hbar \sum_{\vec{F}} \vec{F} |\vec{F}\rangle \langle \vec{F}| \quad (22)$$

107 VI. $\vec{\mathcal{P}}$ in an off-Diagonal Energy Representation

The momentum operator, shown in the Heisenberg picture is:

$$\vec{\mathcal{P}}_{\langle E \rangle} = \sum_{E, E'} \exp \left[\frac{i(E - E')t}{\hbar} \right] \langle E | \vec{\mathcal{P}} | E' \rangle |E\rangle \langle E'|. \quad (23)$$

In a similar manner to the previous calculations it can be shown that:

$$[\mathcal{H}, \vec{\mathcal{P}}_{\langle E \rangle}] = \sum_{E, E'} (E - E') \exp \left[\frac{i(E - E')t}{\hbar} \right] \langle E | \vec{\mathcal{P}} | E' \rangle |E\rangle \langle E'| \quad (24)$$

Or,

$$[\mathcal{H}, \vec{\mathcal{P}}_{\langle E \rangle}] = -i\hbar \frac{d\vec{\mathcal{P}}_{\langle E \rangle}}{dt}. \quad (25)$$

Defining the parameter

$$\vec{P}_{E, E'}(t) = \exp \left[\frac{i(E - E')t}{\hbar} \right] \langle E | \vec{\mathcal{P}} | E' \rangle \quad (26)$$

we obtain that

$$[\mathcal{H}, \vec{\mathcal{P}}_{\langle E \rangle}] = -i\hbar \frac{d}{dt} \sum_{E, E'} \vec{P}_{E, E'}(t) |E\rangle \langle E'|. \quad (27)$$

108 In this representation t appears explicitly in the momentum operator

109 VII. Representations and Terminology

Let us observe the classical equation

$$\dot{\vec{P}} = -\vec{\nabla}H \quad (28)$$

110 On the right side of eq. 28, we represent the classical Hamiltonian as a function of coordinates and
 111 momentum where in energy conserving system, time is explicitly missing despite the fact that \vec{r} and \vec{p}
 112 depend on time. Time dependence appear explicitly on the left side of eq. 28 where the momentum is
 113 now expressed as a function of time. Following eqs. 17 and 27 we find the same terminology in eq. 17
 114 where the system is represented through momentum and space only while time appears explicitly in
 115 the momentum description in eq. 27.

116 VIII. Common Representation

117 In order to utilize the trivial identity $[\mathcal{H}, \vec{P}] = -[\vec{P}, \mathcal{H}]$ in equations 17 and 27, we need
 118 to projects all operators onto the same basis of states. As in classical mechanics where bodies are
 119 distinguishable, we choose the common basis of states to be the labeling set, $|s\rangle, s = 1, 2, \dots$, [5] where
 120 the index s labels the distinguishable bodies.

Defining the corresponding complete and orthogonal basis of states, $|s\rangle$, we can define the identity operators,

$$\mathcal{I} = \sum_s |s\rangle \langle s|. \quad (29)$$

Substituting the relations $\mathcal{I} \vec{F} \mathcal{I}^\dagger$ and $\mathcal{I} \vec{P} \mathcal{I}^\dagger$, in eqs. 15 and 27 we obtain:

$$\vec{F}_{\langle s)} = -[\vec{P}, \mathcal{H}_{\langle p)}]_{\langle s)} = i\hbar \vec{\nabla} \sum_{s,s'} \sum_{\vec{p}, \vec{p}'} V_{p,p'}(\vec{r}) \langle s | \vec{p} \rangle \langle \vec{p}' | s' \rangle |s\rangle \langle s'| \quad (30)$$

and,

$$\dot{\vec{P}}_{\langle s)} = [\mathcal{H}, \vec{P}_{\langle E)}]_{\langle s)} = -i\hbar \frac{d}{dt} \sum_{s,s'} \sum_{E,E'} \vec{P}_{E,E'}(t) \langle s | E \rangle \langle E' | s' \rangle |s\rangle \langle s'|. \quad (31)$$

Implementing eq. 1 for the common representation we obtain the operator equation:

$$\vec{F}_{\langle s)} = \dot{\vec{P}}_{\langle s)} \quad (32)$$

121 This equation that was considered as a postulate in ref. [5] is now derived from basic quantum
 122 principles.

123 IX. Diagonal Representation of Momentum in the Labeling Basis of States

With regards to individual particles, classical mechanics allows particles to share forces (through mutual interactions) while each particle can still be associated with a distinguishable momentum. In quantum terminology, this means that the momentum operator is diagonal under the labeling basis of states. Assuming:

$$\sum_{E,E'} \vec{P}_{E,E'}(t) \langle s | E \rangle \langle E' | s' \rangle = \vec{p}_s(t) \delta_{s,s'} \quad (33)$$

we obtain from eq. 31 the momentum operator's diagonal form:

$$\vec{P}_{\langle s)} = \sum_s \vec{p}_s(t) |s\rangle \langle s| \quad (34)$$

124 X. Internal and External Forces

For simplicity we define the following variable:

$$\vec{F}_{s,s'} \stackrel{\text{def}}{=} -\vec{\nabla} \sum_{\vec{p},\vec{p}'} V_{p,p'}(\vec{r}) \langle s | \vec{p} \rangle \langle \vec{p}' | s' \rangle \quad (35)$$

to obtain from eq. 30 the force operator,

$$\vec{\mathcal{F}}_{\langle s \rangle} = \sum_{s,s'} \vec{F}_{s,s'} |s\rangle \langle s'|. \quad (36)$$

To distinguish between solitary and mutual interaction forces, we separate $\vec{\mathcal{F}}_{\langle s \rangle}$ into the diagonal terms $|s'\rangle \langle s| = \delta_{s,s'}$ and the off-diagonal terms. This yields:

$$\vec{\mathcal{F}}_{\langle s \rangle} = \sum_s \vec{\mathbf{F}}_s, \quad (37)$$

where $\vec{\mathbf{F}}_s$, the net force operator that acts upon a s -particle is:

$$\vec{\mathbf{F}}_s = \vec{F}_s |s\rangle \langle s| + \sum_{s'} \vec{F}_{s \neq s'} |s'\rangle \langle s|. \quad (38)$$

125 with \vec{F}_s being the s -diagonal term and $\vec{F}_{s' \neq s}$ is the off-diagonal term. Focusing on a single particle with
 126 a label s , we identify the diagonal term $\vec{F}_s |s\rangle \langle s|$ with an external force operator while the off-diagonal
 127 sum, $\sum_{s'} \vec{F}_{s' \neq s} |s'\rangle \langle s|$ represents the interaction of the s particles with other particles.

128 XI. Interactions, Newton Third Law and Hermiticity

With the purpose of reconstructing Newton's third law, we obtain the relation

$$\vec{F}_{s' \neq s} = -\vec{F}_{s \neq s'} \quad \text{Note that on both sides, } s \text{ and } s' \text{ were switched.} \quad (39)$$

Defining the interaction force magnitude as $|\vec{F}_{s,s'}| = |\vec{F}_{s' \neq s}|$ we obtain:

$$\vec{\mathbf{F}}_s = \vec{F}_s |s\rangle \langle s| + \sum_{s'} |\vec{F}_{s',s}| (|s'\rangle \langle s| - |s\rangle \langle s'|) \quad (40)$$

Following the convention of an Hermit observable and noticing that $\vec{\mathbf{F}}_s$ of eq. 40 is not such, we suggest the replacement, $|\vec{F}_{s',s}| (|s'\rangle \langle s| - |s\rangle \langle s'|) \rightarrow i |\vec{F}_{s',s}| (|s'\rangle \langle s| - |s\rangle \langle s'|)$ to obtain the Hermit operator:

$$\vec{\mathbf{F}}_s = \vec{F}_s |s\rangle \langle s| + i \sum_{s'} |\vec{F}_{s',s}| (|s'\rangle \langle s| - |s\rangle \langle s'|) \quad (41)$$

129 Note that $\sum_s \vec{\mathbf{F}}_s = \sum_s \vec{F}_s |s\rangle \langle s|$.

130 XII. Second Quantization

Classical mechanics includes particles interactions and is therefore useful to introduce our system with multi-particle algebra. Now the states belong to the Fock space.

Following ref. [5] we use the transformation

$$\begin{aligned} |s\rangle \langle s| &\rightarrow a_s^\dagger a_s \\ |s'\rangle \langle s| &\rightarrow a_s^\dagger a_{s'} \\ |s\rangle \langle s'| &\rightarrow a_{s'}^\dagger a_s, \end{aligned} \quad (42)$$

where a and a^\dagger behave like spin $\frac{1}{2}$ fermionic annihilation and creation operators,

$$\begin{aligned} a_s |0\rangle_{s'} &= 0, & a_s^\dagger |0\rangle_{s'} &= |1\rangle_s \delta_{ss'} \\ a_s |1\rangle_{s'} &= |0\rangle_s \delta_{ss'}, & a_s^\dagger |1\rangle_{s'} &= 0. \end{aligned} \quad (43)$$

The momentum operator becomes:

$$\vec{P}_{\langle s \rangle} = \sum_s \vec{p}_s(t) a_s^\dagger a_s. \quad (44)$$

and Eq. 41 is now modified to,

$$\vec{F}_s = \vec{F}_s a_s^\dagger a_s + i \sum_{s'} \left| \vec{F}_{s',s} \right| \left(a_{s'}^\dagger a_s - a_s^\dagger a_{s'} \right) \quad (45)$$

whereas, the total force operator was:

$$\vec{F}_{\langle s \rangle} = \sum_s \vec{F}_s \quad (46)$$

131 XII.1. Eigenstates of the Force Operator

132 Here, we introduce two states that serve as the eigenstates of $\vec{F}_{\langle s \rangle}$:

133

Vacuum state The state in which no particle is observed:

$$|\emptyset\rangle_{\langle s \rangle} = \prod_{s=1} |0\rangle_s \quad (47)$$

For that state we obtain

$$\vec{F}_{\langle s \rangle} |\emptyset\rangle = 0. \quad (48)$$

Close system state The state in which all s -states are occupied with a single fermionic particle:

$$|C.S.\rangle = \prod_{s=1} |1\rangle_s \quad (49)$$

Inserting $|C.S.\rangle$ into Eqs. 45 and 46, and noticing that the asymmetric term is canceled, we obtain

$$\vec{F}_s |C.S.\rangle = \left(\sum_s \vec{F}_s \right) |C.S.\rangle. \quad (50)$$

Implementing eqs. 44 and 32, we obtain the known classical relation

$$\sum_s \vec{F}_s = \dot{\vec{P}}_{total}, \quad (51)$$

134 where $\vec{P}_{total} \stackrel{\text{def}}{=} \sum_s \vec{P}_s$.

135 XII.2. Single Particle States

In a single particle state, only one state (s) is occupied while all others are empty:

$$|s\rangle = |1\rangle_s \prod_{s' \neq s} |0\rangle_{s'}, \quad (52)$$

Substituting in eq. 46 we obtain,

$$\vec{F}_{\langle s \rangle} |s\rangle = \vec{F}_s |s\rangle + i \sum_{s'} \left| \vec{F}_{s,s'} \right| |s'\rangle. \quad (53)$$

136 Considering that $\forall s' \neq s, \langle s | s' \rangle = 0$, we obtain that $\forall s', \vec{F}_{s' \rightarrow s} = 0$. This means that the single particle
 137 approach cannot represent internal forces and consequently Newton's third law. In section XIII, we
 138 resolve this difficulty by introducing entangled states.

139 XIII. Entanglement

We implement single particle states of the particles pair s_1 and s_2 to generate doubles of entangled states:

$$\text{For } s_1 \neq s_2, \begin{cases} |\psi_+\rangle = |s_1\rangle + i|s_2\rangle \\ |\psi_-\rangle = |s_2\rangle - i|s_1\rangle \end{cases}, \quad |\langle \psi_+ | \psi_+ \rangle|^2 = |\langle \psi_- | \psi_- \rangle|^2 = 2, \quad (54)$$

with the single particle states:

$$\begin{aligned} |s_1\rangle &= |1\rangle_{s_1} \prod_{s \neq s_1} |0\rangle_s && \text{The observer detects only} \\ &&& \text{the particle which is labeled as } s_1 \\ |s_2\rangle &= |1\rangle_{s_2} \prod_{s \neq s_2} |0\rangle_s && \text{The observer detects only} \\ &&& \text{the particle which is labeled as } s_2 \end{aligned} \quad (55)$$

According to eq. 45, each s_1 or s_2 particle is associated with a force operators \vec{F}_{s_1} or \vec{F}_{s_2} , as follows:

$$\begin{aligned} \vec{F}_{s_1} &= \vec{F}_{s_1} a_{s_1}^\dagger a_{s_1} + i \sum_s \left| \vec{F}_{s,s_1} \right| (a_s^\dagger a_{s_1} - a_{s_1}^\dagger a_s) && \text{The force acting} \\ &&& \text{on particle } s_1 \\ \vec{F}_{s_2} &= \vec{F}_{s_2} a_{s_2}^\dagger a_{s_2} + i \sum_s \left| \vec{F}_{s,s_2} \right| (a_s^\dagger a_{s_2} - a_{s_2}^\dagger a_s) && \text{The force acting} \\ &&& \text{on particle } s_2 \end{aligned} \quad (56)$$

where we replaced the dummy index s' with s .

We then obtain:

$$\vec{F}_{s_1} |\psi_+\rangle = \vec{F}_{s_1} |1\rangle_{s_1} \prod_{s \rightarrow s_1} |0\rangle_s + i \sum_\sigma \left| \vec{F}_{\sigma,s_1} \right| |1\rangle_\sigma \prod_{s \neq \sigma} |0\rangle_s \quad (57)$$

and in eq. 55's symbol terminology,

$$\vec{F}_{s_1} |\psi_+\rangle = \vec{F}_{s_1} |s_1\rangle + i \sum_\sigma \left| \vec{F}_{\sigma,s_1} \right| |s_\sigma\rangle \quad (58)$$

and in the same manner we obtain:

$$\begin{aligned} \vec{F}_{s_2} |\psi_-\rangle &= \vec{F}_{s_2} |s_2\rangle + i \sum_\sigma \left| \vec{F}_{\sigma,s_2} \right| |s_\sigma\rangle \\ \vec{F}_{s_1} |\psi_-\rangle &= -i \vec{F}_{s_1} |s_1\rangle + \sum_\sigma \left| \vec{F}_{\sigma,s_1} \right| |s_\sigma\rangle \\ \vec{F}_{s_2} |\psi_+\rangle &= i \vec{F}_{s_2} |s_2\rangle - \sum_\sigma \left| \vec{F}_{\sigma,s_2} \right| |s_\sigma\rangle \end{aligned} \quad (59)$$

To retrieve classical results, we consider an observer that by performing measurements, collapses the entanglement into the distinguishable (labeled) particles. Consequently, they detect only average quantities. Implementing eqs. 54, 58 and 59 we obtain:

$$\begin{aligned} \text{The net force} & \Rightarrow \begin{cases} \langle \psi_+ | \vec{F}_{s_1} | \psi_+ \rangle = \vec{F}_{s_1} + \left| \vec{F}_{s_1,s_2} \right| \\ \langle \psi_- | \vec{F}_{s_1} | \psi_- \rangle = \vec{F}_{s_1} + \left| \vec{F}_{s_1,s_2} \right| \end{cases} \\ \text{acting on particle 1 :} & \\ \text{The net force} & \Rightarrow \begin{cases} \langle \psi_- | \vec{F}_{s_2} | \psi_- \rangle = \vec{F}_{s_2} - \left| \vec{F}_{s_1,s_2} \right| \\ \langle \psi_+ | \vec{F}_{s_2} | \psi_+ \rangle = \vec{F}_{s_2} - \left| \vec{F}_{s_1,s_2} \right| \end{cases} \\ \text{acting on particle 2 :} & \end{aligned} \quad (60)$$

140 Comparing the terms $\langle \psi_+ | \vec{F}_{s_1} | \psi_+ \rangle$ and $\langle \psi_+ | \vec{F}_{s_2} | \psi_+ \rangle$ or $\langle \psi_- | \vec{F}_{s_1} | \psi_- \rangle$ and $\langle \psi_- | \vec{F}_{s_2} | \psi_- \rangle$ we find that
 141 Newton's third law has been reformulated. The third law is what is left from a primary entanglement
 142 of classical particles.

143 XIV. Classical Observer

144 In the previous section we associated measurement outputs with averages. This means that the
 145 so-called single observation in fact includes a multitude of measurements forming an ensemble that
 146 allows the detection of average quantities.

147 XV. Results

148 In this paper, we reformulated classical mechanics laws from pure quantum principles. We
 149 showed that the identity $[\mathcal{H}, \vec{P}] = -[\vec{P}, \mathcal{H}]$ can be implemented to reformulate the laws of classical
 150 mechanics provided that the following two-step procedure is to be followed: First, the right side
 151 operator of a commutator is to be pictured with the corresponding unitary operators of the left side
 152 operator. For example, right side operator $\vec{P}_{\langle E \rangle}$ in $[\mathcal{H}, \vec{P}_{\langle E \rangle}]$ is represented by the left side states and
 153 phases of the Hamiltonian. The second step is to project both commutators (such as in eqs. 4 and 5)
 154 onto a common basis. Then, can we equate both commutators to obtain a physical law. We suggest
 155 that by following this procedure and by implementing other operators, it may be possible to extend
 156 this approach to find or reconstruct other physical laws.

157 As shown earlier, using this approach to reconstruct classical mechanics laws, Newton's third
 158 law corresponds with entanglement. That is to say, prior to a classical mechanical measurement, the
 159 labeling states were defined as quantum states that cannot be factored as a product of the states of its
 160 local constituents. We showed that the remaining evidence for this coherence is the internal forces.
 161 Philosophically speaking, we realize that beyond classical mechanics, there is another reality in which
 162 even classical particles are entangled. As the classical observer can measure only a limited basis of
 163 states, entangled states become impossible to measure.

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168 **Sample Availability:** Samples of the compounds are available from the authors.

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