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Article

Kabirian-based Optinalysis: Its Paradigm, Theorems, and Properties

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Abstract: The conceptual and theoretical backbones of symmetry/asymmetry detections or similarity/dissimilarity, and identity/unidentity study are automorphism or isomorphism respectively. However, the development of equations and methods for symmetry/asymmetry detections, similarity/dissimilarity, and identity/unidentity measures deviates from these backbones. In this article, an equation was proposed for symmetry/asymmetry detections, similarity/dissimilarity, and identity/unidentity measures, and proved that its isoreflective pairs-points are functionally bijective and inverse. The proposal, called Kabirian-based optinalysis, is based on the conceptual and theoretical frameworks of automorphism and isomorphism. The Kabirian-based optinalysis is also proven and characterized as invariant (robust) under translation (i.e., scaling and location shift), and rotation or reflection. Computing codes were written in python language for Kabirian-based optinalysis to serve as working codes for application and verification.

Keywords: isoreflectivity; autoreflectivity; Kabirian bi-coefficients; Kabirian-probability translation models; properties; computing codes.

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1. Introduction

The notion of isometry (as a congruence mapping) is a general phenomenon commonly accepted in Mathematics. It means a mapping that preserves distances. It is a bijective mapping, characterized as the one-to-one mapping of a group onto itself or another in various transformational ways such as reflections, translation, or rotations [1].

Two graphs or sets are *isomorphic* if there is a bijection between their vertices or elements that preserves adjacency; such a bijection is called an *isomorphism*. In other terms, two graphs or sets, subsets A and B are isomorphic if they have the same structure, but their elements or vertices may be different. An isomorphism from a graph onto itself is called an *automorphism*, and the set of all automorphisms of a given graph G forms a group under composition [2]. However, automorphism and isomorphism conceptually and theoretically framed the backbone of the study of symmetry and similarity or identity respectively.

In this paper, Kabirian-based optinalysis is proposed which looks at two mathematical structures as autoreflective or isoreflective as a mirror-like reflection of each other about a centre, and express their degree of symmetry, identity, and similarity. Kabirian-based optinalysis is not a method for deciding that two mathematical structures are isomorphic or automorphic, but extends to an estimation. Kabirian-based optinalysis is conceptually and theoretically backboned by isomorphism and automorphism.

2. Preliminary definitions and theorems

Definition I. Injections, surjections, and bijections of functions between sets, and subsets [3].

These are words that describe certain functions $f : A \to B$ from one set to another.

An injection also called a *one-to-one* function is a function that maps distinct elements to distinct elements, that is, if $x \neq y$, then $f(x) \neq f(y)$. Equivalently, if $f(x) = f(y)$ then, $x = y$.

A surjection also called an *onto function* is one that includes all of B in its image, that is, if $y \in B$, then there is an $x \in A$ such that $f(x) = y$.

A bijection also called a *one-to-one and onto correspondence*, is a function that is simultaneously injective and surjective. Another way to describe a bijection $f : A \to B$ is to say that there is an inverse function $g : B \to A$ so that

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the composition $g\ o\ f : A \to A$ is the identity function on A while $f\ o\ g : B \to B$ is the identity function on B. The usual notation for the function inverse to f is f^{-1}.

If f and g are inverse to each other, that is, if g is the inverse of f, $g = f^{-1}$, then f is the inverse of g, $f = g^{-1}$ Thus, $(f^{-1})^{-1} = f$.

An important property of bijections is that you can convert equations involving f to equations involving f^{-1}:

$$f(x) = y \text{ if and only if } x = f^{-1}(y).$$

Definition II. Isometry (or congruence or congruent transformation) is a distance-preserving transformation between metric spaces, usually assumed to be bijective. Let A and B be metric spaces with metrics d_A and d_B. A map $f : A \to B$ is called an isometry or distance preserving if, for any $a, b \in A$ one has

$$d_B(f(a), f(b)) = d_A(a, b)$$

[1], [3].

Definition III. Isomorphism is a vertex bijection that preserves the mathematical structures (e.g., vertices, edges, non-edges, and connections) between two spaces and graphs that can be reversed by inverse mapping. Two mathematical structures A and B are isomorphic if they have the same structure, but their elements may be different [2], [3]. In some sense, it is defined as the similarity or identity between two objects.

$$f : A \to B$$

$$A \cong B$$

Definition IV. Automorphism is an isomorphism from a mathematical object to itself. It is, in some sense; define as the symmetry of the object, and a way of mapping the object to itself while preserving all of its mathematical structure (e.g., vertices, edges, non-edges, and connections)[2], [3].

$$f : A \to Aut(A')$$

$$A \cong A'$$

Definition V. Scale can be defined as the system of marks at fixed intervals, which define the relationship between the units being used and their representation on the graph.

Theorem I. An isomorphism maps

(i) straight lines to straight lines;

(ii) segments to congruent segments;

(iii) triangles to congruent triangles;

(iv) angles to congruent angles.

[2], [3]

Theorem II. Any isomorphism of the plane is a composition of at most three reflections [2], [3].

Theorem III. A symmetry about a point is an isomorphism [2], [3].

3. Concept and Proposition of Kabirian-based Optinalysis

3.1. Conceptual Definitions

Definition 1: Optinalysis

Optinalysis is a function that autoreflectively or isoreflectively compares the symmetry, similarity, and identity between two mathematical structures as a mirror-like (optic-like) reflection of each other about a symmetrical line or mid-point. In other words, it is a function that compares isoreflective or autoreflective pairs of mathematical structures.

Definition 2: Optinalysis

Optinalysis is a function that is comprised of an assigned optiscale (R) that bijectively re-maps (a symbol ↠ indicates re-mapping) isoreflective or autoreflective pairs of mathematical structures. Figure 1-2 illustrates how isoreflective and autoreflective pairs of points are mapped and also re-mapped by an optiscale.

Optinalysis is expressed in *optinalytic construction*. An optinalytic construction is the mathematical representation of optinalysis between isoreflective or autoreflective pairs.

Optinalysis is defined in two broad types: automorphic (shape) and isomorphic (comparative) optinalysis.

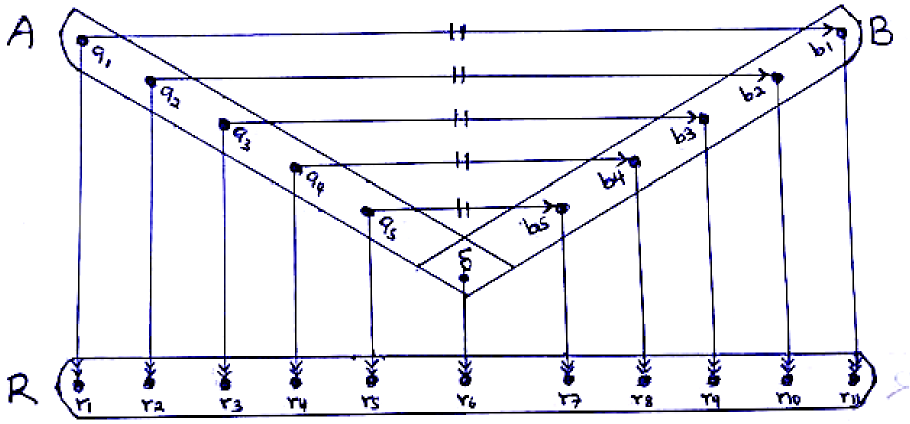

Figure I: Mapping between isoreflective pairs of points and *re-mapping (pair-mapping)* with the optiscale. A is a domain; B is a co-domain of A; δ is a mid-point, and R is the optiscale. The symbol ⤀ indicates a bijective mapping between the isoreflective pairs around a midpoint, and ↠ indicates a bijective re-mapping (pair-mapping) by the optiscale R.

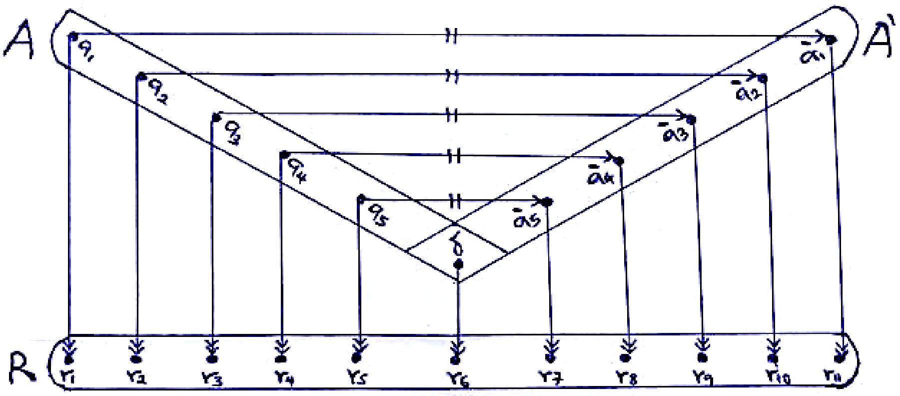

Figure 2: Mapping between autoreflective pairs of points and *re-mapping (pair-mapping)* with the optiscale. A is a domain; A' is a co-domain of A; δ is a mid-point or symmetrical line, and R is the optiscale. The symbol ⤀ indicates a bijective mapping between the autoreflective pairs around a midpoint, and ↠ indicates a bijective re-mapping (pair-mapping) by the optiscale R.

Definition 3: Optiscale

An optiscale refers to a symmetric (with equal and same spread of intervals) scale $R = (r_1, r_2, r_3 \dots r_n)$ or $R = (r_1, r_2, r_3 \dots r_{2n+1})$ such that $R \in \mathbb{R}$, $r_1 \neq 0$, $n \in \mathbb{N}$. Because the elements of the optiscale belong to the set of non-zero real numbers, the complexity of the arithmetic operations in optinalysis can be regulated especially when dealing with numerical values that are decimally low or high.

Definition 4: Conceptual ordering

Conceptual ordering refers to concept-based structuring or arrangement of terms and items. For instance, the arrangement of DNA, RNA, and amino acid sequences is based on the concept of molecular transcription and translation.

Definition 5: Theoretical ordering

Theoretical ordering refers to theory-based structuring or arrangement of terms and items. For instance, the arrangement of real numbers in ascending or descending order is theory-based.

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Definition 6: Autoreflective pairs

Autoreflective pairs describes splitting parts of a single mathematical structure or points under reflection about a central midpoint. An autoreflectivity refers to the logical and meaningful essence or property of being autoreflective.

$$A = (a_1, a_2, a_3, \dots, a_n) \underset{\twoheadleftrightarrow}{\delta = (\hat{a})} B = (a'_n, \dots, a'_3, a'_2, a'_1)$$

Such that $\delta \in A \,\&\, A'$

Definition 7: Isoreflective pairs

Isoreflective pairs describes two mathematical structures or points under reflection about a central midpoint. An isoreflectivity refers to the logical and meaningful essence or property of being isoreflective.

$$A = (a_1, a_2, a_3, \dots, a_n) \underset{\twoheadleftrightarrow}{\delta} B = (b_n, \dots, b_3, b_2, b_1)$$

Such that $\delta \notin A \,\&\, B$

Definition 8: Shape (Automorphic or Intrametric) Optinalysis:

Shape or automorphic or intrametric optinalysis refers to the analysis (of symmetry) between autoreflective pairs of a mathematical structure under optinalysis. It is a method of symmetry detection. Shape optinalysis is defined by its optinalytic constructions:

$$f\colon A \underset{\twoheadleftrightarrow}{\delta} A' \twoheadrightarrow R$$

$$f\colon A = (a_1, a_2, a_3, \dots, a_{\frac{n-1}{2}}) \underset{\twoheadleftrightarrow}{\delta = \left(\hat{a}_{\frac{n+1}{2}}\right)} A' = \left(a'_{\frac{n-1}{2}}, \dots, a'_3, a'_2, a'_1\right) \twoheadrightarrow R = (r_1, r_2, r_3, \dots, r_n)$$

$$f\colon \begin{bmatrix} A = \left(a_1, a_2, a_3, \dots, a_{\frac{n-1}{2}}\right) & \underset{\twoheadleftrightarrow}{\delta = \left(\hat{a}_{\frac{n+1}{2}}\right)} & A' = \left(a'_{\frac{n-1}{2}}, \dots, a'_3, a'_2, a'_1\right) \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, r_2, r_3, \dots, r_{\frac{n-1}{2}}, & r_{\frac{n+1}{2}}, & r_{\frac{n+3}{2}}, \dots, r_{n-2}, r_{n-1}, r_n) \end{bmatrix}$$

Such that $\delta \in A \,\&\, A'; A, A', \delta \,\&\, R \in \mathbb{R}$; $r_1 \neq 0$; $n \in \mathbb{N}$; and $A \,\&\, A'$ is an autoreflective pair on a chosen pairing about a midpoint δ.

Definition 9: Definition 9: Comparative (Isomorphic) Optinalysis:

Comparative or isomorphic optinalysis refers to the analysis (of similarity and identity measures) between isoreflective pairs of mathematical structures under optinalysis. It is a method of similarity and identity measures. Comparative optinalysis is defined by its optinalytic constructions:

$$f\colon A \underset{\twoheadleftrightarrow}{\delta} B \twoheadrightarrow R$$

$$f\colon A = (a_1, a_2, a_3, \dots, a_n) \underset{\twoheadleftrightarrow}{\delta} B = (b_n, \dots, b_3, b_2, b_1) \twoheadrightarrow R = (r_1, r_2, r_3, \dots, r_{2n+1})$$

$$f\colon \begin{bmatrix} A = (a_1, a_2, a_3, \dots, a_n) & \underset{\twoheadleftrightarrow}{\delta} & B = (b_n, \dots, b_3, b_2, b_1) \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, r_2, r_3, \dots r_n, & r_{n+1}, & r_{n+2} \dots, r_{2n-1}, r_{2n}, r_{2n+1}) \end{bmatrix}$$

Such that $\delta \notin A \,\&\, B; A, B, \delta \,\&\, R \in \mathbb{R}$; $r_1 \neq 0$; $n \in \mathbb{N}$; and $A \,\&\, B$ are isoreflective pairs.

Definition 10: Head-to-head Reflection or Pairing

In isomorphic optinalysis, a reflection (pairing) is said to be head-to-head, $Opt(\overleftarrow{A}, \overrightarrow{B})$, if the lower order elements (observations) of the isoreflective pairs (of two mathematical structures) are extreme away from the midpoint.

$$A = (a_1, a_2, a_3, \dots, a_n) \underset{\twoheadleftrightarrow}{\delta} B = (b_n, \dots, b_3, b_2, b_1)$$

Definition 11: Tail-to-tail Reflection or Pairing

In comparative optinalysis, a reflection or pairing is said to be tail-to-tail, $Opt(\vec{A}, \overleftarrow{B})$, if the lower order elements (observations) of the isoreflective pairs (of two mathematical structures) are extreme towards the midpoint.

$$A = (a_n, \dots, a_3, a_2, a_1) \overset{\delta}{\twoheadrightarrow} B = (b_1, b_2, b_3, \dots, b_n)$$

Definition 12: Scalement

A *scalement*, refers to the product of any member of isoreflective or autoreflective pairs of a mathematical structure and its assigned optiscale.

Suppose we have an optinalytic construction of isoreflective pairs of mathematical structures A and B with an assigned optiscale (R) as follows:

$$f: \begin{bmatrix} A = (a_1, a_2, a_3, \dots, a_n) & \overset{\delta}{\twoheadrightarrow} & B = (b_n, \dots, b_3, b_2, b_1) \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, r_2, r_3, \dots r_n, & r_{n+1}, & r_{n+2} \dots, r_{2n-1}, r_{2n}, r_{2n+1}) \end{bmatrix}$$

Such that $\delta \notin A \,\&\, B$; $A, B, \delta \,\&\, R \in \mathbb{R}$; $r_1 \neq 0$; $n \in \mathbb{N}$; and $A \,\&\, B$ are isoreflective pairs on a chosen pairing about a midpoint δ.

Then, the sum of scalements S, for instance, between the isoreflective pairs A and B is defined as:

$$S(A, B) = (r_1.a_1) + \dots + (r_{n+1}.\delta) + \dots + (r_{2n+1}.b_1) = \sum_{i=1}^{n} \sum_{j=k=n+2}^{2n+1} (r_i a_i + r_{n+1}\delta + r_j b_i)$$

Definition 13: Kabirian coefficient of isomorphic optinalysis

Kabirian coefficient, of isoreflective pairs, is expressed as the quotient of the product of the median optiscale and the summation of all elements (of the isoreflective pairs) by the summation of all scalements (of the isoreflective pairs).

Suppose we have an optinalytic construction of isoreflective pairs of mathematical structures A and B with an assigned optiscale (R) as follows:

$$f: A \overset{\delta}{\twoheadrightarrow} B \twoheadrightarrow R$$

$$f: A = (a_1, a_2, a_3, \dots, a_n) \overset{\delta}{\twoheadrightarrow} B = (b_n, \dots, b_3, b_2, b_1) \twoheadrightarrow R = (r_1, r_2, r_3, \dots, r_{2n+1})$$

$$f: \begin{bmatrix} A = (a_1, a_2, a_3, \dots, a_n) & \overset{\delta}{\twoheadrightarrow} & B = (b_n, \dots, b_3, b_2, b_1) \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, r_2, r_3, \dots r_n, & r_{n+1}, & r_{n+2} \dots, r_{2n-1}, r_{2n}, r_{2n+1}) \end{bmatrix}$$

Such that $\delta \notin A \,\&\, B$; $A, B, \delta \,\&\, R \in \mathbb{R}$; $r_1 \neq 0$; $n \in \mathbb{N}$; and $A \,\&\, B$ are isoreflective pairs on a chosen pairing about a midpoint δ.

Then, the Kabirian coefficient of identity and similarity between the isoreflective pairs is expressed in **Eq. 1**.

$$KC_{Sim./Id.}(A, B) = \frac{r_{n+1}(a_1 + a_2 + a_3 + \dots + a_n + \delta + b_n + \dots + b_3 + b_2 + b_1)}{(r_1.a_1) + (r_2.a_2) + (r_3.a_3) + \dots + (r_n.a_n) + (r_{n+1}.\delta) + (r_{n+2}.b_n) + \dots + (r_{2n-1}.b_3) + (r_{2n}.b_2) + (r_{2n+1}.b_1)} \quad \textbf{(Eq.1)}$$

$$KC_{Sim./Id.}(A, B) = \frac{r_{n+1}[\sum_{i=1}^{n}(a_i + \delta + b_i)]}{\sum_{i=1}^{n} \sum_{j=n+2}^{2n+1}(r_i a_i + r_{n+1}\delta + r_j.b_i)} \quad \textbf{(Eq.1)}$$

$$\begin{cases} & if\ g(A) = g(B); \quad KC_{Sim./Id.}(A,B) = 1 \\ if\ g(A) = -g(B), or & -\ g(A) = g(B); \quad KC_{Sim./Id.}(A,B) = 0 \\ & if\ g(A) < g(B); \quad 0 \leq\ KC_{Sim./Id.}(A,B) \leq 1 \\ & if\ g(A) > g(B); \quad 1 \leq KC_{Sim./Id.}(A,B) \leq n+1 \\ & if\ g(A) > g(B); \quad KC_{Sim./Id.}(A,B) \geq n+1, < 0 \end{cases}$$

Where $g(A)$ and $g(B)$ are the optical moments of A and B respectively about the mid-point through a symmetric distance D started from the center. It is expressed by **Eq. 2.1** and **Eq. 2.2**.

$$g: \begin{bmatrix} A = (a_1, a_2, a_3, \dots, a_n) & \overset{\delta}{\twoheadrightarrow} & B = (b_n, \dots, b_3, b_2, b_1) \\ \downarrow & \downarrow & \downarrow \\ D = (d_n, d_{n-1}, d_{n-2}, \dots, d_1, & d_0, & d_1, \dots, d_{n-2}, d_{n-1}, d_n) \end{bmatrix}$$

$$g(A) = (d_n.a_1) + (d_{n-1}.a_2) + (d_{n-2}.a_3) + \cdots + (d_1.a_n) = \sum_{i=1}^{n} (d_i a_i) \qquad \textbf{(Eq.2.1)}$$

$$g(B) = (d_n.b_1) + (d_{n-1}.b_2) + (d_{n-2}.b_3) + \cdots + (d_1.b_n) = \sum_{n=1}^{n} (d_i b_i) \qquad \textbf{(Eq.2.2)}$$

Such that: $(a_1, a_2, a_3, \dots, a_n) \in A$; $(b_n, \dots, b_3, b_2, b_1) \in B$; $(d_0, d_1, d_2, d_3, \dots, d_n) \in D$; $A, B, \& D \in \mathbb{R}$; $d_0 = 0$; $d_1 \neq 0$; $n \in \mathbb{N}$; D must have a symmetric interval, and A & B are isoreflective pairs in a chosen pairing about a midpoint δ.

Definition 14: Kabirian coefficient of automorphic optinalysis

Kabirian coefficient, of autoreflective pairs, is expressed as the quotient of the product of the median optiscale and the summation of all elements (of the autoreflective pairs) by the summation of all scalements (of the autoreflective pairs).

Suppose we have an optinalytic construction of autoreflective pairs of a mathematical structure A and A' with an assigned optiscale (R) as follows:

$$f\colon A \overset{\delta}{\twoheadrightarrow} A' \twoheadrightarrow R$$

$$f\colon A = (a_1, a_2, a_3, \dots, a_{\frac{n-1}{2}}) \ \underset{\twoheadrightarrow}{\delta = \left(\hat{a}_{\frac{n+1}{2}}\right)} \ A' = \left(a'_{\frac{n-1}{2}}, \dots, a'_3, a'_2, a'_1\right) \twoheadrightarrow R = (r_1, r_2, r_3, \dots, r_n)$$

$$f\colon \begin{bmatrix} A = \left(a_1, a_2, a_3, \dots, a_{\frac{n-1}{2}}\right) & \underset{\twoheadrightarrow}{\delta = \left(\hat{a}_{\frac{n+1}{2}}\right)} & A' = \left(a'_{\frac{n-1}{2}}, \dots, a'_3, a'_2, a'_1\right) \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, r_2, r_3, \dots, r_{\frac{n-1}{2}}, & r_{\frac{n+1}{2}}, & r_{\frac{n+3}{2}}, \dots, r_{n-2}, r_{n-1}, r_n) \end{bmatrix}$$

Such that $\delta \in A \,\&\, A'$; $A, A', \delta \,\&\, R \in \mathbb{R}$; $r_1 \neq 0$; $n \in \mathbb{N}$; and A & A' are autoreflective pairs on a chosen pairing about a midpoint δ.

Then, the Kabirian coefficient of symmetry between the autoreflective pairs is expressed in **Eq. 4**.

$$KC_{Sym./Id.}(A,A') = \frac{r_{\frac{n+1}{2}}\left(a_1 + a_2 + a_3 + \cdots + a_{\frac{n-1}{2}} + \hat{a}_{\frac{n+1}{2}} + a'_{\frac{n-1}{2}} + \cdots + a'_3 + a'_2 + a'_1\right)}{(r_1.a_1) + (r_2.a_2) + (r_3.a_3) + \cdots + (r_{\frac{n-1}{2}}.a_{\frac{n-1}{2}}) + (r_{\frac{n+1}{2}}.\hat{a}_{\frac{n+1}{2}}) + (r_{\frac{n+3}{2}}.a'_{\frac{n-1}{2}}) + \cdots + (r_{n-2}.a'_3) + (r_{n-1}.a'_2) + (r_n.a'_1)} \qquad \textbf{(Eq.4)}$$

$$KC_{Sym./Id.}(A,A') = \frac{r_{\frac{n+1}{2}}\left[\sum_{i=1}^{\frac{n-1}{2}}\left(a_i + \hat{a}_{\frac{n+1}{2}} + a'_i\right)\right]}{\sum_{i=1}^{\frac{n-1}{2}} \sum_{j=n}^{\frac{n+3}{2}}\left(r_i a_i + r_{\frac{n+1}{2}}\hat{a}_{\frac{n+1}{2}} + r_j a'_i\right)} \qquad \textbf{(Eq.4)}$$

$$\begin{cases} if\ g(A) = g(A'); & KC_{Sim./Id.}(A,A') = 1 \\ if\ g(A) = -g(A'), or\ -g(A) = g(A'); & KC_{Sim./Id.}(A,A') = 0 \\ if\ g(A) < g(A'); & 0 \le KC_{Sim./Id.}(A,A') \le 1 \\ if\ g(A) > g(A'); & 1 \le KC_{Sim./Id.}(A,A') \le n+1 \\ if\ g(A) > g(A'); & KC_{Sim./Id.}(A,A') \ge n+1, < 0 \end{cases}$$

Where $g(A)$ and $g(A')$ are the optical moment of A and A' respectively about the mid-point through a symmetric distance D started from the centre. It is expressed by **Eq. 5.1** and **Eq. 5.2**.

$$g\colon \begin{bmatrix} A = \left(a_1, a_2, a_3, \dots, a_{\frac{n-1}{2}}\right) & \delta = \left(\hat{a}_{\frac{n+1}{2}}\right) & A' = \left(a'_{\frac{n-1}{2}}, \dots, a'_3, a'_2, a'_1\right) \\ & \twoheadrightarrow & \\ \downarrow & \downarrow & \downarrow \\ D = (d_k, d_{k-1}, d_{k-2}, \dots, d_1, & d_0, & d_1, \dots, d_{k-2}, d_{k-1}, d_k) \end{bmatrix}$$

$$g(A) = (d_k.a_1) + (d_{k-1}.a_2) + (d_{k-2}.a_3) + \cdots + (d_0.a_{\frac{n-1}{2}}) = \sum_{i=1}^{k}\sum_{j=1}^{n}(d_i a_j) \quad \textbf{(Eq.5.1)}$$

$$g(A') = (d_k.a'_1) + (d_{k-1}.a'_2) + (d_{k-2}.a'_3) + \cdots + (d_0.a'_{\frac{n-1}{2}}) = \sum_{i=1}^{k}\sum_{j=1}^{n}(d_i a'_j) \quad \textbf{(Eq.5.2)}$$

Such that $A, B, \& D \in \mathbb{R}$; $d_0 = 0$; $d_1 \neq 0$; $k = \frac{n-1}{2}$; $n, k \in \mathbb{N}$; D must have a symmetric interval, and A & B are isoreflective pairs in a chosen pairing about a midpoint δ.

3.2. Propositions (theorems)

Theorem 1: Bijection function of isomorphic optinalysis

Isoreflective pairs of mathematical structures under optinalysis are similar and identical to a certain magnitude by a coefficient, called optinalytic coefficient (i.e., Kabirian coefficient, denoted as KC).

Claim:

Pairs of isoreflective points under optinalysis are bijective (*one-to-one and onto*) to each other functionally.

Prove of theorem 1:

Supposed we have an optinalytic construction between isoreflective pairs of similar or identical mathematical structures A and B as follows:

$$f\colon A \overset{\delta}{\twoheadrightarrow} B \twoheadrightarrow R$$

$$f\colon \begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (y_3, y_2, y_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A$ & B; A, B, δ & $R \in \mathbb{R}$; $n \in \mathbb{N}$; and A & B are isoreflective pairs on a chosen pairing about a midpoint δ.

By Kabirian-based optinalysis (i.e., **Eq. 3**), each element functions as in **Eq. 3.1- Eq.3.7**.

$$K_c(A,B) = \frac{4(x_1 + x_2 + x_3 + \delta + y_3 + y_2 + y_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1} \quad \textbf{(Eq.3)}$$

$$x_1 = \frac{K_c(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_2 + x_3 + \delta + y_3 + y_2 + y_1)}{4 - K_c} \quad \textbf{(Eq.3.1)}$$

$$x_2 = \frac{K_c(x_1 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_2 + y_1)}{4 - 2K_c} \quad \textbf{(Eq.3.2)}$$

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$$x_3 = \frac{K_c(x_1 + 2x_2 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_3 + y_2 + y_1)}{4 - 3K_c} \quad \textbf{(Eq.3.3)}$$

$$\delta = \frac{K_c(x_1 + 2x_2 + 3x_3 + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_2 + x_3 + y_3 + y_2 + y_1)}{4 - 4K_c} \quad \textbf{(Eq.3.4)}$$

$$y_3 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + x_3 + \delta + y_2 + y_1)}{4 - 5K_c} \quad \textbf{(Eq.3.5)}$$

$$y_2 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_2 + x_3 + \delta + y_3 + y_1)}{4 - 6K_c} \quad \textbf{(Eq.3.6)}$$

$$y_1 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_1 + x_2 + x_3 + \delta + y_3 + y_2)}{4 - 7K_c} \quad \textbf{(Eq.3.7)}$$

Recall the definition of bijective mapping (*one-to-one and onto*), such that if $x = y$, then $f(g(x)) = g(f(y))$. To verify that x and y are bijective, three (3) cases of mathematical proves were evaluated (see details of the proves in Appendix A). Based on the analysis of the proven cases, we conclude that isomorphic optinalysis is a construction and function based on bijective mapping which signifies isomorphism of defined mathematical structures. It is interesting to verify that each pair of isoreflective points are bijective with a different optinalysis-derived function. That means the bijectivity of one pair-points is independent of the others.

Theorem 2: Bijection function of automorphic optinalysis

Autoreflective pairs of mathematical structures under optinalysis are symmetrical or identical to a certain magnitude by a coefficient, called optinalytic coefficient (i.e., Kabirian coefficient, denoted as KC).

Claim:

Pairs of autoreflective points under optinalysis are bijective (*one-to-one and onto*) to each other functionally.

Prove of theorem 2:

Supposed we have an optinalytic construction between autoreflective pairs of symmetrical or identical mathematical structure A and A' as follow:

$$f\colon A \underset{\twoheadrightarrow\!\!\!\!\!\twoheadleftarrow}{\overset{\delta}{}} A' \twoheadrightarrow R$$

$$f\colon \begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta = (\hat{x}_0)}{\twoheadleftarrow\!\!\!\!\!\twoheadrightarrow} & A' = (x'_3, x'_2, x'_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \in A \,\&\, A'; A, A', \delta \,\&\, R \in \mathbb{R}$; $n \in \mathbb{N}$; and $A \,\&\, B$ are autoreflective pairs on a chosen pairing about a midpoint δ.

By Kabirian-based optinalysis (i.e., in **Eq. 6**), each element functions in **Eq. 6.1 - Eq.6.7**.

$$K_c(A, A') = \frac{4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1} \quad \textbf{(Eq.6)}$$

$$x_1 = \frac{K_c(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_2 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{4 - K_c} \quad \textbf{(Eq.6.1)}$$

$$x_2 = \frac{K_c(x_1 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{4 - 2K_c} \quad \textbf{(Eq.6.2)}$$

$$x_3 = \frac{K_c(x_1 + 2x_2 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_3 + x'_2 + x'_1)}{4 - 3K_c} \quad \textbf{(Eq.6.3)}$$

$$\delta = \frac{K_c(x_1 + 2x_2 + 3x_3 + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + x_3 + x'_3 + x'_2 + x'_1)}{4 - 4K_c} \quad \textbf{(Eq.6.4)}$$

$$x'_3 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + x_3 + \delta + x'_2 + x'_1)}{4 - 5K_c} \quad \textbf{(Eq.6.5)}$$

$$x'_2 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 7x'_1) - 4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_1)}{4 - 6K_c} \quad \textbf{(Eq.6.6)}$$

$$x'_1 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_2)}{4 - 7K_c} \quad \textbf{(Eq.6.7)}$$

Recall the definition of bijective mapping (*one-to-one and onto*), such that if $x = x'$, then $f(g(x)) = g(f(x'))$. To verify that x and x' are bijective, three (3) cases of mathematical proof were evaluated (see Appendix B). Based on the analysis of the proven cases, we conclude that automorphic optinalysis is a construction and function based on bijective mapping which signifies automorphism of a defined mathematical structure. It is interesting to verify that each pair of autoreflective points are bijective with a different optinalysis-derived function. That means the bijectivity of one pair-points is independent of the others.

3.3. Optinalysis-probability translation models (OpTMs)

Principle of the translation models

The translation models of Kabirian coefficient of optinalysis to a probability model cannot be achieved by the existing probability rules or theorems, such as the product and addition rules of frequency-based probability and the Bayesian probability. In this case, we are trying to draw a probabilistic conclusion by using the primary result obtained from optinalysis (i.e., Kabirian coefficient of optinalysis) which was based on an independent, mutually inclusive (simultaneously occur), and multi-dimensional pattern of events. The multi-dimensional pattern (shape) of the events makes this approach more conditionally special requiring some special considerations. This probability model is not only providing the proportion of chances from the sample space; but it further evaluates the probability of closeness or distantness of and to an independent, mutually inclusive, and multi-dimensional pattern of events.

The real-world phenomena, implication, and application of this uncovered probability model are seen from the probability of unlocking a password. There is only one chance (from the sample space) of unlocking the password, but each trial from the sample space has a certain probability of appearing closer or distant to the true or correct password. This probability of closeness or distantness is expressed by this translation model.

Definition 15: Optinalysis-probability translation models

The optinalysis-probability translation models are bridges that connect the outcomes of Kabirian-based optinalysis (i.e., Kabirian bi-coefficients) to probability. The translation models translate the two possible Kabirian bi-coefficients into a probability model that infers the level of certainty to which the isoreflective or autoreflective pairs of mathematical structures are similar, identical, or symmetrical.

The expectation of the translation models

The expectations of this translation model (of forward and backward translations of Kabirian-based optinalytic outcomes) are described as *Y-rule* (of Kabirian-based isomorphic or automorphic optinalysis). The Y-rule, demonstrated below, is a Y-shaped chain of forward and backward proceedings of Kabirian-based isomorphic or automorphic outcomes.

For automorphic optinalysis

$$\begin{matrix} KC1_{P_{Sym./Id.}}(A, A') \\ \cdots \\ KC2_{P_{Sym./Id.}}(A', A) \end{matrix} \rightleftharpoons P_{Sym./Id.}(A, A'), = P_{Sym./Id.}(A', A) \rightleftharpoons P_{Asym./Uid.}(A, A') = P_{Asym./Uid.}(A', A)$$

For isomorphic optinalysis

$$\begin{matrix} KC1_{P_{Sim./Id.}}(A, B) \\ \cdots \\ KC2_{P_{Sim./Id.}}(B, A) \end{matrix} \rightleftharpoons P_{Sim./Id.}(A, B), = P_{Sim./Id.}(B, A) \rightleftharpoons P_{Dsim./Uid.}(A, B) = P_{Dsim./Uid.}(B, A)$$

Theorem 3: Optinalysis-probability translation models

The actual probability of symmetry, similarity, or identity ($P_{Sym./Sim./Id}$) is isoreflective to the expected probability (equals to unity, $P_{expected} = 1$) about a midpoint of the isoreflective event's dimensions. The number of dimensions here refers to the number of isoreflective or autoreflective pairs during the optinalysis.

Prove of theorem 3:

Phase:1 forward translation: from Kabirian bi-coefficients to probability of symmetry and similarity

$$f\colon P_{Sym./Sim./Id} \overset{\delta}{\nrightarrow} 1 \twoheadrightarrow R$$

$$f\colon \begin{bmatrix} P_{Sym./Sim./Id} & \overset{\delta=0}{\nrightarrow} & 1 \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, & nr_1 + r_1, & 2nr_1 + r_1) \end{bmatrix}$$

Or the optinalytic construction is inversely expressed as:

$$f\colon \begin{bmatrix} 1 & \overset{\delta=0}{\nrightarrow} & P_{Sym./Sim./Id} \\ \downarrow & \downarrow & \downarrow \\ R = (r_1, & nr_1 + r_1, & 2nr_1 + r_1) \end{bmatrix}$$

where r_1 is the first term of the established optiscale and n was the number of isoreflective pairs or autoreflective pairs during the optinalysis or the number of dimensions or the sample size.

Then, Kabirian coefficient (K_c) is defined as:

$$K_c = \frac{(nr_1 + r_1)(P_{Sym./Sim./Id} + 1)}{(r_1 \times P_{Sym./Sim./Id}) + (2nr_1 + r_1)} \quad \textbf{(Eq.7.1)}$$

Or the Kabirian coefficient (K_c) is inversely defined as:

$$K_c = \frac{(nr_1 + r_1)(1 + P_{Sym./Sim./Id})}{r_1 + P_{Sym./Sim./Id}(2nr_1 + r_1)} \quad \textbf{(Eq.7.2)}$$

By making $P_{Sym./Sim./Id}$ the subject of the formula from **Eq. 7.1** and **Eq. 7.2**, we obtain models in **Eq. 7.3** and **Eq. 7.4** respectively. These **Eq. 7.3** and **Eq. 7.4** translate the obtained Kabirian bi-coefficients of symmetry, similarity, and identity to the probability of symmetry, similarity, and identity respectively

$$P_{Sym./Sim./Id.} = \frac{(nr_1 + r_1) - K_c(2nr_1 + r_1)}{r_1 \times K_c - (nr_1 + r_1)}, \forall\, 0 \le K_c \le 1 \quad \textbf{(Eq.7.3)}$$

$$\begin{cases} if\ \dfrac{n+1}{2n+1} \le KC_{Sym./Sim./Id}(A, B\ or\ A, A') \le 1; & 0 \le P_{Sym./Sim./Id.}(A, B\ or\ A, A') \le 1 \\ if\ 0 \le KC_{Sym./Sim./Id}(A, B\ or\ A, A') \le \dfrac{n+1}{2n+1}; & -1 \le P_{Sym./Sim./Id.}(A, B\ or\ A, A') \le 0 \end{cases}$$

Or inversely as:

$$P_{Sym./Sim./Id.} = x = \frac{(nr_1 + r_1) - r_1 K_c}{(2nr_1 + r_1)K_c - (nr_1 + r_1)}, \forall\, 1 \le K_c \le n + 1; K_c \ge n + 1\ \ \&\, \forall\, K_c \le 0 \quad \textbf{(Eq.7.4)}$$

$$\begin{cases} if\ 1 \le KC_{Sym./Sim./Id}(A, B\ or\ A, A') \le n + 1; & 0 \le P_{Sym./Sim./Id.}(A, B\ or\ A, A') \le 1 \\ if\ KC_{Sym./Sim./Id}(A, B\ or\ A, A') \ge n + 1, or\ \le 0; & -1 \le P_{Sym./Sim./Id}(A, B\ or\ A, A') \le 0 \end{cases}$$

Phase 2: forward translation: from the probability of symmetry, similarity, and identity to the probability of asymmetry, dissimilarity, and unidentity

Eq. 8 and **Eq. 9** translate forward the probability of symmetry, similarity, and identity ($P_{Sym./Sim./Id.}$) to the probability of asymmetry, dissimilarity, and unidentity ($P_{Asym./Dsim./Uid.}$) between isoreflective or autoreflective pairs of mathematical structures under Kabirian-based optinalysis. Translation of Kabirian coefficient is valid if and only if the outcomes are within the range of values -1 to 1 (or -100 to 100 of its equivalent percentage).

If $P_{Sym./Sim./Id}(A, B\ or\ A, A') \ge 0$, then

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$$P_{Asym./Dsim./Uid.}(A,B\ or\ A,A') = 1 - P_{Sym./Sim./Id}(A,B\ or\ A,A') \quad \textbf{(Eq.8)}$$

If $P_{Sym./Sim./Id}(A,B\ or\ A,A') \leq 0$, then

$$P_{Asym./Dsim./Uid.}(A,B\ or\ A,A') = -1 - P_{Sym./Sim./Id}(A,B\ or\ A,A') \quad \textbf{(Eq.9)}$$

Phase 3: backward translation: from the probability of asymmetry, dissimilarity, and unidentity to the probability of symmetry, similarity, and identity

These **Eq. 10** and **Eq. 11** translate backward the probability of asymmetry, dissimilarity, and unidentity ($P_{Asym./Dsim./Uid.}$) to the probability of symmetry, similarity, and identity ($P_{Sym./Sim./Id.}$) respectively.

If $P_{Asym./Dsim./Uid}(A,B\ or\ A,A') \geq 0$, then

$$P_{Sym./Sim./id.}(A,B\ or\ A,A') = 1 - P_{Asym./Dsim./Uid}(A,B\ or\ A,A') \quad \textbf{(Eq.10)}$$

If $P_{Asym./Dsim./Uid}(A,B\ or\ A,A') \geq 0$, then

$$P_{Sym./Sim./id.}(A,B\ or\ A,A') = -1 - P_{Asym./Dsim./Uid}(A,B\ or\ A,A') \quad \textbf{(Eq.11)}$$

Phase 4: backward translation: from the probability of symmetry, similarity, and identity to Kabirian bi-coefficients

These **Eq. 12** and **Eq. 13** translate backward the probability of symmetry, similarity, and identity outcomes to its two possible Kabirian bi-coefficients, designated as $KC\_Alt.1$ and $KC\_Alt.2$.

$$KC\_Alt.1 = \frac{(nr_1 + r_1)(P_{Sym./Sim./Id} + 1)}{(r_1 \times P_{Sym./Sim./Id}) + (2nr_1 + r_1)}, \forall\ 0 \leq K_c \leq 1 \quad \textbf{(Eq.12)}$$

$$KC\_Alt.2 = \frac{(nr_1 + r_1)(1 + P_{Sym./Sim./Id})}{r_1 + P_{Sym./Sim./Id}(2nr_1 + r_1)}, \forall\ 1 \leq K_c \leq n+1; K_c \geq n+1\ \ \&\ \forall\ K_c \leq 0 \quad \textbf{(Eq.13)}$$

where r_1 is the first term of the established optiscale and n is the sample size/item length.

3.4. Properties of Kabirian-based optinalysis

i. Kabirian-based optinalysis is bi-coefficients and chain-translative (i.e., forward and reverse chain-translations). It gives two possible coefficients ($KC1_{P_{Sym./Sim./Id.}}, KC2_{P_{Sym./Sim./Id.}}$) due to its inverse property, but each coefficient translates into the same results ($P_{Sym./Sim./Id.}$, and $P_{Asym./Dsim./Nid.}$), which can be used to compute back up to the two bi-coefficients. This phenomenon is called the *Y-rule of Kabirian-based isomorphic or automorphic optinalysis.*

   For automorphic optinalysis

$$\begin{matrix} KC1_{P_{Sym./Id.}}(A,A') \\ \cdots \\ KC2_{P_{Sym./Id.}}(A',A) \end{matrix} \rightleftharpoons P_{Sym./Id.}(A,A'), = P_{Sym./Id.}(A',A) \rightleftharpoons P_{Asym./Uid.}(A,A') = P_{Asym./Uid.}(A',A)$$

   For isomorphic optinalysis

$$\begin{matrix} KC1_{P_{Sim./Id.}}(A,B) \\ \cdots \\ KC2_{P_{Sim./Id.}}(B,A) \end{matrix} \rightleftharpoons P_{Sim./Id.}(A,B), = P_{Sim./Id.}(B,A) \rightleftharpoons P_{Dsim./Uid.}(A,B) = P_{Dsim./Uid.}(B,A)$$

   The two possible Kabirian bi-coefficients function on different optiscales.

ii. The complete symmetry, identity, or similarity between isoreflective or autoreflective pairs of mathematical structures is invariant (remains the same) under transformations such as pericentral rotation (alternate reflection), central rotation (inversion), product translation, additive translation, and central modulation. Find the proof in Appendix C.

iii. The asymmetry, dissimilarity, and unidentity between isoreflective or autoreflective pairs of mathematical structures are invariant (remain the same) under product translation and central rotation (inversion). Find the proof in Appendix D.

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iv. Under optinalytic normalization, a complete symmetry, similarity, and identity (i.e., $KC = 1$) between isoreflective pairs of mathematical structures remains invariant, but asymmetry, dissimilarity, and unidentity are normalized, to a relative extent, toward completeness. Find the details in Appendix E.

3.5. Python codes

The proposed methods of isomorphic and automorphic optinalysis, computing codes were written in python language. Get the python codes at these links:

https://github.com/Abdullahi-KB/Kabirian-based_optinalysis/blob/main/automorphic_optinalysis.ipynb

https://github.com/Abdullahi-KB/Kabirian-based_optinalysis/blob/main/isomorphic_optinalysis.ipynb

3.6. Drawbacks and limitations of Kabirian-based optinalysis

The following are some of the identified drawbacks and limitations of Kabirian-based optinalysis.

i. The given random ordering (sequence) of elements of a list of the variable(s) can either be preserved or otherwise a conceptual or theoretical (i.e., ascend or descend sorting) ordering has to be established.
ii. For isomorphic optinalysis, the variable lengths must be the same, otherwise, a suitable method needs to be used to align them.
iii. For isomorphic optinalysis, a suitable and efficient pairing style or alternate reflection has to be chosen and adopted for repeatability and comparison of results.
iv. The two possible Kabirian bi-coefficients do not function on the same optinalytic scale. For comparison of results, estimates with the mixed Kabirian coefficients should either be translated forward or otherwise uninformed by backward alternate translation its translations. (See *Y-rule of Kabirian-based isomorphic or automorphic optinalysis*.).

4. Discussion

In this paper, Kabirian-based optinalysis expressed an important paradigm shift for symmetry/asymmetry detections, similarity/dissimilarity, and identity/unidentity measures between isoreflective or autoreflective pairs of mathematical structures. The Kabirian-based optinalysis is conceptually and theoretically based on the methodological paradigm of automorphism and isomorphism. The uniform intervals of the optiscale preserve an equidistant relationship (i.e., isometry) between the corresponding pair of mathematical structures. Furthermore, the optinalytic relationship between any pair point of isoreflective or autoreflective pairs of mathematical structures is proven, in this paper, to be a bijective (inverse) function. It is also interesting to verify that each pair of autoreflective or isoreflective points under optinalysis are bijective with a different function. That means the bijectivity of one pair is independent of the others. This makes optinalysis a reliable method for the comparison of mathematical structures.

The outcomes of Kabirian-based optinalysis are invariant under a set of mathematical operations or transformations such as scaling, rotation, and location shift. These invariance properties of Kabirian-based optinalysis are sufficient evidence to prove its robustness for symmetry detection, similarity, and identity measures.

The main drawbacks and limitations of Kabirian-based optinalytic measures include: the ordering of the list of the variable(s) has to be chosen or established, variables lengths must be the same (for the case of pairwise comparison), pairing style or alternate reflection has to be chosen and adopted.

5. Conclusion

Kabirian-based optinalysis is a new paradigm proposed for symmetry/asymmetry detections, similarity/dissimilarity, and identity/unidentity measures between isoreflective or autoreflective pairs of mathematical structures. The paradigm of Kabirian-based optinalysis is the optiscale bijective re-mapping of isoreflective or autoreflective pairs. Kabirian-based optinalysis is characterized as a bijection function and invariant under a set of transformations such as scaling, rotation, and location shift.

6. Recommendation

Further studies look into the applications of this new paradigm in other fields of mathematics, physics, and as well as statistics, and geometry (development of statistical and geometrical estimators).

Supplementary material

The supplementary files are the appendices and the python codes. All the python codes are available at:

https://github.com/Abdullahi-KB/Kabirian-based_optinalysis

Conflict of interest

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Appendix A

Theorem 1: Bijection function of isomorphic optinalysis

Isoreflective pairs of mathematical structures under optinalysis are similar or identical to a certain magnitude by a coefficient, called optinalytic coefficient (i.e., Kabirian coefficient, denoted as KC).

Claim:

Pairs of isoreflective points under optinalysis are bijective (*one-to-one and onto*) to each other functionally.

Prove of theorem 1:

Supposed we have an optinalytic construction between isoreflective pairs of identical or similar mathematical structures A and B as follows:

$$f\colon \begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadleftarrow\!\!\twoheadrightarrow} & B = (y_3, y_2, y_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A \,\&\, B$; $A, B, \delta \,\&\, R \in \mathbb{R}$; and $A \,\&\, B$ are isoreflective pairs on a chosen pairing about a midpoint δ.

By Kabirian-based optinalysis (i.e., as in equation A), each element functions as Equations A1-A7:

$$K_c = \frac{4(x_1 + x_2 + x_3 + \delta + y_3 + y_2 + y_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1} \tag{A}$$

$$x_1 = \frac{K_c(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_2 + x_3 + \delta + y_3 + y_2 + y_1)}{4 - K_c} \tag{A1}$$

$$x_2 = \frac{K_c(x_1 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_2 + y_1)}{4 - 2K_c} \tag{A2}$$

$$x_3 = \frac{K_c(x_1 + 2x_2 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_3 + y_2 + y_1)}{4 - 3K_c} \tag{A3}$$

$$\delta = \frac{K_c(x_1 + 2x_2 + 3x_3 + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_2 + x_3 + y_3 + y_2 + y_1)}{4 - 4K_c} \tag{A4}$$

$$y_3 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + x_3 + \delta + y_2 + y_1)}{4 - 5K_c} \tag{A5}$$

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$$y_2 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_2 + x_3 + \delta + y_3 + y_1)}{4 - 6K_c} \tag{A6}$$

$$y_1 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_1 + x_2 + x_3 + \delta + y_3 + y_2)}{4 - 7K_c} \tag{A7}$$

Recall the definition of bijective mapping (*one-to-one and onto*), such that if $x = y$, then $f(g(x)) = g(f(y))$. We now have three (3) cases evaluated as follows:

Case A1:
Firstly, we verify if the pair of isoreflective points (i.e., x_1 and y_1) are functionally mapped *one-to-one*.

$$x_1 = y_1 \Rightarrow \frac{K_c(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_2 + x_3 + \delta + y_3 + y_2 + y_1)}{4 - K_c} = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_1 + x_2 + x_3 + \delta + y_3 + y_2)}{4 - 7K_c}$$

$$x_1 = y_1 \Rightarrow \frac{K_c(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_2 + x_3 + \delta + y_3 + y_2)}{4 - K_c} + \frac{K_c(7y_1) - 4y_1}{4 - K_c}$$
$$= \frac{K_c(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_2 + x_3 + \delta + y_3 + y_2)}{4 - 7K_c} + \frac{K_c(x_1) - 4x_1}{4 - 7K_c}$$

When two completely similar or identical pairs of mathematical structures are compared optinalytically, then $K_c = 1$.

Therefore, we now have

$$x_1 = y_1 \Rightarrow \frac{(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_2 + x_3 + \delta + y_3 + y_2)}{3} + \frac{7y_1 - 4y_1}{3}$$
$$= \frac{(2x_2 + 3x_3 + 4\delta + 5y_3 + 6y_2) - 4(x_2 + x_3 + \delta + y_3 + y_2)}{-3} + \frac{x_1 - 4x_1}{-3} \tag{Eq1}$$

Let the common factor to both sides of the equation Eq1. be $p_1 = \frac{(2x_2+3x_3+4\delta+5y_3+6y_2)-4(x_2+x_3+\delta+y_3+y_2)}{3}$

$$x_1 = y_1 \Rightarrow \frac{p_1 + 3y_1}{3} = \frac{-p_1 - 3x_1}{-3}$$

Secondly, we verify if the pair of isoreflective points (i.e., x_1 and y_1) are functionally mapped *onto* each other.

By composing $g(y_1)$ onto $f(x_1)$

$$f(g(x_1)) = \frac{p_1 + 3\left(\frac{-p_1 - 3x_1}{-3}\right)}{3} = \frac{-3p_1 + 3p_1 - 9x_1}{-9} = x_1 \Rightarrow f(x_1) = g^{-1}(y_1)$$

Again by composing $f(x_1)$ onto $g(y_1)$

$$g(f(y_1)) \Rightarrow \frac{-p_1 - 3\left(\frac{p_1 + 3y_1}{3}\right)}{-3} = \frac{3p_1 - 3p_1 - 9y_1}{-9} = y_1 \Rightarrow g(y_1) = f^{-1}(x_1)$$

Finally, since x_1 and y_1 are *one-to-one and onto*, we conclude that x_1 and y_1 are functionally bijective and inverse.

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Case A2:
Firstly, we verify if the pair of isoreflective points (i.e., x_2 and y_2) are functionally mapped *one-to-one*.

$$x_2 = y_2 \Rightarrow \frac{K_c(x_1 + 3x_3 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_2 + y_1)}{4 - 2K_c} = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_2 + x_3 + \delta + y_3 + y_1)}{4 - 6K_c}$$

$$x_2 = y_2 \Rightarrow \frac{K_c(x_1 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_1)}{4 - 2K_c} + \frac{K_c(6y_2) - 4y_2}{4 - 2K_c}$$
$$= \frac{K_c(x_1 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_1)}{4 - 6K_c} + \frac{K_c(2x_2) - 4x_2}{4 - 6K_c}$$

When two completely similar or identical pairs of mathematical structures are compared optinalytically, then $K_c = 1$.

Therefore, we now have

$$x_2 = y_2 \Rightarrow \frac{(x_1 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_1)}{2} + \frac{6y_2 - 4y_2}{2}$$
$$= \frac{(x_1 + 3x_3 + 4\delta + 5y_3 + 7y_1) - 4(x_1 + x_3 + \delta + y_3 + y_1)}{-2} + \frac{2x_2 - 4x_2}{-2} \quad \text{(Eq2)}$$

Let the common factor to both sides of the equation Eq2. be $p_2 = \frac{(x_1+3x_3+4\delta+5y_3+7y_1)-4(x_1+x_3+\delta+y_3+y_1)}{2}$

$$x_2 = y_2 \Rightarrow \frac{2p_2 + 2y_2}{2} = \frac{-2p_2 - 2x_2}{-2}$$

Secondly, we verify if the pair of isoreflective points (i.e., x_2 and y_2) are functionally mapped *onto* each other.

By composing $g(y_2)$ onto $f(x_2)$

$$f(g(x_2)) = \frac{p_2 + 2\left(\frac{-p_2-2x_2}{-2}\right)}{2} = \frac{-2p_2 + 2p_2 - 4x_2}{-4} = x_2 \Rightarrow f(x_2) = g^{-1}(y_2)$$

Again by composing $f(x_2)$ onto $g(y_2)$

$$g(f(y_2)) \Rightarrow \frac{-p_2 - 2\left(\frac{p_2+2y_2}{2}\right)}{-2} = \frac{2p_2 - 2p_2 - 4y_1}{-4} = y_2 \Rightarrow g(y_1) = f^{-1}(x_1)$$

Finally, since x_2 and y_2 are *one-to-one and onto*, we conclude that x_2 and y_2 are functionally bijective and inverse.

Case A3:
Firstly, we verify if the pair isoreflective points (i.e., x_3 and y_3) are functionally mapped *one-to-one*.

$$x_3 = y_3 \Rightarrow \frac{K_c(x_1 + 2x_2 + 4\delta + 5y_3 + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_3 + y_2 + y_1)}{4 - 3K_c} = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + x_3 + \delta + y_2 + +y_1)}{4 - 5K_c}$$

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$$x_3 = y_3 \Rightarrow \frac{K_c(x_1 + 2x_2 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_2 + y_1)}{4 - 3K_c} + \frac{K_c(5y_3) - 4y_3}{4 - 3K_c} = \frac{K_c(x_1 + 2x_2 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_2 + +y_1)}{4 - 5K_c} + \frac{K_c(3x_3) - 4x_3}{4 - 5K_c}$$

When two completely similar or identical pairs of mathematical structures are compared optinalytically, then $K_c = 1$.

Therefore, we now have

$$x_3 = y_3 \Rightarrow \frac{(x_1 + 2x_2 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_2 + y_1)}{1} + \frac{5y_3 - 4y_3}{1} = \frac{(x_1 + 2x_2 + 4\delta + 6y_2 + 7y_1) - 4(x_1 + x_2 + \delta + y_2 + +y_1)}{-1} + \frac{3x_3 - 4x_3}{-1} \quad \text{(Eq3)}$$

Let the common factor to both sides of the equation Eq3. be $p_3 = \frac{(x_1+2x_2+4\delta+6y_2+7y_1)-4(x_1+x_2+\delta+y_2+y_1)}{1}$

$$x_3 = y_3 \Rightarrow \frac{p_3 + y_3}{1} = \frac{-p_3 - x_3}{-1}$$

Secondly, we verify if the pair of isoreflective points (i.e., x_3 and y_3) are functionally mapped *onto* each other.

By composing $g(y_3)$ onto $f(x_3)$

$$f(g(x_3)) = \frac{p_3 + \frac{(-p_3 - x_3)}{-1}}{1} = \frac{-p_3 + p_3 - x_3}{-1} = x_3 \Rightarrow f(x_3) = g^{-1}(y_3)$$

Again by composing $f(x_3)$ onto $g(y_3)$

$$g(f(y_3)) \Rightarrow \frac{p_3 - \frac{p_3 + y_3}{1}}{-1} = \frac{p_3 - p_3 - y_3}{-1} = y_3 \Rightarrow g(y_3) = f^{-1}(x_3)$$

Finally, since x_3 and y_3 are *one-to-one and onto*, we conclude that x_3 and y_3 are functionally bijective and inverse.

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Appendix B

Theorem 2: Bijection function of automorphic optinalysis

Autoreflective pairs of mathematical structures under optinalysis are symmetrical or identical to a certain magnitude by a coefficient, called optinalytic coefficient (i.e., Kabirian coefficient, denoted as KC).

Claim:

Pairs of autoreflective points under optinalysis are bijective (*one-to-one and onto*) to each other functionally.

Prove of theorem 2:

Supposed we have an optinalytic construction between autoreflective pairs of identical or symmetrical mathematical structures A and A' as follows:

$$f: \begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadleftarrow\!\!\twoheadrightarrow} & A' = (x'_3, x'_2, x'_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A$ & A'; A, A', δ & $R \in \mathbb{R}$; and A & A' are autoreflective pairs on a chosen pairing about a midpoint δ.

By Kabirian-based optinalysis (i.e., as in equation B), each element functions as Equations B1-B7:

$$K_c = \frac{4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1} \tag{B}$$

$$x_1 = \frac{K_c(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_2 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{4 - K_c} \tag{B1}$$

$$x_2 = \frac{K_c(x_1 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{4 - 2K_c} \tag{B2}$$

$$x_3 = \frac{K_c(x_1 + 2x_2 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_3 + x'_2 + x'_1)}{4 - 3K_c} \tag{B3}$$

$$\delta = \frac{K_c(x_1 + 2x_2 + 3x_3 + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + x_3 + x'_3 + x'_2 + x'_1)}{4 - 4K_c} \tag{B4}$$

$$x'_3 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + x_3 + \delta + x'_2 + x'_1)}{4 - 5K_c} \tag{B5}$$

$$x'_2 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 7x'_1) - 4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_1)}{4 - 6K_c} \quad \text{(B6)}$$

$$x'_1 = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_2)}{4 - 7K_c} \quad \text{(B7)}$$

Recall the definition of bijective mapping (*one-to-one and onto*), such that if $x = y$, then $f(g(x)) = g(f(y))$. We now have three (3) cases evaluated as follows:

Case B1:
Firstly, we verify if the pair of autoreflective points (i.e., x_1 and x'_1) are functionally mapped *one-to-one*.

$$x_1 = x'_1 \Rightarrow \frac{K_c(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_2 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{4 - K_c}$$
$$= \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_2)}{4 - 7K_c}$$

$$x_1 = x'_1 \Rightarrow \frac{K_c(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_2 + x_3 + \delta + x'_3 + x'_2)}{4 - K_c} + \frac{K_c(7x'_1) - 4x'_1}{4 - K_c}$$
$$= \frac{K_c(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_2 + x_3 + \delta + x'_3 + x'_2)}{4 - 7K_c} + \frac{K_c(x_1) - 4x_1}{4 - 7K_c}$$

When two completely symmetrical or identical pairs of mathematical structures are compared optinalytically, then $K_c = 1$.

Therefore, we now have

$$x_1 = x'_1 \Rightarrow \frac{(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_2 + x_3 + \delta + x'_3 + x'_2)}{3} + \frac{7x'_1 - 4x'_1}{3}$$
$$= \frac{(2x_2 + 3x_3 + 4\delta + 5x'_3 + 6x'_2) - 4(x_2 + x_3 + \delta + x'_3 + x'_2)}{-3} + \frac{x_1 - 4x_1}{-3} \quad \text{(Eq1)}$$

Let the common factor to both sides of the equation Eq1. be $p_1 = \frac{(2x_2+3x_3+4\delta+5y_3+6y_2)-4(x_2+x_3+\delta+y_3+y_2)}{3}$

$$x_1 = x'_1 \Rightarrow \frac{p_1 + 3x'_1}{3} = \frac{-p_1 - 3x_1}{-3}$$

Secondly, we verify if the pair of autoreflective points (i.e., x_1 and x'_1) are functionally mapped *onto* each other.

By composing $g(x'_1)$ onto $f(x_1)$

$$f(g(x_1)) = \frac{p_1 + 3\left(\frac{-p_1 - 3x_1}{-3}\right)}{3} = \frac{-3p_1 + 3p_1 - 9x_1}{-9} = x_1 \Rightarrow f(x_1) = g^{-1}(x'_1)$$

Again by composing $f(x_1)$ onto $g(x'_1)$

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$$g\left(f(x'_1)\right) \Rightarrow \frac{-p_1 - 3\left(\frac{p_1+3x'_1}{3}\right)}{-3} = \frac{3p_1 - 3p_1 - 9x'_1}{-9} = x'_1 \Rightarrow g(x'_1) = f^{-1}(x_1)$$

Finally, since x_1 and x'_1 are *one-to-one and onto*, we conclude that x_1 and x'_1 are functionally bijective and inverse.

Case B2:

Firstly, we verify if the pair of autoreflective points (i.e., x_2 and x'_2) are functionally mapped *one-to-one*.

$$x_2 = x'_2 \Rightarrow \frac{K_c(x_1 + 3x_3 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_2 + x'_1)}{4 - 2K_c} = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 5x'_3 + 7yx'_1) - 4(x_1 + x_2 + x_3 + \delta + x'_3 + x'_1)}{4 - 6K_c}$$

$$x_2 = x'_2 \Rightarrow \frac{K_c(x_1 + 3x_3 + 4\delta + 5x'_3 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_1)}{4 - 2K_c} + \frac{K_c(6x'_2) - 4x'_2}{4 - 2K_c} = \frac{K_c(x_1 + 3x_3 + 4\delta + 5x'_3 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_1)}{4 - 6K_c} + \frac{K_c(2x_2) - 4x_2}{4 - 6K_c}$$

When two completely symmetrical or identical pairs of mathematical structures are compared optinalytically, then $K_c = 1$.

Therefore, we now have

$$x_2 = x'_2 \Rightarrow \frac{(x_1 + 3x_3 + 4\delta + 5x'_3 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_1)}{2} + \frac{6x'_2 - 4x'_2}{2} = \frac{(x_1 + 3x_3 + 4\delta + 5x'_3 + 7x'_1) - 4(x_1 + x_3 + \delta + x'_3 + x'_1)}{-2} + \frac{2x_2 - 4x_2}{-2} \quad \text{(Eq2)}$$

Let the common factor to both sides of the equation Eq2. be $p_2 = \frac{(x_1+3x_3+4\delta+5x'_3+7x'_1)-4(x_1+x_3+\delta+x'_3+x'_1)}{2}$

$$x_2 = x'_2 \Rightarrow \frac{2p_2 + 2x'_2}{2} = \frac{-2p_2 - 2x_2}{-2}$$

Secondly, we verify if the pair of autoreflective points (i.e., x_2 and x'_2) are functionally mapped *onto* each other.

By composing $g(x'_2)$ onto $f(x_2)$

$$f\left(g(x_2)\right) = \frac{p_2 + 2\left(\frac{-p_2-2x_2}{-2}\right)}{2} = \frac{-2p_2 + 2p_2 - 4x_2}{-4} = x_2 \Rightarrow f(x_2) = g^{-1}(x'_2)$$

Again by composing $f(x_2)$ onto $g(x'_2)$

$$g\left(f(x'_2)\right) \Rightarrow \frac{-p_2 - 2\left(\frac{p_2+2x'_2}{2}\right)}{-2} = \frac{2p_2 - 2p_2 - 4x'_1}{-4} = y_2 \Rightarrow g(x'_1) = f^{-1}(x_1)$$

Finally, since x_2 and x'_2 are *one-to-one and onto*, we conclude that x_2 and x'_2 are functionally bijective and inverse.

Case B3:

Firstly, we verify if the pair of autoreflective points (i.e., x_3 and x'_3) are functionally mapped *one-to-one*.

$$x_3 = x'_3 \Rightarrow \frac{K_c(x_1 + 2x_2 + 4\delta + 5x'_3 + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_3 + x'_2 + x'_1)}{4 - 3K_c} = \frac{K_c(x_1 + 2x_2 + 3x_3 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + x_3 + \delta + x'_2 + x'_1)}{4 - 5K_c}$$

$$x_3 = x'_3 \Rightarrow \frac{K_c(x_1 + 2x_2 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_2 + x'_1)}{4 - 3K_c} + \frac{K_c(5x'_3) - 4x'_3}{4 - 3K_c} = \frac{K_c(x_1 + 2x_2 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_2 + x'_1)}{4 - 5K_c} + \frac{K_c(3x_3) - 4x_3}{4 - 5K_c}$$

When two completely symmetrical or identical pairs of mathematical structures are compared optinalytically, then $K_c = 1$.

Therefore, we now have

$$x_3 = x'_3 \Rightarrow \frac{(x_1 + 2x_2 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_2 + x'_1)}{1} + \frac{5x'_3 - 4x'_3}{1} = \frac{(x_1 + 2x_2 + 4\delta + 6x'_2 + 7x'_1) - 4(x_1 + x_2 + \delta + x'_2 + x'_1)}{-1} + \frac{3x_3 - 4x_3}{-1} \quad \text{(Eq3)}$$

Let the common factor to both sides of the equation Eq3. be $p_3 = \frac{(x_1+2x_2+4\delta+6x'_2+7x'_1)-4(x_1+x_2+\delta+x'_2+x'_1)}{1}$

$$x_3 = x'_3 \Rightarrow \frac{p_3 + x'_3}{1} = \frac{-p_3 - x_3}{-1}$$

Secondly, we verify if the pair of autoreflective points (i.e., x_3 and x'_3) are functionally mapped *onto* each other.

By composing $g(x'_3)$ onto $f(x_3)$

$$f(g(x_3)) = \frac{p_3 + \frac{(-p_3 - x_3)}{-1}}{1} = \frac{-p_3 + p_3 - x_3}{-1} = x_3 \Rightarrow f(x_3) = g^{-1}(x'_3)$$

Again by composing $f(x_3)$ onto $g(x'_3)$

$$g(f(x'_3)) \Rightarrow \frac{p_3 - \frac{p_3 + x'_3}{1}}{-1} = \frac{p_3 - p_3 - x'_3}{-1} = y_3 \Rightarrow g(x'_3) = f^{-1}(x_3)$$

Finally, since x_3 and x'_3 are *one-to-one and onto*, we conclude that x_3 and y_3 are functionally bijective and inverse.

Appendix C

Property C: Optinalytic invariance under operation (I)

A perfect symmetry or identity and similarity state between isoreflective or autoreflective pairs under optinalysis remain invariant (stable) under transformations such as pericentral rotation (alternate reflection), central rotation (inversion), translation, (scaling and location shift), and central modulation.

Prove of property C1: Location Invariance

Optinalysis is a location invariance if $K_c(A,B) = K_c(A+c, B+c)$.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\!:\left[\begin{array}{ccc} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{array}\right]$$

Let c be a change in location structure. The optinalytic construction becomes:

$$f\!:\left[\begin{array}{ccc} A = [(x_1+c), (x_2+c), (x_3+c)] & \overset{\delta}{\twoheadrightarrow} & B = [(x_3+c), (x_2+c), (x_1+c)] \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{array}\right]$$

Such that $\delta \notin A, B \,\&\, c$; $A, B, \delta, R \,\&\, c \in \mathbb{R}$; and $A \,\&\, B$ are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A,B) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta}{8x_1 + 8x_2 + 8x_3 + 4\delta} = 1$$

Similarly,

$$K_c(A+c, B+c) = \frac{4[(x_1+c) + (x_2+c) + (x_3+c) + (\delta+c) + (x_3+c) + (x_2+c) + (x_1+c)]}{(x_1+c) + (2x_2+c) + (3x_3+c) + (4\delta+c) + (5x_3+c) + (6x_2+c) + (7x_1+c)}$$

$$K_c(A+c, B+c) = \frac{8\,(x_1+c) + 8\,(x_2+c) + 8\,(x_3+c) + 4\delta}{8\,(x_1+c) + 8\,(x_2+c) + 8\,(x_3+c) + 4\delta} = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta + 24c}{8x_1 + 8x_2 + 8x_3 + 4\delta + 24c} = 1$$

It now shows that $K_c(A,B) = K_c(A+c, B+c)$. Therefore, optinalysis is a location invariant.

Prove of property C2: Scale Invariance

Optinalysis is a scale invariance if $K_c(A,B) = K_c(cA, cB)$.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\!:\left[\begin{array}{ccc} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{array}\right]$$

Let c be a change in scale parameter. The optinalytic construction becomes:

$$f\!:\left[\begin{array}{ccc} A = (cx_1, cx_2, cx_3) & \overset{\delta}{\twoheadrightarrow} & B = (cx_3, cx_2, cx_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{array}\right]$$

Such that $\delta \notin A, B$ & c; A, B, δ, R & $c \in \mathbb{R}$; and A & B are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A,B) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta}{8x_1 + 8x_2 + 8x_3 + 4\delta} = 1$$

Similarly,

$$K_c(cA,cB) = \frac{4(cx_1 + cx_2 + cx_3 + \delta + cx_3 + cx_2 + cx_1)}{cx_1 + 2cx_2 + 3cx_3 + 4\delta + 5cx_3 + 6cx_2 + 7cx_1}$$

$$K_c(cA,cB) = \frac{8cx_1 + 8cx_2 + 8cx_3 + 4\delta}{8cx_1 + 8cx_2 + 8cx_3 + 4\delta} = 1$$

It now shows that $K_c(A,B) = K_c(cA,cB)$. Therefore, optinalysis is a scale-invariant.

Prove of property C3: Invariance under central rotation (Inversion)

Optinalysis is invariant under central rotation if $K_c(A,B) = K_c(B,A)$.

Definition: Central rotation refers to the rotation of all the members of two mathematical structures of an isoreflective pairs through 180° around the central mid-point (δ). This rotation is equivalent to an inversion.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\!: \begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

By central rotation, the optinalytic construction becomes:

$$f\!: \begin{bmatrix} B = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & A = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A$, & B; A, B, δ, & $R \in \mathbb{R}$; and A & B are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A,B) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta}{8x_1 + 8x_2 + 8x_3 + 4\delta} = 1$$

Similarly,

$$K_c(B,A) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(B,A) = \frac{4x_1 + 4x_2 + 4x_3 + 4\delta + 4x_3 + 4x_2 + 4x_1}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(B,A) = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta}{8x_1 + 8x_2 + 8x_3 + 4\delta} = 1$$

It now shows that $K_c(A,B) = K_c(B,A)$. Therefore, optinalysis is a rotation invariant.

Prove of property C4: Invariance under pericentral rotation (Alternate reflection)

Optinalysis is invariant under alternate reflection if $K_c(\overleftarrow{A},\overrightarrow{B}) = K_c(\overrightarrow{A},\overleftarrow{B})$.

Definition: Central rotation refers to the rotation of all the members of two mathematical structures of an isoreflective pairs through 180° around the pericentres (A pericentre is a mid-point of each of the two comparing mathematical structures). This corresponds to an alternate reflection (i.e., from the head-to-head to tail-to-tail reflection or otherwise). An alternate reflection is the alternative form of reflection between isoreflective pairs. The alternate reflection can, in some cases, be used to distinguish between two similar structures but not identical to each other.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\colon\begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

By alternate reflection, the optinalytic construction becomes:

$$f\colon\begin{bmatrix} A = (x_3, x_2, x_1) & \overset{\delta}{\twoheadrightarrow} & B = (x_1, x_2, x_3) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A, \& B$; $A, B, \delta, \& R \in \mathbb{R}$; and A & B are isoreflective pairs in an annotated pairing about a midpoint δ.

Then,

$$K_c(\overleftarrow{A},\overrightarrow{B}) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(\overleftarrow{A},\overrightarrow{B}) = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta}{8x_1 + 8x_2 + 8x_3 + 4\delta} = 1$$

Similarly,

$$K_c(\overrightarrow{A},\overleftarrow{B}) = \frac{4(x_3 + x_2 + x_1 + \delta + x_1 + x_2 + x_3)}{x_3 + 2x_2 + 3x_1 + 4\delta + 5x_1 + 6x_2 + 7x_3}$$

$$K_c(\overrightarrow{A},\overleftarrow{B}) = \frac{4x_3 + 4x_2 + 4x_1 + 4\delta + 4x_1 + 4x_2 + 4x_3}{x_3 + 2x_2 + 3x_1 + 4\delta + 5x_1 + 6x_2 + 7x_3}$$

$$K_c(\overrightarrow{A},\overleftarrow{B}) = \frac{8x_3 + 8x_2 + 8x_1 + 4\delta}{8x_3 + 8x_2 + 8x_1 + 4\delta} = 1$$

It now shows that $K_c(\overleftarrow{A},\overrightarrow{B}) = K_c(\overrightarrow{A},\overleftarrow{B})$. Therefore, optinalysis is a rotation invariant.

Prove of property C5: Invariance under central modulation

Optinalysis is invariant under central modulation if $K_c(A, B) = K_c(A, \delta \pm \beta, B)$.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\colon\begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

By central modulation, the optinalytic construction becomes:

$$f\colon\begin{bmatrix} A = (x_1, x_2, x_3) & \overset{\delta \pm \beta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A, \& B$; $A, B, \delta, \beta, \& R \in \mathbb{R}$; and A & B are isoreflective pairs on a chosen pairing about a midpoint δ.

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Then,

$$K_c(A,B) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{8x_1 + 8x_2 + 8x_3 + 4\delta}{8x_1 + 8x_2 + 8x_3 + 4\delta} = 1$$

Similarly,

$$K_c(A,\delta \pm \beta,B) = \frac{4(x_1 + x_2 + x_3 + (\delta \pm \beta) + x_3 + x_2 + x_1)}{x_1 + 2x_2 + 3x_3 + 4(\delta \pm \beta) + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,\delta \pm \beta,B) = \frac{8x_1 + 8x_2 + 8x_3 + 4(\delta \pm \beta)}{8x_1 + 8x_2 + 8x_3 + 4(\delta \pm \beta)} = 1$$

It now shows that $K_c(A,B) = K_c(A,\delta \pm \beta,B)$. Therefore, optinalysis is invariant under central modulation (normalization).

Appendix D

Property D: Optinalytic invariance under operations (II)

An asymmetrical or dissimilar and unidentical state between autoreflective or isoreflective pairs under optinalysis remains invariant (the same) under product translation and central rotation (inversion).

Prove of property D1: Location Invariance

Optinalysis is a location invariance if $K_c(A,B) = K_c(A+c, B+c)$.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\colon\begin{bmatrix} A = (tx_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

Let c be a change in location structure. The optinalytic construction becomes:

$$f\colon\begin{bmatrix} A = [(tx_1+c), (x_2+c), (x_3+c)] & \overset{\delta}{\twoheadrightarrow} & B = [(x_3+c), (x_2+c), (x_1+c)] \\ \downarrow & \downarrow & \downarrow \\ R = \ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

Such that $\delta \notin A, B \;\&\; c$; $A, B, \delta, R \;\&\; c \in \mathbb{R}$; and $A \;\&\; B$ are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A,B) = \frac{4(tx_1 + x_2 + x_3 + \delta + x_3 + \ x_2 + x_1)}{tx_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{4tx_1 + 4x_1 + 8x_2 + 8x_3 + 4\delta}{tx_1 + 7x_1 + 8x_2 + 8x_3 + 4\delta} \neq 1$$

Similarly,

$$K_c(A+c, B+c) = \frac{4[(tx_1+c) + (x_2+c) + (x_3+c) + (\delta+c) + (x_3+c) + (\ x_2+c) + (x_1+c)]}{(tx_1+c) + (2x_2+c) + (3x_3+c) + (4\delta+c) + (5x_3+c) + (6x_2+c) + (7x_1+c)}$$

$$K_c(A+c, B+c) = \frac{4(tx_1+c) + 4(x_1+c) + \ 8(x_2+c) + 8(x_3+c) + 4\delta}{(tx_1+c) + (7x_1+c) + \ 8(x_2+c) + 8(x_3+c) + 4\delta} \neq 1$$

It now shows that $K_c(A,B) \neq K_c(A+c, B+c)$. Therefore, optinalysis is a location variant (not location invariant).

Prove of property D2: Scale Invariance

Optinalysis is a scale invariance if $K_c(A,B) = K_c(cA, cB)$.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\colon\begin{bmatrix} A = (tx_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

Let c be a change in scale parameter. The optinalytic construction becomes:

$$f\colon\begin{bmatrix} A = (ctx_1, cx_2, cx_3) & \overset{\delta}{\twoheadrightarrow} & B = (cx_3, cx_2, cx_1) \\ \downarrow & \downarrow & \downarrow \\ R = \ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

Such that $\delta \notin A, B \;\&\; c$; $A, B, \delta, R \;\&\; c \in \mathbb{R}$; and $A \;\&\; B$ are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A,B) = \frac{4(tx_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{tx_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{4tx_1 + 4x_1 + 8x_2 + 8x_3 + 4\delta}{tx_1 + 7x_1 + 8x_2 + 8x_3 + 4\delta} \neq 1$$

Similarly,

$$K_c(cA,cB) = \frac{4(ctx_1 + cx_2 + cx_3 + \delta + cx_3 + cx_2 + cx_1)}{ctx_1 + 2cx_2 + 3cx_3 + 4\delta + 5cx_3 + 6cx_2 + 7cx_1}$$

$$K_c(cA,cB) = \frac{c(4tx_1 + 4x_1 + 8x_2 + 8x_3) + 4\delta}{c(tx_1 + 7x_1 + 8x_2 + 8x_3) + 4\delta} = \frac{4tx_1 + 4x_1 + 8x_2 + 8x_3 + 4\delta}{tx_1 + 7x_1 + 8x_2 + 8x_3 + 4\delta} \neq 1$$

It now shows that $K_c(A,B) = K_c(cA,cB)$. Therefore, optinalysis is a scale-invariant.

Prove of property D3: Invariance under central rotation (Inversion)

Optinalysis is invariant under central rotation if $K_c(A,B) = K_c(A,B)$.

Definition: Central rotation refers to the rotation of all the members of two mathematical structures of an isoreflective pairs through 180° around the central mid-point (δ). This rotation is equivalent to an inversion.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f: \begin{bmatrix} A = (tx_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

By central rotation, the optinalytic construction becomes:

$$f: \begin{bmatrix} B = (x_1, x_2, x_3) & \overset{\delta}{\twoheadrightarrow} & A = (x_3, x_2, tx_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A, \& B$; $A, B, \delta, \& R \in \mathbb{R}$; and A & B are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A,B) = \frac{4(tx_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{tx_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{4tx_1 + 4x_1 + 8x_2 + 8x_3 + 4\delta}{tx_1 + 7x_1 + 8x_2 + 8x_3 + 4\delta} \neq 1$$

Similarly,

$$K_c(A,B) = \frac{4(x_1 + x_2 + x_3 + \delta + x_3 + x_2 + tx_1)}{tx_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A,B) = \frac{4x_1 + 4x_2 + 4x_3 + 4\delta + 4x_3 + 4x_2 + 4tx_1}{x_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7tx_1} = \frac{4x_1 + 4tx_1 + 8x_2 + 8x_3 + 4\delta}{x_1 + 7tx_1 + 8x_2 + 8x_3 + 4\delta} \neq 1$$

It now shows that $K_c(A,B) = K_c(A,B)$. Therefore, optinalysis is a rotation invariant.

Prove of property D4: Invariance under pericentral rotation (Alternate reflection)

Optinalysis is invariant under alternate reflection if $K_c(\overleftarrow{A},\vec{B}) \neq K_c(\vec{A},\overleftarrow{B})$.

Definition: Central rotation refers to the rotation of all the members of two mathematical structures of an isoreflective pairs through 180° around the pericentres (A pericentre is a mid-point of each of the two comparing mathematical structures). This corresponds to an alternate reflection (i.e., from the head-to-head to tail-to-tail reflection or otherwise). An alternate reflection

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is the alternative form of reflection between isoreflective pairs. The alternate reflection can, in some cases, be used to distinguish between two similar structures but not identical to each other.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\colon\begin{bmatrix} A=(tx_1,x_2,x_3) & \overset{\delta}{\nrightarrow} & B=(x_3,x_2,x_1) \\ \downarrow & \downarrow & \downarrow \\ R=\ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

By alternate reflection, the optinalytic construction becomes:

$$f\colon\begin{bmatrix} B=(x_3,x_2,tx_1) & \overset{\delta}{\nrightarrow} & A=(x_1,x_2,x_3) \\ \downarrow & \downarrow & \downarrow \\ R=\ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

Such that $\delta \notin A, \& B$; $A, B, \delta, \& R \in \mathbb{R}$; and A & B are isoreflective pairs in an annotated pairing about a midpoint δ.

Then,

$$K_c(\overleftarrow{A},\vec{B})=\frac{4(tx_1+x_2+x_3+\delta+x_3+x_2+x_1)}{tx_1+2x_2+3x_3+4\delta+5x_3+6x_2+7x_1}$$

$$K_c(\overleftarrow{A},\vec{B})=\frac{4tx_1+4x_1+8x_2+8x_3+4\delta}{tx_1+7x_1+8x_2+8x_3+4\delta}\neq 1$$

Similarly,

$$K_c(\vec{A},\overleftarrow{B})=\frac{4(x_3+x_2+tx_1+\delta+x_1+x_2+x_3)}{x_3+2x_2+3tx_1+4\delta+5x_1+6x_2+7x_3}$$

$$K_c(\vec{A},\overleftarrow{B})=\frac{4x_3+4x_2+4tx_1+4\delta+4x_1+4x_2+4x_3}{x_3+2x_2+3tx_1+4\delta+5x_1+6x_2+7x_3}$$

$$K_c(\vec{A},\overleftarrow{B})=\frac{4tx_1+4x_1+8x_2+8x_3+4\delta}{3tx_1+5x_1+8x_2+8x_3+4\delta}\neq 1$$

It now shows that $K_c(\overleftarrow{A},\vec{B})\neq K_c(\vec{A},\overleftarrow{B})$. Therefore, optinalysis is a rotation variant (not rotation invariant).

Appendix E

Property E: Optinalytic normalization

An asymmetrical, dissimilar, or unidentical state between autoreflective or isoreflective pairs of given mathematical structures under optinalysis, can be transformed near-symmetrical or similar states by central modulation. A central modulation refers to the deliberate increase or decrease in quantity at the central mid-point (δ). The quantity affected is called the normalization unit or value, β.

Prove of property E:

Optinalysis is invariant under central modulation if $K_c(A,B)=K_c(A,\delta\pm\beta,B)$.

Suppose we have an optinalytic construction of isoreflective pairs with an assigned optiscale (R = 1, 2, 3, 4, 5, 6, 7) as follows:

$$f\colon\begin{bmatrix} A=(tx_1,x_2,x_3) & \overset{\delta}{\nrightarrow} & B=(x_3,x_2,x_1) \\ \downarrow & \downarrow & \downarrow \\ R=\ (1,2,3, & 4, & 5,6,7) \end{bmatrix}$$

By central rotation, the optinalytic construction becomes:

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$$f\colon \begin{bmatrix} A = (tx_1, x_2, x_3) & \overset{\delta \pm \beta}{\nleftrightarrow} & B = (x_3, x_2, x_1) \\ \downarrow & \downarrow & \downarrow \\ R = (1, 2, 3, & 4, & 5, 6, 7) \end{bmatrix}$$

Such that $\delta \notin A, \& B$; $A, B, \delta, \beta, \& R \in \mathbb{R}$; and A & B are isoreflective pairs on a chosen pairing about a midpoint δ.

Then,

$$K_c(A, B) = \frac{4(tx_1 + x_2 + x_3 + \delta + x_3 + x_2 + x_1)}{tx_1 + 2x_2 + 3x_3 + 4\delta + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A, B) = \frac{4tx_1 + 4x_1 + 8x_2 + 8x_3 + 4\delta}{tx_1 + 7x_1 + 8x_2 + 8x_3 + 4\delta} \neq 1$$

Similarly,

$$K_c(A, \delta \pm \beta, B) = \frac{4(tx_1 + x_2 + x_3 + (\delta \pm \beta) + x_3 + x_2 + x_1)}{tx_1 + 2x_2 + 3x_3 + 4(\delta \pm \beta) + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A, \delta \pm \beta, B) = \frac{4tx_1 + 4x_2 + 4x_3 + 4(\delta \pm \beta) + 4x_3 + 4x_2 + 4x_1}{tx_1 + 2x_2 + 3x_3 + 4(\delta \pm \beta) + 5x_3 + 6x_2 + 7x_1}$$

$$K_c(A, \delta \pm \beta, B) = \frac{4tx_1 + 4x_1 + 8x_2 + 8x_3 + 4(\delta \pm \beta)}{tx_1 + 7x_1 + 8x_2 + 8x_3 + 4(\delta \pm \beta)} \neq 1$$

It now shows that $K_c(A, B) \neq K_c(A, \delta \pm \beta, B)$. Therefore, optinalysis is variant (not invariant) under central modulation (normalization).

Let $\beta \to \infty$, than $K_c(A, \delta \pm \beta, B) \cong 1$. Therefore, A and B are normalized similar, or symmetrical.