PREPRINT
RESEARCH TITLE

Information Theory optimization algorithm for efficient service orchestration in distributed systems

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ABSTRACT

Distributed Systems architectures are becoming the standard computational model for processing and transportation of information, especially for Cloud Computing environments. The increase in demand for application processing and data management from enterprise and end-user workloads continues to move from a single-node client-server architecture to a distributed multitier design where data processing and transmission are segregated. Software development must considerer the orchestration required to provision its core components in order to deploy the services efficiently in many independent, loosely coupled - physically and virtually interconnected - data centers spread geographically, across the globe. This network routing challenge can be modeled as a variation of the Travelling Salesman Problem (TSP). This paper proposes a new optimization algorithm for optimum route selection using Algorithmic Information Theory. The Kelly criterion for a Shannon-Bernoulli process is used to generate a reliable quantitative algorithm to find a near optimal solution tour. The algorithm is then verified by comparing the results with heuristic solutions in 3 test cases. A statistical analysis is designed to measure the significance of the results between the algorithms and the entropy function can be derived from the distribution. The tested results shown an improvement in the solution quality by producing routes with smaller length and time requirements. The quality of the results proves the flexibility of the proposed algorithm for problems with different complexities without relying in nature-inspired models such as Genetic Algorithms and Simulated Annealing. This algorithm can be used by orchestration applications to deploy services across large cluster of nodes by making better decision in the route design.

Keywords
Traveling Salesman Problem, Information Theory, Artificial Intelligence, Computational Complex Theory, Kolmogorov-Complexity, Kelly criterion and Logarithmic utility

INTRODUCTION
Distributed Information Systems (DS) are growing in popularity across the software industry as it provides more computational and data transmission capacity for applications and become an essential infrastructure that is needed to address the increase in demand for data processing.

DS are used as a cost-efficient way to obtain higher levels of performance by using a cluster of low-capacity machines instead of a unique – single point of failure - large node. A DS is more tolerant to individual machine failures and provides more reliability than a monolithic system.

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Parallel computation such as Cloud Computing and High-Performance Computing (HPC) are applications of distributed computing. (Marinescu, 2013)

The Cloud Computing market is very consolidated as the cost to deploy, expand and operate a global infrastructure and network is very large. As of 2020 there are 3 major companies: Amazon AWS, Microsoft and Azure. Companies can reduce their IT costs by orchestrating efficiently their workloads across different data centers by their respective weight impact, defined as a utility function with the Euclidian distance between nodes (and its respective influence on network latency) or the financial utilization time-rate cost for a given set of machines. The Figure 1 from Atomia (Alguacil, 2016) illustrate the data centers coverage for the major cloud providers across the globe as of 2016.

Figure 1 A map for the 3 largest Cloud Computing providers by market share of 2016.

The components of a DS are located in many different machines over a network. The communication and orchestration of process are done by sending and receiving messages. The service exposed are defined by the aggregation of components and its interactions provide the software functionality. Systems such as Service-Oriented Architecture (SOA), peer-to-peer(P2P) and Micro-Services are examples of distributed applications.

Deploying and synchronizing components over many distributed cluster of nodes can be very complex due to multiple variables that can affect the quality of the solution such as network latency between data centers at business hours and at on-demand; cost of renting machines from different Computing Providers; shared servers resources utilization (“noisy neighbor effect”- at both virtual machines and bare metal); valorization of the dollar due to macro and micro economics factors; change in processing time due to model of nodes available for a given time period and operating complexity of the technology stack.

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There are many algorithms proposed in the literature to solve the routing problem such as 2opt, ant colony, greedy algorithm, genetic algorithm (GA) and simulated annealing (SA) but very limited work is found using Algorithmic Information Theory to find the boundaries of decision problems in Turing Machines. In this paper we propose a variation of the TSP by defining the decision problem for the candidate solution as a Shannon-Bernoulli process that follows a log-normal distribution for the cost distance variable (i.e. created by a utility function for the TSP).

An orchestration job has to deploy efficiently S types of services (or tasks) in many different computing resources such as a cluster of containers or a pool of (physical or virtual) machine nodes connected over a distributed network with different weights (or costs) between each pair of nodes. This job is a process that needs to run on all M (unique) resources points. The cost to use each node can be defined as the (round-trip) network latency between the nodes in the network or the financial cost associated to the proportional quantitative utilization rate for each resource in a given time period. As more Computational Capacity is added, choosing the shortest route to multiple target nodes will be more computationally complex. Figure 2 demonstrate the deployment of services {A, B, C} for nodes {1, 2, 3, 4, 5} in a cluster of machines connected over a common network.

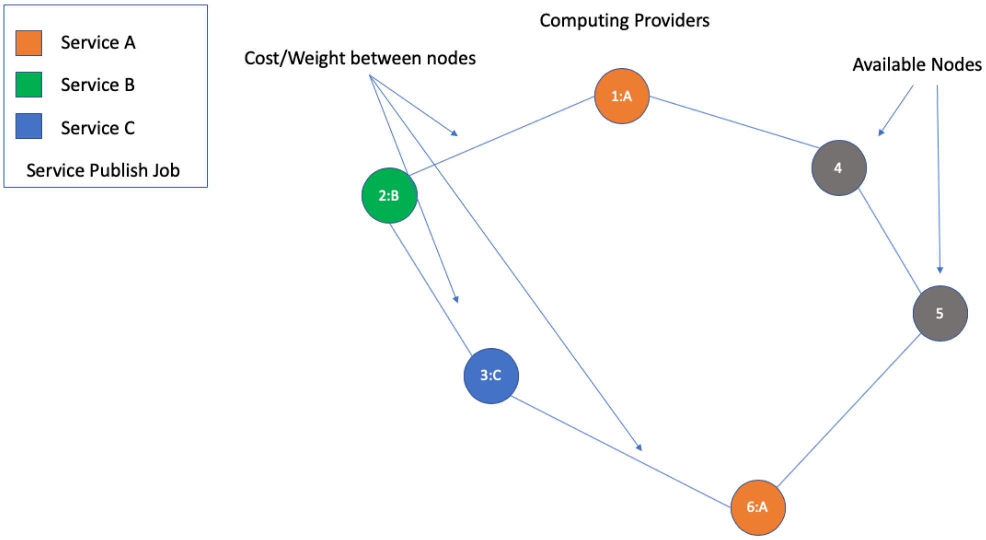

Figure 2 Computing Provider's distributed over a network of machines with the respective published Service's and Available nodes

BACKGROUND

PROBLEM

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The Traveling Salesman Problem was introduced by William Hamilton and Thomas Kirkman. It also known as the *messenger problem*. The problem asks given a list of cities and the distance between each pair of nodes what is the shortest route that visits all cities exactly once and returns to the original city at the end. There are several researches dedicated to address this routing problem and it has applications in mathematics, computer science, statistics and logistics.

The computational complexity of an algorithm shows how much resources are required to apply an algorithm such as how much time and memory are required by a Turing machine to complete execution and can be interpreted as a measure of difficulty of computing functions. A measurement of computation complexity is the big *O* notation. It can be defined as: Let two functions *f* and *g* such as *f(n)* is *O(g(n))* if there are positive numbers *c* and N such that

$$f(n) <= cg(n) \text{ for all } n >= N$$

and is used to estimate the function growth tax (asymptotic complexity).

The TSP problem is an important combinatorial optimization problem. As most of the decision problems, it is in the class of NP-hard problems.

Consider a salesman traveling from city to city and some of them are connected. His goal is to visit each city exactly once and go back to the first city when he finishes. The salesman can choose any path as long as its valid (i.e. visit each city once and finish at the city it has started the tour) he also wants to minimize his cost by taking the shortest route. This problem can be described as a weighted graph G where each city is a node (or vertex) and is connected by a weighted edge only if the two cities are connected by any kind of road, and this road do not cross any other city. The utility function in the TSP is the Euclidian distance. Figure 3 demonstrate the cost matrix between each pair of nodes of a set defined by valid (non-repeating) permutations in a language L with symbols {A, B, C, D}. The cost/weight between points is calculated as a Euclidian distance in a 2D graph. Table 1 demonstrate a sample of valid and invalid strings created from L.

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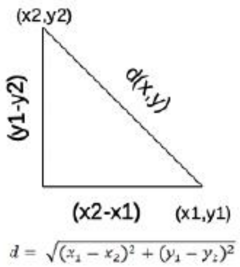

Dist(i,j)	A	B	C	D
A	Ø	Dist(A,B)	Dist(A,C)	Dist(A,D)
B	Dist(B,A)	Ø	Dist(B,C)	Dist(B,D)
C	Dist(C,A)	Dist(C, B)	Ø	Dist(C, D)
D	Dist(D,A)	Dist(D, B)	Dist(D,C)	Ø

A sample graph for the traveling salesman problem and the corresponding cost matrix.

Figure 3 A sample cost matrix for 4 nodes and the Euclidian distance between each pair of nodes.

Table 1 Solution sample of valid and invalid candidates for a given string schema.

Word in Language	Path Sequence	Total cost/weight distance
Valid	A->B->C->D->E->A	Sum of total Euclidian Cost 1
Valid	D->B->C->A->E->D	Sum of total Euclidian Cost 2
Valid	D->B->A->C->E->D	Sum of total Euclidian Cost 3
Invalid	A->B->A->D->E->A	Sum of total Euclidian Cost 4

The graph can be represented as a matrix where each cell value is defined as the respective cost (distance) *w* between nodes *v* and *u*. For N nodes the distance matrix is defined as $D = w(v,u)$ for all (unique) pair of N. The goal of the TSP is to find a permutation π that minimizes the distance between nodes. For symmetric instances the distance between two nodes in the graph is the same in each direction, forming an undirected graph. For asymmetric instances the weights for the edges between nodes can be dynamic or non-existent.

The weigh value of the edge is defined as the distance of the tour (roads) between cities.
For symmetric TSP, as the number of nodes (or cities) increases in the graph G, the number of possible tours growth choice also increase and is exponential. If we consider N nodes, the function of the input size is

$$f\_size = ((N - 1)!/2)$$

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This is the number of elements (states) an algorithm must evaluate to decide (to halt) the problem and is very large thus requiring considerable time and computational resources even for small instances of the problem.

A strategy to address this limitation is to accept near-optimal solutions by setting constrains in the problem using heuristics methods (approximations). Algorithms such as 2opt, GA and SA define $a\text{-}priori$ knowledge about the distribution of the solution space and then repetitively try to improve the quality. It works by following some heuristic (approximation) function schema while trying to avoid a local minimal. As the heuristics (for TSP and NP problems in general) are a best effort strategy to find a (good) near-optimal solution (by enforcing space and time boundaries), it does not guarantee that the solution found is the best candidate to the problem and therefore an program can never be sure that if by running more time the overall solution cost could be improved, unless the entire solution space to the problem is evaluated. This limitation is set by the definition of NP-hard class.

Computational Complexity Theory Problems in the NP class can be solved by a non-deterministic polynomial algorithm. Any given class of algorithms such as P, NP, coNP, regular etc. must have a lower bound that index the best performance any problem in the class can have. This bound can be described as the total amount of input items (or symbols) a machine must process (before halting), and the respective output items produced following a Probability Distribution Function and a given finite Alphabet. A strategy to find a solution to the decision problem is to find a function that reduce or transform a problem from a domain in which there is no know solution to a constrained domain with a known solution.

This allows the algorithm to search the solution space and decide if any solution is a valid (yes-instances) or invalid (no-instances) and its computable by a polynomial-time algorithm.
This strategy allows us to map instances of the Hamiltonian circle problem to a decision version of the Traveling Salesman Problem and can be described as a decision problem to determine if exists a Hamiltonian circuit in a given complete graph with positive integer weights hose length is not greater than a given positive integer m. Each valid (yes-instance) in the TSP problem is mapped to a valid instance in the Hamiltonian problem space and this transformation can be done in polynomial time.

In Figure 4 reproduced from (Raskhodnikova, 2016) we have a visual representation of computational complexities.

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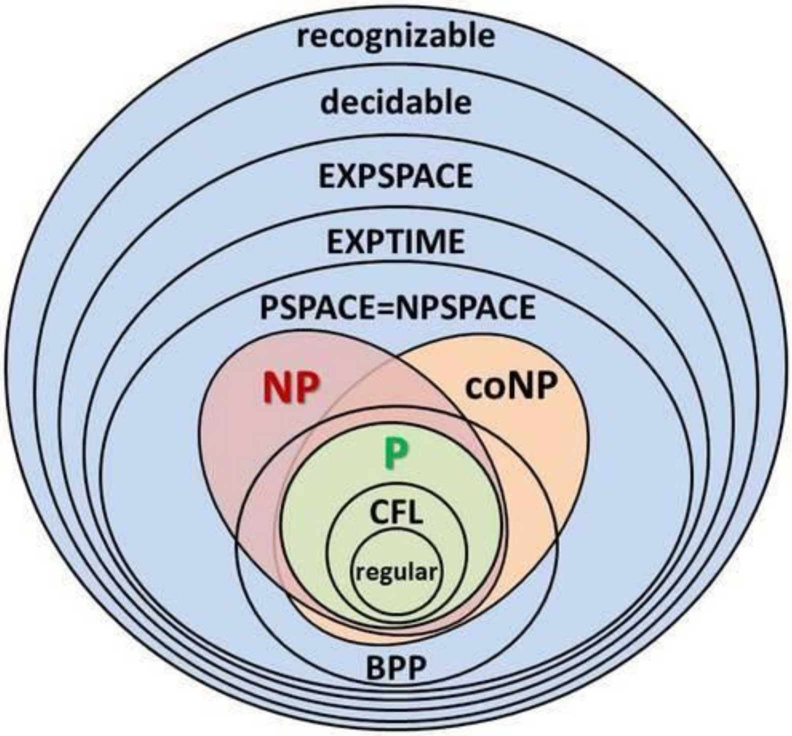

Figure 4 Diagram Representation for the many categories of Computational Complexity

Hamiltonian Graph. A Hamiltonian cycle (or circuit) can described as a "path" that contains all nodes and the elements in this set are not repeated, with exception to the final vertex. This means that a Hamiltonian cycle in G with start node v has all other nodes exactly once and them finishes at node v. A graph G is Hamiltonian if it has a Hamiltonian cycle. A Hamiltonian cycle with minimum weight is an optimal circuit and therefore is the shortest tour in the TSP Problem.

The Figure 5 provides an example of Hamiltonian circuit for a Graph G with 5 nodes {A, B, C, D, E}. The Table 2 shows the cost matrix for the super graph G

Table 2 Cost Matrix for Graph G with 5 nodes

Cost Matrix	A	B	C	D	E
A	0	2	2	1	1
B	2	0	1	1	1
C	2	1	0	2	1
D	1	1	2	0	2
E	1	1	1	2	0

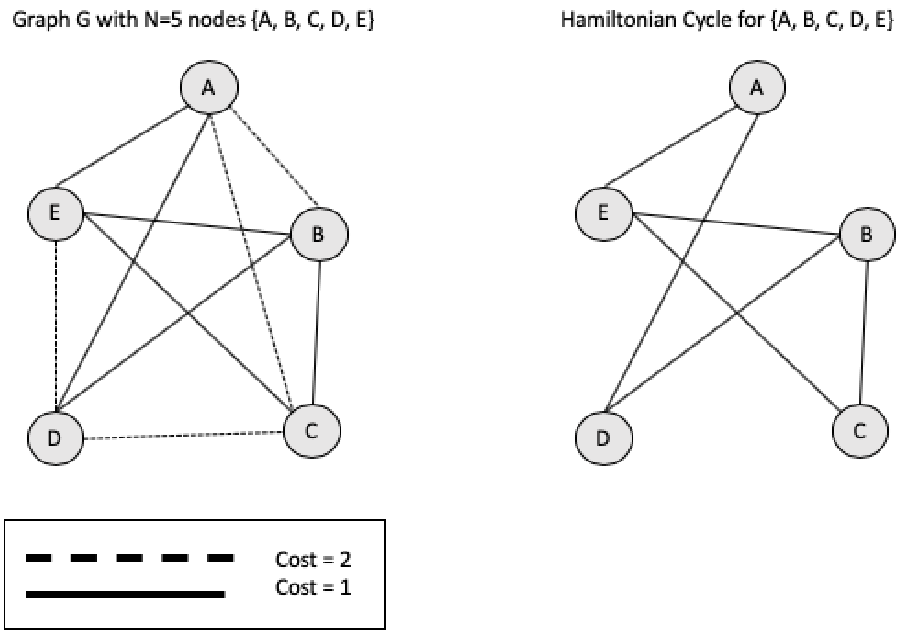

Figure 5 Hamiltonian Cycle from a Super Graph

Although heuristics methods define special cases for the TSP problem and produces near-optimal solutions with short length (weight) Hamiltonian cycles it does not guarantee that the results are the shortest circuit possible. The algorithms to solve the TSP are grouped in 2 categories: exact (Brute-force, greedy) and approximation algorithms (Heuristics such as Simulated Annealing and Genetic Algorithm).

Nature inspired models. Researchers have proposed algorithms inspired by natural events and structures like the heating of metals and the growing behavior of biological organisms. Those methods do not iterate over the entire solution space but rather a portion in order to find the local minimum. They start with an initial random solution and tries to improve the solution quality over each interaction until some input Threshold parameter factor T is reached like a maximum number of interactions; maximum number of candidate solutions; no further improvements found after several iterations; the rate of decay in a dependable temperature probabilistic function or a minimum quality threshold is achieved.

Therefore heuristics (approximation) methods can be interpreted as a non-deterministic way to address the error rate between the known solutions and the unknown solutions in polynomial time (i.e. Entropy reduction methods). Although such algorithms do not have to traverse the entire solution space it must decide - or “bet” - when a random candidate solution with negative gain will be accepted (i.e. candidate with worst solution quality than current know best solution) in the hopes that eventually it would lead to the shortest distance (i.e. a better solution quality).

Nature-inspired models such as Genetic Algorithms (GA) and Simulated Annealing (SA) use prior information to improve the solution results and thus are biased towards this encoding. Alternatively, by modeling the TSP problem as a communication channel with a probability

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density function associated with the stochastic process that generates the solutions at random (following a Bernoulli process), thus we can bound the limits of the search space to a log-normal distribution.

The advantage of this method is that by relying on the statistical analysis of the solution space instead of the computational complexity of the problem we can have equal or better quality than the traditional algorithms without relying on computationally complex implementations that have a higher time and space constrains.

Therefore, this paper attempts to provide an algorithm to solve the TSP using for the decision rule the entropy measured for the solution cost distribution H(X) and by maximizing the expected value of the logarithm of cost/weight/distance variable, defined as the utility function g(X). This is equivalent to maximize the expected geometric growth rate.

CURRENT APPROACH
Literature Review

2opt, k-opt. Croes proposed the 2-opt algorithm (Croes, 1958), a simple local-search heuristic, to solve the optimization problem for the TSP. It works by removing two edges from the tour and reconnects the two paths created. The new path is a valid tour since there is only one way to reconnect the paths. The algorithm continues removing and reconnecting until no further improvements can be found. k-opt implementations are instances of 2-opt function but with $k > 2$ and can lead to small improvements in solution quality. However, as *k* increases so does the time to complete execution.

In his work (Glover, 1998) proposed the Tabu Search method and it can be used to improve the performance of several local-search heuristics such as 2opt. As neighborhood searches algorithms like 2opt can sometimes converge to a local optimum, the Tabu search keeps a list of illegal moves to prevent solutions that provide negative gain to be chosen frequently. In 2opt the two edges removed are inserted in the Tabu list. If the same pair of edges are created again by the 2opt move, they are considered Tabu. The pair is kept in the list until its pruned or it improves the best tour. However, using Tabu searches increases computational complexity to O(n3), as additional computation is required to insert and evaluate the elements in the list.

The Figure 6 show the 2-opt moves from (Emir Zunic, 2017).

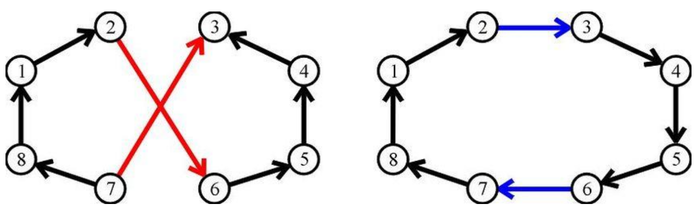

Figure 6 Generating 2-opt moves

(Nilsson, 2003) compared several heuristic strategies for the TSP problem such as Greedy, Insertion, SA, GA, etc. He investigated the performance tradeoff between solution quality and computational time. He classifies the heuristics in two class: Tour construction algorithms and

Tour Improvement algorithms. All algorithms in the first group stops when a solution is found such as brute-force and Greedy Algorithm. In the second group, after a solution is found by some heuristics, it tries to improve that solution (up to certain computation and/or time constraints) such as implemented by 2opt, Genetic Algorithm and Simulated Annealing. He concluded by showing that the computational time required is proportional to the desired solution quality.

Simulated Annealing (SA). Simulated Annealing are heuristics with explicit rules to avoid local minimal. It can be described as a local random search that temporarily accepts moves with negative gain (i.e. were produced by solutions with worst quality than current). These methods simulate the behavior of the cooling process of metals into a minimum energy crystalline structure.

This concept is analogous to the search of global maximum and minimum. The probability of accepting a solution is set by a probability function of a temperature parameter variable *t*. As the temperature decreases over time the probability changes accordingly. Figure 7 demonstrates the simulated decay in the temperature function over the number of interactions in an algorithm.

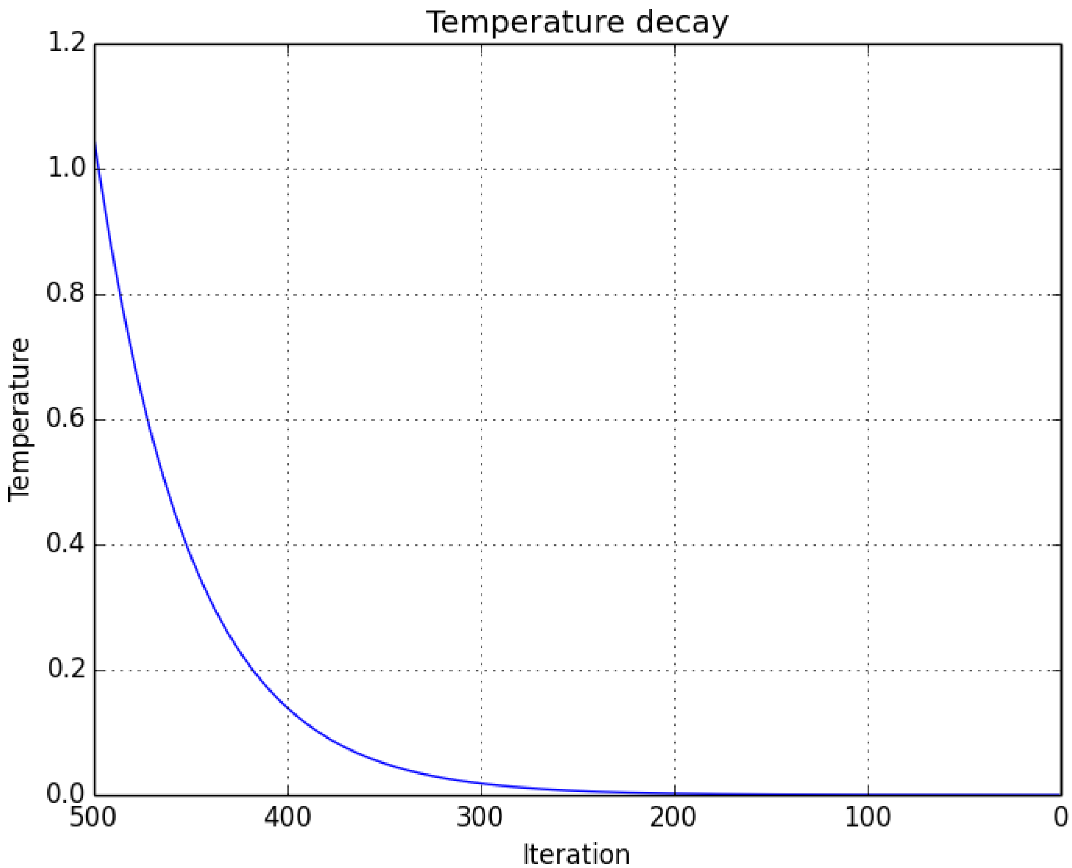

Figure 7 Temperature decay relative to the iterations of the SA algorithm

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The acceptance probability is defined as p(x) = 1 if f (y) <= f (x) and when otherwise

$$p(x) = e^{[-(f(y)-f(x))/t]}$$

where t is the input temperature.

The SA algorithm specifies the neighborhood structure and the cooling function. Figure 8 from (Zhan, 2016) represents the SA algorithm flowchart.

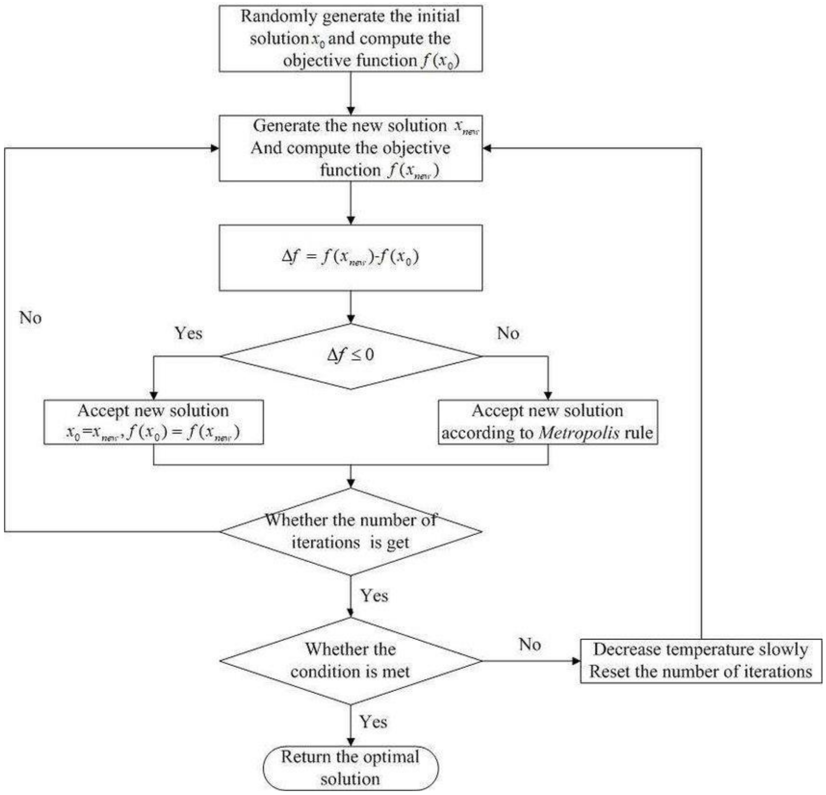

Figure 8 Simulated Annealing Algorithm Flowchart

Metropolis Algorithm. Let f(X) be a function with output proportional to a given target distribution function *r*. The function *r* is the proposal density. At each iteration of the algorithm it attempts to move around the sample space. For each move it decides sometimes to accept a given random solution or stay in place. The probability of the solution of the new proposed candidate is with respect to the current know best solution. If the proposed solution is more probable than the know existing point, we automatically accept the new move. Else if the new proposed solution is less probable, we will sometimes reject the move and the more the decrease the probability, more likely we will accept the new move. Most of the values returned will be around the P(X) but eventually solutions with lower probability will be accepted. This characterizes can be interpreted as a generalization of the methods proposed by Simulated Annealing and Genetic Algorithms.

Other heuristics such as 2-opt, 3-opt, inverse, swap methods can be used to generate candidate solutions. Several researches such as (Nilsson, 2003) have been made to study the performance of different SA operators to solve the TSP problem. (Zhan, 2016) proposed a list-based SA algorithm

using a list-based cooling method to dynamically adjust the temperature decreasing rate. This adaptive approach is more robust to changes in the input parameters. In his work (Kah Huo Leong, 2016) proposed a biological inspired bee system to optimize the routing in Railway systems. They conclude that the average solution results are better or equivalent than the traditional SA and GA methods alone.
The quality of the solution can be improved by allowing more time for the algorithm to run. (Steiglitz, 1968) observed that the performance of 2-opt and 3-opt algorithms can be improved by keeping an ordered list of the closest neighbors for each city-node and thus reducing the amount of solutions to search.

Genetic Algorithm (GA). Genetic Algorithms was first introduced by (Holland, 1975) based on natural selection theory, as a stochastic optimization method in random searches for good (near-optimal) solutions. This approach is analogous to the "survival of the fittest" principle presented by Darwin. This means that individuals that are fitter to the environment are more likely to survive and pass their genetic information features to the next generation.

In TSP the chromosome that models a solution is represented by a "path" in the graph between cities. GA has three basic operations: Selection, Crossover and Mutation. In the Selection method the candidate individuals are chosen for the production of the next generation by following some fittest function In the TSP This function can be defined as the length (weight) of the candidate solutions tour. In Figure 9 we have a representation of genes and Chromosomes.

Solution A	1	0	1	0	1	Gene
Solution B	1	1	1	1	1	Chromosome
Solution C	1	1	0	0	0	Population
Solution D	0	0	1	1	0	

Figure 9 Chromosome for a sample of individual candidate solutions.

In Figure 10 is demonstrated an example of two parents under the Mutation and Crossover operators to generate a new offspring.

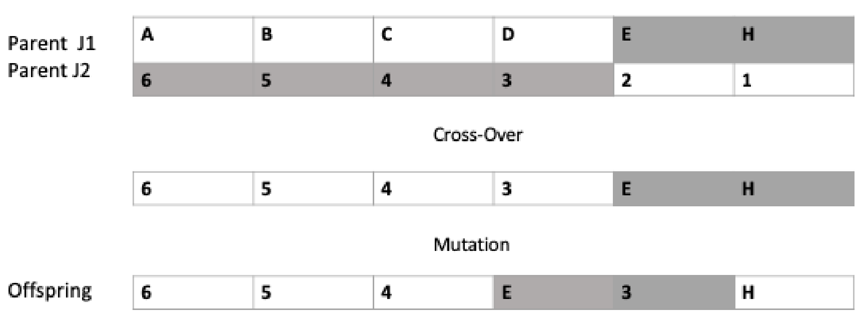

Figure 10 Offspring representation for the Genetic Algorithm Mutation and Crossover operators.

Next those individuals are chosen to mate (reproduction) to produce the new offspring. Individuals that produce better solutions are more fit and therefore have more chances of having offspring. However, individuals that produces worst solutions should not be discarded since they have a probability to improve solution in the future. In other words, the heuristic accepts solutions with negative gain hoping that eventually it may lead to a better solution.

Several researches have studied the performance trade off of selection strategy and how the input parameters affect the quality of solution and the computational time. (Razali, 2011) in his work explores different selection strategies to solve the TSP and compare the performances quality and the number of generations required. It concludes that tournament selection is more appropriate for small instance problems and rank-based roulette wheel can be used to solve large size problems.

(Goldberg, 1991) compared the quality of the solution and the convergence time on many selection methods such as proportional, tournament and ranking. They conclude that ranking and tournament have produced better results that proportional selection, under certain conditions to convergence. In his work (Zhong, 2005) explored proportional roulette wheel and tournament method. He concluded tournament selection is more efficient than proportional roulette selection.

The Figure 11 contains the pseudo-code for a Genetic Algorithm from (K.P. Ferentinos, 2002)

```
generate an initial random population
while iteration <= maxiteration
  iteration = iteration + 1
  calculate the fitness of each individual
  select the individuals according to their fitness
  perform crossover with probability pc
  perform mutation with probability pm
  population = selected individuals after
                      crossover and mutation
end while
```

Figure 11 Basic genetic algorithm.

ALTERNATIVE APPROACH

Information Theory (IT) quantifies the amount of information in a noisy communication channel and is measured in bits of entropy. IT is based in probability theory and statistical distributions. Entropy quantifies the amount of uncertainty in a random Bernoulli variable created by a Bernoulli process thus information can be interpreted as a reduction in the overall uncertainty about a set of finite states. Mutual information is a measure of common information between two random variables and it can be used to maximize the amount of information shared between encoded (sent) and decoded (received) signals. In Table 3 we have the relationship between Information and Entropy. As we increase our knowledge about the states following a probabilistic function distribution, we reduce entropy, as there is less uncertainty about possible state outcomes.

Table 3 Relation between the level of uncertainty and knowledge about possible string outcomes

Information	Entropy	Word	P (X=0)	P (X=1)	E(X)

High	Low	0000	1	0	1*1*1*1 = 1
Medium	Medium	0001	0.75	0.25	0.75*0.75*0.75*0.25 = 0.105
Low	High	0011	0.5	0.5	0.5*0.5*0.5*0.5=0.0625

Information Theory as an approximation method. Information Theory has applications in a range of fields and is used as a mathematical framework for encoding and decoding of information such as in adaptive systems, artificial intelligence, complex systems, network theory, coding theory, etc. IT quantifies the number of bits required to describe a given data using a statistical distribution function for the input data.

Entropy of a random sequence. Entropy is a measure of uncertainty of a random variable. It is the average rate at which information is produced by a stochastic process. (Shannon, 1948) defined the entropy H as a discrete random variable X with possible values as outcomes draw from a probability density function P(X). Figure 12 demonstrate the variation in entropy H(X) vs a Bernoulli distribution. In Equation 1 the entropy function is defined as:

Equation 1 Shannon Entropy Formula

$$\mathrm{H}(X) = -\sum_{i=1}^{n} \mathrm{P}(x_i) \log_b \mathrm{P}(x_i)$$

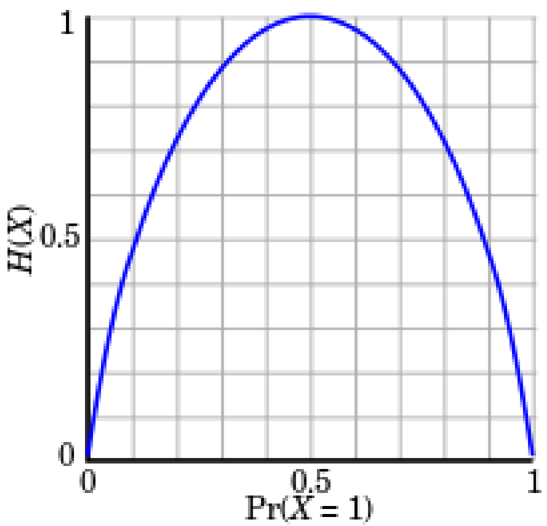

Figure 12 Entropy H(X) vs Probality Pr(X=1)

Random variables and utility function. Let X be an independent random variable with alphabet L: {001, 010, 100. . .}. A utility function *g* is used to model worth or value and is defined by g: X -> R (Real) and it represents a preference of relation between states. The utility function Y=g(X) of a random variable X express the preference of a given order of possible values of X. This order can be a logical evaluation of the value against a given threshold or constant parameter. The g(X) is defined by a normal distribution with given mean and variance under some degrees of freedom (i.e. confidence level).

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As an example if g(X1 = 001 = 1) = c1 and g(X2 = 010 = 2) = c2 are the costs of two routes between a set of nodes in a super-graph G*, we can use this function to determine the arithmetical and logical relationship between them and decide if c1 is worst, better, less, greater or equal to c2. Therefore g(X1) < g(X2). The probability density function pdf(Y) can be used to calculate the entropy of the distribution of the cost values. An exponential utility is a special case used to model when uncertainty (or risks) in the outcome between binary states and in this case the expected utility function is maximized depending on the degree of risk preference.

Figure 14 demonstrate an example of a normal distribution for cost function g(X) = {50, 51, 49, 49.3, 50.5, 50.3, 49.1, 48} and the probability P(X<48). Figure 13 shows the histogram for g(X). Table 4 demonstrate the calculation for the mean, standard deviation and variance for g(X), thus we have:

Table 4 Statistical metrics for function g(X) distribution

Parameters	**Output**
Standard Deviation, σ	0.91241437954473
Count, N	8
Sum, Σx	397.2
Mean, μ	49.65
Variance, σ^2	0.8325

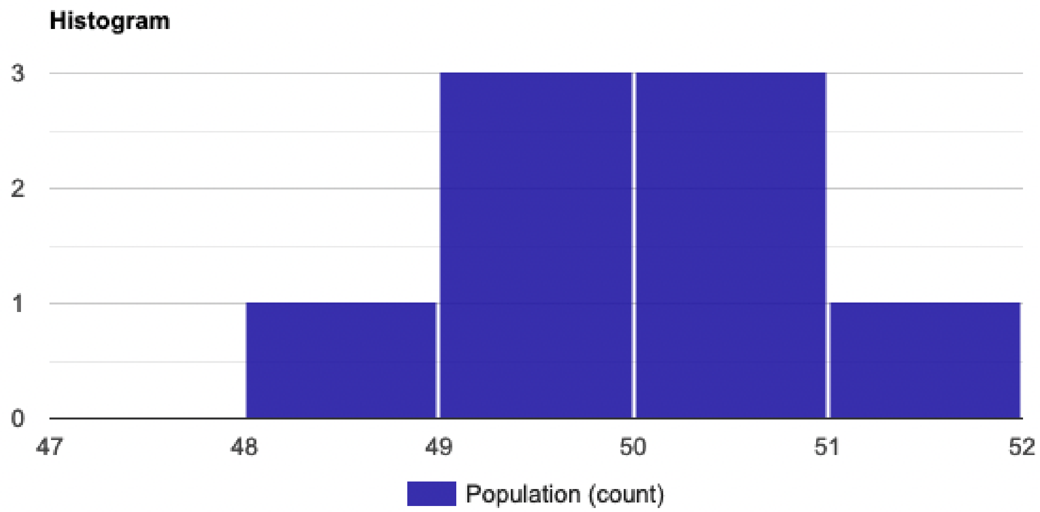


Figure 13 The probability density function for cost function g(X).

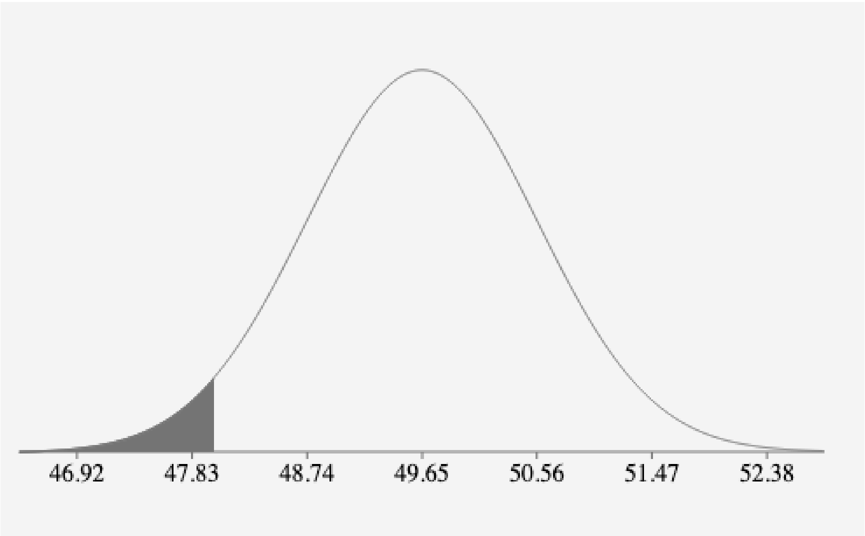


Figure 14 The normal distribution with mean 49.65 and standard deviation 0.91. The P (X<48; X < min g(X)) = 0.0349.

Kolmogorov Complexity. The Kolmogorov complexity (Kolmogorov, 1968) of a string w from language L denoted by Kc_L(w) is the shortest program from alphabet L which produces *w* as output and halts. The conditional Kolmogorov complexity of string *x* relative to word w is defined by Kc_L(w|x) and is the length of the shortest program that receives *x* as input and produces *w* as output.

Complexity of a string and shortest description length. Let U be a Universal computer (Universal Turing Machine). The Kolmogorov complexity Kc(x) of a string x of a computer U is

$$Kc\ (x) = \min \text{length}(p)$$

When **p: U (p) = x**

It is the minimum length program p that output variable x and halts. It's the small possible program. Let C be another computer. If this complexity is general there is a universal computer U that simulates C for any string x by a constant c on computer C. Thus

$$Kc\ (x)\ U <= Kc\ (x)\ C + c$$

Measuring the randomness of a string. Let Kc (x|y): The conditional Kolmogorov complexity of Xn given Y. Consider for example we want to find the binary string with higher complexity between three variables X1(010101010101010), X2(0111011000101110) and Y (01110110001011). In Table 5 we have the representation and minimal encoding using Xn and Y. Therefore, we can see X1 and X2 can be encoded as combinations of Y and thus Kc(X1|Y) > Kc (X2| Y). This relationship is defined as

$$Kc\ (X, Y) = Kc(X|Y) + Kc(Y) = Kc(Y|X) + Kc(X)$$

Table 5 Example of representation of strings X1 and X2 using substring Y.

Variable	Binary Sequence	Minimal String Representation Schema

Y	01110110001011	Substring
X1	010101010101010	01 (#7 pairs of bits) + 0 (#1 single bit)
X2	011101100010110	Y + 0

In Figure 15 from (Maier, 2014) we can see a comparison between a series of strings and the correspondent automata state machine and the regular expression patterns (i.e. regex).

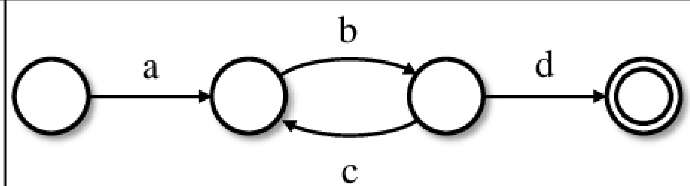
Input strings	1. abcbcbcbcbcbd 2. abd 3. abcbcbcbd 4. abcbcbcbcbcbcbcbcbcbcbcbcbcbd
Regular expression	a(bc)$^{\{0,13\}}$bd more generalized: a(bc)*bd
Automaton	

Figure 15 Examples of representations of a given input string set using regex and an automaton.

The expected value of the Kolmogorov complexity of a random sequence is close to the Shannon entropy. This relationship between complexity and entropy can be described as a stochastic process drawn to a i.i.d on variable X following a probability mass function pdf(x). The symbol x in variable X is defined by a finite alphabet. This expectation is

$$E(1/n)\ Kc(X^n|n) => H(X)$$

Kelly criterion and the uncertainty in random outcomes. The Kelly strategy is a function for optimal size of an allocation in a channel. It calculates the percentage of a resource that should be allocated for a given random process. It was created by John Kelly (Kelly, 1956) to measure signal noise in a network. The bit can be interpreted as the amount of entropy in an expected event with two possible (binary) outcome and even odds. This model maximizes the expectation of the logarithm of total resource value rather than the expected improvement for the utility function from each trial (in each clock unit iteration in a Turing Machine)

The Kelly criterion has applications in gambling and investments in the securities market (Thorp, 1997). In those special cases, the resource (communication) channel is the gambler's financial capital wealth and the fraction is the optimal bet size. The gambler wants to reduce the risk of ruin and maximize the growth rate of his capital. This value is found by maximizing the expected value of logarithm of wealth which is equal to maximize the expected geometric growth rate.

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Similarly, the log-normal Salesman's can improve his strategy in the long run by quantifying the total of available inside information in the channel (or a tape in the Turing machine) and maximizing the expected value of the logarithm of the value function (defined by Traveled Euclidean distance) for each execution clock. Using this approach, he can reduce his uncertainty (entropy H(X)) while optimizing his rate of distance reduction (solution quality improvement) at each execution time.

The Figure 16 demonstrate the Kelly criterion value over the Expected Growth Rate

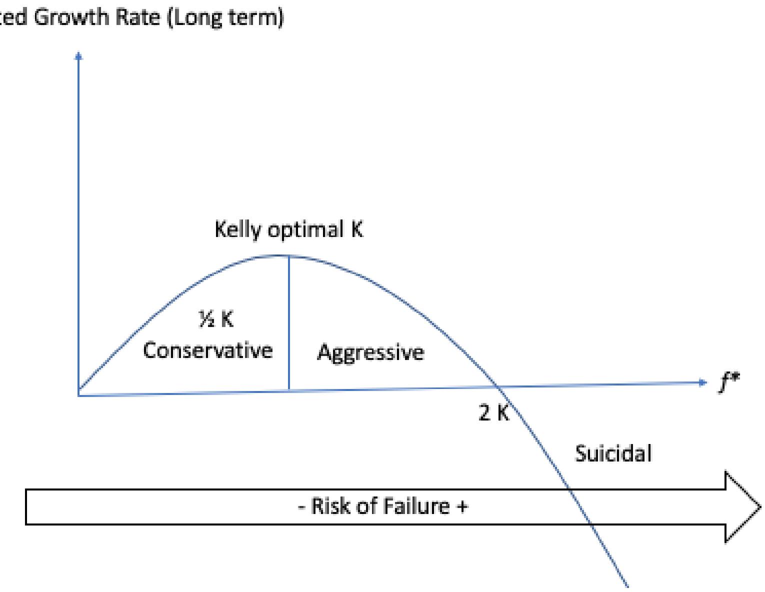

Figure 16 Maximization of entropy in random events

Kelly uncertainty distribution. Let E(Y) be the expected value of random variable Y, H(Y) be the measured entropy for the pdf(Y) distribution and K*(E(Y),H(Y)) = *f** be the maximization of the expected value of the logarithm of the entropy of the utility function Y=g(X) . This fraction is known as the Kelly criterion and can be understood as the level of uncertainty about a given data distribution of the random variable X relative to a probability density function pdf(Y) of a measured respective cost distribution found at the sample. It's a measurement of the amount of useful encoded information.

The value of *f** is a fraction of the cost-value of g(X) on an outcome that occurs with probability p and odds b. Let the probability of finding a value which improves g(X) be p and in this case the resulting improvement is equal to 1 cost-unit plus (1+) the fraction f. The probability of decreasing quality for Y is (1-p). Therefore, the expected value for log variable (E) is differed in Equation 2 as:

Equation 2 Expected value for the cost variable

$$E = p\log(1 + fb) + (1 - p)\log(1 - f)$$

The maximization of the expected value *f** is defined by the Kelly criterion formula in Equation 3

Equation 3 Kelly criterion formula

$$f^* = \frac{pb + p - 1}{b}$$

Where *f** is the optimal fraction, b is the net odds, p is the probability of improving quality (win) in Y=g(X) and the q is the probability of decreasing (loss) quality q=(1-p).

For example, consider a program with a 60% chance of improving the utility function g(X) thus p=0.6 (win) and q=0.4 (lose). Consider the program has a 1-to-1 odds of finding a sequence which improves g(X) and thus b=1(1 quality-unit increase divided by 1 quality-unit decrease). For these parameters the program has a 20%(f*=0.20) of certainty that the outcomes produce values that improve the expected value of g(X) over many trials.

Consider another sample case for a fair coin with probability of success (winning) P (1) = 0.5(50%) and failure(losing) P (0) = 0.5(50%). Table 6 demonstrate the amount improved (+) and worsen (-) for each scenario:

Table 6 Yield Returns from Bernoulli process P(1) and P(0)

Probability	Output	Value
Success (1)	Gain (True)	+ 2 units
Failure (0)	Lose (False)	- 1 unit

The Total Expected Outcome (TEO) is defined by:

TEO = (50% * 2 units) + (50% * 1 unit) = 0.5 units

The Kelly criterion can be calculated alternatively as:

Total Value Allocated (TVA) = Edge/Odds

TVA = TEO/Amount Earned if Success = 0.5 units / 2 units = 25%

Therefore, there are 25% bits of useful information in the noisy channel.

 # METHOD

Quantitative Algorithm Theory. The algorithm is designed to find the near optimal best route to multiple service nodes before returning to the original point. This problem is a variation of TSP.

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Tour improvement – heuristics - algorithms such as 2opt and Simulated Annealing (SA) are used as a benchmark for the proposed Quantitative Algorithm (QA). 3 test cases are used to analyses the solutions generated by each algorithm. The 2opt algorithms produces solutions with smaller total distance but required more time units as the number of nodes increase. SA and QA have a maximum number of allowed interactions, but QA produces better solution quality than SA for the same time period.

The test samples are grouped in 10, 30 and 50 nodes. Each point represents a machine in a data center (i.e. computing and network provider) that can deploy a given service S. The distance cost in this case is the illustrative round-trip network and processing delay. This weight is the length of time to send a signal t(s*) plus the time to reply acknowledging of that the same signal t(s*) was received.

To avoid bias and miss interpretation in the research, the first tour loaded in the computer memory is randomly flushed using a statistical function in Python programming language. The function swaps all elements (using a normal distribution) of the initial tour list, created after reading the list of input nodes.

The solutions found from SA and QA heuristics algorithms were analyzed for accuracy and reliability of the output. We have compared the required time and solution improvement between each program. Each algorithm was measured with a trial with sample N=60.

 # RESULTS – PROPOSED MODEL

In Figure 17 we have the flowchart design for the Quantitative TSP Algorithm (QA). The constraints for the Kelly criterion and the Bernoulli trials are presented in Figure 18 and Figure 20. Table 7 and Table 8 demonstrates the mathematical model and the pseudo-code for the proposed Quantitative Algorithm (QA).

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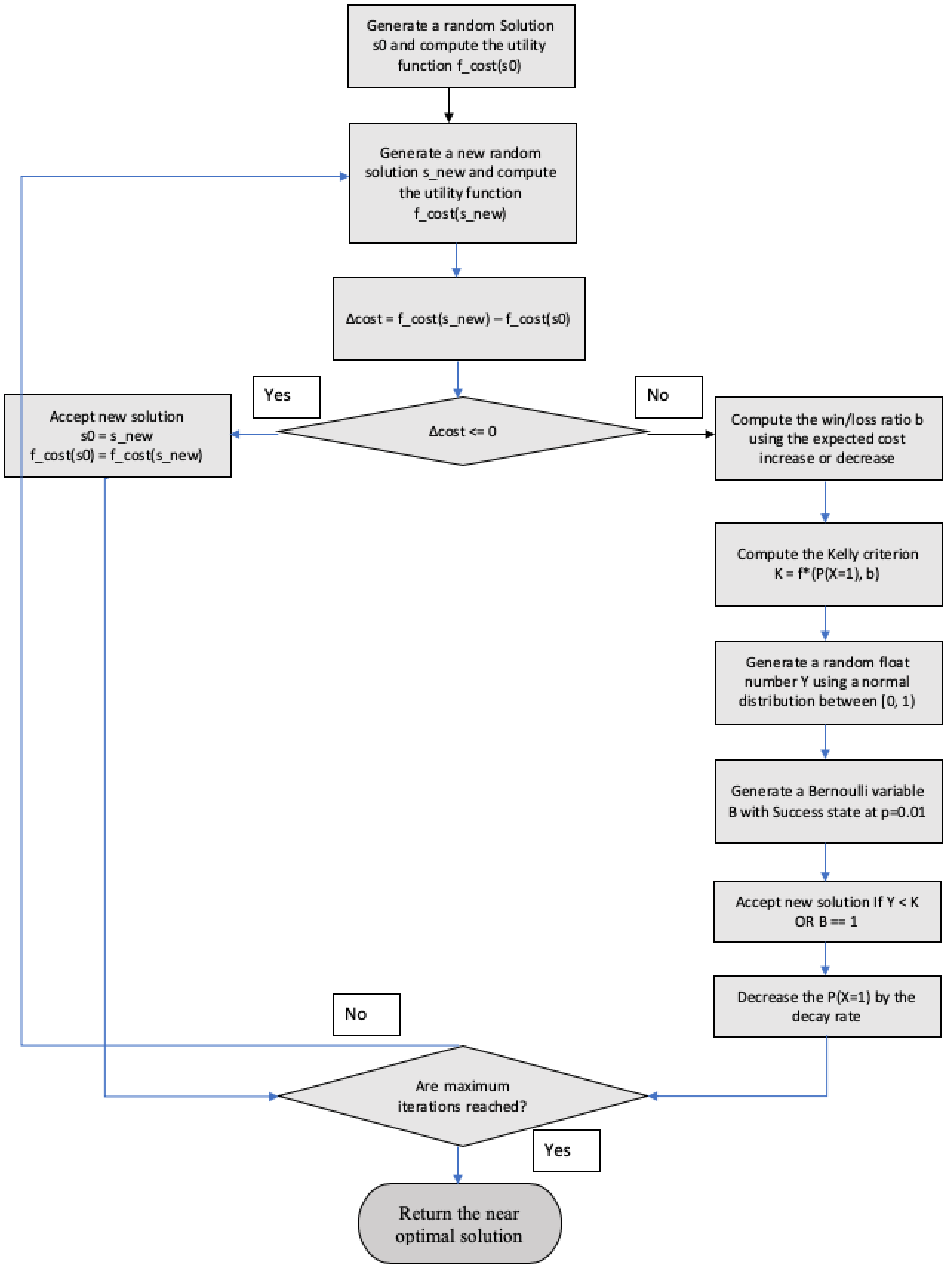

Figure 17 Flowchart for the Quantitative TSP Algorithm interpretation using Information Theory

Table 7 Proposed algorithm to solve the TSP

Step	Procedure	Block
0	List of nodes $L[] <= Parse TSP input file	Parameter
1	Set maximum number of iterations $max_int	Parameter
2	Set s0 as the initial tour created with $L[]	Parameter
3	Set Probability of Success P(1)=$p=1(100%)	Parameter

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	Set Probability of Failure P(0)=$q=1-p	
4	Set Decay rate value to $decay_rate=0.01(1%)	Parameter
5	Set expected cost quality increase E_Q(+Cost)=$e1	Parameter
6	Set expected cost quality decrease E_Q(-Cost)=$e0	Parameter
7	Randomly Flush solution $s0	Parameter
8	Set $first_cost = tourDistance($s0)	Parameter
9	Set $min_Tour = $s0	Parameter
10	Set $minCost = tourDistance($s0)	Parameter
11	The following steps will be repeted for $max_int times ($i=0; $i<=$max_int)	Main
11	Set $s_new = Create new random solution from $s0	Main
11	if (tourDistance($s_new) < tourDistance($s0)) then { if(tourDistance($s_new) < $minCost) then $minCost = tourDistance($s_new); $minTour = $s_new) }	Main - Decision Module - New shortest solution found
11	elif kelllyCriterion($b) == True then $s0 = $s_new	Main - Decision Module - New solution cost is greater than current know solution - Call Decision Module
11	Calculate the ratio $b = $e1/$e0	Decision Module - Kelly Criterion Calculate the net-odds
11	Calculate the Kelly fraction $f_k = ($p*($b+1) - 1)/$b	Decision Module - Kelly Criterion
11	Calculate $p = $p - $decay_rate	Decision Module - Kelly Criterion - Decrease Success Probability
11	Pick number $uniformRandom = random_float(0,1)	Decision Module - Kelly Criterion - Pick random float number with function
11	Pick number $bernoulli_trial = Bernoulli_Sequence(Simbols=[0,1], $p_1=0.01, $p_0=0.999)	Decision Module - Kelly Criterion - Pick random number with weighted distribution. The weigh is the possibility of each output result.
11	if ($uniformRandom < $f_k or $bernoulli_trial == 1) then Return True Else Return False	Decision Module - Kelly Criterion Accept new solution if True

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12	Output the near optimal solution found	Halt Execution

Table 8 Overview of the mathematical model with parameters and constraints.

Mathematical Model	**Category**
L : List of nodes	Parameter
max_int: Maximum number of iterations (positive)	Parameter
s0: Initial tour created with L (path sequence)	Parameter
P(1)=p: Probability of Success	Parameter
P(0)=1-q: Probability of Failure	Parameter
decay_rate: Decay probability rate (for each iteration)	Parameter
e1: Expected cost quality increase (total cost reduction)	Parameter
e0: Expected cost quality decrease (total cost increase)	Parameter
first_cost: Sarting solution cost	Parameter
min_Tour: Minimal tour solution	Parameter
minCost: Minimal cost	Parameter
b: Ratio of quality improvement, b=e1/e2	Parameter
Z: Standard normal variable with u and v > 0 be two real numbers with parameters u and v	Parameter
f* = (p*(b+1) - 1)/b with parameters p=P(1), decay_rate and b	Constraint 1 - Kelly Distribution
X = e^(u+(v*Z))	Constraint 2 - Log-Normal Distribution
P(X=1) = p = 1 - P(X=0) = 1-q	Constraint 3 - Bernoulli Distribution

Algorithmic Information method. The two major components are the simulated Kelly fraction f* (describing the overall uncertainty spread) and the Bernoulli process distribution of the underlining random event between states (estimated as the mean and standard variance for the weight function for each solution). The combination of those factors will be evaluated to decide the start of a neighborhood search (following a Probability density function) when the new

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alternative solution has a negative gain (i.e. new proposed solution is worse than current best-known encoded candidate).

Solutions to the TSP routing problem are explored by algorithms such as 2opt, Simulated Annealing (SA), Greedy and Genetic Algorithm (GA). In this paper we proposed a Quantitative Algorithm (QA) that does not rely on naturally inspired schemas but rather provides a statistical interpretation as a distribution of signals by a stochastic (log normal) process.
This stochastic process is defined as an ordered list of random variables $\{X_n\}$ for a given trial of length N. N is a set of non-negative integers and X_n is defined as a target measurement for a specific instance of time.

The utility function is used to find the near optimal route that have the minimum traveling distance to multiple target node destination while returning to the starting node at the end. There are 2 constraints to be considered in the model presented in this paper: Simulated Entropy Uncertainty and Bernoulli Process.

CONSTRAINT 1

Simulated Uncertainty. The first constraint is limited by the entropy. The input parameters for the Kelly function f^* are the Wining probability P_W and the expected net-odds b for the Bernoulli trial B. The value of P_W is decreased by a fixed rate of 1% (0.01) at each interaction. The value b is measured as the ratio of average improvements of the positive interactions $i+$ divided by the average reduction of the negative interactions $i-$. The result of this function is the percentage of the useful side information available in a noisy channel.

In Table 9 we have an example for the Kelly criterion calculation

Table 9 Example calculation for the Kelly criterion for P (1) = 90, 50 and 10 and net-odds b=1

P (Win/Cost Decrease)	**P (Lose/Cost Increase)**	**Expected (Averaged) Cost Gain (Quality Improvement)**	**Expected (Averaged) Cost Loss (Quality Worsening)**	**Win/Loss Ratio b**	**(Output) Kelly Percentage *f****
90	10	+100	-100	+100/|-100|=1	0.8
50	50	+100	-100	+100/|-100|=1	0
10	90	+100	-100	+100/|-100|=1	-0.8

In Figure 18 we have a circuit representation of the first constraint:

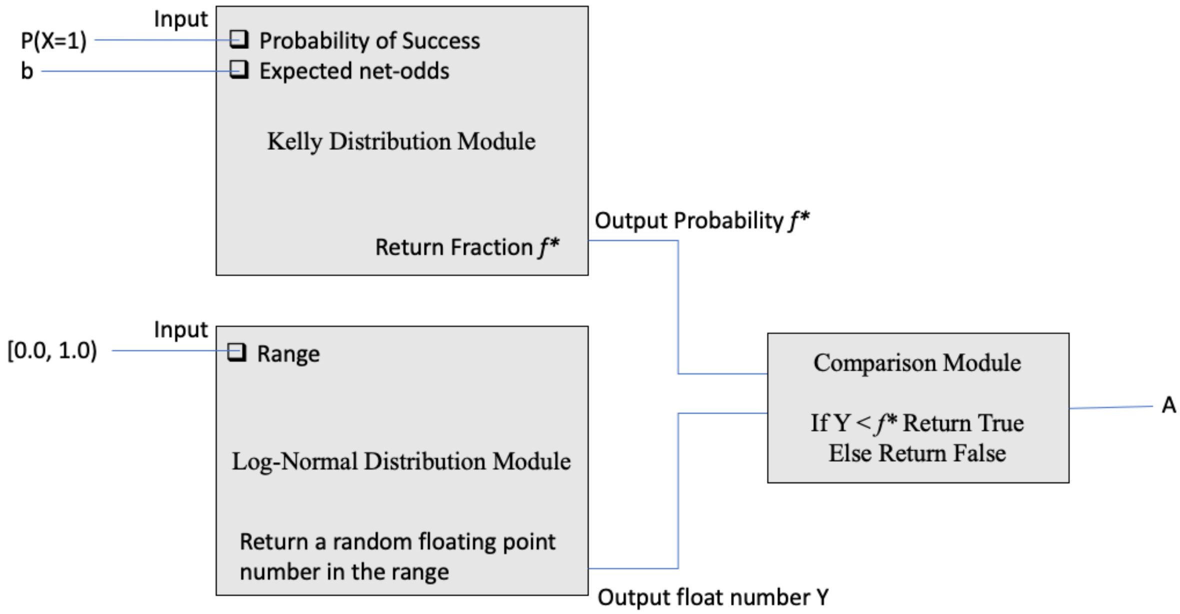

Figure 18 Decision Criterion 1 is implemented as the IF statement of the – simulated - Kelly Criterion and the Bernoulli Log-Normal Distribution with output A.

CONSTRAINT 2

Bernoulli Process. The second constraint is defined by a Bernoulli process as a finite sequence of independent and random Bernoulli variables (i.i.d). This module will return as output the value "True" at 1% of the time for 100 interactions (N=100) and it will accept unlikely (risky) solutions with negative gain to eventually provide improvements bets for the solution quality under some degree of freedom. The process is defined as a trial with two binary states either "True/Success" (1) or "False/Failure" (0) with domain $0 <= p <= 1$

$$P(X_i = 1) = p$$
$$P(X_i = 0) = 1 - p$$
$$E(X) = p$$
$$\text{Variance } Var(X) = p - p^2$$

In Figure 19 we can see a Bernoulli distribution with P (0) = 80% probability of output a Failure state

$$P(X=0) = 0.8$$
$$P(X=1) = 0.2$$
$$\text{Otherwise } P(X) = 0$$

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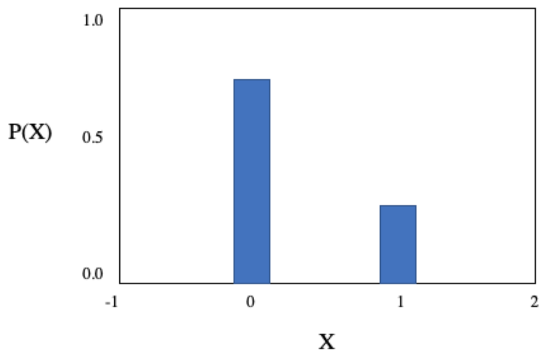

Figure 19 P(X) for the Bernoulli distribution with P(1)=0.2 and P(0) = 1 - P(1) = 0.8

Examples of Bernoulli trials are show in Table 10

Table 10 Examples of random events and the respective binary outcome

Event	**Outcome**
Play a game	Win/Lose
Coin toss	Head/Tail
Processing a request	On time/Late
Defect in equipment’s	Good/Defect
Buy-Sell an asset	Profit/Deficit
Optimizing traveling cost	Reduction/Increase

In Figure 20 we have the circuit representation of the second constraint:

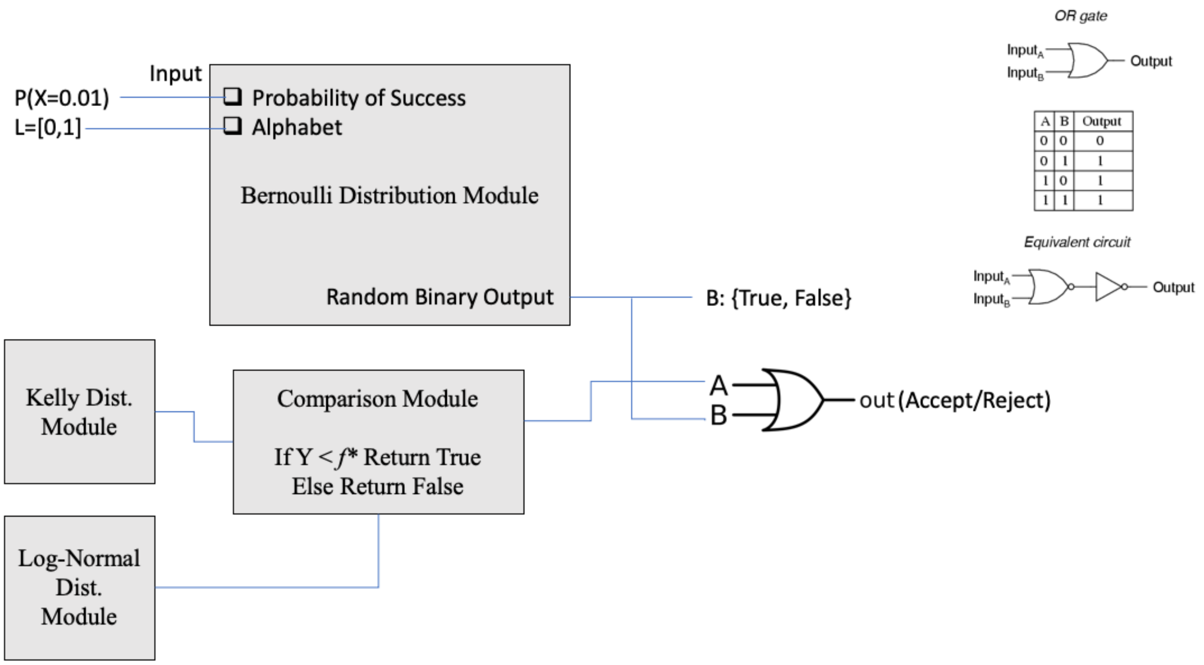

Figure 20 Decision Criterion 2 is implemented as a OR gate using for the input (a) Bernoulli Distribution Trial (B) and (b) The IF statement of the – simulated - Kelly Criterion and the Bernoulli Log-Normal Distribution (A).

DISCUSSION - COMPUTATIONAL RESULTS

The research evaluates the performance of the proposed algorithm through a series of test cases and statistical analysis. The new method is tested against the traditional method Simulated Annealing. Each test case was run for a trial with population size of N=60. In the statistical analysis section, the t-test was used to compare the means between the sample groups. The null hypothesis is that there is no difference between the means of the two populations.

TEST CASES

The test cases are a set with 20, 30 and 50 nodes. The network latency between each node is bounded by the geographic distances between the machines made available by the computing service provider. The proposed algorithm was used to find the near optimal tour required before returning to the first service endpoint. In Table 11, Table 12, Table 13 we present one sample from the 180 trials to show the comparisons between QA and SA. Each table shows the first initial tour, the optimal solution found, and the time resource required (in seconds) for each algorithm for a sample single execution.

Table 11 Test case with 20 nodes

Node#20 (x, y): 1 288 149 2 288 129 3 270 133

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4 256 141
5 256 157
6 246 157
7 236 169
8 228 169
9 228 161
10 220 169
11 212 169
12 204 169
13 196 169
14 188 169
15 196 161
16 188 145
17 172 145
18 164 145
19 156 145
20 148 145

Quantitative Algorithm

First solution path: ['1', '20', '8', '7', '17', '13', '2', '3', '16', '9', '15', '11', '14', '10', '6', '19', '4', '5', '18', '12']

Total distance cost from the first(starting) solution: 1145.7617186058662

Best solution Path: ['6', '5', '1', '2', '3', '4', '9', '15', '16', '17', '18', '19', '20', '14', '13', '12', '11', '10', '8', '7']

Total distance cost for the best solution found: 332.1144088148687

Table 12 Test case with 30 nodes

Node#30 (x, y):

1 288 149
2 288 129
3 270 133
4 256 141
5 256 157
6 246 157
7 236 169
8 228 169
9 228 161
10 220 169
11 212 169

12 204 169
13 196 169
14 188 169
15 196 161
16 188 145
17 172 145
18 164 145
19 156 145
20 148 145
21 140 145
22 148 169
23 164 169
24 172 169
25 156 169
26 140 169
27 132 169
28 124 169
29 116 161
30 104 153

Quantitative Algorithm

First solution path: ['20', '24', '23', '9', '2', '6', '21', '15', '27', '1', '11', '14', '29', '25', '5', '28', '13', '7', '18', '22', '26', '4', '19', '12', '3', '17', '10', '30', '8', '16']

Total distance cost from the first(starting) solution: 2114.8616643887417

Best solution Path: ['4', '9', '12', '13', '14', '24', '23', '25', '22', '26', '27', '28', '29', '30', '21', '20', '19', '18', '17', '16', '15', '11', '10', '8', '7', '6', '5', '1', '2', '3']

Total distance cost for the best solution found: 423.26765018746386

Table 13 Test case with 50 nodes

Node#50 (x, y):

1 288 149
2 288 129
3 270 133
4 256 141
5 256 157
6 246 157
7 236 169

```
 8 228 169
 9 228 161
10 220 169
11 212 169
12 204 169
13 196 169
14 188 169
15 196 161
16 188 145
17 172 145
18 164 145
19 156 145
20 148 145
21 140 145
22 148 169
23 164 169
24 172 169
25 156 169
26 140 169
27 132 169
28 124 169
29 116 161
30 104 153
31 104 161
32 104 169
33  90 165
34  80 157
35  64 157
36  64 165
37  56 169
38  56 161
39  56 153
40  56 145
41  56 137
42  56 129
43  56 121
44  40 121
45  40 129
46  40 137
47  40 145
48  40 153
49  40 161
50  40 169

# Quantitative Algorithm
```

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```
First solution path: ['38', '48', '39', '40', '9', '26', '28', '29', '33', '43', '41', '14', '12', '23', '1', '17', '20', '5', '49', '7', '35', '22', '18', '31', '6', '19', '13', '36', '2', '16', '42', '15', '4', '32', '25', '46', '8', '47', '50', '27', '37', '24', '34', '3', '10', '30', '45', '21', '44', '11']

Total distance cost from the first(starting) solution: 4919.680538441

Best solution Path: ['49', '48', '47', '41', '42', '43', '44', '45', '46', '40', '39', '38', '37', '36', '35', '34', '33', '32', '31', '30', '29', '21', '20', '19', '18', '17', '16', '9', '5', '1', '2', '3', '4', '6', '7', '8', '10', '11', '12', '15', '13', '14', '24', '23', '25', '22', '26', '27', '28', '50']

Total distance cost for the best solution found: 700.0706523445637
```

In Figure 21 we have the line charts with the results for each test case. Figure 21 demonstrate that the QA algorithms produces expected results with smaller – best quality - cost than SA method with smaller time requirements. In all cases the initial path loaded from the input files was randomly flushed before the algorithm execution.

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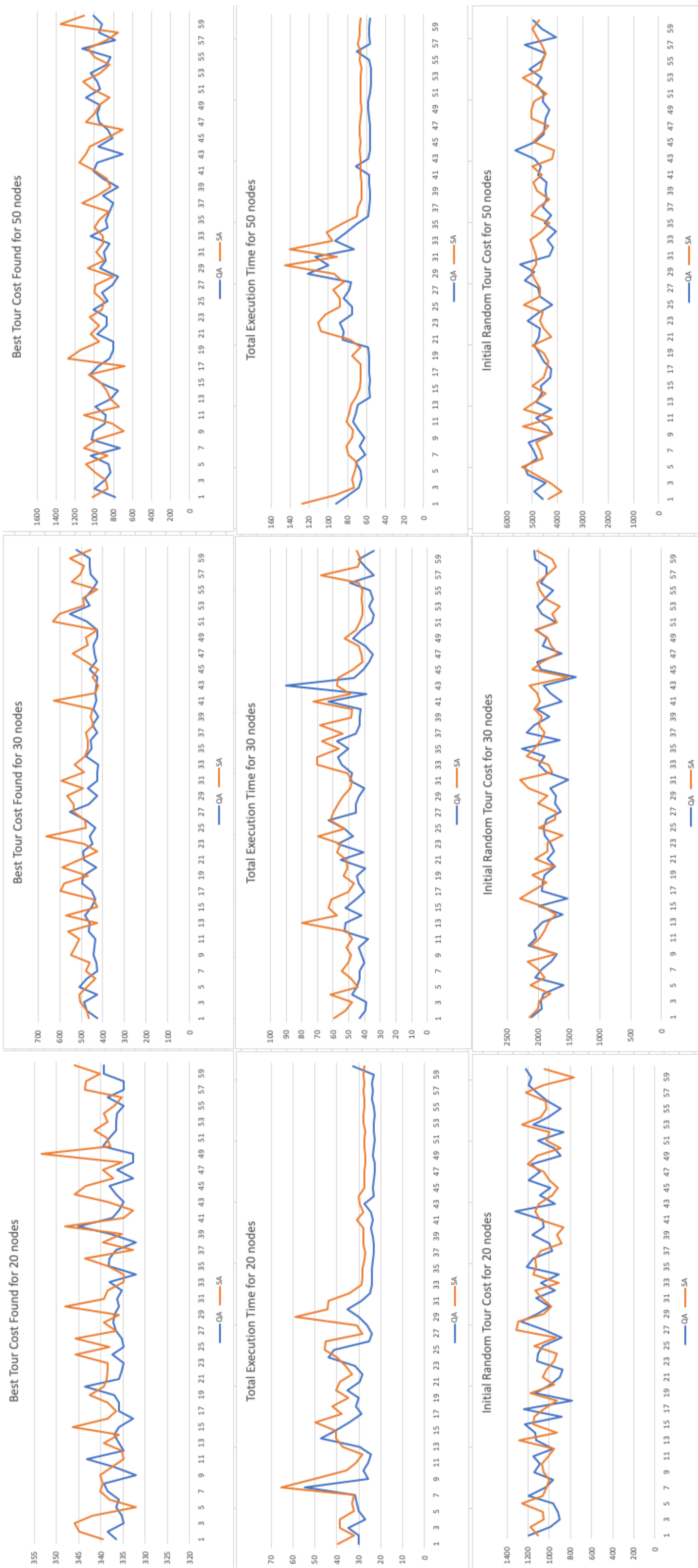

Figure 21 Chart with results for test cases with 20, 30 and 50 nodes with sample size N=60

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Table 14, Table 15, Table 16 compares the results between the proposed algorithm and the heurists methods. They demonstrate that the result generated by the QA algorithm achieves solutions with better quality than the SA heuristic algorithm by finding optimal solutions with smaller total cost. The QA algorithm converges more quickly than the 2opt algorithm for large instances of the problem. The results suggest the proposed Quantitative Algorithm is likely to perform better and generate consistent results as other traditional heuristics methods applied to the TSP. Additionally it provides an Information Theory modelling for Computationally Complex domains such as the NP class of problems in Turing Machines.

In the Appendix sections we have the Table 18, Table 19, Table 20 for the 3 test cases with nodes n={20, 30, 50}, with the trial input variables for best final tour cost, total execution time and the initial tour cost for a sample size of N=60. In Table 14, Table 15, Table 16 we have the t-test p-value for each test case for the cost and execution time variable. Table 14 demonstrate the T-test calculation for 2 independent means for n=20.

Table 14 Difference Scores Calculations for n=10 Nodes and N=60 trials.

t-Test: Two-Sample Assuming Unequal Variances
Cost Variable

	QA	*SA*
Mean	336.552862	339.645729
Variance	6.35307736	18.1161823
Observations	60	60
Hypothesized Mean Difference	0	
df	96	
t Stat	-4.8431338	
P(T<=t) one-tail	2.4468E-06	
t Critical one-tail	1.66088144	
P(T<=t) two-tail	4.8935E-06	
t Critical two-tail	1.98498431	

The result is significant at $p < .05$.

The 60 iterations who executed the QA algorithm (M = 336.552, *Var* = 6.353) compared to the 60 iterations in the SA algorithm (M = 339.645, *Var* = 18.116) demonstrated significantly better cost reduction, with value p = 4.8935E-06.

t-Test: Two-Sample Assuming Unequal Variances
Execution Time Variable

	QA	*SA*
Mean	28.1821012	34.06591929
Variance	43.5437993	69.06678708
Observations	60	60
Hypothesized Mean Difference	0	
df	112	
t Stat	-4.2948228	
P(T<=t) one-tail	1.8689E-05	
t Critical one-tail	1.65857263	
P(T<=t) two-tail	3.7379E-05	
t Critical two-tail	1.98137181	

The result is significant at $p < .05$.

The 60 iterations who executed the QA algorithm (M = 28.182, *Var* = 43.543) compared to the 60 iterations in the SA algorithm (M = 34.065, *Var* = 69.066) demonstrated significantly better time to complete execution, with value p = 3.7379E-05.

Table 15 demonstrate the T-test calculation for 2 independent means for n=30. Table 16 demonstrate the T-test calculation for 2 independent means for n=50.

Table 15 Difference Scores Calculations for n=30 Nodes and N=60 trials

t-Test: Two-Sample Assuming Unequal Variances

Cost Variable

	QA	*SA*
Mean	455.9177842	500.1505
Variance	917.1905283	3396.48698
Observations	60	60
Hypothesized Mean Difference	0	
df	89	
t Stat	- 5.216694386	
P(T<=t) one-tail	5.88396E-07	
t Critical one-tail	1.662155326	
P(T<=t) two-tail	1.17679E-06	
t Critical two-tail	1.9869787	

The result is significant at $p < .05$.

The 60 iterations who executed the QA algorithm (M = 455.917, *Var* = 917.190) compared to the 60 iterations in the SA algorithm (M = 500.150, *Var* = 3396.486) demonstrated significantly better cost reduction, with value p = 1.17679E-06.

t-Test: Two-Sample Assuming Unequal Variances

Execution Time Variable

	QA	*SA*
Mean	45.3019303	53.1616159
Variance	83.6019137	82.7521257
Observations	60	60
Hypothesized Mean Difference	0	
df	118	
t Stat	-4.7202404	
P(T<=t) one-tail	3.2683E-06	
t Critical one-tail	1.65786952	
P(T<=t) two-tail	6.5365E-06	
t Critical two-tail	1.98027225	

The result is significant at $p < .05$.

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The 60 iterations who executed the QA algorithm (M = 45.301, Var = 83.601) compared to the 60 iterations in the SA algorithm (M = 53.161, Var = 82.752) demonstrated significantly better time to complete execution, with value p = 6.5365E-06.

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Table 16 Difference Scores Calculations for n=50 Nodes and N=60 trials.

t-Test: Two-Sample Assuming Unequal Variances

Cost Variable

	QA	*SA*
Mean	901.7845936	960.3116942
Variance	8917.028193	18189.17941
Observations	60	60
Hypothesized Mean Difference	0	
df	106	
t Stat	-2.753583529	
P(T<=t) one-tail	0.003468798	
t Critical one-tail	1.659356034	
P(T<=t) two-tail	0.006937597	
t Critical two-tail	1.982597262	

The result is significant at $p < .05$.

The 60 iterations who executed the QA algorithm ($M = 901.784$, $Var = 8917.028$) compared to the 60 iterations in the SA algorithm ($M = 960.311$, $Var = 18189.179$) demonstrated significantly better cost reduction, with value $p = 0.006937597$.

t-Test: Two-Sample Assuming Unequal Variances

Execution Time Variable

	QA	*SA*
Mean	67.6962824	78.4087974
Variance	215.812669	334.607097
Observations	60	60
Hypothesized Mean Difference	0	
df	113	
t Stat	-3.5368778	
P(T<=t) one-tail	0.00029419	
t Critical one-tail	1.65845022	
P(T<=t) two-tail	0.00058839	
t Critical two-tail	1.98118036	

The result is significant at $p < .05$.

The 60 iterations who executed the QA algorithm (M = 67.696, Var = 215.812) compared to the 60 iterations in the SA algorithm (M = 78.408, Var = 334.607) demonstrated significantly better time to complete execution, with value p = 0.00058839.

These findings can be expanded to other complex problems and it scales linearly to the search space sample. The proposed method is also resistant to the time and space constraints and it has a constant number of maximum iterations. In this paper we have introduced a new interpretation for the entropy rate for a binary program that implements a given NP problem.

The results can be used by many real-world applications such as the optimization of routing messages over a network and the orchestration of services across a distributed system such as provided by Cloud Computing environments and micro-services-oriented architectures. Besides it is also a future reference on the subject of Information Theory, Computational Complex Theory and Logarithmic utility in optimization routing, deployment, scheduling and planning. The research demonstrated that the proposed concepts have statistically significant results with better solution quality in tour planning. The model provides a new interpretation of entropy in problems encoded in Turing Machines and has the potential to change the traditional interpretation of the limits of Computing Theory.

The results are statistically significant (with p-value < 0.05), and we can conclude the proposed algorithm has better solution quality with reduced computational requirements and better cost improvement.

STATISTICAL CASE STUDY

In order to test the performance in solving the TSP we have created trials with sample size N=60 for each test case with different number of nodes n={10, 30, 50} and then compared the results obtained from a traditional heuristic (SA) and the proposed algorithm (QA).
Table 17 demonstrate the two-tailed t-test for two independent samples of costs with Significance Level of 0.05. This is a two-sided test for the null hypothesis with two independent means have the identical expected value. This test measures if the average expected cost value differs significantly across the measured samples. If the p-value is small than the significance level of 0.05 (5%) then we can reject the null hypothesis of equal average means.

Table 17 Comparison matrix for the two-tailed t-test independent means p-values for the test cases with nodes with n=20, 30, 50 and sample size N=60

	20-node	**30-node**	**50-node**
Cost variable t-test for N=60	**QA-SA.**	**QA-SA.**	**QA-SA.**
p-value	4.89351E-06	1.17679E-06	0.0069376

	20-node	**30-node**	**50-node**

Time variable t-test for N=60	QA-SA.	QA-SA.	QA-SA.
p-value	3.73787E-05	6.53652E-06	0.00058839

The Table 17 shows that results are statistically significant (with p-value < 0.05) for test cases n={20, 30, 50}, and we can conclude the proposed Quantitative Algorithm (QA) has better solution quality with reduced computational requirements and better cost improvement than heuristic Simulated Annealing (SA).

CONCLUSION

Service scheduling and network routing has many applications and is related to the optimization problem modeled by the Traveling Salesman Problem. It's possible to improve performance by reducing the cost of transmission of information across many distributed systems locations. A new interpretation and verified optimization algorithm and statistical model based in Information Theory is presented in this paper and it demonstrated how it can be used to solve the TSP. The results support the idea that the proposed method can be used reliably to generate solution under a given degree of freedom. The algorithm can be expanded to large scale problems without the high requirements of computational resources utilization imposed by the brute-force and traditional heuristics methods such as Simulated Annealing and Genetic Algorithms. The algorithm can be adapted to any case of the routing problem. The research can be used as a framework for future works and extend the implications of Information Theory and Kolmogorov-Complexity in solving TSP and NP Problems in general.

The advantage of this approach is that it is independent of the computer encoding the problem and the time and space complexities are additive up to a limit that is linearly proportional to the input size. Other heuristics methods assume a predefined knowledge about the data structure and thus are biased towards this encoded schema. The implications of this interpretation is that by reducing the NP problems to a matter of modularization of encoded and decoded random signals in an communication noisy channel, we can find near optimal solutions that are statistically significant and are guaranteed to produce the best rate of improvement in the long run over many trials(i.e. simulation iterations) even though the problems itself is computationally complex and the alternative sequential brute force algorithm would require exponential time to solve. This mathematical model can be interpreted as a generalization of heuristics methods.

The findings in this paper unifies critical areas in Computing Science, Mathematics and Statistics that many researchers have not explored and provided a new interpretation that advances the understanding of the role of entropy in decision problems encoded in Turing Machines.

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 # Appendix - Supporting Data

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Table 18 Execution Results for 20 nodes (n=20) with samples size N=60

20-node						
Trial N=60	Initial Random Tour Cost		Best Tour Cost Found		Total Execution Time	
0	QA	SA	QA	SA	QA	SA
1	1195.634925	1108.393775	336.620892	339.439482	30.12948538	39.6077362
2	986.9352715	1177.732299	338.5615264	344.758321	30.26399596	32.3809037
3	900.0458995	1054.1641	334.8573413	345.966652	34.91276873	38.7632731
4	916.5463351	1057.927051	335.3011739	341.855037	27.23480984	38.4763941
5	966.1553232	1253.494224	336.620892	332.114409	29.91621914	32.1631082
6	1194.232179	1064.387485	335.8914965	338.044106	31.12498264	33.1915936
7	1036.191977	1029.253756	338.5615264	340.135251	31.81935215	32.060559
8	963.2410995	999.9887981	339.3638244	339.439482	54.49795967	65.4112282
9	1145.761719	1050.78347	332.1144088	340.059594	25.79352437	50.170809
10	1085.090655	1073.381069	337.2219503	337.316661	27.91745513	35.5128518
11	1147.417441	1012.068389	343.0294931	334.857341	25.94175636	31.4301689
12	965.8459042	958.2459652	334.8573413	335.301174	24.29773869	28.3900081
13	1128.489942	1283.781151	335.9022323	339.231552	29.54066219	37.4386245
14	1131.818667	928.6586146	337.0140207	335.902232	47.45874454	40.282868
15	1229.658241	1153.404679	335.9022323	346.41498	40.86180719	40.4422965
16	884.1357153	1138.13307	332.7154672	338.561526	34.48849958	49.9608527
17	1235.918025	1052.377384	335.9022323	336.620892	28.82108637	37.7635394
18	783.6900544	929.1934876	335.8914965	338.561526	31.23878077	42.0187519
19	1130.320419	1174.181378	337.0140207	342.439171	30.00009195	34.7658014
20	989.3865301	951.2563694	343.5134562	339.363824	35.36053606	40.5407622
21	904.2377617	1063.30281	335.9022323	338.741826	29.65111817	38.7390394
22	869.7243065	1029.595623	335.3011739	338.561526	28.38350387	32.86863
23	1116.176649	953.0821731	334.8573413	338.561526	31.87757207	36.2553472
24	1110.200886	931.6046509	337.3166611	345.685585	43.94790548	40.1406831
25	1064.184077	1139.258709	334.8573413	338.119763	41.20329418	45.7096829
26	887.1252702	940.9695223	335.3011739	345.685585	25.25219875	45.1240136
27	1040.447342	1308.425282	336.4925548	336.620892	23.90627663	28.4887706
28	1263.036989	1297.030551	337.2219503	339.231552	26.12993897	31.0484482
29	1115.022094	1055.582523	337.0140207	335.902232	29.95108272	58.9338429
30	998.9318306	981.3689839	335.9022323	348.044562	35.33063453	44.3207047
31	1127.712215	1066.313009	336.4129623	339.342884	28.56060964	43.9690071
32	945.3399049	1133.277072	335.3011739	338.561526	25.08187038	34.241621
33	1079.60359	913.4123438	338.0441064	334.857341	23.84379763	28.6076721
34	913.3758632	1151.462443	332.1144088	334.932998	24.00100185	28.4874175
35	1210.249512	1120.714135	338.3112039	338.561526	24.2204857	28.3244191

36	1147.723302	1130.778705	338.0441064	343.649497	23.43391117	27.3482798
37	971.3903556	1076.686391	336.620892	332.715467	23.04826988	27.2579731
38	998.2103312	887.1398159	332.1144088	339.439482	23.30005102	27.9801567
39	1162.280089	924.1853956	337.9177195	335.301174	23.55994665	28.0173068
40	1054.008142	869.5783019	345.095262	348.044562	24.4069206	28.0580326
41	1062.958346	1092.610068	337.3166611	334.857341	23.58406409	30.9849236
42	1316.550471	1132.915517	335.9022323	332.715467	24.49907442	27.7148466
43	944.5629418	1096.900034	334.8573413	338.561526	27.42394148	29.689707
44	1090.841136	973.9952212	336.620892	345.879822	23.22344759	30.1561858
45	985.9883167	915.8312807	338.0441064	343.428117	23.68698814	27.5134631
46	1193.337578	1001.967642	332.7154672	337.22195	23.15058773	27.6503922
47	1085.046538	1056.207841	336.4129623	339.439482	22.89809065	27.6979532
48	1170.592904	1204.896611	332.7154672	335.301174	22.811833	27.1470865
49	888.3716219	1119.310235	332.7154672	353.222205	23.53460078	27.1795784
50	957.744169	896.1779984	339.4394815	337.917719	22.97475352	27.4758247
51	1105.010107	1023.615603	338.0441064	338.710086	22.86918458	27.2797519
52	868.7296417	1001.961386	336.4925548	341.484758	23.09712953	27.1193013
53	1147.110698	1261.361139	336.620892	338.561526	23.20182619	27.8441451
54	1007.496906	1076.417337	336.4129623	339.231552	22.91703718	27.7154546
55	888.3254017	1028.532428	334.8573413	337.014021	23.08894057	27.652091
56	1015.389387	1037.982865	338.5615264	335.301174	24.19610564	27.4061656
57	1118.083211	1218.690552	334.8573413	343.500897	23.50864635	27.9921575
58	1196.871246	1056.936272	334.8573413	343.428117	23.84533973	27.6875045
59	1172.5209	765.331396	339.3638244	340.059594	23.19705539	27.9977026
60	1225.40291	1041.148332	339.3638244	345.962516	32.50677825	27.3577434

Table 19 Execution Results for 30 nodes (n=30) with samples size N=60

30-node						
Trial N=60	Initial Random Tour Cost		Best Tour Cost Found		Total Execution Time	
0	QA	SA	QA	SA	QA	SA
1	2114.861664	2138.958342	424.5873682	464.700899	42.98706438	59.0712779
2	1941.575116	2007.049171	475.6796776	472.590934	39.46565883	52.120306
3	1950.142483	1989.063029	488.989852	504.453757	38.60825704	47.8920127
4	1902.474244	1796.927917	424.3794386	506.801158	48.13539106	61.4913984
5	1580.54924	2121.678062	510.318469	472.980497	44.92054756	44.2866884
6	2040.501524	1887.043741	474.6886065	433.933129	43.33821167	48.5092938
7	1933.189429	2005.284134	426.0105826	478.033092	43.31067901	54.8387858
8	1791.060214	2174.057473	429.9694289	462.536112	40.27118746	50.435126
9	1697.688572	1712.42956	443.4357473	548.033561	41.870426	48.4660826

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10	2159.279821	2101.511052	439.9631361	526.162767	43.91304872	51.6482356
11	2037.032939	1964.471053	436.517103	507.74012	37.68801916	48.1148758
12	2057.901751	1892.015046	467.0236214	562.704654	52.46498072	51.1041011
13	1930.750408	1831.163179	460.014352	426.528003	52.34054281	79.7551805
14	1596.34284	1709.164989	484.0191023	571.764133	41.91429676	57.815061
15	1989.991866	1932.746628	453.6891305	428.101727	52.06071529	63.1093559
16	1532.819374	2294.07567	432.8197583	443.083125	45.44243764	61.380042
17	1933.611183	2091.996596	453.529067	595.347046	40.35533853	50.0510586
18	1928.551184	1860.347264	495.9653506	580.368162	43.90844072	46.7043453
19	1892.952203	2098.647235	491.0938606	468.450763	45.20533055	53.5513851
20	1727.389612	1746.213002	428.8730412	587.027872	39.56974309	50.744991
21	1840.460231	2052.640373	486.8539675	510.278944	55.44163731	51.324384
22	1734.505095	1843.716782	491.0156727	427.122371	40.87608984	57.7772876
23	1804.725607	1840.418372	447.2951117	486.01134	57.19451948	55.1076989
24	1900.969989	1597.14205	460.1507211	662.168972	47.51163171	69.8538226
25	1902.159287	1982.433073	433.0617383	477.980165	52.9507134	52.5828033
26	1861.379571	1732.215165	478.1275476	481.88559	63.18074776	61.506958
27	1632.125705	1705.527663	551.6573018	534.398358	45.71926259	59.901718
28	1724.804881	2004.34791	464.0476771	533.121258	45.68037277	57.0611448
29	1711.276107	1851.488894	426.6765623	565.016049	43.55173205	53.7347876
30	1798.24361	2147.552334	467.8403795	491.540871	39.89643796	48.881184
31	1505.540858	2286.236266	427.122371	592.196033	48.98996233	48.1386252
32	1868.889925	1772.226931	427.4357473	488.765595	47.68942928	50.6868412
33	1986.199157	1831.914237	423.8687085	530.861945	54.60683943	70.0300942
34	1893.774487	2191.409802	476.6245181	482.210099	56.77093291	69.9386231
35	2254.034678	2057.794281	450.6222049	470.823818	50.39574996	56.2931122
36	1641.972013	1951.754928	456.9229002	470.823818	57.52063273	67.2678736
37	2190.641527	1892.120281	427.1980282	479.691631	45.58968535	54.0853191
38	2059.180754	1991.897847	446.6225652	443.410875	43.33406437	68.1481031
39	1816.734006	1938.160932	423.2676502	457.649743	43.01746808	48.3186617
40	2009.362159	2057.941367	442.9039608	449.980838	42.63856604	48.0714487
41	1619.209653	1977.29523	430.5704872	625.975132	63.20465673	72.4023792
42	1803.072488	2008.969722	436.3291255	434.210099	38.81750628	49.0113726
43	1905.327122	2139.748939	431.6159736	422.823818	90.17018865	57.4507498
44	1396.063861	1510.800171	427.122371	446.937846	46.57496965	56.7508254
45	1961.970303	2085.66334	461.2238488	423.868709	40.88945298	47.0390539
46	2016.906558	1945.01613	431.6159736	482.392057	37.00943786	41.2652409
47	1615.747074	1679.767432	438.5243978	540.930956	34.94234454	41.9965412
48	1919.853209	1791.446699	442.7060685	468.628522	39.62350488	43.5543638

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49	1864.485907	1841.077381	427.122371	478.851995	47.30976915	52.8924325
50	2033.030066	2044.268536	427.1980282	427.122371	41.87760103	45.4800351
51	1717.798426	1690.966193	474.7739314	629.184876	35.09427462	42.9087137
52	1943.528232	1778.978193	553.2831374	599.182846	34.25907436	41.3755334
53	2011.609188	1652.077927	460.8365081	489.159339	36.82620546	41.3364099
54	1893.324832	1873.394732	483.4554998	491.107552	34.69045473	41.6829564
55	1761.65708	1971.263332	448.7644393	426.010583	36.35978929	41.3863896
56	1948.645928	2020.716704	428.0260698	542.400253	48.97381794	43.9572916
57	1867.947796	1850.588061	455.8265125	505.167828	34.33525243	67.9172457
58	1858.698459	1717.237261	461.1244452	485.823316	39.20625309	44.2034129
59	2046.855774	1778.212757	461.5046079	554.4155	43.44987886	42.5260492
60	2054.476294	2022.162447	522.5612263	457.556595	34.17459443	44.7598619

Table 20 Execution Results for 50 nodes (n=50) with samples size N=60

50-node						
Trial N=60	Initial Random Tour Cost		Best Tour Cost Found		Total Execution Time	
0	QA	SA	QA	SA	QA	SA
1	4584.994704	4350.530615	783.6135781	1013.45381	91.94147284	127.121441
2	4931.04388	3829.72755	1002.266343	856.714194	79.49352819	93.1328427
3	4459.545641	4285.493079	892.4679561	878.335327	67.90769484	73.4395111
4	5189.508014	4873.122362	825.1186081	986.814761	65.57152077	74.8066559
5	5303.265261	5389.224823	846.448402	1085.77975	65.92321619	72.4844698
6	4792.633045	4595.016823	1036.163983	855.195737	71.40535116	70.3046693
7	4951.587728	4711.457297	735.82339	1110.36984	61.43196151	78.8009844
8	5127.650419	4851.865628	1028.714485	974.535522	67.16026587	80.7510912
9	4213.123791	4227.131367	1006.163651	693.015657	61.80173641	75.0614487
10	4388.805181	5361.415343	887.0345646	813.130685	68.36587932	73.5915055
11	4862.966377	4203.513913	877.8178844	1104.47353	73.62834893	81.1984111
12	4265.85426	5334.298362	986.2523487	742.17075	70.93162192	78.2357104
13	4830.792434	4901.318711	826.6711289	827.602189	68.93372081	76.0991019
14	4619.684576	4464.911989	749.4744233	871.360054	56.4622385	71.1095197
15	4672.945496	5012.768994	930.1932444	930.647017	57.45928662	66.9842984
16	4301.693121	4563.371791	1048.809952	1059.51057	56.40269137	66.0322041
17	4240.292119	4453.55102	960.9380585	679.490015	57.05427107	66.3409535
18	4546.900924	4368.046961	837.1380532	1269.71181	57.57915765	66.263964
19	4736.784109	4563.713271	803.5690309	1149.69257	57.39472165	75.1020043
20	4927.370618	4985.626756	802.0707333	949.201069	58.56337916	66.5018131
21	4745.962526	4268.210716	971.2161657	1035.84231	84.97886224	77.278123

22	4704.060283	4530.930051	871.0047479	946.415563	83.75003637	107.235943
23	5181.527463	4709.96614	869.6858203	1049.75812	88.00241065	110.964508
24	4718.091802	4570.219583	1003.026828	917.555919	74.77390515	102.335962
25	4226.097192	5336.282205	863.4354307	897.940395	74.88039926	88.0454088
26	4706.978399	4680.602383	919.2626516	999.01329	84.28058593	87.6437221
27	4782.240302	4696.840091	805.5766865	988.048877	77.9414429	94.365899
28	5287.422498	4883.631664	746.8961081	797.72096	76.3188141	84.3477808
29	4916.756496	5036.485112	934.4805999	1063.60839	121.2328974	94.1907275
30	5467.182912	4862.394357	881.0823054	898.962836	99.58652921	145.672337
31	4306.529303	4833.371655	902.1714403	982.659335	113.7672365	90.4546303
32	4183.027842	4974.432819	838.7087408	909.242445	72.86395342	140.711226
33	4392.939578	5062.464887	1037.318628	912.246168	93.25543078	96.0741311
34	4047.530913	4624.516897	855.2813664	997.411612	80.5475141	101.864948
35	4529.583132	4333.230947	877.4782972	948.148568	70.2493321	85.4985131
36	4255.799236	5016.586688	844.0019669	851.668489	58.39587275	69.6642768
37	4591.115727	4742.024106	801.0967517	1122.03704	57.4895984	68.7700118
38	4446.710076	4328.66242	908.2987592	972.825796	56.3946521	65.6056279
39	4463.651251	4803.549729	750.8183686	831.071231	55.89729563	65.1255863
40	4420.655007	4951.243552	909.1278545	866.473154	57.14852512	65.213147
41	4760.124614	4634.478576	1011.834532	987.252055	57.32113831	66.0356184
42	4644.224719	4979.032088	964.2360348	1157.92977	71.29050156	66.2798815
43	4919.680538	4208.101093	700.0706523	1085.31132	57.79070051	66.251104
44	5651.239742	4145.447347	955.4124378	1052.01851	56.53862412	66.9480465
45	4868.697543	4959.720826	813.4887026	872.689612	56.64083376	66.3630462
46	4548.266669	4667.805502	857.9641925	698.448853	56.46022322	67.1403534
47	4526.748229	4364.07524	944.2137708	1083.92177	56.08612165	67.1104689
48	4477.883901	5024.492103	972.2009554	999.856492	57.63071399	66.3520515
49	4317.125097	5044.001873	933.6392304	948.603177	58.23602575	65.5457691
50	4575.872243	4903.155907	1087.920943	839.863471	57.96136195	66.182039
51	4563.047938	4424.909203	934.2368709	1019.90042	56.72733254	65.8799768
52	4820.853628	4875.868205	970.4719961	1112.21415	55.71390442	66.183776
53	4615.052748	5386.727246	1034.389805	939.401344	55.77318577	65.8460782
54	5105.564284	4715.986396	873.7197202	836.813936	55.55562469	65.2542301
55	4633.267827	4585.63924	825.4093226	996.826956	57.06582138	66.729305
56	4487.584383	4467.312127	1123.628464	1071.80932	70.06335915	66.4194826
57	5282.957016	4620.83047	779.0921158	865.162998	56.55935544	69.5257076
58	4045.585905	4825.07628	944.2176677	747.969855	56.93291826	67.0270058
59	4633.980055	5006.414463	918.037855	1358.45442	57.70211316	66.7727485
60	4957.692367	4740.435311	1006.171011	1104.39786	56.55972443	66.2560434

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