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Conformal symmetry and supersymmetry in Rindler space

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Abstract

This paper addresses the fate of extended space-time symmetries, in particular conformal symmetry and supersymmetry, in two-dimensional Rindler space-time appropriate to a uniformly accelerated non-inertial frame in flat 1+1-dimensional space-time. Generically, in addition to a conformal co-ordinate transformation, the transformation of fields from Minkowski to Rindler space is accompanied by local conformal and Lorentz transformations of the components, which also affect the Bogoliubov transformations between the associated Fock spaces. I construct these transformations for massless scalars and spinors, as well as for the ghost and super-ghost fields necessary in theories with local conformal and supersymmetries, as arising from coupling to 2-D gravity or supergravity. Cancellation of the anomalies in Minkowski and in Rindler space requires theories with the well-known critical spectrum of particles arising in string theory in the limit of infinite strings, and is relevant for the equivalence of Minkowski and Rindler frame theories.

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1 Introduction

Conformal field theories have become an important tool in our theories of nature, both in the context of particle physics and quantum gravity and in the context of condensed matter [1, 2, 3]. Conformal field theories in two dimensions are especially relevant in string theory [4], the AdS_3-CFT correspondence and condensed matter [5, 6], and as a model for quantum field theories also in higher dimensions.

Effectively conformal field theories in two space-time dimensions can be formulated as theories of massless fields in a Minkowski background. This is the point of view used for practical purposes in the present paper. In fact our focus is on the description of these theories in a non-standard frame: that of a uniformly accelerated observer [7]. The interest in this frame stems from the non-inertial effects which manifest themselves in the presence of a horizon and finite-temperature behaviour: the Unruh effect [8, 9, 10, 11], a counterpart of the Hawking effect in black-hole space-times [12, 13] and in cosmology [14]. More generally it has been found that free field theories in Rindler space describe the near-horizon behaviour of quantum field theories in the presence of black holes, which is particularly relevant in the context of AdS-CFT correspondence [15]. Extensive reviews of field theories in Rindler space can be found in refs. [16, 17].

In two space-time dimensions local conformal transformations can be used to cast the line element in the form

$$ds^2 = \rho^2(x)\,\eta_{\mu\nu}dx^\mu dx^\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is the flat Minkowski metric and $\rho^2(x)$ is the conformal factor, which is constrained by the topology. In the present paper we restrict ourselves to inertial observers in infinite flat Minkowski space for which $\rho^2(x) = 1$, and to accelerated observers in the associated Rindler space, a subset of Minkowski space consisting of two branches: the right and left Rindler wedges R and L, defined by

$$\begin{aligned} &\text{R:} \qquad x > 0 \quad \text{and} \quad -x < t < x; \\ &\text{L:} \qquad x < 0 \quad \text{and} \quad x < t < -x. \end{aligned} \tag{2}$$

Each wedge is parametrized by a conformally flat metric in terms of co-ordinates (τ, ξ) associated with an accelerated observer:

$$\begin{aligned} R: &\quad x = \frac{1}{a}\,e^{a\xi}\cosh a\tau, \qquad t = \frac{1}{a}\,e^{a\xi}\sinh a\tau, \\ L: &\quad x = -\frac{1}{a}\,e^{-a\xi}\cosh a\tau, \quad t = \frac{1}{a}\,e^{-a\xi}\sinh a\tau. \end{aligned} \tag{3}$$

In this parametrization the line element is of the form (1) with conformal factor $\rho = e^{\pm a\xi}$:

$$\begin{aligned} R: &\quad ds^2 = e^{2a\xi}\left(-d\tau^2 + d\xi^2\right), \\ L: &\quad ds^2 = e^{-2a\xi}\left(-d\tau^2 + d\xi^2\right). \end{aligned} \tag{4}$$

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The parameter a in the co-ordinate transformation represents the proper acceleration in the Minkowski frame of a body with Rindler co-ordinate $\xi = 0$ at the moment it crosses the x-axis. It can be absorbed in a further rescaling of the Rindler co-ordinates (τ, ξ) by $1/a$ and a shift of ξ by $\xi_0 = \ln a$; as a result the line element reduces to (4) with $a = 1$. This is our standard choice in the following. In all of this paper units are chosen such that $c = \hbar = 1$.

2. Massless fields in 2-D Minkowski space

To fix conventions and for later reference, in this section we briefly review free field theories of massless scalar fields (bosons) and massless spinor fields (fermions) in 2-D Minkowski space. The theory of a real, free scalar field $\varphi(x)$ is defined by the action

$$S_b = \frac{1}{2} \int_M d^2x \left[(\partial_t \varphi)^2 - (\partial_x \varphi)^2 \right]. \tag{5}$$

The solutions of the associated Klein-Gordon equation

$$\left(\partial_t^2 - \partial_x^2 \right) \varphi = 0, \tag{6}$$

are superpositions of left- and right-moving fields

$$\varphi(t, x) = \varphi_+(t + x) + \varphi_-(t - x). \tag{7}$$

For definiteness in the following we concentrate on left-moving fields φ_+. Its conjugate field momentum is $\pi_+ = \partial_t \varphi_+$, and the time-evolution of all functionals of the field is governed by the hamiltonian

$$H_b = \frac{1}{2} \int_{-\infty}^{\infty} dx \left[\pi_+^2 + (\partial_x \varphi)^2 \right], \tag{8}$$

using the equal-time commutation relation

$$[\varphi_+(t, x), \pi_+(t, y)] = i\delta(x - y). \tag{9}$$

In a plane-wave basis the field is expanded as

$$\varphi_+(t + x) = \int_0^{\infty} \frac{dk}{2\pi\sqrt{2k}} \left(a_k e^{-ik(t+x)} + a_k^* e^{ik(t+x)} \right), \tag{10}$$

the domain of momenta k for left-moving fields being $[0, \infty)$. The plane-wave mode operators are defined inversely by

$$a_k = \frac{i}{\sqrt{2k}} \int_{-\infty}^{\infty} dx\, e^{ik(t+x)} \overset{\leftrightarrow}{\partial}_t \varphi_+, \qquad a_k^* = -\frac{i}{\sqrt{2k}} \int_{-\infty}^{\infty} dx\, e^{-ik(t+x)} \overset{\leftrightarrow}{\partial}_t \varphi_+. \tag{11}$$

The commutation relation (9) is equivalent to

$$\left[a_k, a_q^* \right] = 2\pi\delta(k - q), \tag{12}$$

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and the hamiltonian after normal-ordering is

$$: H_b := \int_0^\infty \frac{dk}{2\pi}\, k a_k^* a_k. \tag{13}$$

Finally the ground state of the field is the bosonic Minkowski vacuum $|0\rangle_b$:

$$\forall k: \quad a_k|0\rangle_b = 0. \tag{14}$$

Next we discuss spinor fields, using the conventions collected in appendix A. A general Dirac spinor is a complex 2-component object; however, 2-D Minkowski space-time allows Majorana-Weyl spinors which are real single-component objects representing left- or right-moving solutions of the massless Dirac equation. Indeed, in the Majorana representation making use of the charge conjugation operator C the Dirac action takes the diagonal form

$$S_f = \frac{i}{2}\int_M d^2x\, \lambda^T C\partial\!\!\!/\, \lambda, \qquad C\partial\!\!\!/ = \begin{pmatrix} \partial_t - \partial_x & 0 \\ 0 & \partial_t + \partial_x \end{pmatrix}. \tag{15}$$

and for a 2-component Majorana spinor

$$\lambda = C\bar{\lambda}^T = \begin{bmatrix} \lambda_+ \\ \lambda_- \end{bmatrix},$$

the associated Dirac equation splits into independent equations

$$(\partial_t - \partial_x)\,\lambda_+ = 0, \qquad (\partial_t + \partial_x)\,\lambda_- = 0, \tag{16}$$

representing separate left- and right moving fields. Again, for definiteness we concentrate on left-moving fields $\lambda_+(t+x)$. The time-evolution of functionals of this field is governed by the hamiltonian

$$H_f = \frac{i}{2}\int_{-\infty}^\infty dx\, \lambda_+ \partial_x \lambda_+, \tag{17}$$

and the equal-time anticommutation relation

$$\{\lambda_+(t,x), \lambda_+(t,y)\} = \delta(x-y). \tag{18}$$

Defining the plane-wave expansion by

$$\lambda_+(t+x) = \int_0^\infty \frac{dk}{2\pi}\left(\alpha_k e^{-ik(t+x)} + \alpha_k^* e^{ik(t+x)}\right), \tag{19}$$

with inverse

$$\alpha_k = \frac{i}{2k}\int_{-\infty}^\infty dx\, e^{ik(t+x)} \overset{\leftrightarrow}{\partial}_t \lambda_+, \qquad \alpha_k^* = -\frac{i}{2k}\int_{-\infty}^\infty dx\, e^{-ik(t+x)} \overset{\leftrightarrow}{\partial}_t \lambda_+, \tag{20}$$

the anti-commutation relations translate to

$$\left\{\alpha_k, \alpha_q^*\right\} = 2\pi\delta(k-q). \tag{21}$$

After normal-ordering the hamiltonian (17) becomes

$$: H_f := \int_0^\infty \frac{dk}{2\pi}\, k\alpha_k^*\alpha_k, \tag{22}$$

with ground state the fermionic Minkowski vacuum $|0\rangle_f$:

$$\forall k: \quad \alpha_k|0\rangle_f = 0. \tag{23}$$

The theory of a combined massless scalar and spinor possesses supersymmetry. The combined hamiltonian

$$H = : H_b : + : H_f := \int_0^\infty \frac{dk}{2\pi}\, k\left(a_k^* a_k + \alpha_k^*\alpha_k\right), \tag{24}$$

allows a square root

$$Q = \int_0^\infty \frac{dk}{2\pi}\sqrt{k}\left(a_k^*\alpha_k + \alpha_k^* a_k\right), \tag{25}$$

such that $Q^2 = H$ and $[Q, H] = 0$. The supersymmetry transformations of the mode operators have the standard form

$$\begin{aligned} &[a_k, Q] = \sqrt{k}\,\alpha_k, \quad [a_k^*, Q] = -\sqrt{k}\,\alpha_k^*, \\ &\{\alpha_k, Q\} = \sqrt{k}\,a_k, \quad \{\alpha_k^*, Q\} = \sqrt{k}\,a_k^*. \end{aligned} \tag{26}$$

The common Minkowski vacuum $|0\rangle_M = |0\rangle_b \otimes |0\rangle_f$ is supersymmetric:

$$Q|0\rangle_M = 0. \tag{27}$$

It is straightforward to generalize the contructions to theories of a larger number of bosons and fermions. With equal numbers of N bosons and N fermions there are in general $N!$ supersymmetries, associating every boson with each of the fermions, as at this point the various boson and fermion fields are indistinguishable and related by an $O(N) \otimes O(N)$ symmetry.

3. Massless fields in 2-D Rindler space

Two-dimensional Rindler space is the space-time described by the metric (1), with the conformal factor specified for the left and right wedges by (4). It is therefore obvious, that in Rindler space

$$\sqrt{-g}\, g^{\mu\nu} = \eta^{\mu\nu}.$$

Moreover the scalar field in the Rindler frame is related to the original Minkowski field by the co-ordinate transformation (3): $\varphi_R(\tau,\xi) = \varphi[t(\tau,\xi), x(\tau,\xi)]$. It follows that the action for scalar fields in Rindler space is formally identical to that in Minkowski space:

$$S_{Rb} = \frac{1}{2}\int_R d\tau d\xi \left[(\partial_\tau\varphi)^2 - (\partial_\xi\varphi)^2\right]. \tag{28}$$

which holds both in the right and left wedge of Rindler space, independent of the different conformal factors $\rho = e^{\pm a\xi}$. Concentrating again on the left-moving solutions of the associated Klein-Gordon equation and following the same procedures as described for bosons in section 2, we can repesent them in terms of the plane-wave expansion

$$\varphi_{R+}(\tau+\xi) = \int_0^\infty \frac{d\kappa}{2\pi\sqrt{2\kappa}}\left(b_\kappa e^{-i\kappa(\tau+\xi)} + b_\kappa^* e^{i\kappa(\tau+\xi)}\right), \tag{29}$$

where

$$b_\kappa = \frac{i}{\sqrt{2\kappa}}\int_{-\infty}^\infty d\xi\, e^{i\kappa(\tau+\xi)} \overset{\leftrightarrow}{\partial}_\tau \varphi_{R+}, \quad b_\kappa^* = -\frac{i}{\sqrt{2\kappa}}\int_{-\infty}^\infty d\xi\, e^{-i\kappa(\tau+\xi)} \overset{\leftrightarrow}{\partial}_\tau \varphi_{R+}. \tag{30}$$

Free-field dynamics is generated by introducing the hamiltonian

$$H_{Rb} = \int_0^\infty \frac{d\kappa}{4\pi}\,\kappa\,(b_\kappa^* b_\kappa + b_\kappa b_\kappa^*), \tag{31}$$

together with the commutation relation

$$[b_\kappa, b_\sigma^*] = 2\pi\delta(\kappa-\sigma). \tag{32}$$

Standard procedure would replace H_{Rb} by its normal-ordered form, but normal ordering is now ambiguous as the result is different in terms of the mode operators (b_κ, b_κ^*) or in terms the original mode operators (a_k, a_k^*). Indeed, the Rindler vacuum $|0\rangle_{Rb}$ defined by

$$b_\kappa|0\rangle_{Rb} = 0, \tag{33}$$

is not the same as the bosonic vacuum $|0\rangle_b$ in the original Minkowski space, as discussed below.

Translating the free fermion theory to Rindler space is somewhat more involved. Starting from the Dirac-Majorana action on a general space-time manifold $\mathcal{M}$

$$S_{\mathcal{M}f} = \frac{i}{2}\int_{\mathcal{M}} d^2x\,\sqrt{-g}\,\lambda^T C \not{D}\lambda, \qquad \not{D} = \gamma^a e_a^\mu\left(\partial_\mu - \frac{1}{2}\omega_\mu^{ab}\sigma_{ab}\right), \tag{34}$$

where e_a^μ is the inverse 2-bein, ω_μ^{ab} the associated spin connection and σ_{ab} the generators of 2-D Lorentz transformations in tangent spinor space; as explained

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in appendix A we find that in a conformally flat space-time (1) the Dirac operator in the Majorana representation takes the form

$$\sqrt{-g}\, C\not{D} = \sqrt{\rho} \begin{pmatrix} \partial_\tau - \partial_\xi & 0 \\ 0 & \partial_\tau + \partial_\xi \end{pmatrix} \sqrt{\rho}. \tag{35}$$

In addition, the transformation from Minkowski to Rindler co-ordinates also requires a τ-dependent Lorentz transformation of the spinor fields; this is also explained in appendix A. The upshot is, that upon redefining the spinor fields in the R- and L-wedge of Rindler space by

$$\begin{aligned} R: &\qquad \psi_\pm(\tau,\xi) = e^{\pm a(\tau\pm\xi)/2}\lambda_\pm[t(\tau,\xi), x(\tau,\xi)], \\ L: &\qquad \psi_\pm(\tau,\xi) = e^{\mp a(\tau\pm\xi)/2}\lambda_\pm[t(\tau,\xi), x(\tau,\xi)], \end{aligned} \tag{36}$$

the Dirac-Majorana action in a conformally flat space-time like Rindler space reduces to

$$S_{Rf} = \frac{i}{2}\int_R d\tau d\xi \left[\psi_+ \left(\partial_\tau - \partial_\xi\right)\psi_+ + \psi_- \left(\partial_\tau + \partial_\xi\right)\psi_-\right]. \tag{37}$$

The spinor fields $\psi_\pm$ have the same formal properties in Rindler space as the spinor fields $\lambda_\pm$ in Minkowski space; concentrating on the left-moving field we introduce the plane-wave expansion

$$\psi_+(\tau+\xi) = \int_0^\infty \frac{d\kappa}{2\pi}\left(\beta_\kappa e^{-i\kappa(\tau+\xi)} + \beta^*_\kappa e^{i\kappa(\tau+\xi)}\right), \tag{38}$$

where

$$\beta_\kappa = \frac{i}{2\kappa}\int_{-\infty}^\infty dx\, e^{i\kappa(\tau+\xi)} \overset{\leftrightarrow}{\partial}_\tau \psi_+, \qquad \beta^*_\kappa = -\frac{i}{2\kappa}\int_{-\infty}^\infty dx\, e^{-i\kappa(\tau+\xi)} \overset{\leftrightarrow}{\partial}_\tau \psi_+. \tag{39}$$

In terms of mode operators the dynamics is defined by the hamiltonian

$$H_{Rf} = \int_0^\infty \frac{d\kappa}{4\pi}\,\kappa\left(\beta^*_\kappa\beta_\kappa - \beta_\kappa\beta^*_\kappa\right), \tag{40}$$

and the anti-commutator

$$\{\beta_\kappa, \beta^*_\sigma\} = 2\pi\delta(\kappa-\sigma). \tag{41}$$

Again, in the pure-fermion theory we renounce normal ordering in view of its ambiguity. However, in the special case of a supersymmetric theory with equal numbers of bosons and fermions the normal-ordering contributions of bosons and fermions cancel, and therefore normal ordering becomes unambiguous. In that case we can take the full hamiltonian to be

$$H_R = H_{Rb} + H_{Rf} = \int_0^\infty \frac{d\kappa}{2\pi}\,\kappa\left(b^*_\kappa b_\kappa + \beta^*_\kappa\beta_\kappa\right). \tag{42}$$

Clearly it is unique owing to the existence of the supercharge

$$Q_R = \int_0^\infty \frac{d\kappa}{2\pi}\sqrt{\kappa}\,(b^*_\kappa\beta_\kappa + \beta^*_\kappa b_\kappa)\,, \tag{43}$$

with the algebraic properties

$$Q_R^2 = H_R, \qquad [Q_R, H_R] = 0. \tag{44}$$

4. Relating Rindler and Minkowski fields

In the Rindler wedges the field theories of section 2 and section 3 are related by the co-ordinate transformation (3); they describe the same physics in different frames: with respect to an inertial observer using the Minkowski metric, or with respect to an accelerated observer using the conformally flat metrics (4). Therefore also the Hilbert spaces and the observables in the two descriptions are related; the relation actually is provided by a Bogoliubov transformation between the mode operators [8, 13, 16, 17, 18, 19].

For the scalar fields the relation is established starting from (30) and inserting for φ_{R+} its Minkowski space expression, eq. (10). Choosing units to fix $a = 1$ in the following and introducing the Minkowski light-cone co-ordinate

$$z = t + x = e^{\tau+\xi}, \tag{45}$$

the Rindler mode operators in the R-wedge are expressed in terms of the Minkowski ones by

$$\begin{aligned} b_{R\kappa} &= \frac{1}{2\pi\sqrt{2\kappa}}\int_0^\infty \frac{dk}{\sqrt{2k}}\int_0^\infty \frac{dz}{z}\, z^{i\kappa}\left[(\kappa+kz)a_k e^{-ikz} + (\kappa-kz)a^*_k e^{ikz}\right], \\ b^*_{R\kappa} &= -\frac{1}{2\pi\sqrt{2\kappa}}\int_0^\infty \frac{dk}{\sqrt{2k}}\int_0^\infty \frac{dz}{z}\, z^{-i\kappa}\left[(\kappa-kz)a_k e^{-ikz} + (\kappa+kz)a^*_k e^{ikz}\right]. \end{aligned} \tag{46}$$

By appropriate choice of contour the integrals over z can be performed in the complex plane; this leads to

$$\begin{aligned} b_{R\kappa} &= -\frac{i}{2\pi\sqrt{\kappa}}\,\Gamma(1+i\kappa)\int_0^\infty dk\, k^{-\frac{1}{2}-i\kappa}\left(a_k e^{\pi\kappa/2} + a^*_k e^{-\pi\kappa/2}\right), \\ b^*_{R\kappa} &= \frac{i}{2\pi\sqrt{\kappa}}\,\Gamma(1-i\kappa)\int_0^\infty dk\, k^{-\frac{1}{2}+i\kappa}\left(a_k e^{-\pi\kappa/2} + a^*_k e^{\pi\kappa/2}\right). \end{aligned} \tag{47}$$

A similar calculation for the L-wedge, in which $z = -(t+x) = e^{-(\tau+\xi)}$ gives

$$\begin{aligned} b_{L\kappa} &= \frac{i}{2\pi\sqrt{\kappa}}\,\Gamma(1-i\kappa)\int_0^\infty dk\, k^{-\frac{1}{2}+i\kappa}\left(a_k e^{\pi\kappa/2} + a^*_k e^{-\pi\kappa/2}\right), \\ b^*_{L\kappa} &= -\frac{i}{2\pi\sqrt{\kappa}}\,\Gamma(1+i\kappa)\int_0^\infty dk\, k^{-\frac{1}{2}-i\kappa}\left(a_k e^{-\pi\kappa/2} + a^*_k e^{\pi\kappa/2}\right). \end{aligned} \tag{48}$$

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Applying these operators to the bosonic Minkowski vacuum one establishes that

$$\begin{aligned} e^{\pi\kappa/2} b_{R\kappa}|0\rangle_b = e^{-\pi\kappa/2} b^*_{L\kappa}|0\rangle_b = -\frac{i}{2\pi\sqrt{\kappa}}\,\Gamma(1+i\kappa)\int_0^\infty dk\, k^{-\frac{1}{2}-i\kappa} a^*_k|0\rangle_b, \\ e^{-\pi\kappa/2} b^*_{R\kappa}|0\rangle_b = e^{\pi\kappa/2} b_{L\kappa}|0\rangle_b = \frac{i}{2\pi\sqrt{\kappa}}\,\Gamma(1-i\kappa)\int_0^\infty dk\, k^{-\frac{1}{2}+i\kappa} a^*_k|0\rangle_b. \end{aligned} \tag{49}$$

It follows that the Minkowski vacuum contains Rindler excitations, and that annihilation of an excitation in the right Rindler wedge has the same effect modulo a factor $e^{-\pi\kappa}$ as creation of one in the left Rindler wedge, and vice versa.

To relate the excitations of the fermion field in Minkowski and Rindler space we proceed along similar lines, but with some slight modifications to keep track of the conformal and local Lorentz transformations relating the fields in the different observer frames. The starting point is provided by the equations (39), in which we replace the Rindler spinor field ψ_+ by its Minkowski counterpart (36). In the R-wedge of Rindler space, using the light-cone variable (45), we get

$$\begin{aligned} \beta_{R\kappa} = \frac{i}{4\pi\kappa}\int_0^\infty dk \int_0^\infty \frac{dz}{z}\, z^{\frac{1}{2}+i\kappa}\Big[\Big(\frac{1}{2}-i\kappa-ikz\Big)\,\alpha_k e^{-ikz} \\ + \Big(\frac{1}{2}-i\kappa+ikz\Big)\,\alpha^*_k e^{ikz}\Big], \\ \beta^*_{R\kappa} = -\frac{i}{4\pi\kappa}\int_0^\infty dk \int_0^\infty \frac{dz}{z}\, z^{\frac{1}{2}-i\kappa}\Big[\Big(\frac{1}{2}+i\kappa-ikz\Big)\,\alpha_k e^{-ikz} \\ + \Big(\frac{1}{2}+i\kappa+ikz\Big)\,\alpha^*_k e^{ikz}\Big]. \end{aligned} \tag{50}$$

Performing the integrations over z in the compex plane this yields

$$\begin{aligned} \beta_{R\kappa} = e^{-i\pi/4}\Gamma\Big(\frac{1}{2}+i\kappa\Big)\int_0^\infty \frac{dk}{2\pi}\, k^{-\frac{1}{2}-i\kappa}\left(\alpha_k e^{\pi\kappa/2} + i\alpha^*_k e^{-\pi\kappa/2}\right), \\ \beta^*_{R\kappa} = e^{-i\pi/4}\Gamma\Big(\frac{1}{2}-i\kappa\Big)\int_0^\infty \frac{dk}{2\pi}\, k^{-\frac{1}{2}+i\kappa}\left(\alpha_k e^{-\pi\kappa/2} + i\alpha^*_k e^{\pi\kappa/2}\right). \end{aligned} \tag{51}$$

Similarly in the L-wedge of Rindler space we get after the appropriate modifications

$$\begin{aligned} \beta_{L\kappa} = e^{i\pi/4}\Gamma\Big(\frac{1}{2}-i\kappa\Big)\int_0^\infty \frac{dk}{2\pi}\, k^{-\frac{1}{2}+i\kappa}\left(\alpha_k e^{\pi\kappa/2} - i\alpha^*_k e^{-\pi\kappa/2}\right), \\ \beta^*_{L\kappa} = e^{i\pi/4}\Gamma\Big(\frac{1}{2}+i\kappa\Big)\int_0^\infty \frac{dk}{2\pi}\, k^{-\frac{1}{2}-i\kappa}\left(\alpha_k e^{-\pi\kappa/2} - i\alpha^*_k e^{\pi\kappa/2}\right). \end{aligned} \tag{52}$$

From this we derive the following relations for the action on the fermionic Minkowski vacuum:

$$e^{\pi\kappa/2}\beta_{R\kappa}|0\rangle_f = -ie^{-\pi\kappa/2}\beta^*_{L\kappa}|0\rangle_f = e^{i\pi/4}\Gamma\left(\frac{1}{2}+i\kappa\right)\int_0^\infty \frac{dk}{2\pi}\,k^{-\frac{1}{2}-i\kappa}\alpha^*_k|0\rangle_f,$$

$$e^{-\pi\kappa/2}\beta^*_{R\kappa}|0\rangle_f = ie^{\pi\kappa/2}\beta_{L\kappa}|0\rangle_f = e^{i\pi/4}\Gamma\left(\frac{1}{2}-i\kappa\right)\int_0^\infty \frac{dk}{2\pi}\,k^{-\frac{1}{2}+i\kappa}\alpha^*_k|0\rangle_f. \tag{53}$$

Again, the fermionic Minkowski vacuum contains fermionic Rindler excitations, with creation of one in the R-wedge being accompanied by annihilation of a similar one in the L-wedge, and vice versa.

As concerns correlation functions, in free field theories the only non-trivial correlators are two-point functions. In Minkowski space the elementary ones are

$${}_b\langle 0|a_k a^*_q|0\rangle_b = {}_f\langle 0|\alpha_k\alpha^*_q|0\rangle_f = 2\pi\delta(k-q). \tag{54}$$

Using the results (49) and (53) in a recursive way [17] one establishes that the correlators of the Rindler fields in the Minkowski vacuum are

$${}_b\langle 0|b_{R\kappa}b^*_{R\sigma}|0\rangle_b = \frac{2\pi\delta(\kappa-\sigma)}{1-e^{-2\pi\kappa}}, \qquad {}_f\langle 0|\beta_{R\kappa}\beta^*_{R\sigma}|0\rangle_f = \frac{2\pi\delta(\kappa-\sigma)}{1+e^{-2\pi\kappa}}, \tag{55}$$

with identical results for the correlators in the L-wedge. These relations imply the complementary identities

$${}_b\langle 0|b^*_{R\kappa}b_{R\sigma}|0\rangle_b = \frac{2\pi\delta(\kappa-\sigma)}{e^{2\pi\kappa}-1}, \qquad {}_f\langle 0|\beta^*_{R\kappa}\beta_{R\sigma}|0\rangle_f = \frac{2\pi\delta(\kappa-\sigma)}{e^{2\pi\kappa}+1}, \tag{56}$$

stating that the occupation numbers of Rindler excitations in the Minkowski vacuum are of thermal Bose-Einstein and Fermi-Dirac type, corresponding to a state with temperature $T = 1/2\pi$; upon restoring the acceleration parameter by rescaling of the Rindler co-ordinates and momenta the temperature of Minkowski space observed in a Rindler frame with proper acceleration a is $T = a/2\pi$. Details of the derivation are presented in appendix B.

5. Supersymmetry

As we have seen the supersymmetric theory has a unique Rindler hamiltonian (42), which is the square of the supercharge (43). In view of the results (56) it is easily seen to have a divergent Minkowski vacuum expectation value. It follows automatically that the Rindler supercharge does not annihilate the Minkowski vacuum: Rindler supersymmetry is broken by finite temperature effects in the Minkowski vacuum. In contrast, Minkowski supersymmetry generated by the supercharge (25) does annihilate the Minkowski vacuum as stated by (27).

One way to deal with this situation it is to regularize the Rindler supercharge using a symmetric regulator function $g(\kappa,\sigma) = g(\sigma,\kappa)$ and define

$$Q_{Rg} = \int_0^\infty \frac{d\kappa}{2\pi} \int_0^\infty \frac{d\sigma}{2\pi}\, g(\kappa,\sigma)\, (b^*_\kappa \beta_\sigma + \beta^*_\sigma b_\kappa)\,. \tag{57}$$

A straightforward calculations shows that

$$Q^2_{Rg} = \int_0^\infty \frac{d\kappa}{2\pi} \int_0^\infty \frac{d\sigma}{2\pi}\, g^2(\kappa,\sigma)\, (b^*_\kappa b_\sigma + \beta^*_\kappa \beta_\sigma) \equiv H_{Rg}, \tag{58}$$

where

$$g^2(\kappa,\sigma) = \int_0^\infty \frac{d\eta}{2\pi}\, g(\kappa,\eta) g(\eta,\sigma). \tag{59}$$

This properly regularizes the Rindler supercharge and hamiltonian if g is taken from a class of functions having a limit

$$g(\kappa,\sigma) \to \sqrt{\kappa}\,\delta(\kappa-\sigma), \qquad g^2(\kappa,\sigma) \to \kappa\,\delta(\kappa-\sigma). \tag{60}$$

With such a regulator one easily finds that

$$\begin{aligned} {}_M\langle 0|Q^2_{Rg}|0\rangle_M &= {}_M\langle 0|H_{Rg}|0\rangle_M \\ &= \int_0^\infty \frac{d\kappa}{2\pi}\, g^2(\kappa,\kappa) \left(\frac{1}{e^{2\pi\kappa}-1} + \frac{1}{e^{2\pi\kappa}+1}\right) = \int_0^\infty \frac{d\kappa}{2\pi}\, \frac{g^2(\kappa,\kappa)}{2\sinh 2\pi\kappa}, \end{aligned} \tag{61}$$

which is a positive number for any acceptable regulator $g(\kappa,\sigma)$.

6. Conformal symmetries

In addition to the hamiltonian and the supercharge, which are conserved for the theories in the Minkowski frame, one can construct an infinite set of conformal charges annihilating the vacuum state which define a continuum generalization of the Virasoro and super-Virasoro algebra.

The charges, which can be decomposed in contributions from bosons and from fermions, are labeled by the momentum variable k; for $k \geq 0$ they annihilate the Minkowski vacuum, being defined for a single boson and fermion field by

$$\begin{aligned} L^b_k &= \int_0^\infty \frac{dq}{2\pi} \sqrt{q(k+q)}\, a^*_q a_{k+q} + \int_0^k \frac{dq}{4\pi} \sqrt{q(k-q)}\, a_q a_{k-q}, \\ L^f_k &= \int_0^\infty \frac{dq}{2\pi} \left(q + \frac{k}{2}\right) \alpha^*_q \alpha_{k+q} - \int_0^k \frac{dq}{4\pi}\, q\, \alpha_q \alpha_{k-q}, \end{aligned} \tag{62}$$

whilst those labeled by a negative momentum $-k$ are the hermitean conjugates $L_{-k} = L^*_k$. Introducing the general notation $a_{-k} = a^*_k$, $\alpha_{-k} = \alpha^*_k$, it is possible to

write these expressions in a short-hand notation using normal ordering:

$$L_k^b = \int_{-\infty}^{\infty} \frac{dq}{4\pi} \sqrt{|q(q-k)|} : a_q a_{k-q} :, \qquad L_k^f = \int_{-\infty}^{\infty} \frac{dq}{4\pi} \left(-q + \frac{k}{2}\right) : \alpha_q \alpha_{k-q} : . \tag{63}$$

But apart from the fact that the natural domain of the momentum labels k, q is $[0, \infty)$, the normal ordering in Rindler space is ambiguous due to the mixing of creation an annhihilation operators by the Bogoliubov tranformations, as pointed out before. This is especially relevant as we intend to compute correlation functions of Rindler operators in the Minkowski vacuum. Therefore we prefer to write out the explicit expansions (62) rather than using the normal-ordered ones.

The operators (62) satisfy the commutation relations of the Virasoro algebra with a central charge:

$$\begin{aligned} \left[L_k^b, L_l^b\right] &= (k-l) L_{k+l}^b + \frac{1}{12} k^3 \, \delta(k+l), \\ \left[L_k^f, L_l^f\right] &= (k-l) L_{k+l}^f + \frac{1}{24} k^3 \, \delta(k+l). \end{aligned} \tag{64}$$

The expressions for the central charges are easily verified by the standard procedure of evaluating the vacuum expectation value of the commutator

$$\frac{1}{2} {}_b\langle 0| \left[L_k^b, L_l^b\right] |0\rangle_b = {}_f\langle 0| \left[L_k^f, L_l^f\right] |0\rangle_f = \frac{1}{24} k^3 \, \delta(k+l).$$

In supersymmetric Minkowski-frame theories the Virasoro algebra can be extended by supercharges G_k, defined for a single boson plus fermion field and for $k \geq 0$ by

$$G_k = \int_0^{\infty} \frac{dq}{2\pi} \left(\sqrt{q}\, a_q^* \alpha_{k+q} + \sqrt{k+q}\, \alpha_q^* a_{k+q}\right) + \int_0^k \frac{dq}{2\pi} \sqrt{q}\, a_q \alpha_{k-q}; \tag{65}$$

again for negative index $G_{-k} = G_k^*$. In a theory with a single boson and fermion the complete set of (anti-)commutation relations with

$$L_k = L_k^b + L_k^f,$$

is found to have the standard super-Virasoro form

$$\begin{aligned} [L_k, L_l] &= (k-l)\, L_{k+l} + \frac{1}{8} k^3 \delta(k+l), \\ [L_k, G_l] &= \left(\frac{k}{2} - l\right) G_{k+l}, \\ \{G_k, G_l\} &= 2 L_{k+l} + \frac{1}{2} k^2 \delta(k+l). \end{aligned} \tag{66}$$

Note that $L_0 = H$ is the hamiltonian (24), and $G_0 = Q$ is the supercharge (25). They define a closed anomaly-free subalgebra of the super-Virasoro algebra (66). Note also that their vacuum expectations vanish:

$$\begin{aligned} \forall k \geq 0: \quad L_k|0\rangle_M = 0 \quad &\Rightarrow \quad {}_M\langle 0|L_k|0\rangle_M = {}_M\langle 0|L_{-k}|0\rangle_M = 0, \\ G_k|0\rangle_M = 0 \quad &\Rightarrow \quad {}_M\langle 0|G_k|0\rangle_M = {}_M\langle 0|G_{-k}|0\rangle_M = 0. \end{aligned} \tag{67}$$

The same construction also works in the Rindler frame, where in the R-wedge of Rindler space

$$\begin{aligned} L^b_{R\kappa} &\equiv \int_0^\infty \frac{d\sigma}{2\pi}\sqrt{\sigma(\kappa+\sigma)}\, b^*_{R\sigma} b_{R\,\kappa+\sigma} + \int_0^\kappa \frac{d\sigma}{4\pi}\sqrt{\sigma(\kappa-\sigma)}\, b_{R\sigma} b_{R\,\kappa-\sigma}, \\ L^f_{R\kappa} &= \int_0^\infty \frac{d\sigma}{2\pi}\left(\sigma+\frac{\kappa}{2}\right)\beta^*_{R\sigma}\beta_{R\,\kappa+\sigma} - \int_0^\kappa \frac{d\sigma}{4\pi}\,\sigma\,\beta_{R\sigma}\beta_{R\,\kappa-\sigma}, \\ G_{R\kappa} &= \int_0^\infty \frac{d\sigma}{2\pi}\left(\sqrt{\sigma}\, b^*_{R\sigma}\beta_{R\,\kappa+\sigma} + \sqrt{\kappa+\sigma}\,\beta^*_{R\sigma} b_{R\,\kappa+\sigma}\right) + \int_0^\kappa \frac{d\sigma}{2\pi}\sqrt{\sigma}\, b_{R\sigma}\beta_{R\,\kappa-\sigma}, \end{aligned} \tag{68}$$

with analogous definitions in the L-wedge. The essential difference with the Minkowski charges, is that the Rindler operators (68) admit a non-vanishing expectation value in the Minkowski frame:

$$\begin{aligned} {}_M\langle 0|L^b_{R\kappa}|0\rangle_M &= \delta(\kappa)\int_0^\infty d\sigma\,\frac{\sigma}{e^{2\pi\sigma}-1} = \frac{1}{24}\,\delta(\kappa), \\ {}_M\langle 0|L^f_{R\kappa}|0\rangle_M &= \delta(\kappa)\int_0^\infty d\sigma\,\frac{\sigma}{e^{2\pi\sigma}+1} = \frac{1}{48}\,\delta(\kappa), \end{aligned} \tag{69}$$

whilst in the superconformal case

$${}_M\langle 0|G_{R\kappa}|0\rangle_M = 0.$$

Although one could remove these vacuum expectation values by shifting the ground-state energy, this would introduce an extra contribution to the central charge, which is generally not desirable. The singular nature of the expectation value of the hamiltonian $H = L_0$ in practice requires regularization, as we discussed in section 5.

7. Ghosts and local conformal symmetry

As is well-known, the line element in any topologically trivial 2-D space-time can be cast in the form (1) by appropriate co-ordinate transformations. As 2-D gravity is conformally invariant, but non-dynamical, one can therefore interpret the boson and fermion field theories discussed above as the gauge-fixed version

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of a gravitational theory with local conformal symmetry. This is the key to a consistent quantum-theory of strings and superstrings [4]. Local conformal symmetry turns the generators of the Virasoro algebra into operators imposing first-class constraints:

$$\forall k > 0: \quad L_k|phys\rangle = 0. \tag{70}$$

These conditions are consistent only if the anomaly vanishes, but the anomaly now includes a contribution from the Faddeev-Popov ghosts introduced by quantization of the gravitational background as well [20].

In the sector of left-moving fields the dynamics of the anti-commuting gravity ghosts (C, D) follows from the action

$$S_g = i \int_M d^2x\, D\,(\partial_t - \partial_x)\, C. \tag{71}$$

Following standard procedures one derives the hamiltonian

$$H_g = i \int_{-\infty}^{\infty} dx\, D\partial_x C, \tag{72}$$

supplemented by the equal-time anti-commutation relations

$$\{C(t,x), D(t,y)\} = \delta(x-y). \tag{73}$$

From the mode expansions

$$\begin{aligned} C(t,x) &= \int_0^\infty \frac{dk}{2\pi} \left(c_k e^{-ik(t+x)} + c_k^* e^{ik(t+x)}\right), \\ D(t,x) &= \int_0^\infty \frac{dk}{2\pi} \left(d_k e^{-ik(t+x)} + d_k^* e^{ik(t+x)}\right), \end{aligned} \tag{74}$$

we derive the anti-commutators of the mode operators:

$$\left\{c_k, d_q^*\right\} = 2\pi\delta(k-q), \qquad \left\{d_k, c_q^*\right\} = 2\pi\delta(k-q). \tag{75}$$

The corresponding Virasoro generators for $k \geq 0$ are

$$L_k^g = \int_0^\infty \frac{dq}{2\pi} \left[(q-k)\, c_q^* d_{k+q} + (q+2k)\, d_q^* c_{k+q}\right] - \int_0^k \frac{dq}{2\pi}\, (q+k)\, c_q d_{k-q}, \tag{76}$$

with $L_{-k}^g = L_k^{g\,*}$. The ghost Virasoro algebra then becomes

$$[L_k^g, L_l^g] = (k-l)\, L_{k+l}^g - \frac{13}{6}\, k^3 \delta(k+l). \tag{77}$$

For a theory with N_b massless scalars and N_f massless chiral Majorana fermions the full first-class contraints then are

$$\forall k > 0: \quad L_k|phys\rangle = 0, \qquad L_k = \sum_{i=1}^{N_b} L_k^{bi} + \sum_{j=1}^{N_f} L_k^{fj} + L_k^g, \tag{78}$$

with the algebra

$$[L_k, L_l] = (k-l)\, L_{k+l} + \frac{(2N_b + N_f - 52)}{24}\, k^3 \delta(k+l). \tag{79}$$

Thus the central charge vanishes provided

$$2N_b + N_f = 52. \tag{80}$$

The analysis in the right-moving sector proceeds entirely analogously.

In the case of a supersymmetric theory of massless scalars and femions the local conformal symmetry can be extended to local superconformal symmetry. Such a theory is effectively a gauge-fixed 2-D supergravity theory and requires an additional set of commuting supersymmetry ghosts (S, U) with action

$$S_{sg} = \int_M d^2x\, U\, (\partial_t - \partial_x)\, S. \tag{81}$$

The associated hamiltonian and equal-time commutation relations are

$$H_{sg} = \int_{-\infty}^{\infty} dx\, U \partial_x S, \qquad [S(t,x), U(t,y)] = i\delta(x-y). \tag{82}$$

In this case we take the plane-wave expansions to be

$$\begin{aligned} S(t,x) &= \int_0^\infty \frac{dk}{2\pi} \left(s_k e^{-ik(t+x)} + s_k^* e^{ik(t+x)} \right), \\ U(t,x) &= -i \int_0^\infty \frac{dk}{2\pi} \left(u_k e^{-ik(t+x)} - u_k^* e^{ik(t+x)} \right), \end{aligned} \tag{83}$$

which results in mode commutation relations

$$\left[s_k, u_q^*\right] = 2\pi\, \delta(k-q), \qquad \left[u_k, s_q^*\right] = 2\pi\, \delta(k-q). \tag{84}$$

The super-Virasoro operators of the full set of ghosts (C, D) and superghosts (S, U) then is defined by (76) and for $k \geq 0$:

$$\begin{aligned} L_k^{sg} &= \int_0^\infty \frac{dq}{2\pi} \left[\left(q - \frac{k}{2}\right) s_q^* u_{k+q} + \left(q + \frac{3k}{2}\right) u_q^* s_{k+q} \right] \\ &\quad - \int_0^k \frac{dq}{2\pi} \left(q + \frac{k}{2}\right) s_q u_{k-q}, \\ G_k^{sg} &= \int_0^\infty \frac{dq}{2\pi} \left[s_q^* b_{k+q} + b_q^* s_{k+q} + (q+3k) u_q^* c_{k+q} + (q-2k) c_q^* u_{k+q} \right] \\ &\quad + \int_0^k \frac{dq}{2\pi} \left[s_q b_{k-q} - (q+2k)\, c_q u_{k-q} \right], \end{aligned} \tag{85}$$

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with $L^{sg}_{-k} = L^{sg\,*}_{k}$ and $G^{sg}_{-k} = G^{sg\,*}_{k}$. The complete super-Virasoro algebra of the ghosts is given by (77) and

$$\begin{aligned} [L^{sg}_k, L^{sg}_l] &= (k-l)\, L^{sg}_{k+l} + \frac{11}{12}\, k^3 \delta(k+l), \\ \{G^{sg}_k, G^{sg}_l\} &= 2\left(L^{g}_{k+l} + L^{sg}_{k+l}\right) - 5k^2\delta(k+l). \end{aligned} \tag{86}$$

With these results a 2-D supersymmetric theory of N massless scalars and fermions has local superconformal symmetry if

$$\forall k > 0: \quad L_k|phys\rangle = G_k|phys\rangle = 0, \tag{87}$$

where

$$L_k = \sum_{i=1}^{N}\left(L^{bi}_k + L^{fi}_k\right) + L^{g}_k + L^{sg}_k, \qquad G_k = \sum_{i=1}^{N} G^{i}_k + G^{sg}_k, \tag{88}$$

with the algebra

$$\begin{aligned} [L_k, L_l] &= (k-l)\, L_{k+l} + \frac{(N-10)}{8}\, k^3\delta(k+l), \\ [L_k, G_l] &= \left(\frac{k}{2} - l\right) G_{k+l}, \\ \{G_k, G_l\} &= 2L_{k+l} + \frac{(N-10)}{2}\, k^2\delta(k+l). \end{aligned} \tag{89}$$

Thus all superconformal anomalies vanish for $N = 10$. Of course all results derived here are in agreement with corresponding string and superstring theories in the limit of infinite strings.

8. Conformal and superconformal ghosts in Rindler space

The local conformal and superconformal symmetries arising in a gauge-fixed 2-D gravity or supergravity theory with scalar and spinor matter can be extended to the Rindler frame. It requires solving the ghost and superghost field equations in Rindler space, and finding the appropriate Bogoliubov tranformations.

Consider a theory in an arbitrary 2-D space-time with reparametrization and locally Lorentz-invariant action

$$S[F, G] = \int d^2x\, e\, F^{a_1\ldots a_{n+1}} e^{\mu}_{a_{n+1}} D_\mu G_{a_1\ldots a_n}. \tag{90}$$

Here (F, G) is a system of commuting or anti-commuting local Lorentz tensors or (chiral) spinor-tensors of rank $(n, n+1)$, e^{μ}_{a} is the inverse 2-bein field as explained in appendix A; D_μ is the Lorentz-covariant derivative with spin connection $\omega_{\mu}{}^{a}{}_{b}$

defined in (117) and $e = \det e_\mu^{\ a} = \sqrt{-g}$. Consider only purely left- or right-handed components associated with purely self-dual or anti-self-dual tensors:

$$G_{a_1\ldots a_n} = \pm\,\varepsilon_{a_1 b} G^{b}{}_{a_2\ldots a_n} = \ldots = \pm\,\varepsilon_{a_n b} G_{a_1\ldots a_{n-1}}{}^{b}, \tag{91}$$

and similarly for $F^{a_1\ldots a_{n+1}}$. Fixing the Minkowski gauge, the action (90) can be decomposed into actions of the type (71) or (81) for single pairs of components $(F_M^{(n+1)}, G_M^{(n)})$ with gauge-fixed action (considering left-movers for definiteness)

$$S_M^{(n)} = \int_M d^2x\, F_M^{(n+1)}\,(\partial_t - \partial_x)\, G_M^{(n)}. \tag{92}$$

When transforming the action from the Minkowski to the Rindler frame by the transformations (3) with $a = 1$ and following the same arguments as described for spinor components in appendix A, we get a Rindler-frame action in the R-wedge of Rindler space

$$S_R^{(n)} = \int_R d\tau d\xi \left[F_R^{(n+1)}\,(\partial_\tau - \partial_\xi)\, G_R^{(n)}\right], \tag{93}$$

where the field components have been transformed from the Minkowski to the Rindler frame by an extension of (36):

$$\begin{aligned} F_R^{(n+1)}(\tau,\xi) &= e^{(n+1)(\tau+\xi)} F_M^{(n+1)}[t(\tau,\xi), x(\tau,\xi)], \\ G_R^{n}(\tau,\xi) &= e^{-n(\tau+\xi)} G_M^{n}[t(\tau,\xi), x(\tau,\xi)]. \end{aligned} \tag{94}$$

Actually n can be integer or half-integer, depending on whether (F, G) are tensors or spinor-tensors. The aim is now to connect the Rindler plane-wave expansions in the R-wedge

$$\begin{aligned} F_R^{(n+1)} &= \int_0^\infty \frac{d\kappa}{2\pi}\left(f_{R\kappa} e^{-i\kappa(\tau+\xi)} + f_{R\kappa}^* e^{i\kappa(\tau+\xi)}\right) \\ G_R^{(n)} &= \int_0^\infty \frac{d\kappa}{2\pi}\left(g_{R\kappa} e^{-i\kappa(\tau+\xi)} + g_{R\kappa}^* e^{i\kappa(\tau+\xi)}\right), \end{aligned} \tag{95}$$

with the Minkowski ones

$$\begin{aligned} F_M^{(n+1)} &= \int_0^\infty \frac{dk}{2\pi}\left(f_k e^{-ik(t+x)} + f_\kappa^* e^{i\kappa(t+x)}\right) \\ G_M^{(n)} &= \int_0^\infty \frac{dk}{2\pi}\left(g_k e^{-k(t+x)} + g_\kappa^* e^{ik(t+x)}\right). \end{aligned} \tag{96}$$

Inverting the equations (95) using the light-cone variable z defined in (45):

$$\begin{aligned} f_{R\kappa} &= \frac{i}{2\kappa}\int_{-\infty}^{\infty} d\xi\, e^{i\kappa(\tau+\xi)} \overset{\leftrightarrow}{\partial}_\tau F_R^{(n+1)}(\tau,\xi) \\ &= \frac{i}{2\kappa}\int_0^{\infty} \frac{dz}{z}\, z^{-n-1+i\kappa}\,(-i\kappa+n+1+z\partial_z)\, F_M^{(n+1)}(z). \\ g_{R\kappa} &= \frac{i}{2\kappa}\int_{-\infty}^{\infty} d\xi\, e^{i\kappa(\tau+\xi)} \overset{\leftrightarrow}{\partial}_\tau G_R^{(n)}(\tau,\xi) \\ &= \frac{i}{2\kappa}\int_0^{\infty} \frac{dz}{z}\, z^{n+i\kappa}\,(-i\kappa-n+z\partial_z)\, G_M^{(n)}(z), \end{aligned} \tag{97}$$

Substitutions of the Minkowski-frame plane-wave solutions yields

$$\begin{aligned} f_{R\kappa} &= i^{(n+1)}\Gamma(n+1+i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{-n-1-i\kappa}\left((-1)^{n+1}e^{\pi\kappa/2}f_k + e^{-\pi\kappa/2}f_k^*\right), \\ g_{R\kappa} &= i^{n}\Gamma(-n+i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{n-i\kappa}\left(e^{\pi\kappa/2}g_k + (-1)^{n}e^{-\pi\kappa/2}g_k^*\right), \end{aligned} \tag{98}$$

with conjugates

$$\begin{aligned} f_{R\kappa}^* &= i^{(n+1)}\Gamma(n+1-i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{-n-1+i\kappa}\left((-1)^{n+1}e^{-\pi\kappa/2}f_k + e^{\pi\kappa/2}f_k^*\right), \\ g_{R\kappa}^* &= i^{n}\Gamma(-n-i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{n+i\kappa}\left(e^{-\pi\kappa/2}g_k + (-1)^{n}e^{\pi\kappa/2}g_k^*\right). \end{aligned} \tag{99}$$

In the L-wedge of Rindler space one finds by analogous calculations for the left-moving fields

$$\begin{aligned} f_{L\kappa} &= i^{(n+1)}\Gamma(n+1-i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{-n-1+i\kappa}\left(e^{\pi\kappa/2}f_k + (-1)^{n+1}e^{-\pi\kappa/2}f_k^*\right), \\ g_{L\kappa} &= i^{n}\Gamma(-n-i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{n+i\kappa}\left((-1)^{n}e^{\pi\kappa/2}g_k + e^{-\pi\kappa/2}g_k^*\right). \end{aligned} \tag{100}$$

and conjugates

$$\begin{aligned} f_{L\kappa}^* &= i^{(n+1)}\Gamma(n+1+i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{-n-1-i\kappa}\left(e^{-\pi\kappa/2}f_k + (-1)^{n+1}e^{\pi\kappa/2}f_k^*\right), \\ g_{L\kappa}^* &= i^{n}\Gamma(-n+i\kappa)\int_0^{\infty}\frac{dk}{2\pi}\, k^{n-i\kappa}\left((-1)^{n}e^{-\pi\kappa/2}g_k + e^{\pi\kappa/2}g_k^*\right). \end{aligned} \tag{101}$$

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Upon application to the Minkowski vacuum it follows that

$$e^{\pi\kappa/2} f_{R\kappa}|0\rangle_M = (-1)^{n+1} e^{-\pi\kappa/2} f^*_{L\kappa}|0\rangle_M, \quad e^{-\pi\kappa/2} f^*_{R\kappa}|0\rangle_M = (-1)^{n+1} e^{\pi\kappa/2} f_{L\kappa}|0\rangle_M,$$

$$(-1)^n e^{\pi\kappa/2} g_{R\kappa}|0\rangle_M = e^{-\pi\kappa/2} g^*_{L\kappa}|0\rangle_M, \qquad (-1)^n e^{-\pi\kappa/2} g^*_{R\kappa}|0\rangle_M = e^{\pi\kappa/2} g_{L\kappa}|0\rangle_M. \tag{102}$$

Applying these results to the ghost system (71), which fits the system with $n = 1$ [4, 20, 21], and obeying the anti-commutation rules

$$\{c_{R\kappa}, d^*_{R\sigma}\} = \{c_{L\kappa}, d^*_{L\sigma}\} = \{d_{R\kappa}, c^*_{R\sigma}\} = \{d_{L\kappa}, c^*_{L\sigma}\} = 2\pi\delta(\kappa - \sigma), \tag{103}$$

we can compute the two-point correlations of the Rindler ghosts in the Minkowski vacuum:

$$\begin{aligned} {}_M\langle 0|b_{R\sigma} c^*_{R\kappa}|0\rangle_M &= {}_M\langle 0|c_{R\sigma} b^*_{R\kappa}|0\rangle_M = \frac{2\pi\delta(\kappa-\sigma)}{1 - e^{-2\pi\kappa}}, \\ {}_M\langle 0|c^*_{R\kappa} b_{R\sigma}|0\rangle_M &= {}_M\langle 0|b^*_{R\kappa} c_{R\sigma}|0\rangle_M = -\frac{2\pi\delta(\kappa-\sigma)}{e^{2\pi\kappa} - 1}. \end{aligned} \tag{104}$$

Note that these results are independent of the value of n, but do reflect the ghost statistics leading to the minus sign in the last expression.

Finally we turn to the commuting superghosts (S, U). The analysis proceeds parallel to the ghost system (C, D), except that the appropriate value Lorentz and conformal weight is $n = 1/2$ [4, 22] and the anti-commutation rules are replaced by commutation rules. The relevant results are

$$e^{\pi\kappa/2} u_{R\kappa}|0\rangle_M = (-1)^{n+1} e^{-\pi\kappa/2} u^*_{L\kappa}|0\rangle_M, \quad e^{-\pi\kappa/2} u^*_{R\kappa}|0\rangle_M = (-1)^{n+1} e^{\pi\kappa/2} u_{L\kappa}|0\rangle_M,$$

$$(-1)^n e^{\pi\kappa/2} s_{R\kappa}|0\rangle_M = e^{-\pi\kappa/2} s^*_{L\kappa}|0\rangle_M, \qquad (-1)^n e^{-\pi\kappa/2} s^*_{R\kappa}|0\rangle_M = e^{\pi\kappa/2} s_{L\kappa}|0\rangle_M, \tag{105}$$

supplemented by the commutation rules

$$[s_{R\kappa}, u^*_{R\sigma}] = [s_{L\kappa}, u^*_{L\sigma}] = [u_{R\kappa}, s^*_{R\sigma}] = [u_{L\kappa}, s^*_{L\sigma}] = 2\pi\delta(\kappa - \sigma). \tag{106}$$

From these equations one derives the Minkowski two-point functions for the superghosts:

$$\begin{aligned} {}_M\langle 0|u_{R\sigma} s^*_{R\kappa}|0\rangle_M &= {}_M\langle 0|s_{R\sigma} u^*_{R\kappa}|0\rangle_M = \frac{2\pi\delta(\kappa-\sigma)}{1 + e^{-2\pi\kappa}}, \\ {}_M\langle 0|s^*_{R\kappa} u_{R\sigma}|0\rangle_M &= {}_M\langle 0|u^*_{R\kappa} s_{R\sigma}|0\rangle_M = -\frac{2\pi\delta(\kappa-\sigma)}{e^{2\pi\kappa} + 1}. \end{aligned} \tag{107}$$

9. Local conformal invariance in Rindler space

With the results (104) and (107) we can now compute the Minkowski expectation values of the (super-)Virasoro operators including the ghost contributions

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required for local conformal and superconformal invariance. First, the results (68) are supplemented for $\kappa \geq 0$ by

$$\begin{aligned}
L^g_{R\kappa} &= \int_0^\infty \frac{d\sigma}{2\pi}\,[(\sigma-\kappa)\,c^*_\sigma d_{\kappa+\sigma} + (\sigma+2\kappa)\,d^*_\sigma c_{\kappa+\sigma}] - \int_0^\kappa \frac{d\sigma}{2\pi}\,(\sigma+\kappa)\,c_\sigma d_{\kappa-\sigma},\\
L^{sg}_{R\kappa} &= \int_0^\infty \frac{d\sigma}{2\pi}\left[\left(\sigma-\frac{\kappa}{2}\right)s^*_\sigma u_{\kappa+\sigma} + \left(\sigma+\frac{3\kappa}{2}\right)u^*_\sigma s_{\kappa+\sigma}\right]\\
&\quad - \int_0^\kappa \frac{d\sigma}{2\pi}\left(\sigma+\frac{\kappa}{2}\right)s_\sigma u_{\kappa-\sigma},\\
G^{sg}_{R\kappa} &= \int_0^\infty \frac{d\sigma}{2\pi}\,[s^*_\sigma b_{\kappa+\sigma} + b^*_\sigma s_{\kappa+\sigma} + (\sigma+3\kappa)u^*_\sigma c_{\kappa+\sigma} + (\sigma-2\kappa)c^*_\sigma u_{\kappa+\sigma}]\\
&\quad + \int_0^\kappa \frac{d\sigma}{2\pi}\,[s_\sigma b_{\kappa-\sigma} - (\sigma+2\kappa)\,c_\sigma u_{\kappa-\sigma}]\,,
\end{aligned} \tag{108}$$

with $(L^g_R)_{-\kappa} = L^{g*}_{R\kappa}$, $(L^{sg}_R)_{-\kappa} = L^{sg*}_{R\kappa}$ and $(G^{sg}_R)_{-\kappa} = G^{sg*}_{R\kappa}$. Together with the operators (68) for each scalar and spinor field they define the same algebra (89) in Rindler space. Thus the first-class constraints requiring the cancellation of the anomalies are also satisfied for $2N_b + N_f = 52$ bosons and fermions coupled to 2-D conformal gravity and $N_b = N_f = 10$ bosons and fermions coupled to 2-D conformal supergravity.

As in the case of global conformal invariance, the Minkowski expectation values of the conformal charges are modified due to finite temperature effects. With the help of equations (105) and (107) the results (69) are now extended by

$${}_M\langle 0|L^g_{R\kappa}|0\rangle_M = -\frac{1}{12}\,\delta(\kappa), \qquad {}_M\langle 0|L^{sg}_{R\kappa}|0\rangle_M = -\frac{1}{24}\,\delta(\kappa). \tag{109}$$

For the full Virasoro charges, simultaneously requiring cancellation of the anomalies:

$$\begin{aligned}
{}_M\langle 0|\left(N_b L^b_{R\kappa} + N_f L^f_{R\kappa} + L^g_{R\kappa}\right)|0\rangle_M &= \delta(\kappa),\\
{}_M\langle 0|\left(N_b L^b_{R\kappa} + N_f L^f_{R\kappa} + L^g_{R\kappa} + L^{sg}_{R\kappa}\right)|0\rangle_M &= \frac{1}{2}\,\delta(\kappa).
\end{aligned} \tag{110}$$

These expectation values, restricted to $\kappa = 0$, do not contradict the first-class constraints, at the same time showing that the full hamiltonian in Rindler space has a non-vanishing Minkowski expectation value as a result of finite-temperature correlations (104), (107). As discussed before, a regularization procedure is required to deal with the δ-function singularity of the expectation values in applications.

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10. Conclusions

Two-dimensional conformal field theories can be formulated in an inertial frame with a Minkowski metric, or in a uniformly accelerated frame with a local conformal Rindler metric defined by $\rho = e^{\pm a\xi}$; here the sign differentiates between the right and left wedge of Rindler space. Thus in each wedge one obtains two different descriptions of the same theory. The switch between the frames not only changes the co-ordinates, but Fock-space quantization can proceed parallel in both frames, provided an appropriate conformal transformation is applied to the field components as well. The Fock spaces in the two frames are related by Bogoliubov transformations of the mode operators, effectively giving rise to finite-temperature correlations of the Rindler modes for states build on the Minkowski vacuum, which is the true ground state of the theory. The temperature is proportional to the acceleration, an effect well-known from the original work of Unruh, Davies, Wald, Fulling and others for accelerated frames in Minkowski space, and the work of Hawking in the context of black hole space-times.

The switch to the Rindler frame not only creates an effective finite-temperature description, but changes the construction and action of symmetry operators like conformal and supersymmetry charges on the ground state. The naive conformal and supersymmetries in the Rindler frame are broken by finite-temperature effects in the Minkowski vacuum, but the underlying Minkowski symmetries are still present even though hidden by the Bogoliubov transformations.

An additional, different issue is the breaking of conformal and supersymmetries by anomalies, extensively studied in the context of string theory. In the field-theory view taken here they break the local conformal and supersymmetry of arising from coupling conformal matter to conformally invariant gravity or supergravity. Thus consistent coupling to gravity or supergravity is possible only in the case of specific critical theories with a strongly limited spectrum of boson and fermion matter fields, reflecting the critical dimensions of quantum string theories. This is also relevant as the Minkowski-frame and Rindler-frame formulations of massless QFT's are related by a conformal transformation; if this transformation is jeopardized by the conformal anomaly in theories with a non-critical spectrum it may require compensating Wess-Zumino type dynamics [23], e.g. involving a Liouville field.

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Appendix A: 2-D spinor conventions

In this appendix and in the main body of the paper we denote by greek indices $\mu, \nu, ...$ vector en tensor components in an arbitrary space-time manifold with metric $g_{\mu\nu}$; latin indices $a, b, ...$ refer to vector and tensor components in a tangent Minkowski space with mertric η_{ab}. The Majorana representation of the Dirac algebra in 2-dimensional Minkowski space is defined in terms of the 2×2 Pauli matrices as follows:

$$\gamma^0 = i\sigma_2, \qquad \gamma^1 = \sigma_1, \qquad \gamma_3 = \gamma^0\gamma^1 = \sigma_3. \tag{111}$$

In 2-dimensional Minkowski space a Dirac spinor is a 2-component object transforming under Lorentz transformations with parameter $\omega^{ab} = \omega\varepsilon^{ab}$ as

$$\lambda' = e^{\omega^{ab}\sigma_{ab}/2}\lambda = e^{\omega\gamma_3/2}\lambda, \qquad \sigma_{ab} = \frac{1}{4}\left[\gamma_a, \gamma_b\right] = -\frac{1}{2}\,\varepsilon_{ab}\gamma_3. \tag{112}$$

Dirac-conjugate spinors are defined by $\bar{\lambda} = \lambda^\dagger\gamma_0$. The charge-conjugation operator is $C = -C^T = \gamma_0$ with the defining property

$$C\gamma^a C^{-1} = -\gamma^{a\,T}. \tag{113}$$

A self-conjugate Majorana spinor $\lambda = \lambda^c = C\bar{\lambda}^T = \lambda^*$ is therefore a spinor with real components:

$$\lambda = \lambda^c = \begin{bmatrix} \lambda_+ \\ \lambda_- \end{bmatrix}, \qquad \lambda_\pm^* = \lambda_\pm. \tag{114}$$

The subscripts $\pm$ define the eigenvalues under the chiral operator γ_3. Evidently

$$\bar{\lambda} = \lambda^T C. \tag{115}$$

In a general 2-D space-time is the spinors are defined as representations of the local Lorentz group, transforming like (112) with a space-time dependent parameter $\omega(x)$. At the same time a spinor transforms like a scalar under diffeomorphisms: $\lambda'(x') = \lambda(x)$. To construct the Dirac-operator one uses the 2-bein fields $e_\mu^a(x)$ such that the metric is factorized as

$$g_{\mu\nu} = e_\mu^a e_\nu^b\,\eta_{ab}. \tag{116}$$

In terms of the 2-bein fields and their inverse components e_a^μ the spin connection, acting as the gauge field for local Lorentz transformations, is defined by an extension of the metric postulate

$$\nabla_\mu e_\nu^a = \partial_\mu e_\nu^a - \Gamma_{\mu\nu}^{\ \lambda} e_\lambda^a = \omega_\mu^{\ a}{}_b e_\nu^b, \tag{117}$$

implying that

$$\begin{aligned} \omega_\mu^{\ ab} = -\omega_\mu^{\ ba} \;\; = \;\; & \frac{1}{2}\left[e^{\nu a}\left(\partial_\nu e_\mu^{\ b} - \partial_\mu e_\nu^{\ b}\right) - e^{\nu b}\left(\partial_\nu e_\mu^{\ a} - \partial_\mu e_\nu^{\ a}\right)\right. \\ & \left. + e^{\lambda a} e^{\nu b} e_{\mu c}\left(\partial_\lambda e_\nu^{\ c} - \partial_\nu e_\lambda^{\ c}\right)\right]. \end{aligned} \tag{118}$$

In terms of these objects the full space-time and Lorentz covariant derivative of a spinor field is

$$D_\mu \lambda = \left(\partial_\mu - \frac{1}{2}\, \omega_\mu^{\ ab} \sigma_{ab} \right) \lambda, \tag{119}$$

the Dirac operator being defined by $\not{D} = \gamma^a e_a^{\ \mu} D_\mu$.

Next turn to the special class of conformally flat metrics (1). For these metrics (in a hybrid notation)

$$e_\mu^{\ a} = \rho\, \delta_\mu^a, \qquad e_a^{\ \mu} = \frac{1}{\rho}\, \delta_a^\mu. \tag{120}$$

It follows that the components of the spin connection are

$$\omega_\mu^{\ ab} = \left(\delta_\mu^b \eta^{a\nu} - \delta_\mu^a \eta^{b\nu} \right) \partial_\nu \ln \rho. \tag{121}$$

Specifically in 2-D space-time with conformally flat co-ordinates (τ, ξ):

$$\omega_\tau^{\ 01} = -\omega_\tau^{\ 10} = -\partial_\xi \ln \rho, \quad \omega_\xi^{\ 01} = -\omega_\xi^{\ 10} = -\partial_\tau \ln \rho. \tag{122}$$

Combining this result with (119) and (112), while noting that

$$\sqrt{-g} = \det e_\mu^{\ a} = \rho^2,$$

one derives the result (35):

$$\sqrt{-g}\, C \not{D} = \sqrt{\rho} \begin{pmatrix} \partial_\tau - \partial_\xi & 0 \\ 0 & \partial_\tau + \partial_\xi \end{pmatrix} \sqrt{\rho}.$$

But this is not yet the final result; starting form the Minkowski 2-bein $e_{M\mu}^{\ \ a} = \delta_\mu^a$, the Rindler 2-bein is obtained by

$$e_{R\mu}^{\ \ a} = \frac{\partial x_M^\lambda}{\partial x_R^\mu} e_{M\lambda}^{\ \ b}\, \Omega_b^{\ a}, \tag{123}$$

where in the R-wedge of Rindler space

$$\frac{\partial x_M^\lambda}{\partial x_R^\mu} = e^{a\xi} \begin{pmatrix} \cosh a\tau & \sinh a\tau \\ \sinh a\tau & \cosh a\tau \end{pmatrix}$$

is the Jacobian of the transformation from Minkowski to Rindler co-ordinates, and $\Omega_b^{\ a}$ is a local Lorentz transformation. Clearly the Jacobian is not diagonal, but this can be restored by taking the local Lorentz transformation

$$\Omega = \begin{pmatrix} \cosh a\tau & -\sinh a\tau \\ -\sinh a\tau & \cosh a\tau \end{pmatrix}.$$

This puts the 2-bein back to the diagonal form $e_{R\mu}^{\ \ a} = e^{a\xi} \delta_\mu^a$. For the L-wedge a similar procedure can be followed. As a result, the use of a diagonal vierbein in

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Rindler space must be accompanied by a corresponding local Lorentz transformation on spinor fields; in the present case this takes the form

$$\lambda' = e^{a\tau\gamma_3/2}\lambda. \tag{124}$$

This explains why in the R- and L-wedges of Rindler space the redefinition (36) of the spinor fields by

$$\begin{aligned} R: \qquad & \psi_\pm(\tau,\xi) = e^{\pm a(\tau\pm\xi)/2}\lambda_\pm[t(\tau,\xi),x(\tau,\xi)], \\ L: \qquad & \psi_\pm(\tau,\xi) = e^{\mp a(\tau\pm\xi)/2}\lambda_\pm[t(\tau,\xi),x(\tau,\xi)], \end{aligned}$$

reduces the Rindler Dirac-Majorana action formally to the Minkowski one.

Appendix B: Thermal correlations in Rindler space

The computation of the two-point correlation functions of the Rindler fields in the Minkowski vacuum start from equations (49) and (53):

$$\begin{aligned} b_{R\kappa}|0\rangle_b &= e^{-\pi\kappa}b^*_{L\kappa}|0\rangle_b, \qquad b^*_{R\kappa}|0\rangle_b = e^{\pi\kappa}b_{L\kappa}|0\rangle_b, \\ \beta_{R\kappa}|0\rangle_f &= -ie^{-\pi\kappa}\beta^*_{L\kappa}|0\rangle_f, \quad \beta^*_{R\kappa}|0\rangle_f = ie^{\pi\kappa}\beta_{L\kappa}|0\rangle_f, \end{aligned} \tag{125}$$

and their conjugates:

$$\begin{aligned} {}_b\langle 0|b^*_{R\kappa} &= e^{-\pi\kappa}{}_b\langle 0|b_{L\kappa}, \qquad {}_b\langle 0|b_{R\kappa} = e^{\pi\kappa}{}_b\langle 0|b^*_{L\kappa}, \\ {}_f\langle 0|\beta^*_{R\kappa} &= ie^{-\pi\kappa}{}_f\langle 0|\beta_{L\kappa}, \quad {}_f\langle 0|\beta_{R\kappa} = -ie^{\pi\kappa}{}_f\langle 0|\beta^*_{L\kappa}. \end{aligned} \tag{126}$$

The following chain of commutation relations and substitutions leads to the desired result:

$$\begin{aligned} {}_b\langle 0|b_{R\kappa}b^*_{R\sigma}|0\rangle_b &= 2\pi\delta(\kappa-\sigma) + {}_b\langle 0|b^*_{R\sigma}b_{R\kappa}|0\rangle_b \\ &= 2\pi\delta(\kappa-\sigma) + e^{-\pi(\kappa+\sigma)}{}_b\langle 0|b_{L\sigma}b^*_{L\kappa}|0\rangle_b \\ &= 2\pi\delta(\kappa-\sigma)\,(1+e^{-2\pi\kappa}) + e^{-\pi(\kappa+\sigma)}{}_b\langle 0|b^*_{L\kappa}b_{L\sigma}|0\rangle_b \\ &= 2\pi\delta(\kappa-\sigma)\,(1+e^{-2\pi\kappa}) + e^{-2\pi(\kappa+\sigma)}{}_b\langle 0|b_{R\kappa}b^*_{R\sigma}|0\rangle_b. \end{aligned} \tag{127}$$

The solution of this equation is (55) and implies (56):

$$\begin{aligned} {}_b\langle 0|b_{R\kappa}b^*_{R\sigma}|0\rangle_b &= 2\pi\delta(\kappa-\sigma) + {}_b\langle 0|b^*_{R\sigma}b_{R\kappa}|0\rangle_b = \frac{2\pi\delta(\kappa-\sigma)}{1-e^{-2\pi\kappa}}, \\ {}_b\langle 0|b^*_{R\sigma}b_{R\kappa}|0\rangle_b &= \frac{2\pi\delta(\kappa-\sigma)}{e^{2\pi\kappa}-1}. \end{aligned} \tag{128}$$

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The derivation of the fermion two-point function proceeds entirely parallel, using anti-commutation relations instead of commutation relations, which results in a change of sign in the denominator.

The relations (125), (126) also imply a direct relation between the common Rindler vacuum state $|0\rangle_R = |0\rangle_{RR} \otimes |0\rangle_{RL}$:

$$b_{R\kappa}|0\rangle_{RR} = \beta_{R\kappa}|0\rangle_{RR} = 0, \qquad b_{L\kappa}|0\rangle_{RL} = \beta_{L\kappa}|0\rangle_{RL} = 0, \tag{129}$$

and the Minkowski vacuum:

$$|0\rangle_M = \exp\left[\int_0^\infty \frac{d\kappa}{2\pi}\, e^{-\pi\kappa}\, (b^*_{R\kappa} b^*_{L\kappa} - i\beta^*_{R\kappa}\beta^*_{L\kappa})\right] |0\rangle_R. \tag{130}$$

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