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Article

Ultimate compressive strength of stiffened panel: An empirical formulation for flat-bar type

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Abstract: This research aims to study the ultimate limit state (ULS) behaviour of stiffened panel under longitudinal compression by non-linear finite element method (NLFEM). There are different types of stiffeners being used in shipbuilding i.e. T-bar, flat-bar and angle-bar. However, this research focuses on the ultimate compressive strength behaviour of flat-bar stiffened panel. A total of 420 of reliable scenarios of flat-bar stiffened panel are selected for numerical simulation by ANSYS NLFEM. The ultimate strength behaviours obtained were used as data for the development of closed form shape empirical formulation. Recently, Kim et al. [1] proposed for advanced empirical formulation for T-bar stiffened panel and the applicability of the proposed formulation to flat-bar stiffened panel will be confirmed by this study. The accuracy of the empirical formulation obtained for flat-bar stiffened panel has been validated by FE simulation results of statistical analysis (R^2 = 0.9435). The outcome obtained will be useful for ship structural designers in predicting the ultimate strength performance of flat-bar type stiffened panel under longitudinal compression.

Keywords: ocean and shore technology (OST); empirical formula; ultimate limit state; longitudinal compression; stiffened plate; ships and offshore structures; structural design.

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1. Introduction

It is common that the stiffened and unstiffened panels are used for primary structural supporting members in the field of ships and offshore engineering. In general, mild (MS24) and high tensile (AH32 or AH 36) steels are being used as construction materials for voyage in Southern Sea Route (SSR). While in the case of Northern Sea Route (NSR) which was opened due to global warming effect, the different grades of the steels i.e. B, D, E or F are recommended to be used [2-4].

A wide range of studies in assessing and predicting the structural condition of intact and damaged structures have been conducted by many researchers for the robust design of ships and offshore structures. In particular, the Finite Element Method (FEM), one of the famous numerical methods, is considered as powerful technique to solve issues in various fields such as engineering, medical and etc. Moreover, the Computational Fluid Dynamics (CFD) based and Fluid-Structure Interaction (FSI) based numerical simulations are also getting more and more popular in structural design following the development of computer technology [5].

The experimental and analytical methods, meanwhile, are also considered as the useful ways to resolve the engineering issues, provided if there is adequate financial support and time frame given subject to physical limitation such as equipment's size and testing area issues. In the case of analytical

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methods, while it provides absolute solution, it has limitation of not covering complex geometries with actual environmental conditions [6].

In this regards, the design formulation or the empirical formulation approach is adopted in rule book by major classification societies such as Lloyd's Register (LR), American Bureau of Shipping (ABS), Det Norske Veritas Germanischer Lloyd (DNV GL) and followed by major shipyards in general. A number of empirical formulations and simplified techniques were proposed by various researchers in terms of ultimate limit state of intact and damaged hull girders [7-14], unstiffened panel (= plate) [15-17], stiffened panel [1, 18-22] and many others.

Recently, historical and technical reviews on existing empirical formulations in predicting ULS have been conducted for unstiffened and stiffened panels [23-24]. In the case of stiffened panel which is made of steel and aluminium, most of the existing empirical formulations [1, 21, 25-29] were basically developed as a function of two parameters such as plate slenderness ratio (β) and column slenderness ratio (λ) by adopting the plate stiffener combination (PSC) model.

More recently, Kim et al. [21] addressed that ultimate compressive strength of stiffened panel tends to fluctuate in the lower range of the column of slenderness ratio (λ) as shown in Fig. 1. This means that stiffened panel has a higher nonlinearity with the combination of plate and stiffener's geometries. Therefore, simplified or single line shape empirical formulation, such as a function of β and λ, may not be accurate in representing the ultimate strength behaviour of stiffened panel. Moreover, additional parameters should be considered in predicting the accurate ultimate compressive strength behaviour of stiffened panel.

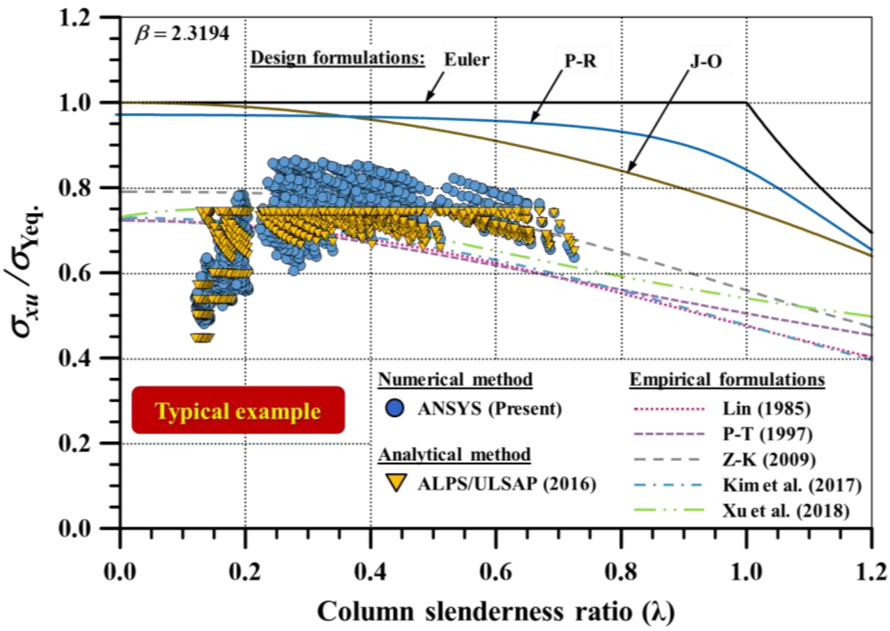

Figure 1. Typical example of the existing design and empirical formulations in predicting the ultimate limit state (ULS) of T-bar stiffened panel [20].

In this regard, Kim et al. [1] introduced a refined empirical formulation in predicting the ultimate compressive strength of T-bar type stiffened panel as shown in Fig. 2 by using conventional data processing technique with four (4) parameters such as plate slenderness ratio (β), column slenderness ratio (λ), web slenderness ratio (h_w/t_w) and moment of inertia of stiffener to moment of inertia of plate ratio in z-direction (vertical) (I_{sz}/I_{pz}) as shown in Table A1. From the wide range of the numerical simulations (in total 10,500 scenarios of T-bar stiffened panel), an advanced empirical formulation has been proposed [1] with reliable accuracy of ULS compared by ANSYS FE numerical simulation results (R^2 = 0.98).

Most of the existing empirical formulations can be used in predicting ULS of T-bar stiffened panel under longitudinal compression. However, there are limited studies conducted on flat- and angle-

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bar stiffened panel. In this regards, 540 numerical simulations have been conducted by ANSYS Non-Linear Finite Element Method (NLFEM) to obtain the ultimate strength of flat-bar type stiffened panel under longitudinal compression. The ULS results of flat-bar stiffened panel have been utilised as the input for the data processing. In addition, the applicability of the empirical formulation proposed by Kim et al. [1] has also been tested whether it can be fitted for flat-bar or otherwise.

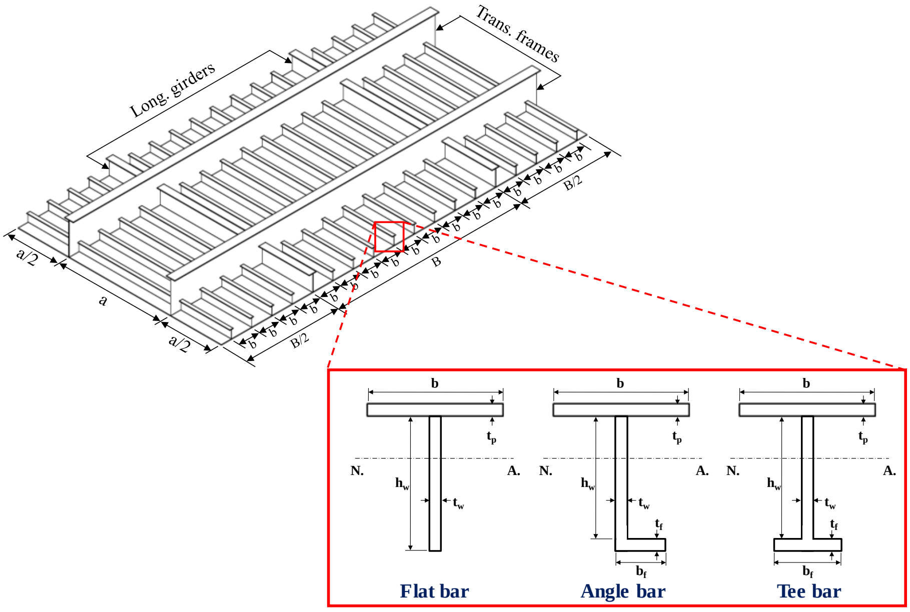

Fig. 2 Schematic view of the stiffened panel with three types of stiffeners.

Finally, the accuracy of the refined empirical formulation for flat-bar stiffened panel obtained in this study has been verified by statistical analysis. The applicability of the outcome obtained from this study has been verified by ANSYS FE numerical simulations as well as existing empirical formulations for flat-bar stiffened panel by adopting single line shape formulations as proposed by Paik [27], Xu et al. [29] and Khedmati et al. [30].

A useful outcome is achieved in predicting the ULS of flat-bar stiffened panel which is one of the primary structural components of ships and offshore structures.

2. Brief review of the existing formulations

As mentioned earlier, recently Zhang [23] and Kim et al. [24] provided detailed technical reviews on existing design and empirical formulations to predict ultimate strength of the stiffened panel. The details of the same can be found in the articles mentioned above.

In this study, the representative existing empirical formulations have been addressed in this section. The formulations introduced in this paper will be used for comparison with FE numerical simulation by statistical analysis in the following section. In general, the empirical formulations in predicting the Ultimate Limit State (ULS) of the stiffened panel formulates as a function of plate slenderness ratio (β) and column slenderness ratio (λ) as shown in Eq. (1).

$$\sigma_{xu} / \sigma_{Yeq.} = f(\beta, \lambda) \tag{1}$$

where, σ_{xu} = ultimate compressive strength in x-axis (= under longitudinal compression), $\sigma_{Yeq.}$ = Equivalent yield strength of plate and stiffener, β = plate slenderness ratio ($=(b_p/t_p)\cdot\sqrt{(\sigma_{Yp}/E)}$), λ = column slenderness ratio ($=[L/(\pi\cdot r)]\cdot\sqrt{(\sigma_{Yeq.}/E)}$), σ_{Yp} = Yield strength of plate, E = Young's

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modulus, L = Length of stiffened panel, r = Radius of gyration ($=\sqrt{(I/A)}$), I and A = moment of inertia and sectional area for plate-stiffener combination (PSC) model's section respectively.

By assuming the basic format of empirical formulations illustrated in Eq. (1), a number of empirical formulations (or closed form shape formulation) have been proposed by experimental and numerical methods. The common design formulations used in shipyard are presented in Eqs. (2.1) to (2.3). Details on existing design formulations may refer to Paik [16].

Euler formulation

$$\sigma_E / \sigma_{Yeq.} = \begin{cases} 1.0 & for \quad \lambda \le 1.0 \\ 1/\lambda^2 & for \quad \lambda > 1.0 \end{cases} \tag{2.1}$$

Johnson-Ostenfeld (J-O) formulation

$$\sigma_{cr} / \sigma_{Yeq.} = \begin{cases} 1/\lambda^2 & for \quad \sigma_E/\sigma_{Yeq.} \le 0.5 \\ 1-\lambda^2/4 & for \quad \sigma_E/\sigma_{Yeq.} > 0.5 \end{cases} \tag{2.2}$$

Perry-Robertson (P-R) formulation under axial compression

$$\sigma_{xu} / \sigma_{Yeq.} = \frac{1}{2}\left(1+\frac{1+\eta}{\lambda^2}\right) - \sqrt{\frac{1}{4}\left(1+\frac{1+\eta}{\lambda^2}\right)^2 - \frac{1}{\lambda^2}} \tag{2.3}$$

Lin [25] also suggested generalised shape of the empirical formulation to predict ultimate strength of stiffened panel subject to longitudinal compression as shown in Eq. (3.1). Thereafter, Paik and Thayamballi [26] proposed for the revised empirical formulation based on test database collected. In addition, they set the upper limit of the empirical formulation of which the ultimate compressive strength of stiffened panel may not exceed the elastic buckling stress ($\sigma_{Yeq.} / \lambda^2$) as shown in Eq. (3.2). The coefficients consisting of the Eq. (3.1) and (3.2) are summarised in Table 1.

General shape of the empirical formulation

$$\sigma_{xu} / \sigma_{Yeq.} = \frac{1}{\sqrt{c_1 + c_2\lambda^2 + c_3\beta^2 + c_4\lambda^2\beta^2 + c_5\lambda^4}} \tag{3.1}$$

where, c_1 - c_5 =coefficients which may be referred to Table 1.

Upper limit of empirical formulation

$$\sigma_{xu} / \sigma_{Yeq.} = \frac{1}{\sqrt{c_1 + c_2\lambda^2 + c_3\beta^2 + c_4\lambda^2\beta^2 + c_5\lambda^4}} \le \frac{1}{\lambda^2} \tag{3.2}$$

Table 1. Coefficients to predict ULS.

Coefficient	Lin [26]	P-T [26]
c_1	0.960	0.995
c_2	0.765	0.936
c_3	0.176	0.170
c_4	0.131	0.188
c_5	1.046	-0.067

Note: P-T = Paik and Thayamballi.

More recent empirical formulations are also summarised in Eq. (4) to (6). Zhang and Khan [28] proposed Eq. (4) with limitation of the range of column slenderness ratio ($\lambda \le \sqrt{2}$). Kim et al. [21] also suggested for empirical formulation to be based on numerical simulations which allows to cover the wide range of the column slenderness ratio. Recently, Xu et al. [29] proposed for empirical formulation in predicting the ULS of all types of stiffened panel i.e. T-bar, angle-bar, and flat-bar as shown in Table 2 subject to longitudinal compression as well as lateral pressure.

Z-K formulation [28]

$$\sigma_{xu} / \sigma_{Yeq.} = \frac{1}{\beta^{0.28}\sqrt{1.0+\lambda^{3.2}}} \quad \text{for } \lambda \le \sqrt{2} \text{ range only} \tag{4}$$

Kim's formulation [21]

$$\sigma_{xu} / \sigma_{Yeq.} = \frac{1}{0.8884+e^{\lambda^2}} + \frac{1}{0.4121+e\sqrt{\beta}} \tag{5}$$

Xu's formulation [29]

$$\sigma_{xu} / \sigma_{Yeq.} = \frac{1}{\sqrt{\begin{pmatrix} X_0 + X_1\lambda + X_2\beta + X_3\lambda\beta \\ +X_4\lambda^2 + X_5\beta^2 + X_6\lambda^2\beta^2 \\ +X_7\lambda^3 + X_8\beta^3 + X_9\lambda^3\beta^3 + X_{10}\lambda^4 \end{pmatrix}}} \le \frac{1}{\lambda^2} \tag{6}$$

Details of existing empirical formulations and its technical review may refer to Zhang [23] and Kim et al. [24]. It is highlighted that the existing empirical formulations in Eqs. (1-6) are presented as a single line shape equation and this is one of the important reason that advanced empirical formulation is required in predicting more accurate ULS results.

Table 2 Coefficients in predicting ULS by Xu et al. [29]

Coefficient	T-bar	Angle-bar	Flat-bar
X_o	3.555	1.192	1.127
X_1	-3.577	-1.583	-4.915
X_2	-3.424	-0.355	0.490
X_3	0.999	0.289	0.773
X_4	4.737	3.407	10.075
X_5	1.812	0.462	-0.109
X_6	-0.220	-0.018	-0.140
X_7	-2.584	-2.260	-7.089
X_8	-0.277	-0.084	0.040
X_9	0.017	-0.002	0.010
X_{10}	0.458	0.456	1.564

Recently, Mei and Wang [31] also proposed single line shape empirical formulation in predicting ULS of stiffened panel which is similar shape by Lin [25] and Paik-Thayamballi [26]. However, they limited the maximum order of plate and column slenderness ratios as 2nd and 3rd, respectively. The obtained FE results were limited to propose empirical formulation so that the outcome has not been compared in this study.

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3. Ultimate strength calculations by non-linear finite element method (NLFEM)

Limit state design (LSD), also known as load and resistance factor design (LRFD) is now well known design method in the field of structural engineering. The LSD [16] includes ultimate limit state (ULS), fatigue limit state (FLS), accidental limit state (ALS), and serviceability limit state (SLS). Among others, a number of studies have been conducted on ULS based design, technique and its applications in terms of ULS application to the stiffened panels [32-35], scaling effect [36-37], stiffened panel with opening [38], dynamic ULS [39], low temperature effect [14] and prediction of ULS by artificial neural network (ANN) [40]. The FLS of offshore riser by ANN and simplified method [41-42], the LSD of non-ice class aged ship [43, 44], steel plated structure [45], ship's hull [46] and FLNG [47] are also investigated.

3.1. Selection of scenarios of flat-bar stiffened panel

Recently, Kim et al. [24] conducted a wide range of technical reviews on existing empirical formulation in predicting the ultimate strength of stiffened panel subject to longitudinal compression. In addition, they have tested the accuracy of the existing empirical formulations by conducting 10,500 cases of T-bar stiffened panel numerical simulations using ANSYS NLFEM. The total number of the stiffened panel scenarios were selected as shown in Eq. (7).

$$\underset{a}{1}\times\underset{b}{1}\times\underset{t_p}{7}\times\underset{h_w}{10}\times\underset{t_w}{6}\times\underset{b_f}{5}\times\underset{t_f}{5}=10{,}500 \text{ scenarios} \tag{7}$$

where, a = plate length, b = plate breadth, t_p = plate thickness, h_w = web height, t_w = web thickness, b_f = flange breadth, t_f = flange thickness.

In this study, the flat-bar stiffened panel which consists of plate and web is targeted. It means that flat-bar stiffened panel does not have flange. In this regards, the 10,500 scenarios as mentioned above in Eq. (7) can be reduced to 420 cases through neglecting flange so that the scenarios of flange breadth (b_f) and flange thickness (t_f) can be not considered in this study. The details of 420 scenarios can be summarised in Eq. (8) and Table 3.

$$\underset{a}{1}\times\underset{b}{1}\times\underset{t_p}{7}\times\underset{h_w}{10}\times\underset{t_w}{6}=420 \text{ scenarios} \tag{8}$$

Table 3. Details of selected flat-bar stiffened panels.

Material properties			
	Yield strength	Elastic modulus	Poisson's ratio
Plate / Stiffener	315 MPa	205.8 GPa	0.3
Geometric properties (Unit = mm)			
Plate	Length	4,150	
	breadth	830	
	Thickness	9.5, 11, 14, 16, 21.5, 32.5 and 44.5	
Stiffener	Web height	200, 284, 300, 360, 425, 460, 500, 700, 800 and 1,000	
	Web thickness	10, 11.5, 12.5, 13.5, 20 and 28	
Summary			
Plate slenderness ratio		3.4181, 2.9620, 2.3194, 2.0295, 1.5103, 0.9991 and 0.7297	
Column slenderness ratio (PSC model)		420 cases	

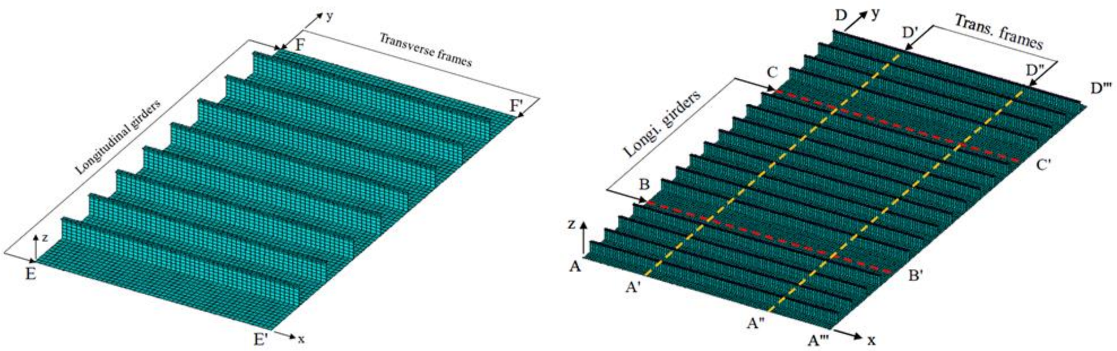
Note: PSC = plate-stiffener combination.

3.2. Structural modelling

It is recognised that ultimate strength behaviour of stiffened and unstiffened panels vary depending on structural modelling technique. It means that the application of appropriate FE structural modelling technique is essentially required in order to obtain realistic outcomes. In particularly, the effect of assumed boundary condition, model size in longitudinal and transverse directions, material modelling technique, mesh size, initial imperfections such as initial deflection and welding-induced residual stress and many other elements should be carefully taken into consideration [48].

ISSC [7] conducted a wide range of parametric studies on ultimate strength of stiffened panel by considering the changes of geometries. They have studied the effect of model size on ULS by selecting One bay/one span and Two bay/two span (1/2-1-1/2 model) stiffened panel models as shown in Fig. 3(a) and 3(b). The details of boundary conditions for both models shown in Fig. 3(a) and 3(b) are summarised in Table 4(a) and 4(b) respectively. As expected, one bay/one span model results in overestimation of the ULS value than two bay/two span model. It is due to the effect of sideways deformation of the stiffeners located at the transverse frames are not allowed for one bay/one span model. This is also caused by the effect of boundary condition assumed in both models. The imperfection sensitivity and geometric effects in stiffened panel is also studied by Ahmer Wadee and Farsi [49].

Based on the findings by ISSC [7], two bay/two span model as shown in Fig. 3(b) and Table 4(b) is adopted in this study with average level of initial deflection for plate and initial distortions for stiffeners. In case of welding-induced residual stress effect, it was not considered in this study. It has reported that 10-13 % decrement of ULS of stiffened panel is expected to be achieved due to the effect of welding-induced residual stress [50]. The number of mesh in plate and web part is 10 and 6 respectively based on mesh convergence study [7, 21].



(a) one bay/one span model (b) two bay/two span model

Figure 3. Modelling of stiffened panel [7].

3.3. Structural analysis and results

A total of 420 numerical simulations by ANSYS were conducted to obtain ULS of flat-bar stiffened panel under longitudinal compression. As shown in Fig. 4, the ULS trends are plotted based on λ and β. As expected, ULS tends to be decreasing when the λ is increasing. When the plate slenderness ratio increases or plate is getting thinner, the ULS tends to have general trend. This is to be discussed further in the following section by comparison with empirical formulation.

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Table 4. Applied boundary condition [7].

(a) One bay/one span model referred to Fig. 3(a)	
E-E′ & F-F′	• Simply supported condition with $R_y = R_z = 0$ and $U_z = 0$ • Uniform displacement in the y-direction (U_y = uniform) coupled with the plate
E-F & E′-F′	• Simply supported condition with $R_x = R_z = 0$ and $U_z = 0$ • Uniform displacement in the x-direction (U_x = uniform) coupled with the longitudinal stiffeners
(b) Two bay/two span model referred to Fig. 3(b)	
A-A′′ & D-D′′′	• Symmetric condition with $R_x = R_y = 0$ • Uniform displacement in the y-direction (U_y = uniform) coupled with the plate
A-D & A′′′-D′′′	• Symmetric condition with $R_y = R_z = 0$ • Uniform displacement in the x-direction (U_x = uniform) coupled with the longitudinal stiffener
A′-D′, A′′-D′′, B-B′& C-C′	• $U_z = 0$

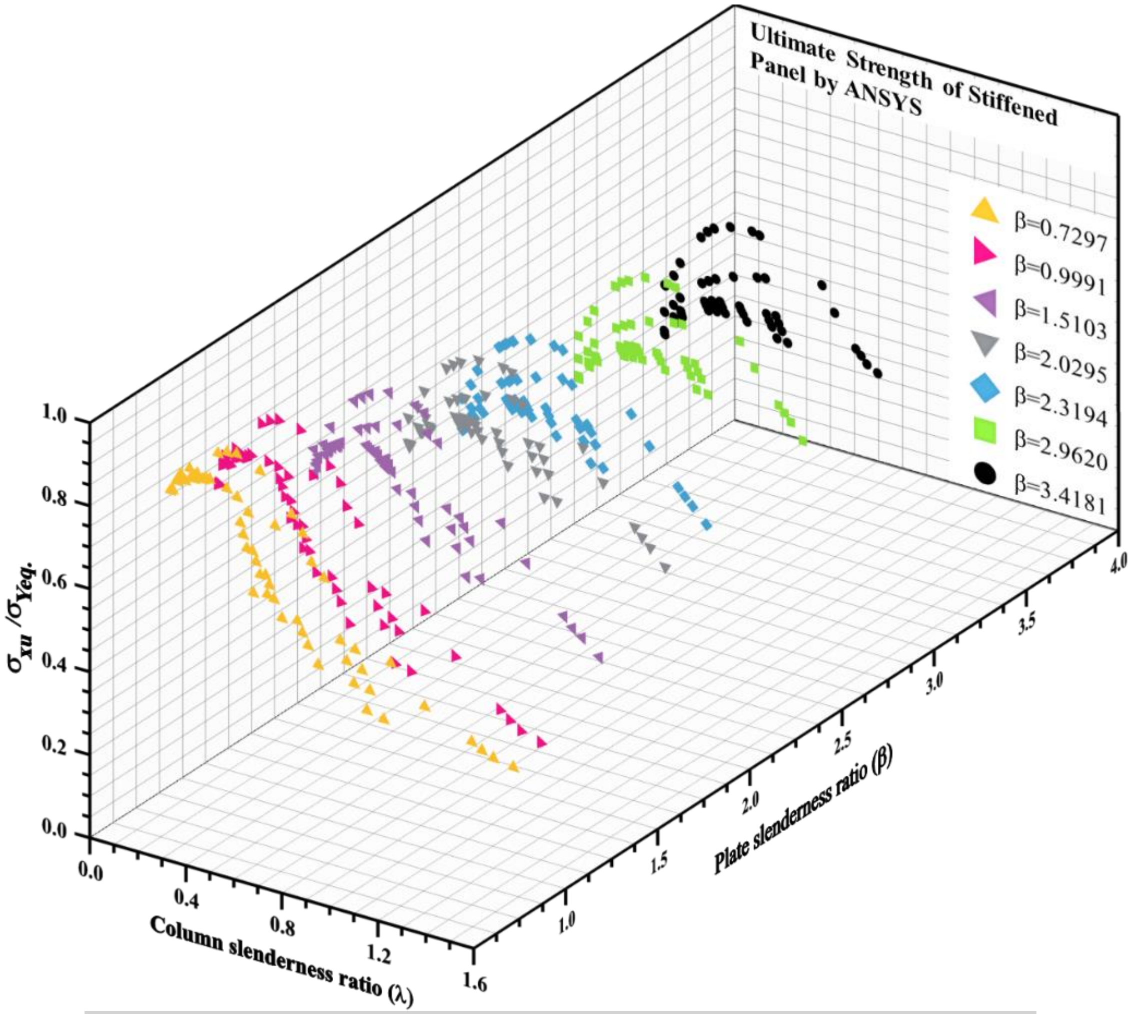


Figure 4. The calculated ULS results by ANSYS nonlinear finite element method.

4. Development of empirical formulation and verification of its applicability

4.1. Empirical formulation for flat-bar stiffened panel

Recently, an advanced empirical formulation was conducted to predict ultimate strength of the T-bar type stiffened panel under longitudinal compression by Kim et al. [1]. A wide range of the numerical simulations of 10,500 cases in total have been conducted by considering the changes of the geometric properties in terms of plate and stiffener.

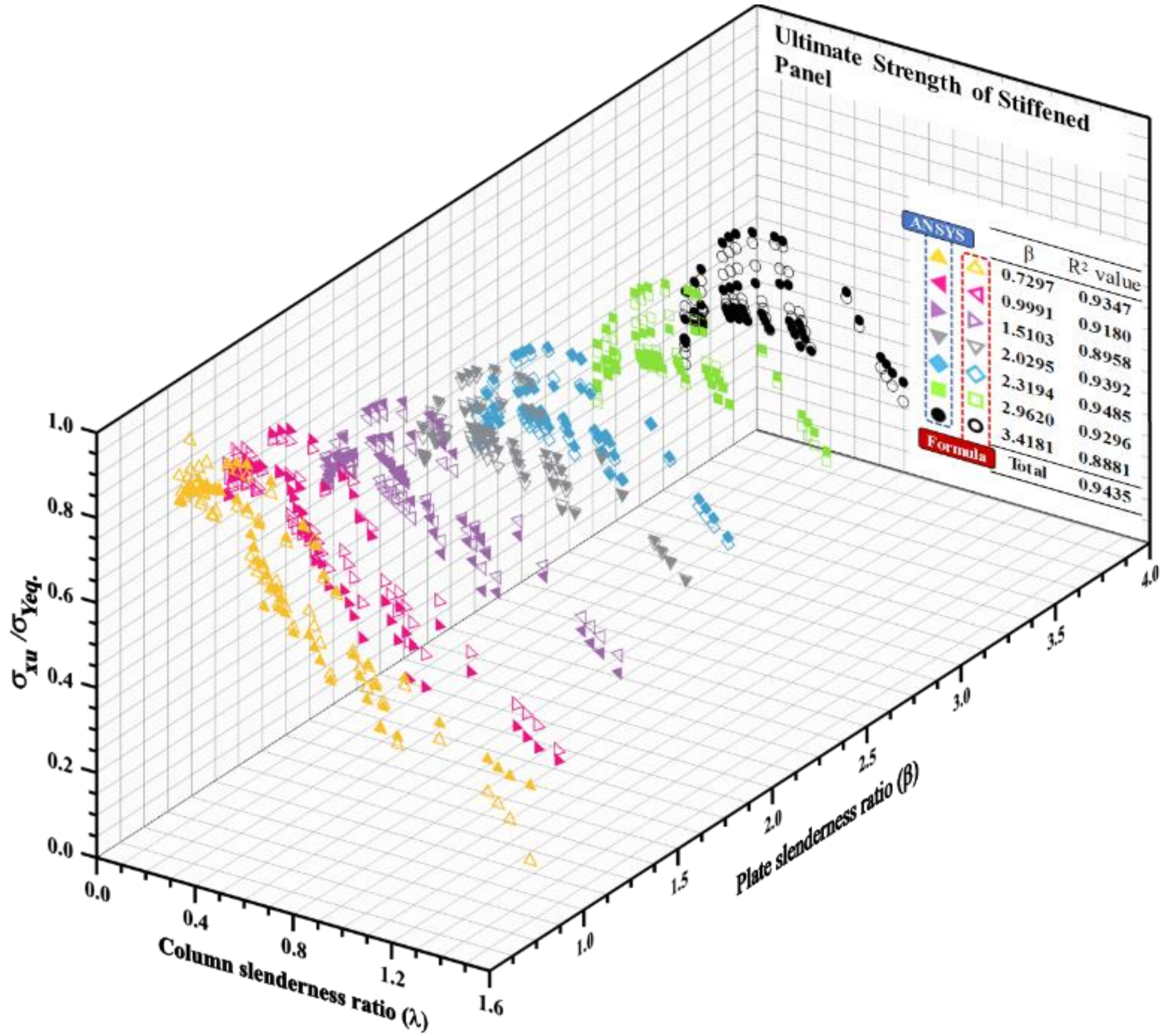
It has confirmed the fluctuation behaviour of ULS was found in the small range of the column slenderness ratio as illustrated in Fig. 1. In order to predict ULS of stiffened panel more accurately, Kim et al. [1] has additional considered two (2) important parameters i.e. web slenderness ratio, h_w/t_w , and Moment of inertia of stiffener to moment of inertia of plate ratio in z-direction, $I_{pz}/I_{sz} = (h_w t_w^3 + t_f b_f^3)/(t_p b^3)$, in addition to the two original basic parameters i.e. β and λ, shown in Eq. (1).

The numerical simulation results obtained were analysed by data processing technique and the polynomial function shape empirical formulation [1] is presented in Eq. (9). By adopting the proposed empirical formulation, fifteen (15) coefficients for Flat-bar are newly obtained based on FE numerical simulation in this study. Table 5 shows the fifteen (15) coefficients for T-bar and Flat-bar consisting of the empirical formulations.

$$\sigma_{xu}/\sigma_{Yeq} = \left[\begin{array}{l} c_0 + \left(c_1 + c_2\sqrt{\lambda} + \dfrac{c_3}{\beta} + c_4\dfrac{h_w}{t_w} + c_5\sqrt{\dfrac{I_{pz}}{I_{sz}}}\right)\sqrt{\lambda} + \left(c_6 + \dfrac{c_7}{\beta} + c_8\dfrac{h_w}{t_w} + c_9\sqrt{\dfrac{I_{pz}}{I_{sz}}}\right)\dfrac{1}{\beta} \\ + \left(c_{10} + c_{11}\dfrac{h_w}{t_w} + c_{12}\sqrt{\dfrac{I_{pz}}{I_{sz}}}\right)\dfrac{h_w}{t_w} \qquad + \left(c_{13} + c_{14}\sqrt{\dfrac{I_{pz}}{I_{sz}}}\right)\sqrt{\dfrac{I_{pz}}{I_{sz}}} \end{array}\right] \le 1.0 \tag{9}$$

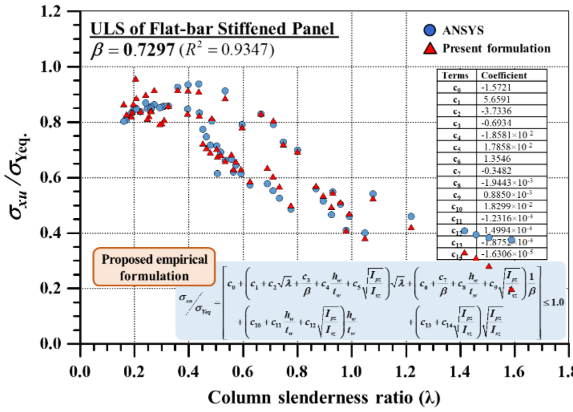
Table 5 Coefficients for empirical formulations to predict ULS of stiffened panel.

Terms	Coefficients	
	T-bar [1]	**Flat-bar (present)**
c_0	-0.1449	-1.5721
c_1	2.9787	5.6591
c_2	-2.6098	-3.7336
c_3	-0.2418	-0.6934
c_4	1.2374×10^{-3}	-1.8581×10^{-2}
c_5	1.3470×10^{-2}	1.7858×10^{-2}
c_6	0.8841	1.3546
c_7	-0.3361	-0.3482
c_8	1.5975×10^{-3}	-1.9443×10^{-3}
c_9	2.7745×10^{-3}	0.8850×10^{-3}
c_{10}	-7.5919×10^{-3}	1.8299×10^{-2}
c_{11}	3.2442×10^{-5}	-1.2316×10^{-4}
c_{12}	4.9670×10^{-5}	1.4994×10^{-4}
c_{13}	1.3267×10^{-2}	-1.8752×10^{-4}
c_{14}	-5.4149×10^{-5}	-1.6306×10^{-5}

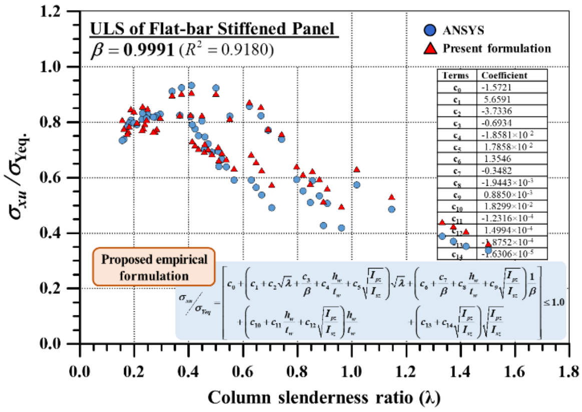


(a) all β

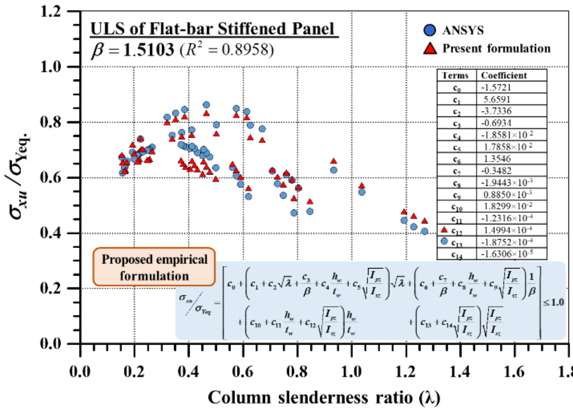
Fig. 5 ULS Comparison between ANSYS and obtained empirical formulation.



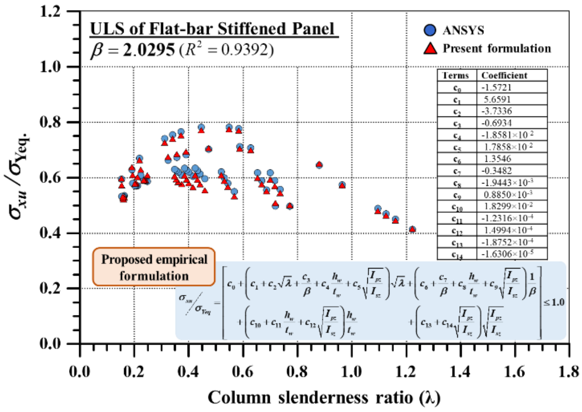
(b) $\beta = 0.7297$



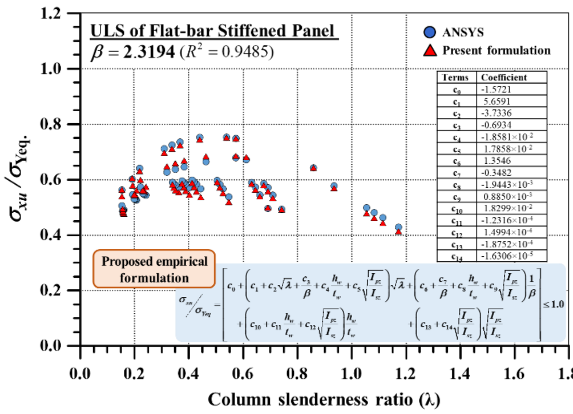
(c) $\beta = 0.9991$



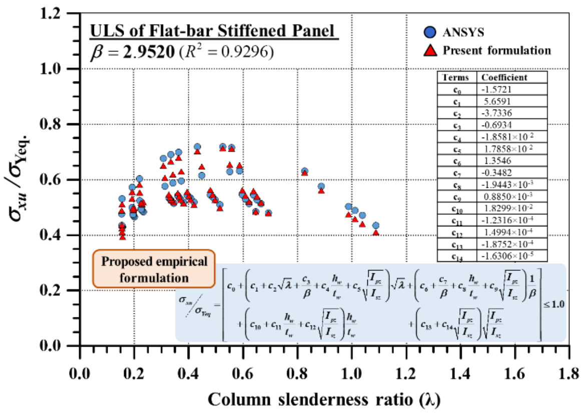
(d) $\beta = 1.5103$



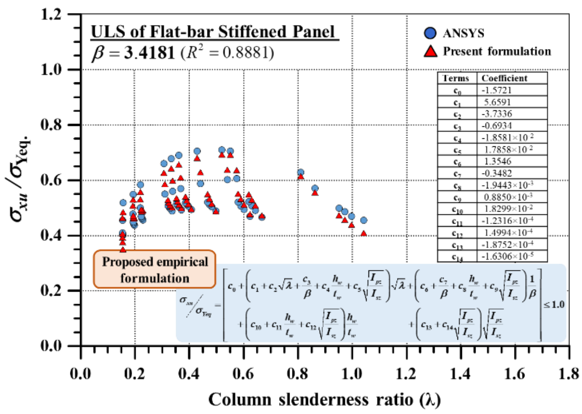
(e) $\beta = 2.0295$



(f) $\beta = 2.3194$



(g) $\beta = 2.9520$



(h) $\beta = 3.4181$

Fig. 5 ULS Comparison between ANSYS and obtained empirical formulation (Continued).

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The ULS results obtained by ANSYS FE simulation in Fig. 4 were directly compared with results obtained by the empirical formula in Fig. 5(a) to (h). In general, the empirical formulation shows relatively good agreement with ANSYS results based on R^2 results. When the β increases, the ULS tends to have more general shape as shown in Fig. 5(e) to (h). In this study, the general shape represents a tendency to regularly decrease diagonally. In particular, this tendency can be observed when the plate is thin. It seems that the buckling of the plate element is affecting the overall collapse behaviour of the stiffened panel. As shown in Fig. 5(b) to (d), when the plate slenderness ratio is less than 1.8 which is generally considered as thick plate, ULS tends to fluctuate greatly based on the variation of the stiffener size.

In this study, we have verified that the applicability of the empirical formulation in Eq. (9) can be extended to flat-bar stiffened panel with new set of the coefficients as summarised in Table 5. It shows relatively good agreement with ANSYS results with the maximum and minimum range of the R^2 values ($0.8881 \le R^2 \le 0.9435$). It can, however, be further improved by studying the phenomenon of the flat-bar stiffened panel under longitudinal compression in future. Particularly, the collapse behaviour of the plate under longitudinal compression should be studied.

4.2. Statistical analysis for verification of developed empirical formulation

The ULS results obtained by various methods such as numerical simulations [51, 52] and empirical formulations [21, 25, 26, 28, 29] together with the proposed refined empirical formulation in this study were plotted in Fig. A1(a) to (g). In the case of Zhang and Khan [28], they have limited it within the range of $\lambda \le \sqrt{2}$. The detailed comparisons were conducted by statistical analysis as summarised in Table 6. The statistical analysis results are also represented in Fig. 6(a) to (h). As expected, design formulations such as J-O, P-R and Euler tend to overestimate the ULS about 55-65% comparing with ANSYS FE numerical simulation as referred to Mean value in "ALL" column shown in Table 6 (1.5463 ≤ Mean ≤ 1.6539& 0.1922 ≤ COV ≤ 0.1932).

In the case of the empirical formulations, improved results of Mean and COV were observed compared to design formulations (1.1225 ≤ Mean ≤ 1.3922 & 0.1395 ≤ COV ≤ 0.1662). Most of the existing empirical formulations were slightly overestimated the ULS values. On the other hand, ALPS/ULSAP which is considered as analytical solution underestimates ULS values about 17-18% than ANSYS numerical simulation results (Mean = 0.8260 & COV = 0.4046). In particular, severe underestimation is observed when the column slenderness ratio (λ) is in the range between 0.1 and 0.3. If this range of λ is excluded, this study shows that the Mean and COV values are significantly improved to 0.9912 and 0.1389 respectively. In this study, we have selected reliable but limited range of flat-bar type stiffened panel. The empirical formulation proposed by this study provides well-fitted ULS results with ANSYS FE simulations (Mean = 1.0024 & COV = 0.0583).

In summary, design formulations which is generally adopted in shipbuilding overestimates ULS values than empirical formulations and analytical solution (ALPS/ULSAP). Most of the existing empirical formulations show good agreement with the refined FEM results by ANSYS. However, single line shaped empirical formulations still have limitation to predict the ULS values accurately. The ALPS/ULSAP which is an analytical method based solution can be considered as reliable way to take into account for nonlinearity of the ULS values. However, ALPS/ULSAP is recommended only when the λ is greater than 0.3. An additional advantage of ALPS/ULSAP is that it enables robust design through pessimistic analysis results.

Lastly, the proposed empirical formulation in this study has considered two (2) more parameters mentioned in section 4.1, so that it enables to predict ULS values and its nonlinearities accurately.

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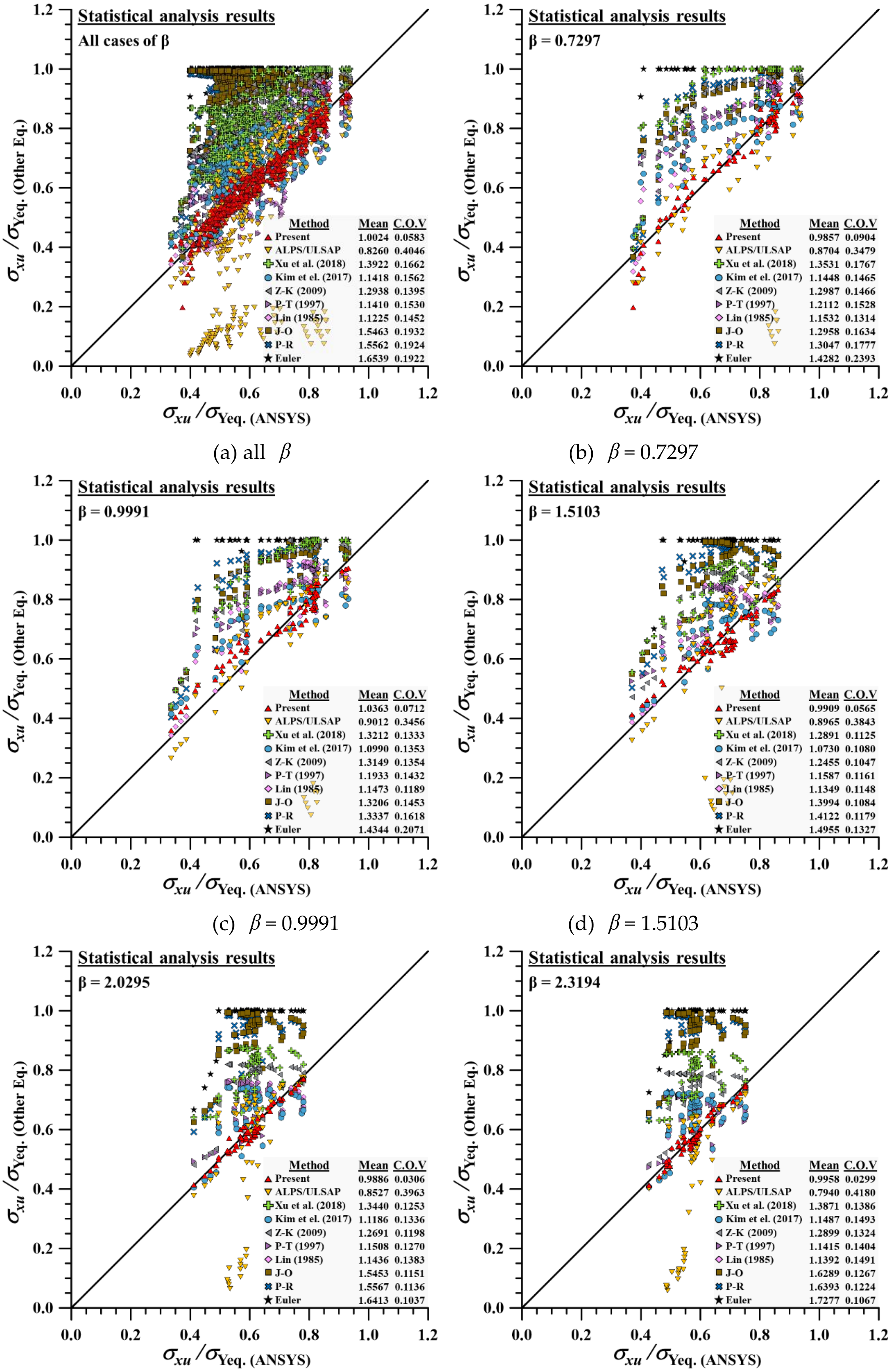
Table 6. Statistical analysis results.

Existing methods		Plate slenderness ratio (β)															
		0.7297		**0.9991**		**1.5103**		**2.0295**		**2.3194**		**2.9520**		**3.4181**		**ALL**	
		Mean	**COV**	**Mean**	**COV**	**Mean**	**COV**	**Mean**	**COV**	**Mean**	**COV**	**Mean**	**COV**	**Mean**	**COV**	**Mean**	**COV**
Design Formulations	**J-O**	1.2958	0.1634	1.3206	0.1453	1.3994	0.1084	1.5453	0.1151	1.6289	0.1267	1.7750	0.1462	1.8591	0.1577	1.5463	0.1932
	P-R	1.3047	0.1777	1.3337	0.1618	1.4122	0.1179	1.5567	0.1136	1.6393	0.1224	1.7825	0.1398	1.8640	0.1511	1.5562	0.1924
	Euler	1.4282	0.2393	1.4344	0.2071	1.4955	0.1327	1.6413	0.1037	1.7277	0.1067	1.8818	0.1212	1.9684	0.1316	1.6539	0.1922
Empirical Formulations	**Lin**	1.1532	0.1314	1.1473	0.1189	1.1349	0.1148	1.1436	0.1383	1.1392	0.1491	1.0939	0.1637	1.0456	0.1723	1.1225	0.1452
	P-T	1.2112	0.1528	1.1933	0.1432	1.1587	0.1161	1.1508	0.1270	1.1415	0.1404	1.0909	0.1621	1.0409	0.1743	1.1410	0.1530
	Z-K	1.2987	0.1466	1.3149	0.1354	1.2455	0.1047	1.2691	0.1198	1.2899	0.1324	1.3159	0.1517	1.3240	0.1626	1.2938	0.1395
	Kim	1.1448	0.1465	1.0990	0.1353	1.0730	0.1080	1.1186	0.1336	1.1487	0.1493	1.1936	0.1717	1.2150	0.1840	1.1418	0.1562
	Xu	1.3531	0.1767	1.3212	0.1333	1.2891	0.1125	1.3440	0.1253	1.3871	0.1386	1.4839	0.1652	1.5668	0.1824	1.3922	0.1662
	Present	**0.9857**	**0.0904**	**1.0363**	**0.0712**	**0.9909**	**0.0565**	**0.9886**	**0.0306**	**0.9958**	**0.0299**	**1.0064**	**0.0364**	**1.0128**	**0.0474**	**1.0024**	**0.0583**
Analytical Solution	**ALPS/ULSAP**	0.8704	0.3479	0.9012	0.3456	0.8965	0.3843	0.8527	0.3963	0.7940	0.4180	0.7342	0.4396	0.7331	0.4618	0.8260	0.4046

Note: J-O = Johnson and Ostenfeld, P-R = Perry and Robertson, Lin = Lin [25], P-T = Paik and Thayamballi [26], Z-K = Zhang and Khan [28], Kim = Kim et al. [21], and Xu = Xu et al. [29].



(a) all β (b) $\beta = 0.7297$

(c) $\beta = 0.9991$ (d) $\beta = 1.5103$

(e) $\beta = 2.0295$ (f) $\beta = 2.3194$

Fig. 6 Statistical analysis between ANSYS and individual formulations.

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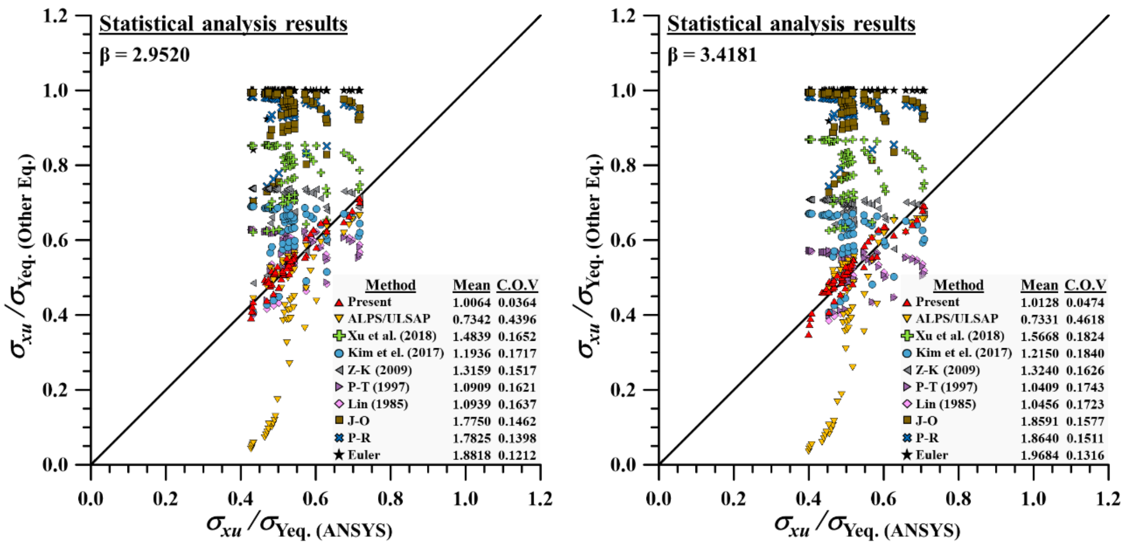

(g) $\beta = 2.9520$ (h) $\beta = 3.4181$

Fig. 6 Statistical analysis between ANSYS and individual formulations (Continued).

Table 7. The obtained R^2 values from Fig. 5(a).

Plate slenderness ratio (β)	R^2
0.7297	0.9347
0.9991	0.9180
1.5103	0.8958
2.0295	0.9392
2.3194	0.9485
2.9620	0.9296
3.4181	0.8881
Total	**0.9435**

5. Conclusions

In this study, the refined empirical formulation was proposed to predict ultimate strength performance or Ultimate Limit State (ULS) of flat-bar type steel stiffened panel under longitudinal compression. In total, 420 cases of numerical simulations by ANSYS Non-Linear Finite Element Method (NLFEM) were conducted and used as an input data to develop empirical formulation. The formulation obtained shows good agreement with ANSYS results, in general ($0.8881 \leq R^2 \leq 0.9485$) as shown in Table 7. In conclusion, it is verified that the obtained empirical formulation obtained is well fitted with ANSYS numerical simulation results (R^2=0.9435).

The detailed results are summarised as follows.

Findings

- When the plate slenderness ratio (β) increases, the ULS tends to be generalised shape which represents a tendency to regularly decrease diagonally as shown in Fig. 5(e) to (h). This may be caused by that buckling of the plate element which is affecting the overall collapse behaviour of the stiffened panel. In addition, this trend has been observed when the plate is considered as thin ($1.8 \leq \beta$)
- As represented in Fig. 5(b) to (d), when the plate slenderness ratio is thick, the Ultimate Limit State (ULS or ultimate strength) tends to fluctuate greatly depending on the variation of the stiffener size.

- Two (2) parameters i.e. column slenderness ratio (λ) and plate slenderness ratio (β) are considered as the main parameters of the existing empirical formulations. As indicated by Kim et al. [21, 24], single line shaped existing empirical formulations may not be able to implement the fluctuation behaviour of ULS. It means that the additional parameters should be considered in predicting the accurate ULS in the region of fluctuation.
- The applicability of the refined empirical formulation proposed by Kim et al. [1] was tested by statistical analysis. It is confirmed that proposed empirical formulation can be applied to flat-type stiffened panel with modified fourteen (14) coefficients.

The limitations of this study are also documented as follows of which should be further studied in future:

Limitations

- The empirical formulation proposed by this study is based on ANSYS numerical simulation results with assumed scenarios in Tables 3 and boundary condition in Table 4(b). It is well recognised that data processing is depending on the input data. This means that other types of input data i.e. ULS values obtained by experimental or analytical method would provide slightly different final outcomes. Nevertheless, the numerical simulation results assumed by simply supported boundary condition with average level initial deflection may help designers in the robust design of ships and offshore structures by maintaining the additional structural safety margin.
- This study is only considering the prediction of ULS of steel stiffened panel i.e. ship's deck or upper side shell stiffened panel under longitudinal compression. Other type of applied loadings such as biaxial compression, lateral pressure and etc. should also be taken into consideration in future.
- With regard to initial imperfection, initial deflection of plate and initial distortion of stiffener are only considered in this study, while welding-induced residual stress was not considered. In general, it is reported that the residual stress may affect 10-13% of ULS decrement [50].
- In addition, other types of stiffener (angle-bar type) should also be studied to develop the empirical formulation.

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Conflicts of Interest: The authors declare no conflict of interest.

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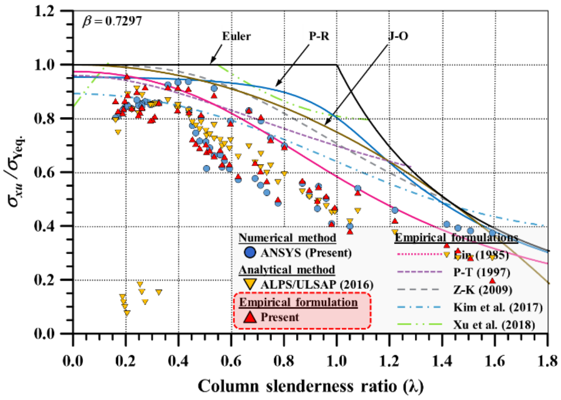
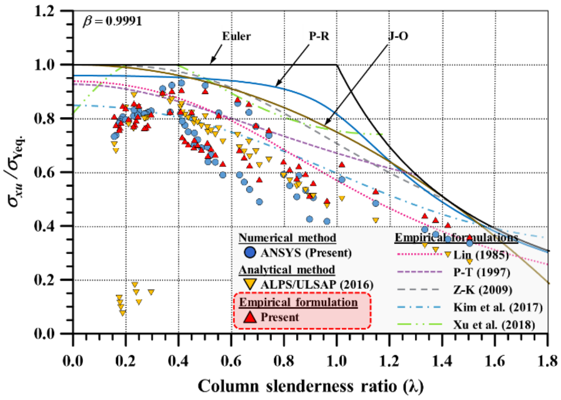
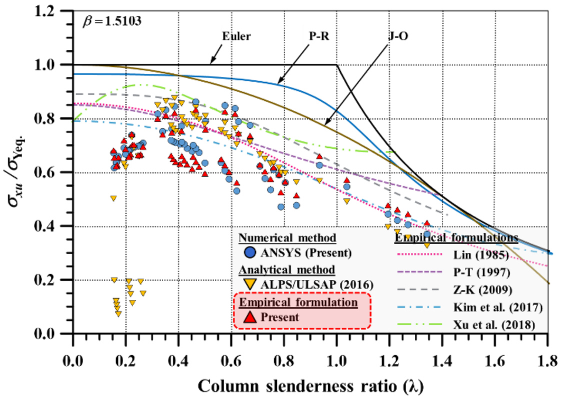
Appendix A



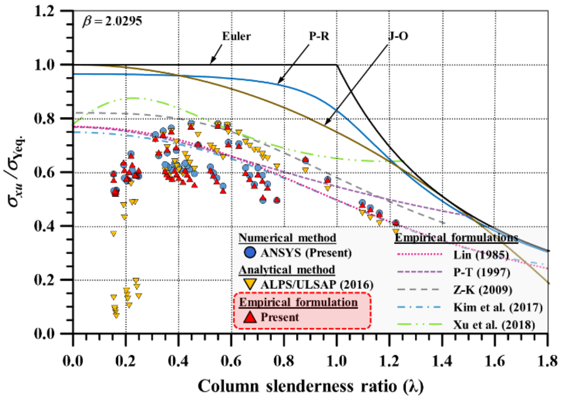
(a) $\beta = 0.7297$



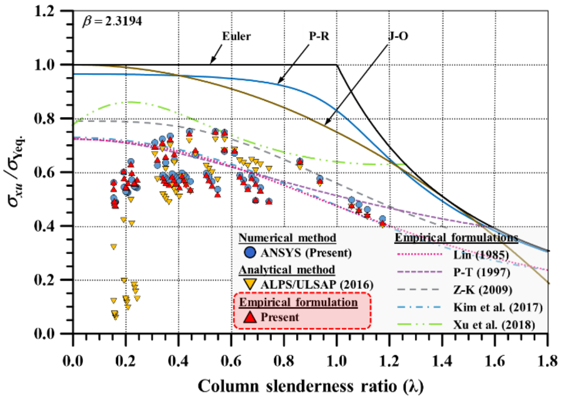
(b) $\beta = 0.9991$



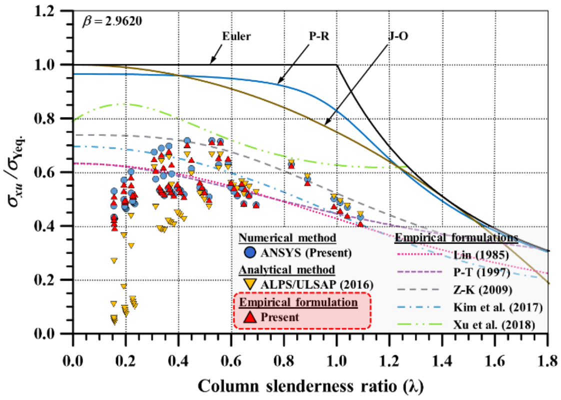
(c) $\beta = 1.5103$



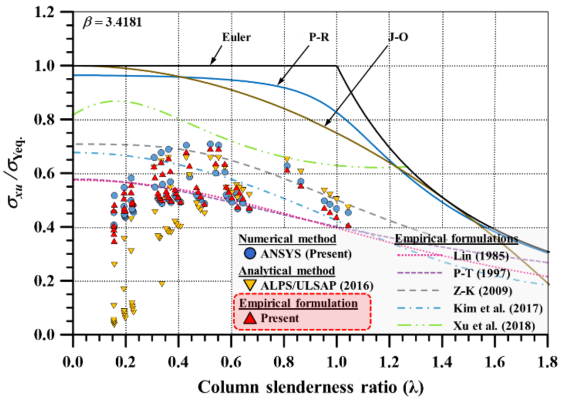
(d) $\beta = 2.0295$



(e) $\beta = 2.3194$



(f) $\beta = 2.9520$



(g) $\beta = 3.4181$

Figure A1. Comparison of ULS results.

References

1. Kim, D.K.; Lim, H.L.; Yu, S.Y. Ultimate strength prediction of T-bar stiffened panel under longitudinal compression by data processing: A refined empirical formulation. *Ocean Engineering* **2019**, *192*, 106522 (https://doi.org/10.1016/j.oceaneng.2019.106522).
2. Park, D.K.; Paik, J.K.; Kim, B.J.; Seo, J.K.; Li, C.G.; Kim, D.K. Ultimate strength performance of Northern Sea going non-ice class commercial ships. *Structural Engineering Mechanics* **2014**, *52*(3), 613-632.
3. Park, D.K.; Kim, D.K.; Park, C.H.; Park, D.H.; Jang, B.S.; Kim, B.J.; Paik, J.K. On the crashworthiness of steel-plated structures in an arctic environment: An experimental and numerical study. *Journal of Offshore Mechanics and Arctic Engineering* **2015**, *137*(5), 051501.
4. Park, D.K.; Kim, D.K.; Seo, J.K.; Kim, B.J.; Ha, Y.C.; Paik, J.K. Operability of non-ice class aged ships in the Arctic Ocean - Part II: Accidental limit state approach. *Ocean Engineering* **2015**, *102*, 206-215.
5. Topa, A.; Kim, D.K.; Kim, Y.T. 3D Numerical simulation of seamless pipe piercing process by fluid-structure interaction method. MATEC Web of Conferences **2018**, *203*, 06016 (https://doi.org/10.1051/matecconf/201820306016).
6. Paik, J.K. *Advanced Structural Safety Studies with Extreme Conditions and Accidents*, Springer: Singapore, 2020 (https://doi.org10.1007/978-981-13-8245-1).
7. ISSC. Ultimate Strength (Committee III.1). The 18th International Ship and Offshore Structures Congress (ISSC 2012), Rostock, Germany; 9-13 September 2012.
8. Paik, J.K.; Kim, D.K.; Park, D.H.; Kim, H.B.; Mansour, A.E.; Caldwell, J.B. Modified Paik-Mansour formula for ultimate strength calculations of ship hulls. *Ships and Offshore Structures* **2013**, *8*(3-4), 245-260.
9. Paik, J.K.; Kim, D.K.; Park, D.H.; Kim, H.B.; Kim, M.S. A new method for assessing the safety of ships damaged by grounding. International Journal of Maritime Engineering 2012, 154(Part A1), 1-20.
10. Kim, D.K., Pedersen, P.T., Paik, J.K., Kim, H.B., Zhang, X.M. and Kim, M.S. Safety guidelines of ultimate hull girder strength for grounded container ships. Safety Science 2013, 59, 46-54.
11. Kim, D.K., Kim, H.B., Mohd Hairil, M. and Paik, J.K. Comparison of residual strength – grounding damage index diagrams for tankers produced by the ALPS/HULL ISFEM and design formula method. International Journal of Naval Architecture and Ocean Engineering 2013, 5(1), 47-61.
12. Kim, D.K., Kim, B.J., Seo, J.K., Kim, H.B., Zhang, X.M. and Paik, J.K. Time-dependent corrosion damage on the development of residual strength – grounding damage index diagram. Ocean Engineering 2014, 76, 163-171.
13. Zhang, S.; Pedersen, P.T.; Villavicencio, R. Probability and Mechanics of Ship Collision and Grounding; Elsevier Inc. 2019 (https://doi.org/10.1016/C2017-0-02422-9)
14. Kim, D.K.; Kim, H.B.; Park, D.H.; Mohd, M.H.; Paik, J.K. A practical diagram to determine the residual longitudinal strength of grounded ship in Northern Sea Route. *Ships and Offshore Structures*, **In-press** (https://doi.org/10.1080/17445302.2019.1667113).
15. Paik, J.K.; Kim, B.J.; Seo, J.K. Methods for ultimate limit state assessment of ships and ship-shaped offshore structures: Part I Unstiffened plates. *Ocean Engineering* **2008**, *35*(2), 261-270.
16. Paik, J.K. *Ultimate Limit State Analysis and Design of Plated Structures*, 2nd ed.; John Wiley & Sons: Chichester, UK, 2018.
17. Kim, D.K.; Poh, B.Y.; Lee, J.R.; Paik, J.K. Ultimate strength of initially deflected plate under longitudinal compression: Part I = An advanced empirical formulation. *Structural Engineering and Mechanics* **2018**, *68*(2), 247-259.
18. Tanaka, Y.; Endo, H. Ultimate strength of stiffened plates with their stiffeners locally buckled in compression. *Journal of the Society of Naval Architects of Japan* **1988**, *164*, 456-467.
19. Tanaka, S., Yanagihara, D., Yasuoka, A., Harada, M., Okazawa, S., Fujikubo, M., Yao, T. Evaluation of ultimate strength of stiffened panels under longitudinal thrust. *Marine Structures* **2014**, *36*, 21-50.
20. Liew, A.; Gardner, L. Ultimate capacity of structural steel cross-sections under compression, bending and combined loading. *Structures* **2015**, *1*, 2-11.
21. Kim, D.K.; Lim, H.L.; Kim, M.S.; Hwang, O.J.; Park, K.S. An empirical formulation for predicting the ultimate strength of stiffened panels subjected to longitudinal compression. *Ocean Engineering* **2017**, 140, 270-280.
22. Ozdemir, M.; Ergin, A.; Yanagihara, D.; Tanaka, S.; Yao, T. A new method to estimate ultimate strength of stiffened panels under longitudinal thrust based on analytical formulas. *Marine Structures* **2018**, *59*, 510-535.

 doi:10.20944/preprints202007.0572.v1

23. Zhang, S.M. A review and study on ultimate strength of steel plates and stiffened panels in axial compression. *Ships and Offshore Structures* **2016**, *11*(1), 81-91.
24. Kim, D.K.; Lim, H.L.; Yu, S.Y. A technical review on ultimate strength prediction of stiffened panels in axial compression. *Ocean Engineering* **2018**, *170*, 392-406.
25. Lin, Y.T. Structural Longitudinal Strength Modelling. Ph.D. Dissertation, University of Glasgow, Glasgow, UK, 1985.
26. Paik, J.K.; Thayamballi, A.K. An Empirical Formulation for Predicting the Ultimate Compressive Strength of Stiffened Panels. The 7th International Offshore and Polar Engineering Conference (ISOPE 1997), Honolulu, USA; 25-30 May 1997 (ISOPE-I-97-444).
27. Paik, J.K. Empirical formulations for predicting the ultimate compressive strength of welded aluminum stiffened panels. *Thin-Walled Structures* **2007**, *45*(2), 171-184.
28. Zhang, S.M.; Khan, I. Buckling and ultimate capability of plates and stiffened panels in axial compression. *Marine Structures* **2009**, *22*(4), 791-808.
29. Xu, M.C.; Song, Z.J.; Zhang, B.W.; Pan, J. Empirical formula for predicting ultimate strength of stiffened panel of ship structure under combined longitudinal compression and lateral loads. *Ocean Engineering* **2018**, *162*, 161-175.
30. Khedmati, M.R.; Zareei, M.R.; Rigo, P. Empirical formulations for estimation of ultimate strength of continuous stiffened aluminium plates under combined in-plane compression and lateral pressure. *Thin-Walled Structures* **2010**, *48*(3), 274-289.
31. Mei, H.; Wang, D. An Empirical Formula for Predicting Elastic Ultimate Buckling Strength of Flat-Bar Stiffened Panels with Initial Imperfections. The 38th International Conference on Ocean, Offshore and Arctic Engineering (OMAE 2019), Glasgow, Scotland, UK; 9-14 June 2019.
32. Anyfantis, K.N. Ultimate strength of stiffened panels subjected to non-uniform thrust. *International Journal of Naval Architecture and Ocean Engineering* **2020**, *12*, 325-342.
33. Silva, H.M.; Wojewoda, J. Determination of the product of inertia of stiffened plates based on Finite Element Method results. *Engineering Structures* **2020**, *207*, 110201.
34. Shi, G.J.; Gao, D.W. Transverse ultimate capacity of U-type stiffened panels for hatch covers used in ship cargo holds. *Ships and Offshore Structures*, **in-press** (https://doi.org/10.1080/17445302.2020.1765691).
35. Kong, X.; Yang, Y.; Gan, J.; Yuan, Y.; Ao, L.; Wu, W. Experimental and numerical investigation on the detailed buckling process of similar stiffened panels subjected to in-plane compressive load. *Thin-Walled Structures* **2020**, *148*, 106620.
36. Wang, Q.; Wang, D. Scaling characteristics of hull girder's ultimate strength and failure behaviors: An empirically modified scaling criterion. *Ocean Engineering* **2020**, *212*, 107595.
37. Song, Z.J.; Xu, M.C.; Moan, T.; Pan, J. Dimensional and similitude analysis of stiffened panels under longitudinal compression considering buckling behaviours. *Ocean Engineering* **2019**, *187*, 106188.
38. Liu, B.; Garbatov, Y.; Wu, W.; Soares, C.G. Strength assessment of aluminium and steel stiffened panels with openings on longitudinal girders. *Ocean Engineering* **2020**, *200*, 107047.
39. Jagite, G.; Bigot, F.; Derbanne, Q.; Malenica, Š.; Le Sourne, H.; Cartraud, P. A parametric study on the dynamic ultimate strength of a stiffened panel subjected to wave- and whipping-induced stresses. *Ships and Offshore Structures*, **in-press** (https://doi.org/10.1080/17445302.2020.1790985).
40. Ahmadi, F.; Ranji, A.R.; Nowruzi, H. Ultimate strength prediction of corroded plates with center-longitudinal crack using FEM and ANN. Ocean Engineering **2020**, 206, 107281.
41. Wong, E.W.C.; Kim, D.K. A simplified method to predict fatigue damage of TTR subjected to short-term VIV using Artificial Neural Network. *Advances in Engineering Software* **2018**, *126*, 100-109.
42. Kim, D.K.; Incecik, A.; Choi, H.S.; Wong, E.W.C.; Yu, S.Y.; Park, K.S. A simplified method to predict fatigue damage of offshore riser subjected to vortex-induced vibration by adopting current index concept. *Ocean Engineering* **2018**, *157*, 401-411.
43. Park, D.K.; Kim, D.K.; Seo, J.K.; Kim, B.J.; Ha, Y.C.; Paik, J.K. Operability of non-ice class aged ships in the Arctic Ocean - Part I: Ultimate limit state approach. *Ocean Engineering* **2015**, *102*,197-205.
44. Park, D.K.; Kim, D.K.; Kim, B.J.; Seo, J.K.; Paik, J.K. Operability of non-ice class aged ships in the Arctic Ocean – Part II: Accidental limit state approach. *Ocean Engineering* **2015**, *102*, 197-205.
45. Park, D.K.; Kim, D.K.; Park, C.H.; Park, D.H.; Jang, B.S.; Kim, B.J.; Paik, J.K. On the crashworthiness of steel-plated structures in an Arctic environment: An experimental and numerical study. *Journal of Offshore Mechanics and Arctic Engineering* **2015**, *137*(5), 051501.

46. Park, D.K.; Paik, J.K.; Kim, B.J.; Seo, J.K.; Li, C.G.; Kim, D.K. Ultimate strength performance of Northern sea going non-ice class commercial ships. *Structural Engineering and Mechanics* **2014**, *52*(3), 613-632.
47. Choi, H.S.; Seo, D.W.; Kim, K.H.; Shin, G.; Choung, J.M.; Kim, K.S.; Wong, E.W.C.; Topa, A.; Kim, D.K. Applicability of high manganese steel to FLNG storage tank considering collision event. *Journal of Marine Science and Engineering*, **under review**.
48. Paik, J.K.; Kim, D.K.; Lee, H.; Shim, Y.L. A method for analyzing elastic large deflection behavior of perfect and imperfect plates with partially rotation-restrained edges. *Journal of Offshore Mechanics and Arctic Engineering* **2012**, *134*(2), 021603.
49. Ahmer Wadee, M.; Farsi, M. Imperfection sensitivity and geometric effects in stiffened plates susceptible to cellular buckling. *Structures* **2015**, *3*, 172-186.
50. Khan, I.; Zhang, S. Effects of welding-induced residual stress on ultimate strength of plates and stiffened panels. *Ships and Offshore Structures* **2011**, *6*(4), 297-309.
51. ANSYS. User's manual version 13.0. ANSYS Inc., Canonsburg, USA, 2014.
52. ALPS/ULSAP. A computer program for ultimate limit state assessment of stiffened panels. Advanced Technology Center, DRS C3 Systems, Parsippany, NJ, USA, 2016 (Available from: www.maestromarine.com).