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An improved GreenOFDM scheme for PAPR reduction

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Abstract: This paper proposes an improvement to a recent Peak-to-Average-Power-Ratio (PAPR) reduction technique for Orthogonal Frequency Division Multiplexing (OFDM), the GreenOFDM. This technique, which is inspired by SeLected Mapping (SLM), generates several waveform candidates using a given number of Inverse Fast Fourier Transforms (IFFT), and selects the one with the lowest PAPR for the transmission of the OFDM symbol. In GreenOFDM, we show that for U IFFTs, it provides better PAPR reduction capabilities than SLM-OFDM as it increases the number of waveform candidates from U (available for SLM-OFDM) to $U^2/4$. In this work, we propose a simple and clever extension of the GreenOFDM that further increases the number of waveform candidates by a factor 4 (from $U^2/4$ to U^2), or equivalently reduces by a factor of 2 the number of IFFTs for a same PAPR performance. Compared to SLM-OFDM, the improved GreenOFDM technique reduces the complexity by requiring only the square-root of the number of IFFTs for a same PAPR reduction performance.

Keywords: OFDM; PAPR; Selected Mapping; GreenOFDM.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a widely used modulation technique for wireless communication systems thanks to its high spectral efficiency and robustness against frequency-selective channel. However, a well-known limitation of OFDM (as well as for other multi-carrier schemes) is its high Peak-to-Average Power Ratio (PAPR) [1]. In addition, the PAPR in OFDM is a random variable. These characteristics of the PAPR in OFDM are troublesome for many reasons:

- (i) the PAPR of OFDM signals needs to be correctly estimated in order to take its dynamic range into consideration for the Digital-to-Analog Converters specification [2,3];
- (ii) in order to correctly estimate the PAPR of OFDM signals in the digital domain, the computational complexity of the digital modulator is increased due to oversampling [4];
- (iii) the high PAPR of these signals generally increases the Power Amplifier's (PA's) power dissipation as its linear region has to be extended to be able to accommodate signals with wide amplitude excursions. Not doing so results in non-linear distortions and out-of-band radiations that degrade the system performances [5].

To counteract the above-mentioned limitations, several PAPR reduction schemes have been proposed in the literature. These schemes could be classified into different and not exhaustive categories as follows [6]:

- Signal distortion techniques: Clipping and Filtering [7,8], Companding [9].
- Multiple signaling and probabilistic techniques: SeLected Mapping OFDM (SLM-OFDM) [10,11], GreenOFDM [12], Tone Injection [13], Tone Reservation [14,15], Active Constellation Extension [16], Partial Transmit Sequence [17].
- Pre-coding techniques: Single Carrier Frequency Division Multiple Access (SC-FDMA) [18], Carrier Interferometry [19,20].

Among these schemes, the SLM-OFDM is a well known one and has very recently been improved into the so-called GreenOFDM technique. The underlying idea of SLM-OFDM is to generate a number of different OFDM symbol waveform candidates from the same data and to select the waveform with the lowest PAPR for transmission. For SLM-OFDM, the PAPR reduction depends on the number of waveform candidates; the more the number of candidates, the lesser the PAPR of OFDM. Essentially, GreenOFDM is a SLM-OFDM variant that increases the number of candidates in a square-scale while using the same number of IFFTs as in conventional SLM-OFDM. Thus, for a given number of computed IFFTs, a better PAPR reduction for GreenOFDM is attained as compared to SLM-OFDM.

GreenOFDM receives attention in the literature [21–24]. Some works compare the original GreenOFDM scheme with PTS (Partial Transmit Sequence) variants but without using the same scheme [23,24]. Other works aim to improve the PAPR reduction capabilities as in [21], but the results seem to come from a not so exhaustive simulation study. Finally, other works like [22] reduce the complexity by relaxing the PAPR reduction capability as we have also proposed in [25].

In contrast to all the previous studies, we propose in this paper a simple yet efficient improvement of the GreenOFDM that allows to obtain a higher number of candidates without increasing the number of IFFTs. As shall become apparent, this ingenious extension of the original GreenOFDM technique permits a reduction by a factor of 2 of the number of IFFTs for a same PAPR performance, providing a relevant gain in terms of performance/complexity trade-off.

This document is organized as follows: First, a brief OFDM and PAPR description is given. Then, SLM-OFDM and GreenOFDM algorithms are succinctly presented before introducing the improved version. In the last part, computer simulations of the PAPR distribution are presented to clearly demonstrate the improvement of the proposed technique and finally conclusions are drawn.

2. OFDM and PAPR

OFDM is a multi-carrier modulation technique in which every time-domain symbol carries the sum of N overlapping orthogonal subcarriers mapped by $\{A_k\}$ symbols (e.g. Quadrature Amplitude Modulation: QAM symbols) with $0 \leq k \leq N - 1$.

The OFDM base-band time-domain symbol is obtained by applying the Inverse Discrete Fourier Transform (IDFT)¹ to the frequency-domain inputs $\{A_k\}$. The digital time-domain symbols are often oversampled to emulate the peaks that might appear in the analog domain and hence obtain a well-approximated digital version. A $L \cdot N$ points IFFT over

$$\{X_k\} = \{A_0 A_1 \cdots A_{\frac{N}{2}-1} \underbrace{0 \cdots 0}_{(L-1) \cdot N} A_{\frac{N}{2}} A_{\frac{N}{2}+1} \cdots A_{N-1}\}$$

¹ Digitally implemented with an Inverse Fast Fourier Transform (IFFT).

achieves the oversampling, where L is the oversampling factor. The OFDM base-band time-domain symbol is hence given by:

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} X_k \cdot \exp \left\{ i2\pi \frac{kn}{LN} \right\} \quad (1)$$

with $0 \leq n \leq N-1$ and $i = \sqrt{-1}$. Generally an oversampling factor $L = 4$ is applied since it is enough to well approximate the analog symbols. Higher values will only add a higher computational complexity without substantial gain in the approximation.

The PAPR of the oversampled OFDM symbol is given by:

$$\text{PAPR} = \frac{\max_{0 \leq n \leq LN-1} \{|x[n]|^2\}}{\mathbb{E} \{|x[n]|^2\}} \quad (2)$$

where $\mathbb{E}\{\cdot\}$ is the expectation and $\mathbb{E} \{|x[n]|^2\} = \mathbb{E} \{|X_k|^2\}$ for X_k i.i.d. random variables ($\{X_k\} \in 2^n$ -QAM with $n = 2m, m \in \mathbb{N}^+$). The PAPR of OFDM symbols is high because the numerator of Eq. (2) is high. This is because *sometimes* many modulated subcarriers $X_k \cdot \exp \left\{ i2\pi \frac{kn}{LN} \right\}$ add themselves in-phase generating high peaks. Furthermore, since the PAPR is a random variable it can be characterized by its Complementary Cumulative Distribution Function (CCDF), i.e. the probability that the PAPR of an OFDM symbol exceeds a predetermined value γ . A semi-empirical approximation is given by [26]:

$$\text{CCDF}_{\text{OFDM}}(\gamma) \approx 1 - (1 - \exp \{-\gamma\})^{2.8N} \quad (3)$$

3. From SLM-OFDM to GreenOFDM

In conventional SLM-OFDM, U different symbol waveforms are generated from the same data set $\{X_k\}$. To do so, the input data set $\{X_k\}$ is repeated U times and each copy is multiplied element-wise by pseudo-random sequences where the coefficients have unitary norm $\phi_{u,k}$, $0 \leq u \leq U-1$; i.e. $X_{u,k} = X_k \cdot \phi_{u,k}$ with $|\phi_{u,k}| = 1$. It is noteworthy to mention that in the following, U represents the total number of computed IFFTs as well as the total number of pseudo-random sequences. The time-domain symbol waveforms $x_u[n]$ are computed by LN -IFFTs with inputs $\{X_{u,k}\}$. The waveform that exhibits the lowest PAPR among the $C_{\text{SLM}} = U$ candidates is selected for transmission. The SLM-OFDM algorithm is summarized in Algorithm 1 and its block diagram depicted in Figure 1.

The CCDF of PAPR of conventional SLM-OFDM is well approximated by [11]:

$$\text{CCDF}_{\text{SLM}}(\gamma) \approx \left(1 - (1 - \exp \{-\gamma\})^{2.8N} \right)^{C_{\text{SLM}}} \quad (4)$$

The function $\text{CCDF}_{\text{SLM}}(\text{PAPR})$ decreases when the number of candidates $C_{\text{SLM}} = U$ increases. However, higher values of C_{SLM} lead to larger values of the computational burden because of the required number of IFFTs.

In order to mitigate this limitation of SLM-OFDM, the key idea of GreenOFDM is to generate the candidates by adding (with a normalisation factor $\sqrt{2}$) two coupleable IFFT-based waveforms. This allows for the generation of more candidates C_{Green} corresponding to all the possible combinations, whilst maintaining the same PAPR reduction behaviour of SLM-OFDM with respect to the number of candidates (i.e. equation (4) with C_{Green} instead of C_{SLM}).

Algorithm 1 The SLM-OFDM algorithm.

Require: $\{X_k\}, \{\phi_{u,k}\}$ with $\phi_{u,k} \in \{\pm 1\}$
 $\text{minPAPR} \leftarrow +\infty;$
for ($u \leftarrow 0$ to $U - 1$) **do**
 $\{X_{u,k}\} \leftarrow \{X_k \cdot \phi_{u,k}\};$
 $\{x_u[n]\} \leftarrow \text{IFFT}\{\{X_{u,k}\}\};$
 if ($\text{PAPR}\{x_u[n]\} < \text{minPAPR}$) **then**
 $\text{minPAPR} \leftarrow \text{PAPR}\{x_u[n]\};$
 $\{x_{\bar{u}}[n]\} \leftarrow \{x_u[n]\};$
 end if
end for
SEND $\{x_{\bar{u}}[n]\};$

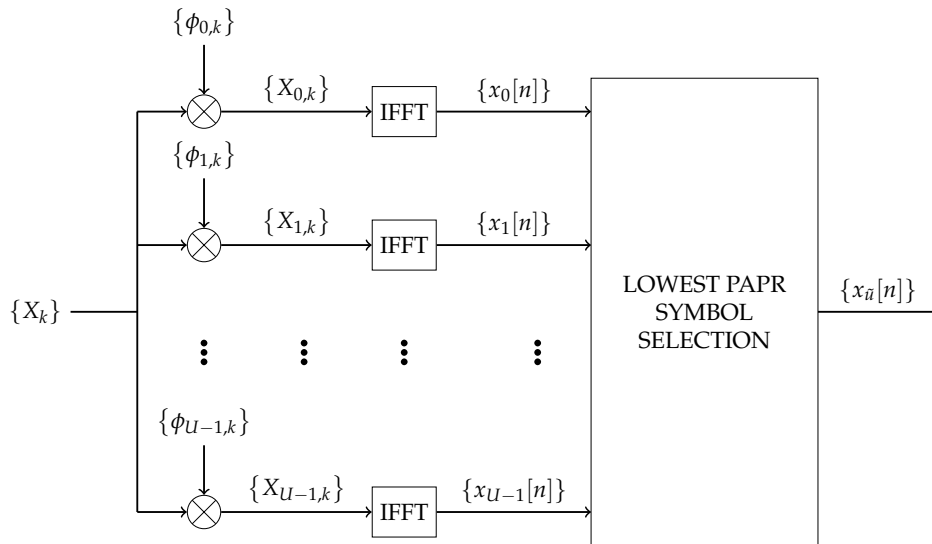


Figure 1. SLM-OFDM block diagram.

The GreenOFDM technique generates $C_{\text{Green}} = U^2/4$ OFDM symbols candidates by first splitting the U set of copied and randomized data $\{X_{u,k}\}$ into two groups: the first one represented by the index g_1 , with $0 \leq g_1 \leq \frac{U}{2} - 1$ and the second one represented by the index g_2 , with $\frac{U}{2} \leq g_2 \leq U - 1$. The first group waveforms $\{x_{g_1}[n]\}$ is obtained, like in SLM-OFDM, through IFFTs over $\{X_{g_1,k} = X_k \cdot \phi_{g_1,k}\}$ (with $\{\phi_{g_1,k}\} = \{\phi_{u,k}\}, u < \frac{U}{2}$ and $\phi_{u,k} \in \{\pm 1\}$). Similarly, the second group waveforms $\{x_{g_2}[n]\}$ are obtained through IFFTs over $\{X_{g_2,k} = X_k \cdot \phi_{g_2,k}\}$ (with $\{\phi_{g_2,k}\} = \{i \times \phi_{u,k}\}, u \geq \frac{U}{2}$ and $\phi_{u,k} \in \{\pm 1\}$). As it can be seen, the difference between the two groups of waveforms is the fact that the sequences $\{\phi_{g_2,k}\}$ use pure imaginary unitary values: $\{\pm i\}$ whereas the sequences $\{\phi_{g_1,k}\}$ use pure real unitary values: $\{\pm 1\}$. In this way, individual elements of the two sequences are orthogonal in order not to cancel out some subcarriers after adding the two IFFT outputs. The last part of the GreenOFDM method, which depicts the way the $\frac{U^2}{4}$ waveform candidates are obtained, is described in Algorithm 2. The equivalent block diagram of the GreenOFDM is depicted in Figure 2.

Algorithm 2 The GreenOFDM algorithm.

Require: $\{x_{g_1}[n]\} = \text{IFFT}\{X_k \cdot \phi_{g_1,k}\}$, $\{x_{g_2}[n]\} = \text{IFFT}\{X_k \cdot i \cdot \phi_{g_2,k}\}$
 $\text{minPAPR} \leftarrow +\infty$;
for ($g_1 \leftarrow 0$ to $\frac{U}{2} - 1$) **do**
 for ($g_2 \leftarrow \frac{U}{2}$ to $U - 1$) **do**
 $\{x_{g_1,g_2}[n]\} \leftarrow \frac{\{x_{g_1}[n]\} + \{x_{g_2}[n]\}}{\sqrt{2}}$;
 if ($\text{PAPR}\{x_{g_1,g_2}[n]\} < \text{minPAPR}$) **then**
 $\text{minPAPR} \leftarrow \text{PAPR}\{x_{g_1,g_2}[n]\}$;
 $\{x_{\tilde{g}_1,\tilde{g}_2}[n]\} \leftarrow \{x_{g_1,g_2}[n]\}$;
 end if
 end for
end for
SEND $\{x_{\tilde{g}_1,\tilde{g}_2}[n]\}$;

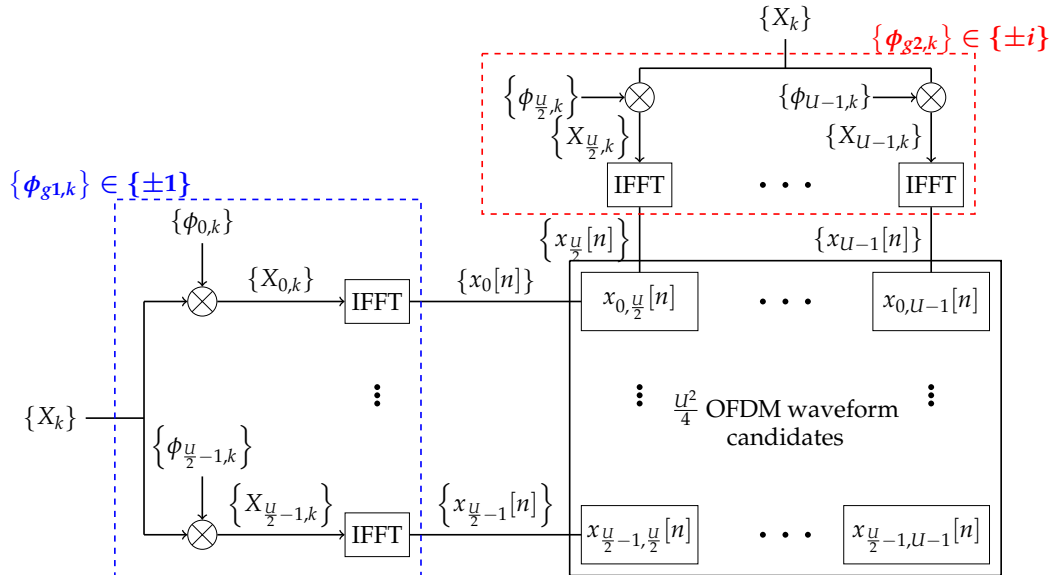


Figure 2. Original GreenOFDM block diagram [12,27].

4. An improved GreenOFDM version

The goal of the improved version is to increase the total number of candidates without increasing the number of IFFTs. As aforementioned, the key idea of the GreenOFDM technique is based on the use of two different groups (respectively generated from $\{\pm 1\}$ or $\{\pm i\}$ orthogonal sequences) of IFFTs. In the original version of the GreenOFDM technique, only the first half of the IFFTs were used for the first group ($\{x_{g_1}[n]\} = \text{IFFT}\{X_k \cdot \phi_{g_1,k}\}$ with $g_1 < \frac{U}{2}$), the second half being reserved for the second group ($\{x_{g_2}[n]\} = \text{IFFT}\{X_k \cdot \phi_{g_2,k}\}$, with $g_2 \geq \frac{U}{2}$). In the proposed improved version, the full set of IFFTs will be used both for the first group, and then for the second group after a slight modification exploiting the linearity of the IFFT and ensuring orthogonality between the two groups.

The IFFTs of the initial sequence $\{X_k\}$ weighted by pure real binary sequences $\{\phi_{u,k}\}$ with values in $\{\pm 1\}$ compose directly the first group :

$$\{x_u[n]\} = \text{IFFT}\{X_k \cdot \phi_{u,k}\}, \text{ with } 0 \leq u \leq U - 1$$

The complex waveforms of the second group correspond conceptually to the IFFTs of the initial sequence $\{X_k\}$ weighted by pure imaginary binary sequences $i \times \{\phi_{u,k}\}$ with values in $\{\pm i\}$, but are generated in a less complex way. Indeed, due to the linearity of the IFFT, the complex waveforms of the second group $\{\tilde{x}_u[n]\}$ are equal to the complex waveforms of the first group $\{x_u[n]\}$, only modified by a factor of i :

$$\begin{aligned}\{\tilde{x}_u[n]\} &= \text{IFFT}\{X_k \cdot i \cdot \phi_{u,k}\} \quad \text{with } 0 \leq u \leq U-1 \\ &= i \times \text{IFFT}\{X_k \cdot \phi_{u,k}\} \\ &= i \times \{x_u[n]\},\end{aligned}$$

So, the second group can be obtained from the IFFT outputs of the first group $\{x_u[n]\}$ by only shifting the complex values $(\Re + i\Im)$ into $(-\Im + i\Re)$, eliminating the need to compute the IFFTs again. For a given number of IFFT, the size of each group is then doubled compared to the original GreenOFDM method.

The improved GreenOFDM method is described in Algorithm 3. It can be observed that the first and second groups are redefined. The first group contains the U IFFTs (the equivalent to the SLM-OFDM generated candidates), and the second group is trivially generated by exploiting the linearity of the IFFT.

Algorithm 3 The improved GreenOFDM algorithm.

```
Require:  $\{x_u[n]\} = \text{IFFT}\{X_k \cdot \phi_{u,k}\}$ 
minPAPR  $\leftarrow +\infty$ ;
for ( $u_1 \leftarrow 0$  to  $U-1$ ) do
  for ( $u_2 \leftarrow 0$  to  $U-1$ ) do
    if  $u_1 == u_2$  then
       $\{x_{u_1, u_2}[n]\} \leftarrow \{x_{u_1}[n]\};$ 
    else
       $\{x_{u_1, u_2}[n]\} \leftarrow \frac{\{x_{u_1}[n] + i \times x_{u_2}[n]\}}{\sqrt{2}};$ 
    end if
    if ( $\text{PAPR}\{x_{u_1, u_2}[n]\} < \text{minPAPR}$ ) then
      minPAPR  $\leftarrow \text{PAPR}\{x_{u_1, u_2}[n]\};$ 
       $\{x_{\tilde{u}_1, \tilde{u}_2}[n]\} \leftarrow \{x_{u_1, u_2}[n]\};$ 
    end if
  end for
end for
SEND $\{x_{\tilde{u}_1, \tilde{u}_2}[n]\};$ 
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In the particular case $u_1 = u_2$, there is no need to perform the additions $\{x_{u_1}[n] + i \cdot x_{u_2}[n]\}$ because it does not change the PAPR of the initial waveform $\{x_{u_1}[n]\}$. In the proposed method, these extra operations are avoided in that particular case by choosing the candidate directly from the IFFT output, i.e. we choose the candidate to be equal to $\{x_{u_1}[n]\}$.

The number of candidates that are generated in the proposed method, here after referred to as the *GreenOFDMv2*, is equal to the number of times the inner loop is run. It corresponds to $C_{\text{Greenv2}} = U^2$. This means that we have 4 times more candidate waveforms as compared to the initial version of the

GreenOFDM by computing the same number of IFFTs, hence a better PAPR reduction is expected. Figure 3 depicts the proposed method block diagram.

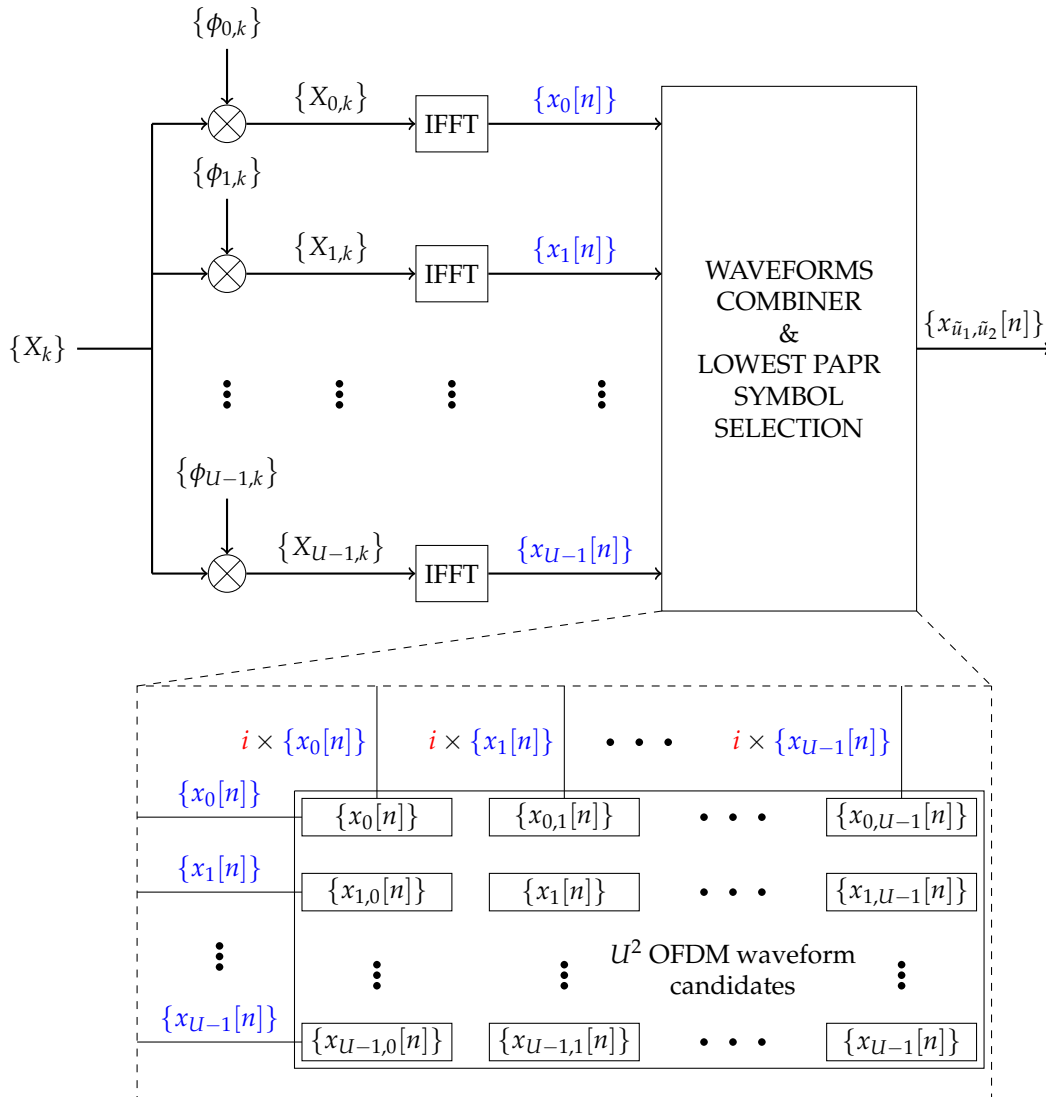


Figure 3. The proposed *GreenOFDMv2* method block diagram.

5. Simulation results and discussion

Computer simulations were carried out to evaluate the CCDF of PAPR of the improved GreenOFDM method proposed in this work and to compare it with the previous methods (SLM-OFDM and the former GreenOFDM) and the conventional OFDM. Also, the PAPR threshold for a given CCDF (fixed in 10^{-3} in the simulations) and its evolution were investigated to know and predict the required number of IFFTs to stay under a given PAPR threshold for the different techniques and different number of subcarriers.

5.1. PAPR Complementary Cumulative Distribution Function (CCDF)

Results for 10^5 symbols with $N = 64$ QPSK-modulated subcarriers, with the oversampling factor $L = 4$ and $U = 16$ IFFTs for the SLM-OFDM, GreenOFDM and the GreenOFDMv2, are depicted in Fig.4 where the dashed lines correspond to the approximated expressions of the CCDF in Eq.(3) and Eq.(4) and the marks correspond to the simulation results.

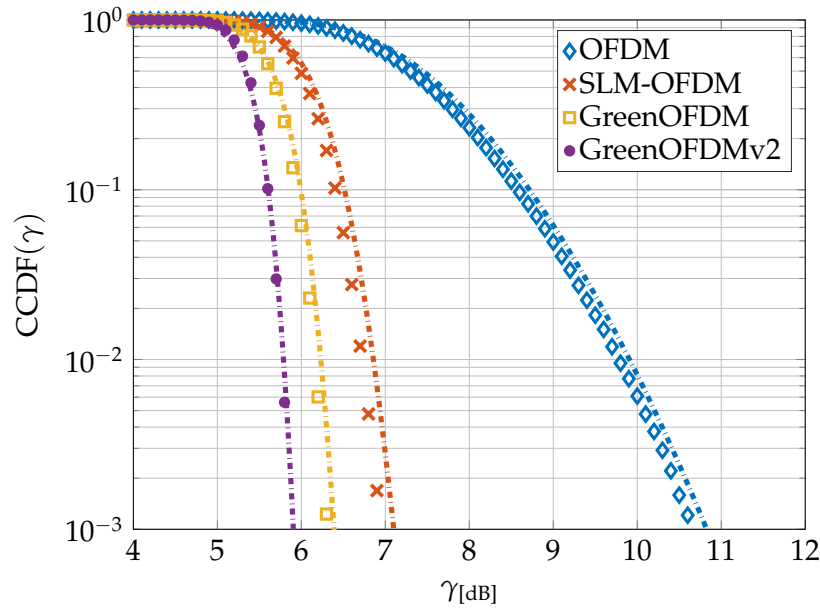


Figure 4. The CCDF of PAPR for conventional OFDM, SLM-OFDM, GreenOFDM and GreenOFDMv2 with the same parameter $U = 16$ and for $N = 64$ subcarriers per symbol, obtained by simulation (marks) and approximated expressions (dashed lines).

The simulation results are well approximated by the CCDFs expressions for OFDM, SLM-OFDM (with $C_{\text{SLM}} = U$), GreenOFDM (by replacing C_{SLM} by $C_{\text{Green}} = U^2/4$) and GreenOFDMv2 (by replacing C_{SLM} by $C_{\text{Greenv2}} = U^2$).

Moreover, it can be seen that the best performance for a fixed probability ($\text{CCDF}(\gamma)$) is obtained for the GreenOFDMv2 method as compared to SLM-OFDM and the previously proposed GreenOFDM, with a gain of 1.2 dB and 0.5 dB, respectively, for a $\text{CCDF}(\gamma) = 10^{-3}$ (the gain versus conventional OFDM is about 5 dB). This is because of the increase in the number of candidates (U^2 for GreenOFDMv2 rather than U for SLM-OFDM or $\frac{U^2}{4}$ for GreenOFDM).

5.2. PAPR threshold : simulations and approximated formula

Different values of N and U were also simulated to highlight the advantage of the proposed method in different configurations. To simplify the representation, the value of γ is calculated against the number of sub-carriers and the number of candidates for a fixed value of $\text{CCDF}(\gamma) = p$. Eqs.(3)&(4) were reformulated in order to compare the approximated expressions with the simulations. In the case of OFDM, Eq.(3) was reformulated as:

$$\gamma \approx -\log \left(1 - (1 - p)^{\frac{1}{2.8N}} \right) \quad (5)$$

and for the other methods, Eq.(4) was reformulated as:

$$\gamma \approx -\log \left(1 - \left(1 - p^{\frac{1}{C}} \right)^{\frac{1}{2.8N}} \right) \quad (6)$$

where $C = U$ for SLM-OFDM, $C = \frac{U^2}{4}$ for GreenOFDM and $C = U^2$ for the proposed method.

Results are summarized in Fig.5 where the performances for different values of U and N are plotted at a fixed probability $\text{CCDF}(\gamma) = 10^{-3}$. For OFDM, SLM-OFDM, GreenOFDM and the

proposed method, the dots represent the simulation results and the dashed lines correspond to the approximated expressions in (5) and (6).

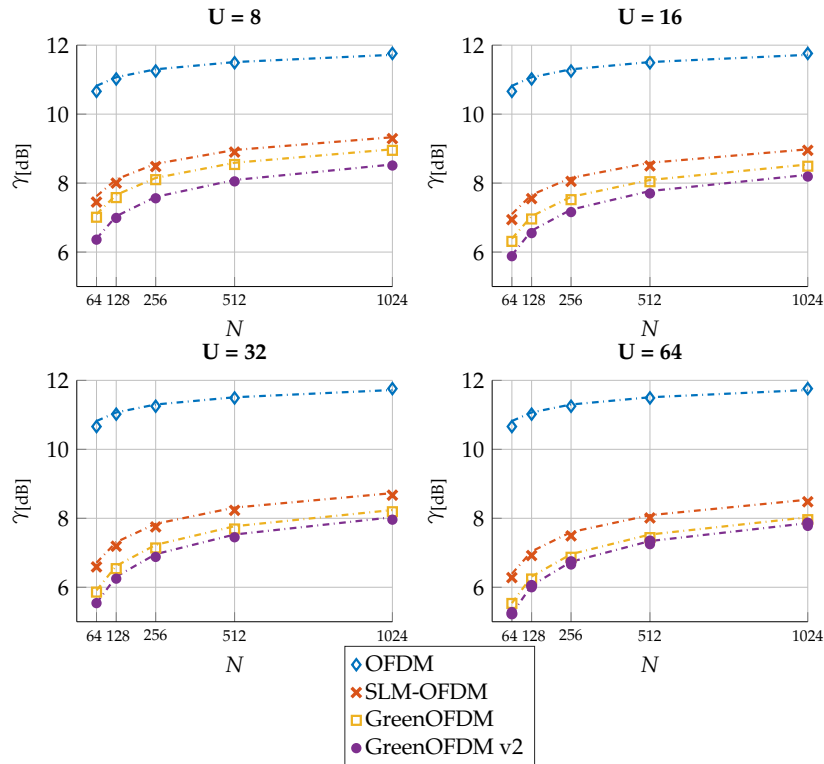


Figure 5. The value of γ [dB] to attain $\text{CCDF}(\gamma) = 10^{-3}$ for different values of U and N for OFDM, SLM-OFDM, GreenOFDM and GreenOFDMv2, obtained by simulation (marks) and approximated expressions (dashed lines).

In all the cases, the value of γ [dB] is always lower for the proposed method as compared to the SLM-OFDM and the GreenOFDM. However, the difference with the original version of GreenOFDM reduces as U increases.

It can also be noted that, for all the tested values of N and U , the CCDF threshold γ obtained by simulation is very close to the one obtained from Eq.(6). It means that the PAPR performance of the proposed technique is also well deterministically linked to the number of candidates and can be accurately predicted by the use of the Eqs.(4) and (6).

Another way to interpret the results is in terms of the reduced number of IFFTs needed to attain the same PAPR performance (and hence the same number of available candidates). To facilitate the observation of this fact, Fig.6 regroups the different configurations with equivalent performance (PAPR threshold for CCDF of 10^{-3}) for a number of subcarriers going from $N = 64$ to 1024. As expected, for all N , the proposed method requires the smallest number of IFFTs, keeping always a reduction factor of 2 with respect to the initial GreenOFDM method. For example, to ensure that the PAPR stays under 8 dB (with a probability $1 - 10^{-3}$), the required number of IFFTs is only 8 for the improved GreenOFDM method, against $8 \times 2 = 16$ for the original GreenOFDM and $8^2 = 64$ for the inspirational SLM technique. And even for a great number of 1024 subcarriers, the improved GreenOFDM technique maintains a quite reasonable complexity of 32 IFFTs contrary to SLM ($32^2 = 1024$ expected IFFTs).

The drawback, as for all the SLM-based methods, is the need of a Side Information (SI) in order to correctly de-randomize the received data symbols on the receiver-side.

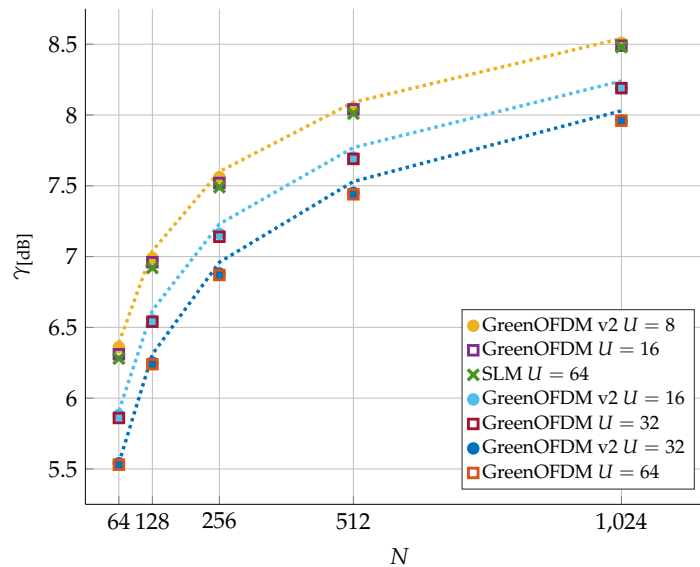


Figure 6. The value of γ [dB] to attain $\text{CCDF}(\gamma) = 10^{-3}$ for different values of U and N for SLM-OFDM, GreenOFDM and GreenOFDMv2 obtained by simulation (marks). Approximated expressions in Eq.(6) are also plotted in dashed lines for GreenOFDMv2.

5.3. Discussion on additional possible complexity reduction

As seen before, the proposed method requires only the square-root of the number of IFFTs of the original SLM method for a same PAPR reduction performance (i.e. a same number of candidates), making it a feasible technique from a complexity point-of-view. In addition to the reduction in the number of expected IFFTs, the strategy proposed in [25,27] to reduce the implementation complexity of GreenOFDM can also be exploited for improved GreenOFDM technique to further reduce the computational complexity.

The probabilistic strategy exploits the knowledge of the CCDF of the PAPR. Indeed, to attain a certain performance in terms of PAPR (at a CCDF of PAPR p , with a value of γ_p deterministically predicted through Eq.(6)), there is no need to find the lowest PAPR symbol candidate among all the possible but the first that satisfies a PAPR below the threshold γ (denoted as γ_{p_0} , p_0 being typically chosen to 10^{-3}). In addition, the idea of finding the first candidate with acceptable PAPR is complemented with the fact that calculating the PAPR requires to compute the instant power for each sample (see Eq.(2)). Indeed, if the Instantaneous-to-Average Power Ratio (IAPR) of a certain candidate exceeds the imposed *threshold*, there is no need to continue with that candidate and it can be directly discarded to continue with another one.

These facts are taken into account and implemented through the *IFFTs-on-demand* and *hierarchical sampling* methods (please refer to [27] sections 3.1 and 3.2 for the details of each method) in order to drastically reduce the computational complexity.

Furthermore, in the previous work it was shown that relaxing the PAPR reduction capability to a slightly higher threshold γ_{p_i} with

$$\gamma_{p_i[\text{dB}]} = \gamma_{p_0[\text{dB}]} + i \times 0.1[\text{dB}] \quad i = 0, 1, 2, \dots$$

attains considerably higher computational reductions with a low impact in the PAPR reduction which provides a good performance-complexity trade-off.

The same methods are implementable in the improved GreenOFDM version as described in Algorithm 4. In this algorithm, the IFFTs are computed step-by-step as the candidates are progressively calculated in the 'Hierarchical-sample' method that returns a flag in addition of the candidate itself. This flag being at '1' means that the corresponding candidate PAPR is below the imposed threshold γ_i and can be chosen for transmission. Otherwise the candidate has at least an IAPR exceeding γ_i and is hence discarded.

Algorithm 4 Computational complexity reduction in the improved GreenOFDM scheme.

Require: $\{X_k\}, \{\phi_{u,k}\}$ with $\phi_{u,k} \in \{\pm 1\}, \gamma_{p_i}$
 $\{x_{0:U-1}[n]\} \leftarrow \text{zeros}(U, LN);$
for ($u_1 \leftarrow 0$ **to** $U - 1$) **do**
 $\{x_{u_1}[n]\} = \text{IFFT}\{X_k \cdot \phi_{u,k}\}$
for ($u_2 \leftarrow u_1$ **to** 0 **step** -1) **do**
if $u_1 == u_2$ **then**
 $\{\text{flag}, \{x_{u_1, u_2}[n]\}\} \leftarrow \text{Hierarchical-sample}\{\gamma_{p_i}, \{x_{u_1}[n]\}\};$
else
 $\{\text{flag}, \{x_{u_1, u_2}[n]\}\} \leftarrow \text{Hierarchical-sample}\{\gamma_{p_i}, \{x_{u_1}[n]\}, \{x_{u_2}[n]\}\};$
end if
if (flag) **then**
 $\{x_{\bar{u}_1, \bar{u}_2}[n]\} \leftarrow \{x_{u_1, u_2}[n]\};$
STOP();
end if
end for
end for
SEND $\{x_{\bar{u}_1, \bar{u}_2}[n]\};$

All these strategies have allowed a relevant computational reduction of the former GreenOFDM in [25], especially in terms of the number of IFFTs around fifty percent (with $N = 64$ subcarriers, $U = 64$ IFFTs, and a back-off of 0.3 dB for the PAPR threshold). The computational reduction results in the improved GreenOFDM are expected to be better than those in the former version of GreenOFDM. The reason lies on the specific way in which the IFFTs are progressively computed and the possible available candidates. Indeed, in the former GreenOFDM method, the available candidates up to the v -th computed IFFT (with $v \leq U$) are $C_v \propto v^2/4$ (section 3.1 in [27]); while for the proposed method, the available candidates up to the v -th computed IFFT are $C_v = v^2$. This means that every additional computed IFFT allows to generate many more candidates for the proposed method and hence more *chances* to find the symbol whose PAPR is below the imposed threshold (reducing the expected total number of operations²).

6. Conclusion

This paper has presented an efficient method to reduce the PAPR in OFDM. This method is a clever simplification of the recent GreenOFDM technique, which is itself an extension of the well-known SLM PAPR reduction technique. In all these techniques, several waveform candidates are generated from a certain number of IFFT to retain the one with the lowest PAPR as actual OFDM

² In the previous work, it was pointed-out that the IFFTs computations are the more representatives in terms of the total number of operations

symbol. The PAPR performance of these methods is shown to be directly a decreasing function of the number of candidates. To increase the number of candidates for a given number of IFFT, the key idea of GreenOFDM was to combine two coupleable IFFT-based waveforms rather than only one in the SLM method. Moreover, the new version proposed in this paper allows to use plainly the full set of IFFTs for each group of waveforms while maintaining their coupleability, which was not the case for the original version [12] that splitted the IFFTs into two smaller separate subsets.

Consequently, compared to the first version of the GreenOFDM method, the proposed method adds a simple yet efficient improvement to reduce the PAPR by increasing the number of available candidates by a factor of 4 without increasing the number of IFFTs. Alternatively, a same PAPR performance is obtained by using only half the number of IFFTs. Compared to the initial well known SLM technique, the improved GreenOFDM technique provides then definitively a drastic complexity reduction while keeping the same idea, requiring only the square-root of the number of IFFTs for a same PAPR reduction performance.

Approximate formula are provided to predict the PAPR performance of the proposed method as a function of the number of sub-carriers and the number of candidates, or alternatively to determine the number of required IFFTs for a given PAPR performance. This prediction is very useful not only to anticipate the complexity of the technique, but also to allow implementation of further complexity reduction methods dedicated to GreenOFDM proposed previously [25,27].

Further studies are needed to propose efficient blind methods adapted to the presented technique to recover the data on the receiver side without the need of implicit side-information.

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Abbreviations

The following abbreviations are used in this manuscript:

CCDF	Complementary Cumulative Distribution Function
IAPR	Instantaneous-to-Average Power Ratio
IFFT	Inverse Fast Fourier Transform
OFDM	Orthogonal Frequency Division Multiplexing
PA	Power Amplifier
PAPR	Peak-to-Average Power Ratio
SLM	SeLected Mapping

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