Energy Vector and Time Vector in the Dirac Theory

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Abstract

We have introduced a sign operator of energy, analogous to the operator helicity, but in the direction of what we call energy vector. However, this energy vector need time vector. For giving physical senses to the components of such time vector we try to explain the time dilation in special relativity in terms of them and try to relate them to the tunnelling times when an electron crosses a potential barrier.

Keywords : Tunnelling time, helicity, time dilation, Dirac equation, superluminal velocity.

Introduction

Mysteries of time incresease as physics penetrate deeper and deeper into nature's secrets. [1] said "The treatment of time in quantum mechanics is one of the important and challenging open questions in the foundations of quantum theory".

The title of the paper make us think immediately to three dimensional time. Three dimensional time theories are not something news. Many literatures have already spoken about them. Among other are [2–6]. At our side we

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have been by chance fallen to this question when we have encountered what we call energy vector. But, many different time quantities : tunnelling times, decay times, dwell times, delay times, arrival times, or jump times in quantum mechanics and proper time, time dilation in special relativity make us dare to introduce time vector in the Dirac theory, a quantum relativistic theory which put time and space on an equal footing.
The resolution of the Dirac equation by using the tensor product or Kronecker product of matrices gives rise an operator [7] whose eigenvalues are the negative energy and the positive energy. We called this operator the "sign operator of energy". Both this operator and the operator helicity are vectors in the Pauli algebra. Their components with respect to the Pauli basis $(\sigma^1, \sigma^2, \sigma^3)$ are, respectively the components of what we call energy vector and the momentum vector. We know that the phase of a wave function solution of the Dirac equation is a combination of the components of the momentum vector coupled to the components of the position vector, that is the scalar product $\vec{p}.\vec{x}$, and the energy coupled with the classical time, that is Et. So, if we consider the energy vector, time vector should be needed in the phase of the wave function. However, we should give senses to the components of this time vector, in order to know in what situations we should consider them.
We will study at first the time vector for a free electron and will try to explain the time dilation in special relativity.

The components of a time vector and any combinations of these components would evolve simultaneously from the begining to the ending of a phenomenon like the passage time and the dwell time in quantum tunnelling, from the entrance to the outrance of a potential barrier. So it is normal to think that we will able to give senses to the components of the time vector by using the tunnelling times in quantum tunnelling.
Our method consist to put forward some hypotheses for the couplings of energies with different combinations of the components of the time vector, for example the magnitude of the energy vector couples with magnitude of the time vector, and try to find out what combinations of the components of the time vector couples with the same energy as couples with such and such tunnelling time. It follows what combination of components of the time vector is equal to the tunneling time.
The paper is organized as follows : in the first section we will show the road which has lead us to an energy vector; in the second section we will introduce the time vectors for free electron and for electron crossing through a potential; in the last section we will try to give senses to the components of time vector compared with the quantum tunnelling times when the electron crosses a potential barrier.

1 Sign Operator of energy in the Dirac Theory

The Dirac equation [8]

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi - mc\psi = 0 \quad (1)$$

is the quantum relativistic equation for a free spin-$\frac{1}{2}$ fermion, where the γ^{μ}'s are the gamma matrices. In this equation $\hbar$ is the Planck constant, c the speed of light, m the mass of the spin-$\frac{1}{2}$ fermion and ψ is its wave function.

Throughout this paper we use the Dirac representation, where the gamma matrices are

$$\gamma^0 = \sigma^3 \otimes \sigma^0, \ \ \gamma^1 = i\sigma^2 \otimes \sigma^1, \ \ \gamma^2 = i\sigma^2 \otimes \sigma^2 \ \ \gamma^3 = i\sigma^2 \otimes \sigma^3$$

with

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ \text{ and } \ \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

are the Pauli matrices and σ^0 the 2×2-unit matrix.
The wave function solution of the Dirac equation may be written as Kronecker product or tensor product (See, for instance [9])

$$\psi(t, \vec{x}) = \xi \otimes s e^{-\frac{i}{\hbar}(\pm Et - \vec{p}.\vec{x})} \quad (2)$$

of the energy state $\xi e^{-\frac{i}{\hbar}(\pm Et-\vec{p}.\vec{x})}$ and helicity state s, where $\xi = |\xi(E,p)\rangle = \sqrt{\frac{E+mc^2}{2E}} \begin{pmatrix} 1 \\ \frac{\epsilon cp}{E+mc^2} \end{pmatrix}$ is the eigenvector associated to the positive energy $E = +\sqrt{c^2p^2 + m^2c^4}$ or $\xi = |\bar{\xi}(E,p)\rangle = \sqrt{\frac{E+mc^2}{2E}} \begin{pmatrix} -\frac{\epsilon cp}{E+mc^2} \\ 1 \end{pmatrix}$, eigenvector associated to the negative energy $-E = -\sqrt{c^2p^2 + m^2c^4}$ of the hamiltonian operator $h_D = \epsilon cp\sigma^1 + mc^2\sigma^3$, and s is the eigenvector of the helicity operator $\frac{\hbar}{2}\vec{\sigma}.\vec{n}$, that is the spin operator in the direction of the momentum vector $\vec{p} = \begin{pmatrix} p^1 \\ p^2 \\ p^3 \end{pmatrix}$, with $\vec{n} = \frac{\vec{p}}{||\vec{p}||} = \frac{\vec{p}}{p} = \begin{pmatrix} n^1 \\ n^2 \\ n^3 \end{pmatrix}$.
In all of that ϵ is the sign of the helicity or the handedness.
We called the operator $h_D = \epsilon cp\sigma^1 + mc^2\sigma^3$ "sign operator of energy" [7,10].
Let us introduce the "energy vector" $\vec{E} = \begin{pmatrix} \epsilon cp \\ 0 \\ mc^2 \end{pmatrix}$. Therefore, the operator $\frac{\hbar}{2}\frac{h_D}{E} = \frac{\hbar}{2}\frac{\vec{\sigma}.\vec{E}}{E}$ is the projection of the spin operator in the direction of the

energy vector $\vec{E}$. Let us call the eigenvalues of this operator "enginity" and this operator the "enginity operator". Therefore there is the probabilities of the particle of having the positive enginity $+\frac{\hbar}{2}$ or the negative enginity $-\frac{\hbar}{2}$. For seeing that more clearly let us compare the enginity operator with the helicity operator.

$h_D = \epsilon cp\sigma^1 + mc^2\sigma^3$ hamiltonian operator	$\vec{\sigma}.\vec{p} = p_1\sigma_1 + p_2\sigma_2 + p_3\sigma_3$
let $\vec{E} = \begin{pmatrix} \epsilon cp \\ 0 \\ mc^2 \end{pmatrix}$ energy vector	with $\vec{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$ momentum vector
$E = \left\|\vec{E}\right\| = \sqrt{m^2c^4 + c^2p^2}$ the energy	$p = \|\vec{p}\| = \sqrt{p_1^2 + p_2^2 + p_3^2}$
$\frac{\hbar}{2E}h_D = \frac{\hbar}{2E}\epsilon cp\sigma^1 + \frac{\hbar}{2E}mc^2\sigma^3$ enginity operator	$\frac{\hbar}{2p}\vec{\sigma}.\vec{p} = \frac{\hbar}{2p}p_1\sigma_1 + \frac{\hbar}{2p}p_2\sigma_2 + \frac{\hbar}{2p}p_3\sigma_3$ helicity operator or
spin operator in the direction of $\vec{E}$	spin operator in the direction of $\vec{p}$
$\frac{h_D}{E} = \frac{\epsilon cp}{E}\sigma^1 + \frac{mc^2}{E}\sigma^3$ enginity sign operator	$\frac{\vec{\sigma}.\vec{p}}{p} = \frac{p_1}{p}\sigma_1 + \frac{p_2}{p}\sigma_2 + \frac{p_3}{p}\sigma_3$ helicity sign operator
Probability for having positive or negative enginity (energy)	Probability for having positive or negative helicity

So, a spin-$\frac{1}{2}$ particle can be in a superposition of a state of positive and a state of negative energy.

But, as we have said in the introduction, energy vector $\vec{E} = \begin{pmatrix} \epsilon cp \\ 0 \\ mc^2 \end{pmatrix}$ need time vector $\vec{t} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$.

2 Components of the Time Vector

Let us at first regard an electron with mass m, moving freely along an x axis, from a point O to a point A of this axis. An observer observes the motion of the electron in a frame where the electron is at rest. So, this observer can mesure the time, the proper time $\tau = t_3$ that takes the electron for moving from O to A. For calculating the energy of the electron this observer use the formula $E = mc^2$.

Now let us see how an observer in a frame fixed at the point O, mesures the time that takes the electron for moving from O to A with velocity v and how he calculates the energy of the electron. For this observer, A is at a distance L from O. The electron takes the impulsion $p = \frac{mv}{\sqrt{1-(v/c)^2}}$, and the observer mesure the time τ' for the passage of the electron from O to A and use the formula $E = \sqrt{m^2c^4 + c^2p^2}$ for calculating the energy of the electron. The energy is the magnitude of the energy vector $\vec{E} = \begin{pmatrix} \epsilon cp, \\ 0 \\ mc^2 \end{pmatrix}$ which need the time vector $\vec{t'} = \begin{pmatrix} t'_1 \\ 0 \\ t'_3 \end{pmatrix}$, where ϵ is the sign of the helicity.

$$mc^2t_3 = mc^2t'_3 + \epsilon cpt'_1 - px \tag{3}$$

$$c^2t_3^2 = c^2t'^2_3 + c^2t'^2_1 - x^2 \tag{4}$$

We can check easily that t'_3 should not be equal to t_3. Then, solving this system of two equations we will have two time vectors. But, according to (4), these two time vectors have the same euclidian norm, and according to the special relativity of Einstein

$$\tau' = \sqrt{t'^2_3 + t'^2_1} = \frac{1}{\sqrt{1 - (v/c)^2}}\tau$$

In this formula, since $t_3 \neq t'_3$, the time t'_1 appeared when the electron takes the impulsion p is not at all the responsible of the time dilation in special relativity. The classical time $\tau' = \sqrt{t'^2_3 + t'^2_1}$ and the component times t'_3, t'_1 evolve from O to A, but only the classical time can be observed. Then, the wave function is of the form (2).

Now let us suppose that from O to A the electron moves in a uniform potential U. For the observer at the frame where the electron is fixed the energy vector is $\vec{E'} = \begin{pmatrix} 0, \\ -U \\ mc^2 \end{pmatrix}$ and the time vector is $\vec{\mathcal{T}} = \begin{pmatrix} 0 \\ \mathcal{T}_2 \\ \mathcal{T}_3 \end{pmatrix}$. Whereas for the observer at the second frame the energy vector is $\vec{\mathcal{E}} = \begin{pmatrix} \epsilon cp, \\ -U \\ mc^2 \end{pmatrix}$ whose components are respectively the energy due to the impulsion, the energy due to the mass and the potential energy, that is the energy due to the space, which makes appear the second component of the time vector, $\vec{\mathcal{T}'} = \begin{pmatrix} \mathcal{T}'_1 \\ \mathcal{T}'_2 \\ \mathcal{T}'_3 \end{pmatrix}$. The minus sign of the second component of the energy vector will be explained later. It follows

$$\phi = \mathcal{T}_3mc^2 - \mathcal{T}_2U = mc^2\mathcal{T}'_3 + \epsilon cp\mathcal{T}'_1 - U\mathcal{T}'_2 - px \tag{5}$$

$$c^2\mathcal{T}_3^2 + c^2\mathcal{T}_2^2 = c^2\mathcal{T}'^2_3 + c^2\mathcal{T}'^2_1 + c^2\mathcal{T}'^2_2 - x^2 \tag{6}$$

and

$$\sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2 + \mathcal{T}_2'^2} = \frac{1}{\sqrt{1-(v/c)^2}}\sqrt{\mathcal{T}_3^2 + \mathcal{T}_2^2}$$

The total energy of the electron is the magnitude

$$\mathcal{E} = \sqrt{m^2c^4 + c^2p^2 + U^2} \tag{7}$$

of the energy vector, which is like the one in [11] for the extension to the Klein-Gordon equation, and the magnitude

$$\mathcal{T}' = \sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2 + \mathcal{T}_2'^2}$$

of the time vector $\vec{\mathcal{T}}'$ is the classical time.
We put forward the following hypotheses for possible couplings of energy with time in the phase of the wave function:

$$\mathcal{E}\mathcal{T}' = \sqrt{m^2c^4 + c^2p^2 + U^2}\sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2 + \mathcal{T}_2'^2} \tag{8}$$

$$E\mathcal{T} = \sqrt{m^2c^4 + c^2p^2}\sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2} \tag{9}$$

$$mc^2\mathcal{T}_3' \tag{10}$$

$$U\mathcal{T}_2' \tag{11}$$

$$\sqrt{c^2p^2 + U^2}\sqrt{\mathcal{T}_2'^2 + \mathcal{T}_1'^2} \tag{12}$$

$$\epsilon cp\mathcal{T}_1' \tag{13}$$

3 Components of Time Vector and tunnelling Times of electron

For giving physical senses of the components of time vector, we think that it is normal to try to find their possible relations with the tunnelling times. But, let us at first search for the Dirac type equation for the electron in a potential less than the kinetic energy of the electron.

3.1 A Dirac equation with parity violation

The Dirac equation we would like to search for is a Dirac equation which has the energy vector $\vec{\mathcal{E}} = \begin{pmatrix} \epsilon cp, \\ -U \\ mc^2 \end{pmatrix}$, that is whose operator enginity is

$$H = \epsilon cp\sigma_1 - U\sigma_2 + mc^2\sigma_3 \tag{14}$$

with $U < \sqrt{c^2p^2 + m^2c^4}$.
The search for a solution of the form $\psi = A(p)e^{-\frac{i}{\hbar}(\mathcal{E}t - \vec{p}\cdot\vec{x})}$ of the Dirac-Sidharth equation [12]

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi - i\sqrt{\alpha}l\hbar\gamma^5\Delta\psi = 0$$

by using the kronecker product leads to the operator enginity

$$H' = \epsilon cp\sigma_1 - c\sqrt{\alpha}p^2\frac{l}{\hbar}\sigma_2 + mc^2\sigma_3$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \sigma^1 \otimes \sigma^0$.

Then, in following the backward way, from the operator enginity (14) we will have as Dirac equation for discribing electron in a potential U the following equation

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi - i\frac{U}{c}\gamma^5\psi = 0 \tag{15}$$

Because of the presence of γ^5, party is violated [13]. Looking for a wave function of the form

$$\psi = A(p)e^{-\frac{i}{\hbar}(\mathcal{E}t - \vec{p}\cdot\vec{x})}$$

that is of the form of (2), by using the kronecker product of matrices, we will have

$$\psi = \sqrt{\frac{\mathcal{E} + mc^2}{2\mathcal{E}}}\frac{1}{\sqrt{2(1+n^3)}}\begin{pmatrix} 1 \\ \frac{-cp-iU}{\mathcal{E}+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -n^1 + in^2 \\ 1 + n^3 \end{pmatrix} e^{-\frac{i}{\hbar}(\mathcal{E}t - \vec{p}\cdot\vec{x})} \tag{16}$$

as solution with positive enginity and negative helicity.

3.2 Components of Time Vector and tunnelling Times

The components of the time vector make us to think to this one of the controversial issues of modern quantum theory, the question of tunnelling time, i.e. the time that takes a particle to move from one side of a barrier of

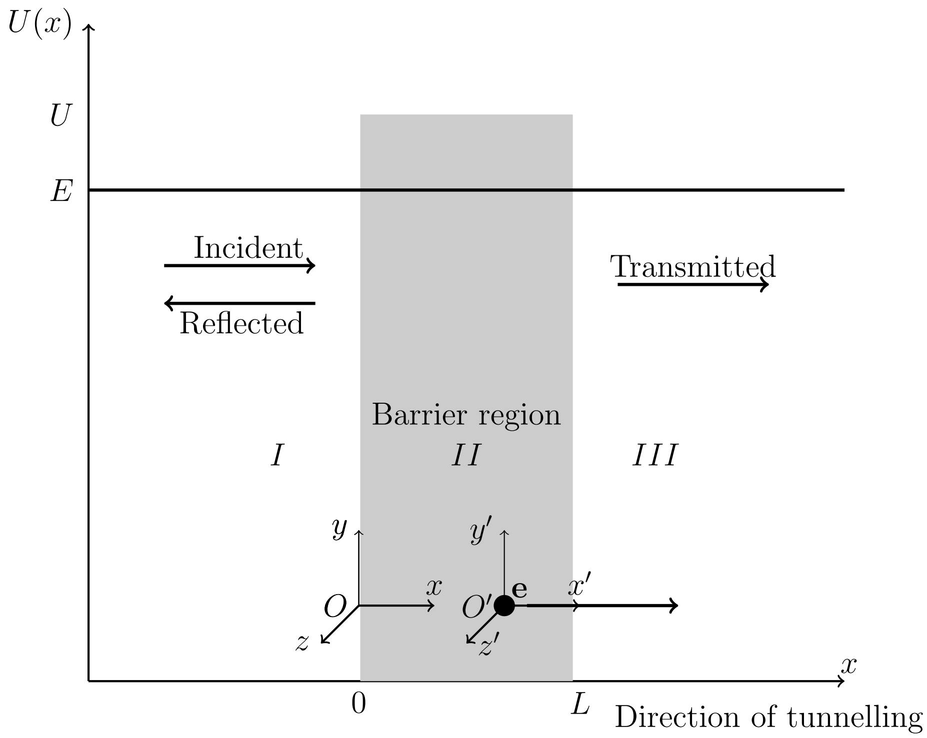

Figure 1: An electron **e** with kinetic energy E moves along the x-axis and interacts with a rectangular barrier with height U, $U > E$, and width L

potential to the other side [14]. Some experimental investigations have supported a nonzero tunnelling time, while others supported a zero tunnelling time , [15, 16].
But for giving senses to the components of the time vector we have to opt to the nonzero tunnelling time and let us consider the case of one dimensional tunnelling of electron through a potential barrier.
Quantum tunnelling is a phenomenon in which particles penetrate a potential energy barrier with a height greater than the total energy of the particles. The phenomenon is interesting and important because it violates the principles of classical mechanics. Suppose an uniform and time-independent beam of electrons with energy E traveling along the x-axis (in the positive direction to the right) encounters a potential barrier (Figure1) described by (See, for instance [17])

$$U(x) = \begin{cases} 0 & \text{when } x < 0 \\ U & \text{when } 0 \leq x \leq L \\ 0 & \text{when } x > L \end{cases}$$

When both the width L and the height U are finite, a part of the quantum wave packet incident on one side of the barrier can penetrate the barrier boundary and continue its motion inside the barrier, where it is gradually attenuated on its way to the other side. A part of the incident quantum wave packet eventually emerges on the other side of the barrier in the form of the transmitted wave packet that tunneled through the barrier. How much of the incident waves can tunnel through a barrier depends on the barrier width L and its height U , and on the energy E of the quantum particle incident on the barrier. For such transmitted waves there are four widely used tunnelling times calculated by finding the transmission amplitude given by: $T = |T|\, e^{i\theta}$ [18].The two of them are : Larmor time [19, 20], τ_{LM} and Eisenbud-Wigner times [21], τ_{EW}. The first has been called resident or dwell time:

$$\tau_{LM} = -\hbar \frac{\partial \theta}{\partial U} \tag{17}$$

The second has been called the passage time,

$$\tau_{EW} = \hbar \frac{\partial \theta}{\partial E} + \frac{L}{k} \tag{18}$$

An additional term, L/k is present in τ_{EW}, where L and k are the barrier width and electron velocity, respectively. This additional term corresponds to the propagation of the electron in the barrier region if that barrier were absent, and has to be added to get the total time [22], since the first term only gives a relative time shift [21].

However, the quantum tunnelling phenomena and the consideration of the time vector make to think that the Dirac equation inside the potential barrier $(E < U)$ is not of the form (15) [24]. So, let us constructe the wave function of the electron inside the barrier in terms of the components of the time vector.

The energy vector $\vec{E} = \begin{pmatrix} \epsilon cp \\ 0 \\ mc^2 \end{pmatrix}$ before the barrier region will become $\vec{\mathcal{E}} = \begin{pmatrix} \epsilon cp \\ -U \\ mc^2 \end{pmatrix}$ when the particle is inside the barrier region.

The time $\mathcal{T} = \sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2}$ and $\mathcal{T}_2'$ can be qualified respectively as passage time $\mathcal{T}_p = \sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2}$ and resident time $\mathcal{T}_r = \mathcal{T}_2'$. These three times, the classical time $\mathcal{T}' = \sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2 + \mathcal{T}_2'^2}$, the passage time $\mathcal{T}_p$ and the resident time $\mathcal{T}_r$ evolve from the entrance to the outrance of the barrier region. But according to the quantum tunnelling phenomena the classical time can not be observed, whereas at least one of the passage time and the resident time can be. Actually,

$$\mathcal{T}' > \mathcal{T}_p \qquad \mathcal{T}' > \mathcal{T}_r$$

All these times evolve from zero to positive values.
Let us search for θ in (17) and (18) in terms of τ_{LM} and τ_{EW}. From (17)

$$\theta = -\frac{1}{\hbar}\tau_{LM}U + K(E)$$

where $K(E)$ is a function of E. Then,

$$\frac{\partial\theta}{\partial E} = K'(E)$$

in substituting in (18)

$$K'(E) = \frac{1}{\hbar}\tau_{EW} - \frac{1}{\hbar}\frac{L}{v}$$

Using the relations $p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$ and $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ (See for instance, [23]), we have

$$K'(E) = \frac{1}{\hbar}\tau_{EW} - \frac{1}{\hbar}\frac{E}{\sqrt{E^2 - m^2c^4}}L$$

and then

$$\theta = \frac{1}{\hbar}\left(E\tau_{EW} - U\tau_{LM} - pL\right) + \lambda(L) \tag{19}$$

with $\lambda(L)$ independant of E and U, such that $U > E$.
We will able to see the value of the constant $\lambda(L)$ if a boundary conditions on the phase difference θ are determined. But, according to the couplings (9) and (11), for giving senses to the components of time vector, define the phase which evolves from the phase at $x = 0$ to $x = L$, inside the potential barrier, as

$$\varphi_{II} = -\frac{1}{\hbar}\left(E\sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2} - U\mathcal{T}_2' - px\right)$$

with at $x = L$, $\sqrt{\mathcal{T}_3'^2 + \mathcal{T}_1'^2} = \tau_{EW}$ and $\mathcal{T}_2' = \tau_{LM}$. Then, we have $\lambda(L) = 0$ and for the case of positive enginity and negative helicity incident, reflected and transmitted wave functions are

$$\psi_I(x) = \begin{pmatrix} 1 \\ \frac{-cp}{E+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(E\sqrt{t_3'^2+t_1'^2}-px\right)} + A \begin{pmatrix} 1 \\ \frac{cp}{E+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(E\sqrt{t_3'^2+t_1'^2}+px\right)} \quad (x<0)$$

$$\psi_{II}(x) = B \begin{pmatrix} 1 \\ \frac{-cp-iU}{\mathcal{E}+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(E\sqrt{\mathcal{T}_3'^2+\mathcal{T}_1'^2}-U\mathcal{T}_2'-px\right)} + C \begin{pmatrix} 1 \\ \frac{cp-iU}{\mathcal{E}+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(E\sqrt{\mathcal{T}_3'^2+\mathcal{T}_1'^2}-U\mathcal{T}_2'+px\right)} \quad (0<x<L) \quad (20)$$

$$\psi_{III}(x) = D \begin{pmatrix} 1 \\ \frac{-cp}{E+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(E\sqrt{t_3'^2+t_1'^2}-px-E\tau_{EW}+U\tau_{LM}+pL\right)} \quad (L<x)$$

The form of each term of the wave function (20) inside the barrier is not like the one has been thought in [24]. It is a wave function solution, not of $(1+1)$ spacetime Dirac equation particular case of (16), but a $(1+2)$ spacetime Dirac equation.
In the case where the energy of the electron is higher than the value of the potential $(E>U)$, the wave function inside the potential will be of the form

$$\psi_{II}(x) = A' \begin{pmatrix} 1 \\ \frac{-cp-iU}{\mathcal{E}+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(\mathcal{E}\sqrt{\mathcal{T}_3'^2+\mathcal{T}_2'^2+\mathcal{T}_1'^2}-px\right)} = A' \begin{pmatrix} 1 \\ \frac{-cp-iU}{\mathcal{E}+mc^2} \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-\frac{i}{\hbar}\left(\mathcal{E}\sqrt{\tau_{EW}^2+\tau_{LM}^2}-px\right)}$$

because according to (8) the classical time t in the wave function (16) is $t=\sqrt{\mathcal{T}_3'^2+\mathcal{T}_2'^2+\mathcal{T}_1'^2}$.

3.3 Discussion

The minus sign before the potential energy U in the phase difference (19) explains our choice of the minus sign before the second component of the energy vector $\vec{\mathcal{E}}$. Thus this second component is a negative energy. Like the first component which may be positive or negative energy depending on whether helicity sign ϵ is positive or negative, the second component may also be positive or negative energy.

If we choose $+U$ as second component of the energy vector $\vec{\mathcal{E}}$, the Larmor time τ_{LM} will be negative.
For both the two choices, according to the Feynman-Stückelberg interpretation of the negative energy in the Dirac theory (See for instance [25]) it is not the electron which spends the resident time τ_{LM} but its antiparticle, a positron.

Conclusion and Outlook

The energy vector in the Dirac theory has come when we would try to show the analogy between sign of helicity and the sign of energy, which we have then called sign of enginity. This energy vector need time vector whose components deserve physical senses.
The component of the time vector which occur when the electron takes an impulsion is not at all the responsible of the time dilation in special relativity.
In the Dirac representation, for the tunnelling of the electron through a potential barrier the passage time can be defined as the magnitude of the projection of the time vector to the plan of first and third components of the time vector, whereas the dwell time can be defined as the second components of the time vector. They are respectively the Eisenbud-Wigner time and the Larmor time, τ_{EW} and τ_{LM}, at the potential barrier outrance. Then, for an electron crossing a potential barrier the classical time $\left\|\vec{\mathcal{T}}'\right\| = \sqrt{\tau_{EW}^2 + \tau_{LM}^2}$ can not be observed, whereas the passage time τ_{EW} can be.
It has been shown from the coupling of the negative energy $-U$ with the Larmor time τ_{LM} in the expression (19) of the phase difference that it is not the electron which spends the resident time τ_{LM} but its antiparticle, a positron.
For a free electron we can not give senses to t_3' or t_1' separately. We think that a possible observability of these two components of time vector would be in a phenomenon of free spin-$\frac{1}{2}$ superluminal particle, then whose wave function would be a solution of a $(3+2)$ spacetime Dirac equation. So, we join the authors of [26, 27] which say :"The problem of representation and localizations of superluminal particles has been solved only by the use of higher dimensional space and it has been claimed that the localization space for tachyons is T^4- space with one space and three times".

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