Two Exact Solutions Illustrating the Schwarzschild Singularity's Resolution

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Abstract: General wisdom thinks that the central of blackholes are singular and featured by mass, charge and spin exclusively; the microstate reflected in the Bekenstein-Hawking entropy can be accounted for only in some quantum gravity. We challenge this wisdom with two exact solution families to the sourceful Einstein equation which describe dust ball with oscillatory inner structure and variety of mass profiles. We attribute the matter content's oscillation across the central point of the system to the forward scattering domination in high energy collisions induced by inter-gravitating acceleration and provide their Penrose-Carter diagram representation and quantum description briefly. Our solution family provides a very simple semi-classic picture for the micorstate definition of black holes.

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1 Introduction

Essential singularity is the key obstacle for our understanding of black hole physics. By Hawking and Penrose's theorem [1], matters contracting into a region less than the horizon determined by their mass will get on the central point in finite proper time and form singularities there. As long as singularity forms, we cannot use any known physics to understand these matter's behavior. Singularity, or the singular point has not any feature except mass, charge and spin. However [2, 3], due to the famous work of Hawking and Bekenstein, it is known that blackhole has entropy and many possible microscopic states. The general wisdom thinks that this entropy and related microstates could be accountable only in some quantum gravities. We challenged this wisdom in a serial of recent works [4–7]. The key idea to these works is the matters(consisting of the black hole)' oscillation inside the horizon.

Although many people refute to talk about, we have not any first principle forbidding such oscillations' occurrance in a linearly-inverse potential well in either Newton Mechanics or General Relativity. Some people may object to the use of terminology "potential well" and "test particle" since GR allows no such thinking way. For those, replacing the "potential well" and "test particle" with "self gravitation field" and "area elements" of a spherically symmetric collapsing shell, the following equations will be equally applicable to describe the system's evolution. Assuming that $r \leqslant a < 2GM$, dynamics of the oscillation reads

$$\mathrm{NM}: \; m\ddot{r} = -\frac{GMm}{r^2} \Rightarrow \dot{r}^2 - \frac{2GM}{r} = -\frac{2GM}{a}(\text{integration constant}) \tag{1}$$

$$\mathrm{GR}: \; \begin{cases} h\dot{t}^2 - h^{-1}\dot{r}^2 = 1, \;\; h = 1 - \frac{2GM}{r} \\ \ddot{t} + \Gamma^t_{tr}\dot{t}\dot{r} = 0 \Rightarrow h\dot{t} = \text{const} \equiv \sqrt{1 - \frac{2GM}{a}} \end{cases} \Rightarrow \begin{cases} \dot{r}^2 - \frac{2GM}{r} = -\frac{2GM}{a} \\ \dot{t} = \frac{\sqrt{1-\frac{2GM}{a}}}{1-\frac{2GM}{r}} \end{cases} \tag{2}$$

h and Γ in the GR equation are metric function and connection coefficients of relevant spherical symmetric spacetime respectively. Both equations (1) and (2) can be integrated explicitly to yield oscillating solutions, whose first 1/4th period of expression read,

$$\mathrm{NM}: \; r(t) \text{ through } t = -\frac{a^{3/2}}{\sqrt{2GM}}\big(\arctan\frac{\sqrt{r}}{\sqrt{a-r}} - \frac{\pi}{2}\big) + \frac{\sqrt{ar(a-r)}}{\sqrt{2GM}} \tag{3}$$

$$\mathrm{GR}: \; r(\tau) \text{ through } \tau = -\frac{a^{3/2}}{\sqrt{2GM}}\big(\arctan\frac{\sqrt{r}}{\sqrt{a-r}} - \frac{\pi}{2}\big) + \frac{\sqrt{ar(a-r)}}{\sqrt{2GM}} \tag{4}$$

$$r(t): \; it = (4GM+a)\big(\arctan\sqrt{\frac{r}{a-r}} - \frac{\pi}{2}\big) - \frac{4GM\big(\arctan\sqrt{\frac{2GM-a}{2GM}\frac{r}{a-r}} - \frac{\pi}{2}\big)}{\sqrt{1-\frac{a}{2GM}}} - \sqrt{(a-r)r} \tag{5}$$

both the NM time t and GR proper time τ in the first two equations are real, but the GR coordinate time t in the last equation is purely imaginary. We display in Figure 1 the full oscillation curves with more details. From the figure, we easily see that, either in NM or GR,

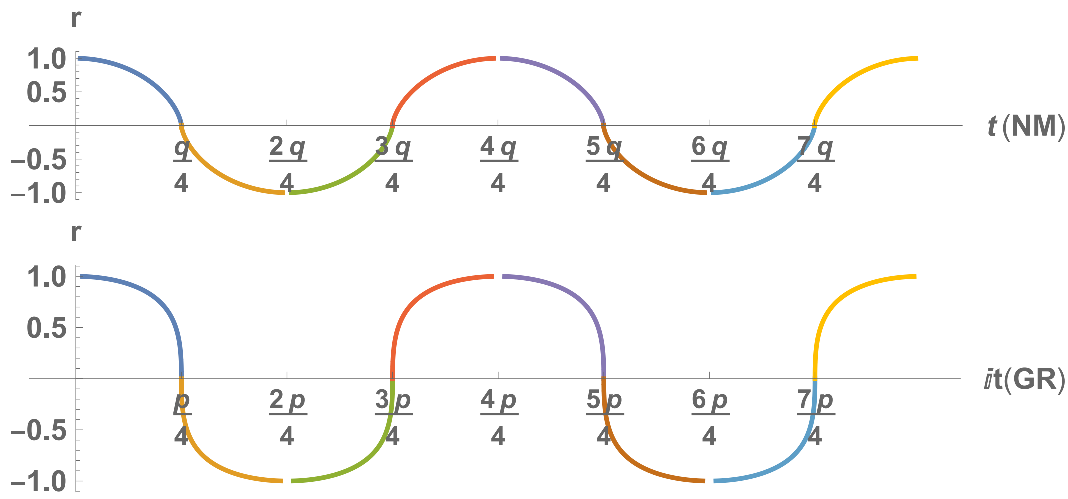

Figure 1. Upper part, the oscillation $r(t)[r(\tau)$ completely the same] curve of particles radially moving in a linear-inverse potential well described by newton mechanics or general relativity. t is newtonian absolute time and τ is the proper time of particles in general relativity. Downer part, the oscillatory $r(it)$ curve of the same particle in general relativity, it is the continuation of coordinate time t defined by infinitely far away observers. For any freely falling particle, $t_{r\to r_h+\epsilon} \to \infty$, $t_{r\to r_h-\epsilon} \to i\infty$. We set $GM = 1$ and $a = 1$, so if the potential well is produced by a Schwarzchild black hole with singular central, then our test particles are released from inside the horizon and the oscillation period equals $q = 4\pi$, $p = (8\sqrt{2} - 10)\pi$. In the case $a \to 2GM = r_h$, $p \to \infty$.

the radial coordinate of particles in linearly-inverse potential wells are periodical functions of the t, τ or it. That is, although the potential well is singular, test particles' trace in it is rather well definable.

Some people may argue against the continuation of time concept into complex field. But we simply look it as an indication that the far away observer's time t is not a well defined physical coordinate to cover the whole life of a black hole. We take the consisting matter's oscillation inside the horizon as true occurrences and find that there are at leas two coordinate systems, [4–6] and [7] respectively, by which we can write out the inner metric of a black hole with oscillation core instead a once-and-for-ever singular center. We explored the functionality of this matters' inside-horizon oscillation in understanding the area law entropy formula and Hawking's information missing puzzle. The current work is a shortened introduction to the most persuasive part in our explorations — two exact solutions to the sourceful Einstein equation, firstly appearing in [7], exhibiting matters' oscillation inside the horizon and their relevance to the area law entropy formula, section 2 and 3. Comparing with previous publishes, the current work provides diagrammatic description for these dynamical space-time and explicit wave function for these matters' inside horizon oscillation. We also response to many common questions or critics from colleagues, section 4.

2 AdS$_{2+1}$-Schwarzschild Black Holes

Our first example is the 2+1 dimensional disk in the asymptotically Ante de-Sitter background. As is well know, in the asymptotically flat background, any collapsing matter with mass less than $(2G)^{-1}$ will lead to conic singularities on the central point with no horizon wrapping around, while those with mass more than $(2G)^{-1}$ always lead to spacetime with "wrongly" signatured metric [8]. However, in asymptotically Anti-de Sitter conditions [9–13], the background cosmological constant provides a parabolic potential with minimal value zero. Adding matter effects which are always lowering the potential everywhere, blackholes with finite size can appear,

$$ds^2 = -hdt^2 + h^{-1}dr^2 + r^2 d\phi^2 \tag{6}$$

$$h = 1 + \frac{r^2}{\ell^2} - 2GM \tag{7}$$

with the horizon radius given by $r_h = \sqrt{2GM-1}\ell$. Killing the term $\frac{r^2}{\ell^2}$ in h, this will become the metric of a mass-point in asymptotical flat spacetime. In either asymptotics, conic singularity occurs as a deficit angle when one walking around the $r = 0$ point along small enough equal-r orbit.

The conic singularity in 2+1D AdS blackholes is a weaker singularity than the essential one in 3+1D Schwarzschlds. However, both singularities are key obstacles for our understanding of blackhole physics. Our resolving of the conic singularity is as follows. At classic level, due to triviality of 2+1 general relativity, the motion of a dust shell [1] under self gravitation in AdS background is just that of harmonic oscillators. While the evolution of a spherically symmetric disk under self gravitation is just the composition of many such concentric shells [7]. Inside the matter occupation region, the spacetime metric can be written as

$$ds^2 = -d\tau^2 + a^2(\tau)\left[\frac{d\varrho^2}{1+\varrho^2\ell^{-2}-2GM(\varrho)} + \varrho^2 d\phi^2\right] \tag{8}$$

$$a[\tau] = \cos[\ell^{-1}\tau],\ 0 \leqslant \varrho \leqslant \varrho_{\rm hor}\ M[\varrho_{hor}] = M_{\rm tot} \tag{9}$$

Outside the matter occupation region, ds^2 can be joined smoothly to (6) by the standard textbook methods [14]. We checked that this inner metric satisfies the sourceful Einstein equation with $\Lambda = -\ell^{-2} < 0$ and $T_{\mu\nu} = \text{diagonal}.\{\rho(\tau,\varrho) = \frac{M'\varrho^{-1}}{8\pi\, a[\tau]^2},\ 0,\ 0\}$. At classic levels $M(\varrho)$ has infinite number of possible forms which provide just the basis for microscopic states evaluated by the Bekenstein-Hawking entropy. By the usual singularity theorem, all matters will get on the $\varrho = 0$ point and form conic singularities there in finite proper time. That point will be the ending of time, essentially an equal time surface. But in our exact solution (8), matters falling on that surface will go across and oscillate there, see Figure 2 for pictorial understandings. This going across could be attributed to the quantum phenomena, when two or more particles collide together at high enough energy, forward amplitude always dominate the total scattering process, little bounding state forms.

[1] We call rings in 2+1D spacetime and shells in 3+1D spacetime as shells uniformly.

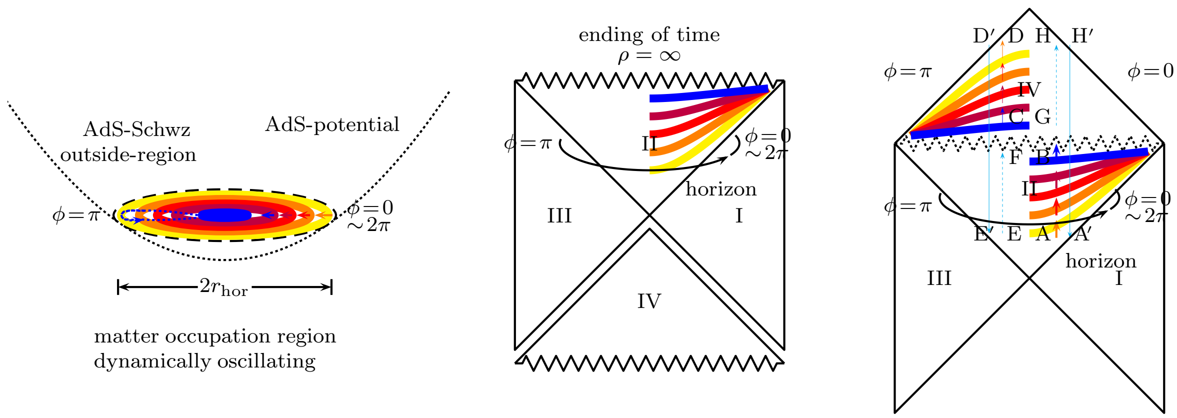

Figure 2. A dust disk (left) with oscillating inner structure in 2+1D AdS background and its Penrose-Carter diagram. By the usual understanding (middle), the $\varrho = 0$ is an energy-density divergent point, essentially an equal-time surface, on which time ends. In our understanding (right), there is no time ending. The dust matter consisting of the disk just oscillate across that point. A complete oscillation period is A-B-C-D-D′-E′-E-F-G-H-H′-A′-A. Region III and IV consisting of antipodal points in region I and II. Colored family of lines on the middle and right figures correspond to various half equal-r shells on the left figure. We can also see the whole family of lines as snapshots of the outmost shell at different instantaneous epochs.

At quantum levels, looking the black hole, or more precisely the dust disk as many concentric shell, each with mass m_i and varying radius r, and introducing a wave function $\psi_i(r)$ to denote the probability amplitude the shell be found at radius r, we will find that $\psi_i(r)$ satisfy equations of the form

$$[(i\hbar m_i^{-1}\partial_r)^2 + 1 + \frac{r^2}{\ell^2} - 2GM_i - \gamma_i^2]\psi_i(r) = 0. \tag{10}$$

This equation follows from the canonical quantization (write the radial momentum $m\dot{r}$ into operators $i\hbar\frac{\partial}{\partial r}$ and the Hamiltonian constraint as an operatorised equality) of the shell's classic equation of motion

$$\dot{r}^2 - \gamma_i^2 + h_i = 0 \Leftarrow \begin{cases} h_i\dot{t}^2 - h_i^{-1}\dot{r}^2 = 1, \;\; h_i = 1 + \frac{r^2}{\ell^2} - 2GM_i \\ \ddot{t} + \Gamma^{(i)t}_{tr}\dot{t}\dot{r} = 0 \Rightarrow h_i\dot{t} = \gamma_i = \text{const} \end{cases} \tag{11}$$

which in turn follows from the geodesic motion of a freely falling object accompanying the freely collapsing dust shell. The mass definition M_i in (10)-(11) is subtle. When a shell is released outside horizon $r_{\rm release} > r_h$, $\dot{r} = 0$, $\dot{t} = 1$, $\gamma_i = h_i(r)_{r=r_{\rm release}} > 0$; when it is released inside the horizon, $\gamma_i^2 < 0$. So at classic levels, $\gamma_i^2 < 0$ will assure the shell's motion inside the horizon. But at quantum levels, the radial coordinate varies in the whole $0 \leqslant r < \infty$ range. $\gamma_i^2 < 0$ does not assure $\psi_i(r)$'s being zero uniformly outside the horizon. Considering the fact that simple square integrability will force the shell's appearing probability density to take maximals inside or although outside but very close to the horizon, we define M_i as the mass summation of all shell i's whose $[r|\psi_{i'}(r)|^2]_{\max}$ occur more close to the origin than $[r|\psi_i(r)|^2]_{\max}$ does. Approximately, $M_i \approx \sum_{i' \leqslant i} m_{i'}$.

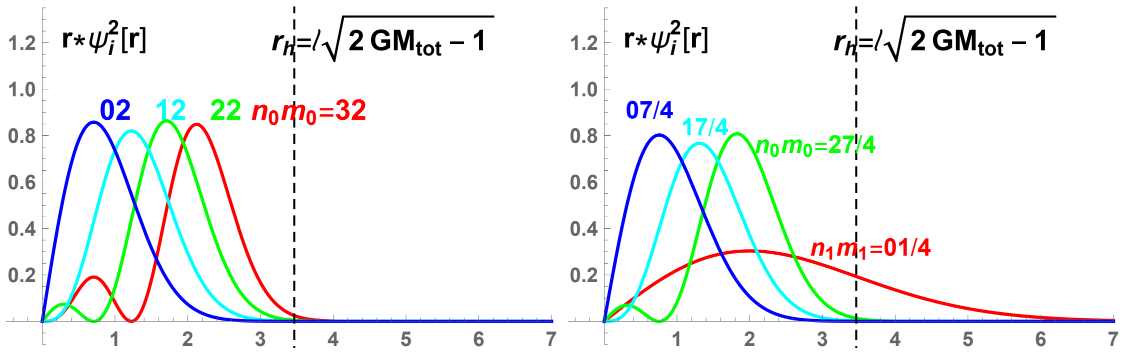

Figure 3. Two examples of shell wave functions for an AdS_{2+1}-Schwarzschild black hole of mass $M_{\rm tot} = 2\ell_{\rm ads}^{-1}$. We let $G = \hbar = 1$ and measure all dimensional quantities by $\ell_{\rm ads}$ or its inverse. If we consider the black hole consisting just of one shell of mass $m_{i(=0)} = 2\ell_{\rm ads}^{-1}$, then the shell has 4 square integrable microstate wave functions, left panel. If we consider the black hole consisting of two concentric shells, right panel, then constrained by the quantizing condition (12), the possible shell mass decomposition could be $\{m_{i(=0)}^{\cdot\ell_{\rm ads}}, m_{i(=1)}^{\cdot\ell_{\rm ads}}\} = \{\frac{7}{4}, \frac{1}{4}\}, \{\frac{7-\epsilon}{4}, \frac{1+\epsilon}{4}\}, \cdots, \{\frac{1}{\sqrt{2}}, 2 - \frac{1}{\sqrt{2}}\}$ which is infinite and in-countable . However, possible quantum numbers $\{n_{i(=0)}, n_{i(=1)}\}$ can only be $\{0/1/2, 0\} = \{\mathrm{Int}[< \frac{49}{16} - \frac{1}{2}], \frac{1}{4}\cdot 2 - \frac{1}{2}\}$, $\{1, 1\}$ and $\{0, 0/1/2\} = \{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{2}, \mathrm{Int}[<(2 - \frac{1}{\sqrt{2}})\cdot 2 - \frac{1}{2}]\}$. We can further consider the case the black hole consisting of more layers. However, constrained by the quantizing condition (12), the number of distinguishable shell quantum states in all cases are finite and countable. All shell wave function displayed in the figure has long tail of nonzero values outside the horizon. This makes the physic horizon of black holes a thick blurring or fuzzing region instead a cut-clear geometric surface.

Neglecting subtleties involved in M_i, eq(10) is nothing but the eigen-state Schrodinger equation of harmonic oscillators. The square integrability of the wavefunction requires that the following combination of mass & γ parameter of the shell be quantized

$$\big(GM_i + \frac{\gamma_i^2 - 1}{2}\big) m_i \ell \hbar^{-1} \equiv \frac{E_i}{\hbar\omega} = n_i + \frac{1}{2} \qquad (12)$$
$$\omega \equiv \ell^{-1},\ n_i = 0, 1, 2 \cdots .$$

n_i here is upper bounded because both the total mass of the disk and the maximal value of γ_i^2 are bounded. Recalling that, at classical level, to assure the outmost shell's moving inside the horizon, $\gamma_{out.most}^2 < 0$ is required; while for those non-outmost shells, γ_i^2 are not necessarily negative but are all upper bounded. Taking all these shells together, we have

$$\sum_i m_i = M_{\rm tot}, \psi_1 \otimes \psi_2 \otimes \cdots \psi_{imax} = \psi_{\rm tot}. \qquad (13)$$

We displayed in Figure 3 examples of wave function of consisting dust shells of an AdS_{2+1}-Schwarzschild black hole of mass $M_{\rm tot} = 2\ell_{\rm ads}^{-1}$. In the left panel, the black hole is considered consisting of just one shell, so each curve in the figure corresponds to a wave function of the system. While in the right panel, the black hole is considered consisting of two shells, the wave function of the system is direct product of the red line and the remaining three color lines. From the figure we easily see that, all shell wave functions are nonzero outside geometric surface $r = 2GM$, thus making the horizon a thickened blurring or fuzzy region instead of cut-clear surface.

The quantizing condition (12) implies that the ways for us to decompose (more precisely, to look or to consider) the disk into concentric shells and setting their release parameter γ_is are not arbitrary. Among these ways, a special class is, choosing all γ_is in such a way that $\big(GM_i + \frac{\gamma_i^2-1}{2}\big)$ equals to 1 uniformly, in this case all $\frac{m_i}{\hbar\ell^{-1}}$ are integers. For this choice, the famous partition-number formula of Ramanujan says that we have

$$W = \exp\big\{2\pi\sqrt{\frac{1}{6}\frac{M_{\rm tot}}{\hbar\ell^{-1}}}\big\} = \exp\big\{\frac{2\pi\sqrt{2GM_{\rm tot}\ell^2}}{4G}\big(\frac{3}{4}\frac{\ell}{G}\big)^{-\frac{1}{2}}\big\} \tag{14}$$

ways to comprise a disk of mass $M_{\rm tot}$ with all consisting shells oscillating inside the horizon. Obviously, $k_B \log W$ is almost the Bekenstein-Hawking formula $S_{BH} = A/4G$ except for a numeric factor $\big(\frac{3}{4}\frac{\ell}{G}\big)^{-1/2}$. Considering subtleties involved in M_i's definition and other possible ways of shell-decomposition and γ_is' setting will not change the basic fact that we have only finite number of ways to construct the disk. Instead such doing may bring us the factor $\big(\frac{3}{4}\frac{\ell}{G}\big)^{-1/2}$ back as the exact Bekenstein-Hawking formula requires. This means that, at quantum levels a 2+1D AdS-Schwarzschild blackhole's inside-horizon matter's oscillation has the potential to account for the area law of Bekenstein-Hawking formula, $S \propto \sqrt{M_{\rm tot}}$, in the absence of any unknown quantum gravitation theories. See Figure 3 and captions there for the example of why the number of distinguishable ways the black hole be considered as concentric shells is finite and countable.

3 3+1 Dimensional Schwarzschild Black Holes

Consider now the evolution of a self-gravitating dust ball in 3+1D asymptotically flat spacetime. Assuming that the dust occupation region's radius is less than the radius $r = 2GM_{\rm tot}$ determined by the mass of the ball, so the dusts all lie inside the horizon of the "black hole". Parallel with (8)-(9), the spacetime metric inside the dust occupation region can be written as

$$ds^2 = -d\tau^2 + \frac{\big[1-\big(\frac{2GM}{\varrho^3}\big)^{\frac{1}{2}}\frac{M'\varrho}{2M}\tau\big]^2 d\varrho^2}{a[t,\varrho]} + a[\tau,\varrho]^2\varrho^2 d\Omega_2^2 \tag{15}$$

$$a[\tau,\varrho] = \big[1 - \frac{3}{2}\big(\frac{2GM[\varrho]}{\varrho^3}\big)^{\frac{1}{2}}\tau\big]^{\frac{2}{3}},\ M[\varrho_{\rm hor} \leqslant \varrho] = M_{\rm tot} \tag{16}$$

$$a[\tau + p^{\varrho}_{\rm eriod}, \varrho] = a[\tau,\rho], p^{\varrho}_{\rm eriod} = \frac{4\cdot 2}{3}\big(\frac{\varrho^3}{2GM[\varrho]}\big)^{\frac{1}{2}} \xrightarrow{\varrho=\varrho_{\rm hor}} \frac{16}{3}GM_{\rm tot} \tag{17}$$

We checked that this metric satisfies the sourceful Einstein equation with $T_{\mu\nu} = \text{diagonal}\{\rho = \frac{M'[\varrho]/8\pi\varrho^2}{a^{\frac{3}{2}} + \frac{3GM'\tau^2}{4\varrho^2} - \left(\frac{GM}{\varrho^3}\right)^{\frac{1}{2}} \frac{M'\varrho\tau}{2M}}, p = 0, 0, 0\}$; while outside the dust occupation region, it can be joined to the Schwarzschild metric smoothly [14].

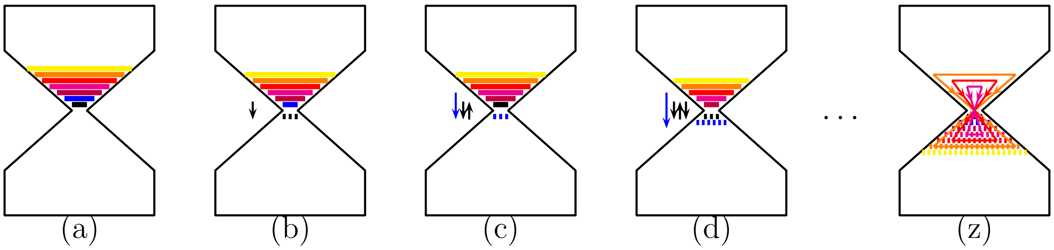

Figure 4. A sandyclock whose sands can be decomposed as many different colored layers, each of which falls to and oscillates across the central point independently. In a time-recording period a-z, each layer experiences independent contraction-singular-antipodal-expansion evolution, different layer have different oscillation periods. In a black hole consisting of dusts without other interaction except gravitation, the dusts can be considered as many concentric shells, each of which oscillates around the central point just the same way as the sand layers do in the sandyclock. The microstates of a Schwarzschild black hole just hinge on the structure of this sandyclock like singularity.

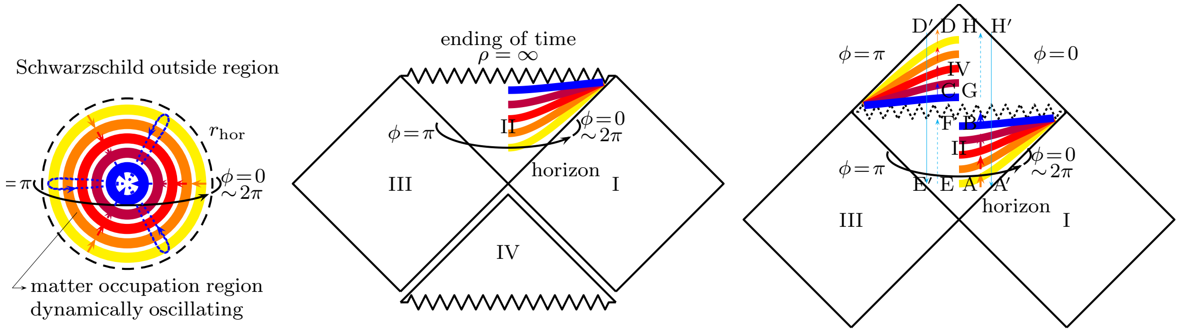

Figure 5. Similar to Figure 1, a dust ball with oscillation inner structure in 3+1D Minkowski background. A complete oscillation period consists of A-B-C-D-D′-E′-E-F-G-H-H′-A′-A. Regions III and IV consisting of antipodal points in regions I and II. Colored family of lines on the middle and right figures correspond to various half equal-r shells on the left figure. We can also see the whole family of lines as snapshots of the outmost shell at different instantaneous epochs.

The essential singularity in (15)-(17) is a periodically, in fact continuously, forming and resolving one, see Figure 4 for a sandyclock analogue. This is totally different from that of Schwarzschild metrics. If we consider the whole dust ball as many concentric dust shells, each with mass m_i and varying radius r, then the radius will evolve with time according to equations of the following form

$$\dot{r}^2 - \gamma_i^2 + h_i = 0 \Leftarrow \begin{cases} h_i \dot{t}^2 - h_i^{-1}\dot{r}^2 = 1, \; h_i = 1 - \frac{2GM_i}{r} \\ \ddot{t} + \Gamma^{(i)t}_{tr}\dot{t}\dot{r} = 0 \Rightarrow h_i \dot{t} = \gamma_i = \text{const} \end{cases} \tag{18}$$

Totally the same as in 2+1D AdS dust ball case (11), the parameter γ_i^2 in eq(18) is also negative when a shell i is released from inside the horizon determined by the mass carried by all shells inside it, or although positive but always upper bounded when a shell i is released from outside the horizon determined by the mass carried by shells inside it but still inside the horizon determined by the total mass the of ball. When the shell is released from inside the horizon, γ_i is pure imaginary and we can set it to i through appropriate choice of affine parameter. Equations (18) has oscillating solutions just as we display in the introduction section, see Figure 1 for references. While in Figure 5 we provide Penrose-Carter diagrammatic description for the whole dynamic space-time, according to which the mass shell firstly falls onto the central point, then expands from that point and then falls back from the other side again. This oscillation's occurrence can also be attributed to the forward scattering domination when the dust particles collide together under inter-gravitations.

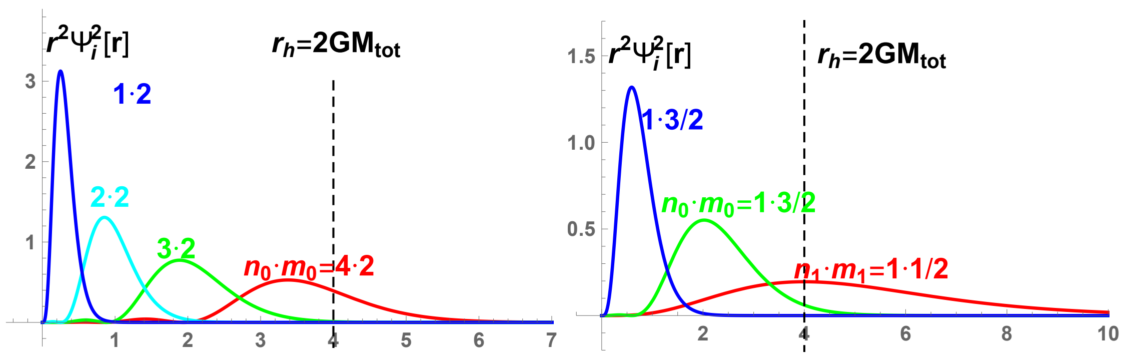

Figure 6. Examples of shell wave function for the consisting dust shell of a 3+1 dimensional Schwarzschild black hole of mass $M_{\rm tot} = 2M_{\rm pl}$. We let $\hbar = 1$ and measure all dimensional quantities by $M_{\rm pl}$ or its inverse. If we consider the black hole consisting of just one shell of mass $m_{i(=0)} = 2M_{\rm pl}$, then the shell has 4 square integrable microstate wave functions, left panel. If we consider the black hole consisting of two concentric shells, right panel for example, then constrained by the quantizing condition (20), the possible shell mass decomposition could be $\{m_{i(=0)}, m_{i(=1)}\} = \{\frac{3}{2}, \frac{1}{2}\}$, $\{\frac{3-\epsilon}{2}, \frac{1+\epsilon}{2}\}$, $\cdots$, $\{1, 1\}$ which is infinite and in-countable . However, possible quantum numbers $\{n_{i(=0)}, n_{i(=1)}\}$ can only be $\{1\text{or}2, 1\} = \{\text{Int}[0, \frac{9}{4}], \frac{1}{2}\cdot 2\}$ and $\{1, 1\}$. We can further consider the black hole consisting of more layers. However, constrained by the quantizing condition (20), the number of distinguishable mass-shell quantum states in all cases are finite and countable. Completely the same as AdS$_{2+1}$-Schwarzschild black holes, the horizon of the Schwarzschild black holes are also blurred by the nonzero tail of shell wave functions.

Quantize equation (18) by the standard canonical method, we have

$$\Big[-\frac{\hbar^2}{2m_i}\partial_r^2 - \frac{GM_i m_i}{r} - E_i \Big]\psi_i(r) = 0, \;\; 0 \leqslant r < \infty \tag{19}$$

Simple square integrability condition will impose that

$$\frac{\gamma_i^2-1}{2}m_i \equiv E_i = -\frac{(GM_i m_i)^2 m_i}{n_i^2\hbar^2},\ n_i = 1, 2, \cdots \tag{20}$$

This implies that the way we look the dust ball as concentric shells and set their release condition $h_i\dot{t}_{r=r_{\rm release}} = \gamma_i$ are not arbitrary. In fact, all possible ways are countable and have the potential to give the Bekenstein-Hawking entropy reasonable interpretation [6]. We display in Figure 6 two examples of shell wave function for a Schwarzschild black hole of $2M_{\rm pl}$ mass. In the caption, we explain why the number of distinguishable ways the whole dust ball be considered as concentric shells are finite and countable. However, to get the area law entropy formula, we have to resort numeric computations. For those readers feel interesting with this topic, please refer to references [5, 6].

Both metric (15)-(17) and (8)-(9) are written in a cosmological style comoving coordinate, they can be connected to the outside Schwarzschild ones smoothly [14]. Many people refuse to talk about oscillations inside the black hole horizon because they occur beyond the time definition of far away observers. However, just as we see in this and previous section, this refusal is irrational because we can build the coordinate system so that the whole space consists of two patches, S- Schwarzschild patch which covers the outside horizon region and C- comoving patch which covers the inside horizon region. The two are bordered on a sphere infinitely near but above the horizon surface so that any particle falling towards the black hole can get to the C-patch from S-patch in finite duration[2]. Exact Schwarzschild black hole with singular center which accumulates all matter contents of the system is an over-extrapolation of outside geometry of spherical symmetric astrophysical object. Microstate of such object is unique, to talk about its entropy is just like to talk about the entropy of single pure state system. Entropy originates from our uncertainty about how matters consisting of the black hole are distributing and oscillating inside the horizon.

4 Discussions on the meaning of matters' inside-horizon oscillation

Many people argue that, it is of nonsense to talk about a spherically symmetric black hole's inner matter distribution and motion modes at classic levels due to the horizon's forbidding of such inner structure's being measurable by outside observers. However, this argument applies only in single body context. When we let two spherically symmetric black holes (one with totally known inner structure and the other unknown) spin around and merge with each other, spherical symmetries of the system is obviously broken and the structure of the previously unknown black hole will be measurable from gravitational waves following from the

[2]Blurring or fuzziness of the horizon caused by quantum effects forms physic basis for our setting the border surface this way. That it is, when we consider quantum effects, the out most mass shell consisting of the black hole have very large probabilities be measured outside its horizon, thus making the horizon a blurring or fuzzing region instead a cut-clear surface. This allows us to set the border surface between S and C some distance above the math horizon defined by $r = 2GM$.

two bodies' merging process. The basic reason here is that, black holes with different inner matter distribution and motion modes, for example two with singular central points and two without such singular centers, will exhibit different tidal deformation and lead to different gravitational waves as the merging proceeds [7]. We display in Figure 7 the quadrupole of two such static binary systems. Although the difference between the two static systems is a diagonal constant matrix which has no observable effects, when they spin around their own central vertical line, the right system will experience different tidal deformation and produce different gravitation waves relative to the left one. In practice, data collected in the LIGO/VIRGO observation may not be sufficient to uncover exactly the inner structure of black holes being observed. But it may have potentials to tell us if black holes have singular centers or not. So to talk about the consisting matters' inside horizon oscillation is not a pure theoretical question, it is an experimental one.

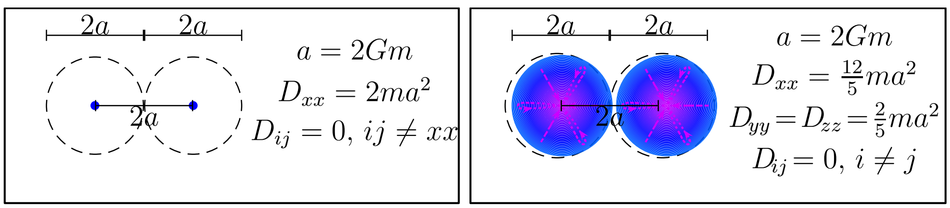

Figure 7. The quadrupole of binary black holes with different inner structure. The left have singular central points, the right have uniform matter distribution. When the systems are static, their quadruple difference is a diagonal constant matrix which has no observable effects. But when they spin around their own central vertical line, the right system will produce different tidal deformation and gravitation waves relative to the left ones.

In single body case, the hint of matter's oscillation inside the horizon could be seen in some theoretical examination. For example, to recover Hawking radiation's unitarity and to get proper interpretation for the Bekenstein Hawking entropy's origin, 't Hooft [15–22] pointed out that extra assumption, such as Antipodal Points Identification (API) about the horizon itself is almost unavoidable. API assumes that points in regions III and IV of the Penrose diagram of the Schwarzschild black hole should be identified with the antipodal ones in regions I and II. Obviously, in black hole with oscillatory inner structures such as those described by our exact solutions, API is not an artificial assumption, but a natural deduction of math formalism. Further more, when we consider quantum effects, matters consisting of the black hole have very large probabilities be measured outside its horizon, thus making the horizon a blurring or fuzzing region instead a cut-clear surface. Observers outside the hole need not waiting infinite durations to see signals unveiling the matters' falling into this region and participating oscillations inside it. When we use coordinate system discussed at the last paragraph of previous section, this can be seen even at pure classical levels.

To emphasize the role of matters consisting of the black hole instead of focusing on pure gravitational or geometric degrees of freedom in the information missing puzzle's resolving is also the key of works [23–28]. However, there is a popular saying that the number of degree of freedom following from the consisting matter would lead to a volume-law entropy instead an area-law type one. This is a very misleading doctrine. Since the number of particles constrained inside the horizon are proportional to the black hole mass $N \propto M$, thus in 3+1D Schwarzschild case proportional to its horizon size $N \propto r_h = 2GM$, simple calculation

$$Z = \int d\vec{x}_1 d\vec{p}_1 \cdots d\vec{x}_N d\vec{p}_N e^{-\beta H(\vec{x}_1,\vec{p}_1,\cdots)}, S \sim (1-\beta\partial_\beta)\ln Z \sim N^{1-\epsilon} \sim r_h^{1-\epsilon}, \epsilon > 0 \tag{21}$$

will give us neither volume-law, nor area-law, but diameter-law entropy at most. Then how do we get area law entropy in the previous section? The key point in our works is, the objects carrying degrees of freedom evaluated by the area-law entropy is not the usual particles consisting of the black hole, but Collective Motion Modes of them. According to our quantizing condition (20), the characteristic mass of such modes is $m_{\text{i=out.most}} \propto (GM_i)^{-1} = GM_{\text{tot}}$. In a black hole of mass M_{tot}, the number of such objects is $N_{CMM} \propto GM_{\text{tot}}^2$, which is obviously proportional to the area of the black hole under consideration.

5 Conclusion

We make a short introduction to two exact inner solution families to the Einstein equation sourced by zero-pressure dust balls in 2+1D AdS and 3+1D Minkowski background spacetime. Both solution families have oscillatory inner cores thus make the eternal singularity of usual Schwarzschild black holes into a periodically forming and resolving one. Comparing with our previous publications, we provide in this work detailed Penrose-Carter diagrammatic representation for the periodic singularity's forming and resolving process, and explain why this inner horizon oscillation is meaningful both theoretically and experimentally. Common to our two solution families is the consisting matter's oscillation across the central point. Such oscillations' occurrence instead of eternal singularities' forming and keeping forever can be attributed to the simple fact that, when two or more particles collide together at high energies following from inter gravitating accelerations around the central point, forward amplitude dominates the scattering process, no bounding state forms. Through simple canonical quantization method, we point out that the number of allowed members in our solution families, thus microstate of the system, is finite and countable, and has the potential to give the Bekenstein-Hawking entropy a reasonable interpretation in the absence of any unknown quantum gravitation theories.

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References

[1] S. Hawking, G. F. Ellis, "The Large Scale Structure of Space Time" Cambridge: Cambridge University Press. ISBN 0-521-09906-4.

[2] S. Hawking, "Black hole explosions", *Nature* **248** (1974) 30; "Particle creation by blackholes", *Comm. Math. Phys.* **43** (1975) 199.

[3] S. Hawking, "Breakdown of Predictability in Gravitational Collapse", *Phys. Rev.* **D14,** 2460 (1976)

[4] Ding-fang Zeng, "Resolving the Schwarzschild singularity in both classic and quantum gravities", *Nucl. Phys.* **B917** 178-192, arXiv: 1606.06178.

[5] Ding-fang Zeng, "Schwarzschild Fuzzball and Explicitly Unitary Hawking Radiations", *Nucl. Phys.***B930** (2018) 533-544, arXiv: 1802.00675.

[6] Ding-fang Zeng, "Information missing puzzle, where is hawking's error?", *Nucl. Phys.***B941** (2018) 665, arXiv: 1804.06726.

[7] Ding-fang Zeng, "Exact inner metric and microscopic state of AdS_3-Schwarzschild BHs", *Nucl. Phys.***B954** (2020) 115001, arXiv: 1812.06777.

[8] S. Carlip, "Conformal Field Theory, (2+1)-Dimensional Gravity, and the BTZ Black Hole", *Class. Quant. Grav.* **22** (2005) R85-R124, gr-qc/0503022.

[9] M. Banados, "Global Charges in Chern-Simons theory and the 2+1 black hole", *Phys. Rev.* **D52** 1996 (5816), hep-th/9405171.

[10] M. Banados, A. Gomberoff, "Black Hole Entropy in the Chern-Simons Formulation of 2+1 Gravity" *Phys. Rev* **D55** (1997) 6162, gr-qc/9611044.

[11] S. Carlip, "The Statistical Mechanics of the Three-Dimensional Euclidean Black Hole", *Phys. Rev.* **D55** (1997) 878, gr-qc/9606043.

[12] J. Baez, D. Wise, A. Crans, "Exotic Statistics for Strings in 4d BF Theory", *Adv. Theor. Math. Phys.* **11** (2007) 707, gr-qc/0603085.

[13] E. Witten, "Three Dimensional Gravity Reconsidered", arXiv: 0706.3359

[14] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, chapter 11, Wiley and Sons, New York, 1972, ISBN-13: 978-0471925675

[15] G. 't Hooft, "Microstates in Black Holes and the Topology of Space-Time", ISSP 2016, 169-192

[16] G. 't Hooft, "The quantum black hole as a theoretical lab, a pedagogical treatment of a new approach", ISSP 2018, 1902.10469

[17] G. 't Hooft, "Discreteness of Black Hole Microstates" 1809.05367

[18] G. 't Hooft, "What happens in a black hole when a particle meets its antipode" 1804.05744

[19] G. 't Hooft, "The Firewall Transformation for Black Holes and Some of Its Implications", *Found. Phys.* **47** (2017) 12, 1503-1542 1612.08640

[20] G. 't Hooft, "Virtual Black Holes and SpaceTime Structure", *Found. Phys.* **48** (2018) 10, 1134-1149

[21] G. 't Hooft, "The Quantum Black Hole as a Hydrogen Atom: Microstates Without Strings Attached", 1605.05119

[22] "Black hole unitarity and antipodal entanglement", *Found. Phys.* **46** (2016) 9, 1185-1198, 1601.03447

[23] T. Vachaspati, D. Stojkovic; "Quantum Radiation from Quantum Gravitational Collapse", *Phys. Lett.* **B663** (2008) 107-110, ePrint: gr-qc/0701096.

[24] A. Saini, D. Stojkovic, "Radiation from a collapsing object is manifestly unitary", *Phys. Rev. Lett.* **114** (2015), 111301; arXiv: 1503.01487.

[25] J. Hutchinson, D. Stojkovic, "Icezones instead of firewalls: extended entanglement beyond the event horizon and unitary evaporation of a black hole", *Class. Quant. Grav.* **33** (2016) no.13, 135006; arXiv: 1307.5861.

[26] E. Greenwood, D. Stojkovic, "Quantum gravitational collapse: Non-singularity and non-locality", *JHEP* **0806** (2008) 042; arXiv: 0802.4087.

[27] J. Wang, E. Greenwood, D. Stojkovic, "Schrodinger formalism, black hole horizons and singularity behavior", *Phys. Rev.* **D80** (2009) 124027; arXiv: 0906.3250.

[28] A. Saini, D. Stojkovic, "Nonlocal (but also nonsingular) physics at the last stages of gravitational collapse", *Phys. Rev.* **D89** (2014) no.4, 044003; arXiv: 1401.6182.