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A Toy Model for Pushing Gravity and Some Related Estimates

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Abstract

The ancient theory of Fatio de Duillier and Lesage on pushing gravity has been mainly criticized because of the extreme heating which would be produced in the case of inelastic shocks, supposed to be necessary to produce a gravity field. Here we investigate in an extremely simplified situation the possibility of creating a virtual acceleration with purely elastic repeated shocks and we derive some estimates on the mass and velocity of gravitons based on this model.

Key words: gravitation; elastic shocks; virtual acceleration

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1 Introduction

The cause of gravitational force has become a basic question right after the discovery by Newton of the gravitational field produced by matter. Around that time, Fatio de Duillier, then Lesage, cf. e.g. [1, 2, 3, 4]) formulated the idea that gravity might be the result of interaction of matter with tiny unseen particles, qualified as "ultra-mundane", pushing, as a consequence of a mutual 3D shield effect, any pair of massive objects towards each other. At that time the atomic theory of matter was not found yet, but today one might think, if we follow this theory, that the "gravitational mass" of a material object is determined by the number of nucleons (protons+ neutrons), since it is not clear at all how the electronic cloud might interact with the particles. Fortunately, the "inertial mass" of an electron is inferior to that of a nucleon by 3 orders of magnitude.

It was explained in [5] how the pushing gravity might provide a solution to the missing mass enigma. It is therefore of interest to try to understand the mechanism leading to the emergence of gravitational acceleration in the case of a single nucleon bombed by ultra-mundane corpuscles in the way assumed by pushing gravity. If no object is interposed in the trajectory of corpuscules, the nucleon will not undergo any notable force, or more precisely the force will change sign at a very small time scale, so that no motion of the nucleon will be detectable. On the other hand it is quite interesting to examine what happens when a single direction is blocked (for instance by another nucleon) on one side only.

This short note is devoted to that purely mathematical, physically unrealistic toy model that might however give some relevant quantitative information about the possibility of pushing gravity in a purely elastic framework, a choice which has been rejected until now by all specialists with the argument that no gravitational field would emerge in this context.

2 Elastic shock of two particles with colinear velocities.

Although this is a simple and rare case, it is interesting, for the very simplified model that we shall consider, to recall the following well known result concerning the purely elastic shock of two point masses. Consider two particles with respective masses m_1, m_2, and velocities u_1, u_2 before collision, v_1, v_2 after collision. The equality of the total linear momentum before and after the collision is expressed by:

$$m_1 u_1 + m_2 u_2 \;=\; m_1 v_1 + m_2 v_2.$$

Because the shock is perfectly elastic, the conservation of the total kinetic energy gives :

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 \;=\; \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2.$$

These equations may be solved directly to find v_1, v_2 when u_1, u_2 are known:

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \tag{2.1}$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 \tag{2.2}$$

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3 A very simple toy model.

We now consider a nucleon, assimilated to a punctual mass M, and a punctual "graviton" of much smaller mass m, both particles being on the real line, with for instance the nucleon placed at the origin. The nucleon has an initial speed v_0 and the graviton meets the nucleon with initial speed u before the shock. For the moment we do not make any hypothesis on the signs and we are only interested in the velocity v_1 of the nucleon after the shock. In the previous set of formulas we therefore take $m_1 = M$, $m_2 = m$, $u_1 = v_0$ and $u_2 = u$. From the first formula (2.1) we find, mutatis mutandis

$$v_1 = \frac{M-m}{M+m}v_0 + \frac{2m}{M+m}u \tag{3.3}$$

3.1 Analysis of the first shock.

Since we want to analyse the effect of successive shocks with the gravitons which are supposed to be high speed particles, we may assume that $v_0 = 0$. In this case we find

$$v_1 = \frac{2m}{M+m}u \tag{3.4}$$

showing that the velocity of the incoming graviton is transferred to the nucleon with a reduction factor $\frac{2m}{M+m}$. The incoming graviton "rebounds" with a velocity

$$v_2 = \frac{m-M}{m+M}u$$

in the opposite direction, the absolute velocity undergoing a relative loss of $\frac{2m}{M+m}$. So the gravitons lose a fixed proportion of their energy each time they collide with a motionless nucleon. But this does not mean anything from a global point of view since they might regain energy when they bump on another nucleon with a velocity opposite to theirs. This is consistent with the conservation of total energy .

3.2 Analysis of the completely isolated case.

One important objection against the Duillier-Lesage pushing gravity theory is the fact that in the "free directions", without obstacles for gravitons, the absence of motion is the result of an equilibrium between forward and backward shocks, those shocks giving rise to inefficient forces which cancel each other and may nevertheless produce a very high temperature. In the case of perfectly elastic shocks this objection vanishes. A simple way to understand that is to consider the simplest case where back and forth shocks are systematically alternated. Of course the alternance of forward and backward shocks can be different with an arbitrary level of complexity in the successions, but intuitively the result will be similar in the long run if the signs are reasonably distributed as expected from the random character of incoming particles. In the simplest case, assuming that all gravitons have speeds $\pm v$, we are led to study the inductive sequence

$$\forall k \geq 0, \quad v_{k+1} = \frac{M-m}{M+m}v_k + (-1)^k \frac{2m}{M+m}v \tag{3.5}$$

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with $v_0 = 0$. Setting

$$v_k = \frac{2m}{M+m} v w_k; \quad \rho = \frac{M-m}{M+m}$$

we are reduced to the simple relation

$$\forall k \geq 0, \quad w_{k+1} = \rho w_k + (-1)^k; \quad w_0 = 0 \tag{3.6}$$

whose solution is immediate

$$w_1 = 1, w_2 = -1 + \rho, w_3 = 1 - \rho + \rho^2, ...w_k = (-1)^{k+1} \sum_{j=0}^{k-1} (-\rho)^j = (-1)^{k+1} \frac{1-(-\rho)^k}{1+\rho}$$

As a consequence we obtain

$$v_k = (-1)^{k+1} \frac{1-(-\rho)^k}{1+\rho} \frac{2m}{M+m} v$$

showing that the nucleon is bound to perform small oscillations of size less than $\frac{2m}{M+m}|v|$. In practice such oscillations will be indiscernable from rest. We can say that the gravitons have no noticeable effect on an isolated nucleon, or more generally produce no significant motion of the nucleon in the directions where no obstacle comes to perturb the incoming particles.

3.3 The case of a unilateral obstacle.

Let us now consider the case where an obstacle (for instance another nucleon) is placed on the right of our original nucleon, prohibiting shocks with gravitons coming from the right. Now incoming gravitons come from the left as a flux of particles with identical speeds $u = V > 0$ which we can, for simplicity, assume separated by a fixed (small) interval of time $h > 0$. The gravitons reach the origin at time kh and they rebound against the nucleon at some time $t_k > kh$ (to be computed later) since this particle has been pushed forward by the previous shocks. Then if no other force comes to, constrain the nucleon, the succesive velocities $v_k = v(t_k)$ are given by

$$\forall k \geq 0, \quad v_{k+1} = \frac{M-m}{M+m} v_k + \frac{2m}{M+m} V \tag{3.7}$$

Now the same reduction as previously gives

$$\forall k \geq 0, \quad w_{k+1} = \rho w_k + 1; \quad w_0 = 0 \tag{3.8}$$

which provides

$$w_k = \sum_{j=0}^{k-1} \rho^j = \frac{1-\rho^k}{1-\rho}$$

and finally

$$v_k = (1-\rho^k) V \tag{3.9}$$

This looks at first sight contrary to our ambition of justifying the emergence of a gravitational acceleration. Indeed from (3.9) it follows at once that

$$\lim_{k \to \infty} v_k = V \tag{3.10}$$

and

$$\lim_{k \to \infty} (v_{k+1} - v_k) = 0 \tag{3.11}$$

precluding once and for all the existence of a fixed induced acceleration for very large times.

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3.4 A fundamental remark.

But this is not the end of the story! First we observe that (3.10) was predictable. Indeed the velocity is increasing after each shock, and if it falls above V, no shock with a graviton coming from the left is possible any longer. Easy to understand ... afterwards. Then we observe that from the equation

$$v_{k+1} - v_k = \frac{2m}{M+m}\rho^{k+1}V$$

Between the sucessive times of impact t_k and t_{k+1} the derivative suddenly jumps from v_k to v_{k+1}, displaying a "virtual acceleration "

$$\frac{v_{k+1} - v_k}{t_{k+1} - t_k} = \frac{2mV}{M+m} \times \frac{\rho^{k+1}}{t_{k+1} - t_k}.$$

As mentioned previously the time interval $t_{k+1} - t_k$ is not exactly equal to kh. In order to compute the times t_k we observe that at time t_{k+1} the abscissa of the nucleon has become

$$x_{k+1} = t_1 v_1 + (t_2 - t_1)v_2 + ...(t_{k+1} - t_k)v_{k+1}$$

so that

$$[t_{k+1} - (k+1)h]V = t_1 v_1 + (t_2 - t_1)v_2 + ...(t_{k+1} - t_k)v_{k+1}$$

Similarly we have

$$[t_k - kh]V = t_1 v_1 + (t_2 - t_1)v_2 + ...(t_k - t_{k-1})v_k$$

and by subtracting both sides of these inequalities we find

$$[t_{k+1} - t_k - h]V = (t_{k+1} - t_k)v_{k+1} = (t_{k+1} - t_k)(1 - \rho^{k+1})V$$

and finally

$$t_{k+1} - t_k = \frac{h}{\rho^{k+1}}$$

Incidentally this yields the formula

$$t_k = h\sum_1^k \frac{1}{\rho^j} = \frac{1-\rho^k}{\rho^k - \rho^{k+1}}h$$

which tends to kh as $\rho \to 1$, but the main point here is the formula

$$\frac{v_{k+1} - v_k}{t_{k+1} - t_k} = \frac{2mV}{M+m} \times \frac{\rho^{2(k+1)}}{h}.$$

displaying a "virtual acceleration "

$$\gamma \sim \frac{2m}{M}\frac{V}{h}$$

as long as k is not too large because ρ is very close to 1 provided m/M is sufficiently small. This suggest that me may recover a gravitational force if the gravitons have sufficiently small masses and V is sufficiently large. Quantifying this idea with real data is the object of next section.

4 Some estimates

We have

$$\rho = 1 - \frac{2m}{M+m} > 1 - \frac{2m}{M}$$

If we wish for instance to secure the inequality $\rho^k > e^{-1/1000} \sim 0,999$ which seems to be comfortable, we require

$$kln(1 - \frac{2m}{M}) > -1/1000$$

which amounts essentially to

$$\frac{2m}{M} < 10^{-3}\frac{1}{k}$$

We would like to reach for k, to fix the ideas, the total number of possible shocks spaced in time by the interval h during a time corresponding to the currently estimated age of the universe. Therefore we choose, just to see what it will imply

$$k = A/h$$

where h is the fraction of a second separating two shocks and A is the age of universe in seconds. this gives the sufficient condition

$$\frac{m}{M} < 10^{-21}h$$

For instance if we assume a flux of 1000 gravitons per second, we must assume that the gravitons are 10^{-24} times lighter than a nucleon. This is very small , much smaller than the present estimate for the mass of neutrinos. But why not?

Now what about the velocity V? The virtual acceleration undergone by the nucleon as a consequence of shocks is

$$\gamma \sim \frac{2m}{Mh}V$$

It seems reasonable to compare this value of γ with the acceleration that would be produced, via Newton, by the interposition of a nucleon on the right of the given one, at the minimal possible distance, corresponding to the radius of an average atom, about one angstrom or $10^{-10}m$. That acceleration is about $10^{-17}m/s^2$. Assuming for instance that m/M has the highest permitted value $10^{-21}h$ we find $2 \times 10^{-21}V > 10^{-17}m/s$, thus $V > 10^4 m/s$ is enough. Even classical sub-luminic particles can fulfill the conditions. On the other hand, there are motions apparently driven by gravitation which are much faster than $10^4 m/s$, so logically the velocity of gravitons, if they exist, is much higher than our lower estimate.

5 Concluding remarks

If our simplified punctual 1D model has some relevance, for macroscopic objects, the "virtual acceleration" will become an acceleration in the classical sense as a consequence of averaging effects. In addition, in reality, even for one nucleon, the "jump" from the velocity v_k to v_{k+1} is certainly not instantaneous. Now the most important, and maybe surprising, information obtained in the above calculations is that, in order for a pushing gravity to function, the gravitons must have a mass smaller than 10^{-20} times the mass of a nucleon. In addition, the model suggests an exponential exhaustion of the gravitational forces for very large time, even if the decrement is extremely weak. At this point, we must insist on the fact that the necessity of a very small mass for gravitons results, in our calculations, from the hypothesis that the gravitational affects remained essentially constant in the past 10^{10} years, which is not at all a certitude. Moreover, in space, the massive objects tend to move, which might have the effect of "reinitializing " their position with respect to the flux of gravitons. As a conclusion, the gravitons might be heavier than what was computed, but in such a case one should be able to detect an important decay of the gravitational constant over very long periods of time. This is, for instance, not contradictory with the evolution scheme based on the Big Bang.

Now we must admit that this very simple toy model is, of course, far from reality, since we condensed all directions into one to see what happens. In many cases of gravitational attraction, the obstacle is viewed from the attracted object under a very small angle, but it is not always the case and in order to rehabilitate the Fatio-Lesage pushing gravity theory in a purely elastic framework, much more has to be done. It seems to be quite difficult to build a continuous model that would allow macroscopic calculations giving more credit to the hypothesis of pushing gravity. And recovering rigorously Newton's formulas on a discrete model is probably even more difficult, with the possible need of powerful computers as a guide to a full understanding of the situation. Finally it is clear that in order to appraise the gravitational constant as a function of the flux of gravitons, the mass and the velocity, a three dimensional model has to be devised.

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