Cosmology with variable G and nonlinear electrodynamics

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In a bid to resolve lingering problems in cosmology, more focus is being tilted towards cosmological models in which physical constants of nature are not necessarily real constants, but varying with cosmic time. In this paper we study cosmology in nonlinear electrodynamics with the Newton's gravitational constant G not a constant but varies with the scale factor of the universe. The evolution of the scale factor $a(t)$ in this model depends on α, which gives an steady universe when $\alpha = 0.5$. As α increases to $\alpha = 1.0, 1.5, 2.0, 3.0$ the universe enter into inflation scenario after that the magnetic monopole field decayed and is converted to radiation. We checked the stability of the model and obtained that it is classically stable with the best condition for the stability at $5/2 \geq \alpha > 7/4$.

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I. INTRODUCTION

Remarkable achievements gained in the field of cosmology over the last decades is attributed to observed Cosmic Microwave Background (CMB) radiation and type-Ia supernovae observations which suggest that cosmic expansion is accelerating [1]. According to the standard cosmological model, the universe evolved from an initial singularity – which is a breakdown in the geometrical structure of space and time. Singularities are always constraints in physics and indeed in cosmology when describing the early universe. To deal with this and other fundamental problems in modern cosmology such as the horizon problem, flatness problem, monopole problem and inflation. Magnetic universe models which portray no initial singularity due to strong electromagnetic field in the modified Nonlinear Electrodynamics (NED) can explain the cosmic inflation period of the universe which is a theory of exponential expansion of space in the early universe [2–18]. Other models such as non-minimal coupling, varying speed of light (VSL), quantum gravity effect, Lagrange with quadratic term, inflation by scalar fields, NED without modification of general relativity have been introduced in the literature to solve the puzzle of cosmology and mystery of inflation [19–40].

Max Born and Leopold Infeld used the idea of Gustav who in 1909 had attempted to construct a purely electromagnetic theory of charge particles to proposed a new theory in 1934, fully relativistic and gauge invariant nonlinear electrodynamics [41]. Born-Infeld proposed a nonlinear Lagrange with an interesting attribute of changing from to Maxwell's theory for low electromagnetic fields. Since there are no new degrees of freedom such as scalar fields or branes, opines that works of cosmology described by NED should have some interesting features of cosmic importance. The sources of cosmic inflation can be trace to nonlinear electromagnetic radiation which is explained by modified Maxwell's equations. When coupled with gravitational field, NED may give negative pressure and also can lead to cosmic inflation [22]. The evolution of the universe when explored with a new NED model such that electromagnetic field coupled to gravitational field prevents cosmic singularity at the big bang. The electromagnetic and gravitational fields were very strong during the evolution of the early universe, thereby leading a quantum correction and giving birth to NED [23–25].

In recent times, interest is geared towards cosmological models in which physical constants of nature are varying with time [42–44]. For instance, in the VSL theory, most pending problems of standard cosmological models are being resolved with considering inflation [45–47]. In the Einstein's field equation, the Newton's gravitational constant G acts as a coupling constant between geometry of spacetime and matter. It is noted that there are significant evidence that gravitational constant G can be varying in a time [48]. Spurred by the discovery of occurrence of large numbers Weyl and Dirac proposed the theory of variable G. In other to unify gravitation and elementary particle Physics, Einstein's theory with time varying G is already in the literature [46, 49–59].

Our main aims is to use the model of nonlinear electromagnetic field with a simple Lagrange density and the Newton's gravitational constant G that varying with the scale factor to study the evolution of the universe and other quantities of cosmic inflation.

The structure of the paper is thus: In section II, we briefly introduced the cosmology of a universe filled with nonlinear magnetic monopole field. In section III, we obtain the evolution of the universe filled with nonlinear magnetic monopole field and variable gravitational constant. In section IV, we checked the stability of the model and give our conclusion.

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II. NON-LINEAR MAGNETIC MONOPOLE FIELDS AND COSMOLOGY

In nonlinear electrodynamics, we define the Lagrangian density by [10]

$$\mathcal{L}_{NED} = -\frac{\mathcal{F}^{\alpha}}{4}, \tag{1}$$

$\mathcal{F}$ denotes an invariant quantity known as the Maxwell invariant . Because the matter Lagrange is independent of the metric's derivatives, in tensorial language the matter energy-momentum defination using (1) is given as [12]

$$T_{\mu\nu} = -K_{\mu\lambda}F_{\nu}^{\ \lambda} + g_{\mu\nu}\mathcal{L}_{NED}, \tag{2}$$

with

$$K_{\mu\lambda} = \frac{\partial \mathcal{L}_{NED}}{\partial \mathcal{F}} F_{\mu\lambda}. \tag{3}$$

Here, it is assumed that on the cosmic Background, there exist a dominant stochastic magnetic field whose wavelengths are less than the curvature. Hence, the mean electromagnetic fields now become the source of Einstein equations [60]. The averaged electromagnetic fields are given as [2]:

$$\langle E \rangle = \langle B \rangle = 0, \langle E_i B_j \rangle = 0, \tag{4}$$

$$\langle E_i E_j \rangle = \frac{1}{3} E^2 g_{ij}, \langle B_i B_j \rangle = \frac{1}{3} B^2 g_{ij},$$

where $\langle\ \rangle$ denotes averaging brackets used for taking mean of volume. The wavelength of radiation is considered to be lower than the volume and the volume smaller than the curvature.

However, the case of real nonlinear magnetic monopole is when $E^2 = 0$. Therefore, as obtained from equation (1), the energy density $\rho = -T^0_0$ and the pressure $p = T^i_i/3$ of the nonlinear monopole magnetic field is [5]

$$\rho_{NED} = -\mathcal{L}_{NED}, \tag{5}$$

$$p_{NED} = \mathcal{L}_{NED} - \frac{2B^2}{3}\frac{\partial \mathcal{L}_{NED}}{\partial \mathcal{F}}, \tag{6}$$

where $\mathcal{L}_{NED}$ is defined in Eq. (1) with $\mathcal{F} = B^2/2$.

From the above equations, we obtained the energy density equation ρ and pressure p as thus:

$$\begin{aligned} \rho = \rho_{NED} &= \tfrac{2^{-\alpha}(B^2)^{\alpha}}{4} \\ p = p_{NED} &= \tfrac{2^{-\alpha}}{12}(B^2)^{\alpha}\,(4\,\alpha - 3) \end{aligned} \tag{7}$$

III. COSMOLOGY WITH VARIABLE G AND NONLINEAR ELECTRODYNAMICS

In varying G theories, the action is still

$$S = \int dx^4 \left(\sqrt{-g} \left(\frac{R}{16\pi G} + \mathcal{L}_{NED} \right) \right) \tag{8}$$

Taking the variation of the action with respect to the metric and ignoring surface terms leads to

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{9}$$

In a cosmological context, the Friedmann Robertson Walker metric for variable speed of light c and the Newtonian gravitational constant G can be written as

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega \right], \tag{10}$$

where $a(t)$ is the scale factor, t the comoving time and $K = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively.

For the case of flat FRW ($K = 0$) and $c = 1$, the Einstein's equations are,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G(t)}{3}\rho \tag{11}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3}(\rho + 3p). \tag{12}$$

Where H represents the Hubble parameter and dot is the differentiation with respect to time.

However, the conservation equation that follows from (11)-(12) is for time variation in $G(t)$ is now [42, 43]:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\rho\frac{\dot{G}}{G}. \tag{13}$$

the above conservation equation can be written in this forms:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) + \rho\frac{\dot{G}}{G} = 0, \tag{14}$$

where the equation of state parameter is $\omega = \frac{p}{\rho}$ denotes the equation of state parameter for the dark energy. [51] gives the speed of light c and the gravitational constant G in form of the power-low of the scale factor as:

$$G = G_0 a^m. \tag{15}$$

where G_0 is a positive constant. Since we know that G increases with time, m must be positive.

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From the conservation equation (13) we obtain

$$-\frac{\partial\mathcal{L}_{NED}}{2\partial\mathcal{F}}\cdot\left(\frac{\mathrm{d}}{\mathrm{d}t}((B(t))^2)+4\,\frac{B(t)^2\dot{a}}{a}\right)-\frac{\mathcal{L}_{NED}\dot{G}}{G}=0. \tag{16}$$

The solution of the above equation gives an important relation beteen $B(t)$ and $a(t)$ as.

$$B(t)=a(t)^{-1/2\,\frac{4\,\alpha+m}{\alpha}}B_0. \tag{17}$$

Conviniently written in terms of the scale factor, the evolution of energy density and pressure is given by:

$$\rho=\frac{2^{-\alpha}}{4}\left({B_0}^2a(t)^{\frac{-4\,\alpha-m}{\alpha}}\right)^{\alpha}, \tag{18}$$

$$p=\frac{2^{-\alpha}}{12}\left({B_0}^2a(t)^{\frac{-4\,\alpha-m}{\alpha}}\right)^{\alpha}(4\,\alpha-3)\,. \tag{19}$$

Then we have

$$\rho+p=\frac{2^{-\alpha}}{3}\left({B_0}^2a(t)^{\frac{-4\,\alpha-m}{\alpha}}\right)^{\alpha}\alpha, \tag{20}$$

$$\rho+3p=2^{-1-\alpha}\left({B_0}^2a(t)^{\frac{-4\,\alpha-m}{\alpha}}\right)^{\alpha}(2\,\alpha-1)\,, \tag{21}$$

and the EoS parameter ω is

$$\omega=\frac{4}{3}\alpha-1. \tag{22}$$

It follows from Eq. (22) that at $\alpha=0$, $\omega=-1$ for de Sitter spacetime and at $\alpha=1$, $\omega=1/3$ for ultra-relativistic case. The matter content of the universe is related to its acceleration via Einstein by:

$$\frac{\ddot{a}}{a}=-\frac{4\pi G(t)}{3}(\rho+3p). \tag{23}$$

To check the singularity on energy density and pressure at $a(t)\to 0$ and $a(t)\to\infty$, we finds that,

$$\lim_{a(t)\to 0}\rho(t)=\lim_{a(t)\to 0}p(t)=0, \tag{24}$$

$$\lim_{a(t)\to \infty}\rho(t)=\lim_{a(t)\to \infty}p(t)=0. \tag{25}$$

By using the Einstein's field equation and energy momentum tensor, the Ricci Scalar R which gives the curvature of spacetime is calculated

$$R=8\,\pi\,G_0\,a(t)^m(\rho-3p). \tag{26}$$

The Ricci tensor squared $R_{\mu\nu}R^{\mu\nu}$and the Kretschmann scalar $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ are also obtained as

$$R_{\mu\nu}R^{\mu\nu}=(8\,\pi\,G_0\,a(t)^m)^2(\rho^2+3p^2), \tag{27}$$

$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}=(8\,\pi\,G_0\,a(t)^m)^2\left(\frac{5}{3}\rho^2+2\rho p+3p^2\right). \tag{28}$$

$$\lim_{a(t)\to 0}R(t)=\lim_{a(t)\to 0}R_{\mu\nu}R^{\mu\nu}=\lim_{a(t)\to 0}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}=0, \tag{29}$$

The nature of the scale factor gives the behaviour of the curvature scalar. By taking of the above equations as the universe accelerates at $a(t)\to 0$, we obtained no singularities in the curvature scalar, Ricci tensor and the Kretschmann scalar.

IV. THE EVOLUTION OF THE SCALE FACTOR OF THE UNIVERSE

The first Friedmann equation with variable $G(t)$ for the flat universe is given by

$$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G(t)}{3}\rho. \tag{30}$$

When a particle moves in one dimension in a potential $V(a)$, the equation of motion is

$$\dot{a}^2+V(a)=0 \tag{31}$$

The potential function

$$V(a)=-1/3\,2^{1-\alpha}\pi\,G_0\,(a(t))^{m+2}\left(B_0^2(a(t))^{\frac{-4\,\alpha-m}{\alpha}}\right)^{\alpha} \tag{32}$$

is negative and possesses a maximum at $a=a_c=-C_1$.
Using the (15) and (5), it becomes

$$\frac{-2^{1-\alpha}\pi\,G_0\,a^{-4\,\alpha+2}B_0^{2\,\alpha}+3\,\dot{a}^2}{3a^2}=0, \tag{33}$$

then we find the scale factor $a(t)$ is equal to

$$a(t)\approx 1/2\,\sqrt{B_0}2^{3/4}2^{3/4\,\alpha^{-1}}\left(G_0\,\alpha^2(C_2-t)^2\right)^{1/4\,\alpha^{-1}}3^{-1/4\,\alpha^{-1}}\pi^{1/4\,\alpha^{-1}}. \tag{34}$$

When $\alpha=1$ and $t=-t_0$, we obtained that $a(t)\approx\sqrt(t)$, this depicts radiation dominated universe [62]. In the early universe, there is a de Sitter phase because of the nonlinear corrections to Maxwells theory. Thus, the model describes inflation at the early epoch as shown in Fig. 1. Moreover, Fig. 1 shows the universe evolves from big bang and is expanding with accelerating expansion. The case in which $\alpha=0.5$ indicates an empty universe and increasing the value of α, the universe inflationary epoch is visible.

Introducing the quantity q (the deceleration parameter) [62], we described the expansion of the universe by:

$$q=-\frac{\ddot{a}a}{(\dot{a})^2}=9/2\,\frac{\rho+3\,p}{a\rho}. \tag{35}$$

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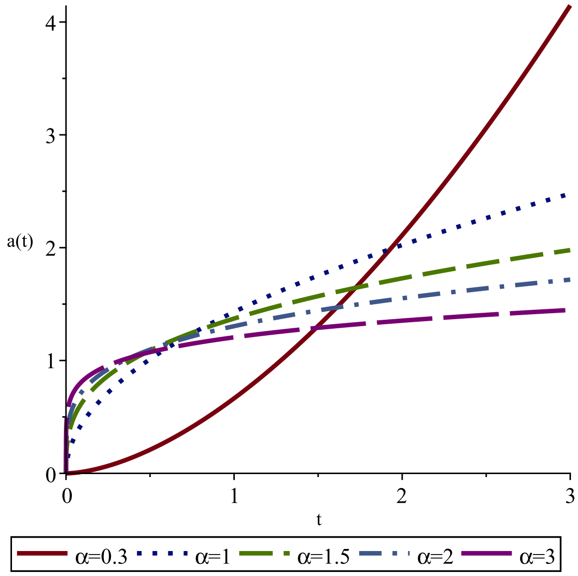

FIG. 1: Plot of the scale factor a versus the time t (for $B_0 = G_0 = C_2 = m = 1$).

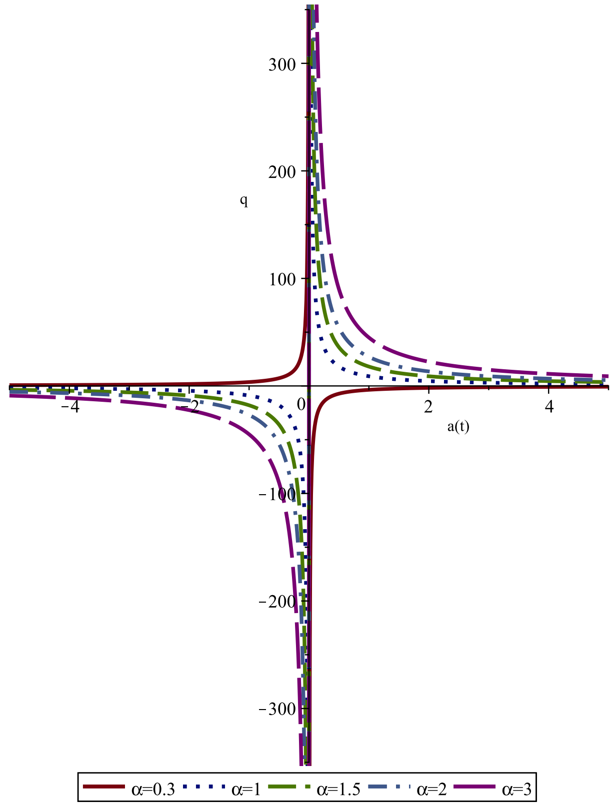

FIG. 2: Plot of the deceleration parameter q versus the scale factor a (for $B_0 = G_0 = C_2 = m = 1$).

Note that $q = 0$ at $\alpha = \frac{1}{2}$ shown in Fig. 2. There is a inflation phase for $q < 0$ and deceleration phase for $q > 0$.

The deceleration parameter represents a two flips. In the first case for $\alpha > 0.5$, the universe transits from the inflationary to decelerated stage. In the second case for $\alpha \leq 0.3$, the universe switches from the decelerated stage to the current accelerating phase.

To estimate the amount of the inflation, we use the definition of e-foldings

$$N = \ln \frac{a(t_{end})}{a(t_{in})} \tag{36}$$

where t_end is the time inflation ends while t_in is the time it begins. For $N \simeq 70$ e-folding, the cosmic flatness and horizon problems can be resolved. Hence, we obtained the scale factor for beginning time of inflation (for $m = 1, \alpha = 1, G_0 = 1, B_0 = 1$)

$$a(t_{in}) = 3.46x10^{-31}. \tag{37}$$

Using the second Friedmann equation Eq. (23) which is known as the acceleration equation for the universe, we plotted the Fig. 3.

$$\frac{\ddot{a}}{a} = \frac{(1 - 2\,\alpha)}{4\alpha^2 t^2}. \tag{38}$$

It is clear that acceleration stops at $\alpha = 0.5$.

V. CONCLUSION

For any cosmological model to survive, it is an established fact that the speed of sound do not exit the local speed of light, $c_s \leq 1$. The second veracity that ensures stability requires that the square of the speed of sound is positive, i.e $c_s^2 > 0$. In case, the model is a classically stable one [61]. At $E = 0$, we obtained:

$$c_s^2 = \frac{dp}{d\rho} = \frac{dp/d\mathcal{F}}{d\rho/d\mathcal{F}} = -\frac{7}{3} + \frac{4}{3}\alpha \tag{39}$$

A requirement of the classical stability $c_s^2 > 0$ is $\alpha > \frac{7}{4}$ and the causality $c_s \leq 1$ is $\alpha \leq \frac{5}{2}$. Hence, the best value of α for both stability conditions are $\frac{5}{2} \geq \alpha > \frac{7}{4}$.

In this work we have studied cosmology with varying gravitational constant G and Nonlinear Electrodynamics in a flat FRW universe. Under change of scale factor, the evolution of magnetic field reduced to $B(t) = a(t)^{-1/2}B_0$ as obtained in [12] when $m = 0$. The evolution of the scale factor shows that the models gives an accelerating expanding universe with

$$a(t) \approx 1/2\,\sqrt{B_0}2^{3/4}2^{3/4\,\alpha^{-1}}\left(G_0\,\alpha^2(C_2-t)^2\right)^{1/4\,\alpha^{-1}} 3^{-1/4\,\alpha^{-1}}\pi^{1/4\,\alpha^{-1}}. \quad (40)$$

, where B_0 represents the magnetic induction field at present time t_0 and β a free parameter presented in Fig. 1. When $\alpha = 1$ and $t = -t_0$, we obtained that $a(t) \approx \sqrt{(t)}$, this depicts radiation dominated universe [62]. Also as observed in the equations (24), (25), and (29), we obtained no singularity in the energy density, pressure and curvature terms respectively. Furthermore, we also studied the stability of the this model and observed that that it depends of the constant α and that it is classically stable. In future study, we will like to study the evolution of the universe with both varying G and c, in nonlinear electrodynamics.

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