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# On the quark scaling theorem and the polarisable dipole of the quark in a scalar field

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**Summary**
In this article the possible impact is discussed of two unrecognized theoretical elements on the present state of particle physics theory. These elements are the awareness that (a) the quark is a Dirac particle with a polarisable dipole moment in a scalar field and that (b) Dirac's wave equation for fermions, if derived from Einstein's geodesic equation, reveals a scaling theorem for quarks. It is shown that the recognition of these elements proves by theory quite some relationships that are up to now only empirically assessed, such as for instance, the mass relationships between the elementary quarks, the mass spectrum of hadrons and the mass values of the Z boson and the Higgs boson.

Keywords: polarisable Dirac dipole; quark scaling; hadron mass spectrum; Z boson; Higgs boson

## 1. Introduction

This article is aimed to discuss the possible impact that the awareness of a polarisable dipole moment of quarks in a scalar potential field of quarks, conceived as Dirac particles in a particular format, would have on the interpretation of particle physics theory. The discussion will be focussed on the bonds between quarks in mesons and baryons. As is well-known, canonical particle physics theory is captured in a rather abstract mathematical formalism. This formalism has been developed under adoption of some axiomatic attributes that have been unknown prior to the development of the Standard model. Among these are, for instance, weak isospin and hypercharge. They show up as quantum numbers attributed to the elementary fermions [1], such as listed in Table I.

**Table I**

| Fermions | u | d | $e^-$ | $\nu_e$ |
|---|---|---|---|---|
| S- spin | 1/2 | 1/2 | 1/2 | 1/2 |
| $m_0$-mass | ? | ? | $m_e$ | $\approx 0$ |
| $I_Z$ -weak isospin charge | 1/2 | -1/2 | -1/2 | 1/2 |
| Y-hypercharge | 1/3 | 1/3 | -1 | -1 |
| $Q = I_Z + Y/2$ | 2/3 | -1/3 | -1 | 0 |

It is my aim to show how these attributes are related to those of a quark that has a polarisable dipole moment in a scalar potential field. Such a dipole moment is a unique

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property of particular non-electron formats of Dirac's particle, while it is absent in electron-type ones. As will be shown, this dipole moment could be the key for assigning reliable figures to the rest masses of elementary quarks and their hadron composites. It will be shown that a re-interpretation of these attributes allows a physical interpretation of isospin and removes the reason to accept the asymmetrical electric charge assignment to quarks. It will be shown as well that the number of elementary fermions would be reduced significantly.

Like all elementary fermions, quarks follow Fermi-Dirac statistics, obey the Pauli exclusion principle, have half integer spin and have distinct antiparticles. They can be modelled with the Dirac equation. The canonic formulation of Dirac's particle equation reads as [2,3],

$$(i\hbar\gamma^{\mu}\partial_{\mu}\psi - \beta m_0 c\psi) = 0\,.$$

In which $\beta$ is a 4 x 4 unity matrix and in which the 4 x 4 gamma matrices have the properties,

$$\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 0 \text{ if } \mu \neq \nu\text{; and } \gamma_0^2 = 1; \gamma_i^2 = -1;\ \beta^2 = 1\,. \tag{1}$$

As usual, $c$ is the vacuum light velocity, $\hbar$ is the reduced Planck constant and $m_0$ is the rest mass of the particle. While this equation captures a basic attribute as mass and attributes as spin state and particle/antiparticle state, it does not include quite some other properties of elementary fermions. It does not even include electric charge as an attribute, while Dirac's theory is originally conceived for electrons. It includes mass $m_0$ and nuclear spin $S$, but the hypercharge and weak isospin are missing. These are rather artificial attributes, conceived in the mathematical standard model, in which empirical phenomena are captured by axiomatic abstraction.

While spin $S$ can be physically understood in terms of the eigen value of an elementary angular moment $\hbar$, weak isospin has no known physical interpretation. Apart from its relationship with the electric charge as shown by the Gell-Mann-Nishijma formula [4,5] at the bottom line of Table I, it plays a role in the classification of hadrons, in interactions between nuclear particles and in the interaction with the omni-present energetic background field, known as the Higgs field. Weak isospin shows the same behaviour as the nuclear spin $S$ in the sense of being subject to the same algebra rules as nuclear spin, thereby establishing an isospin triplet state $|1,\pm 1\rangle$ or $|1,0\rangle$ next to a singlet state $|0,0\rangle$.

Particle physics theory has been developed over many decades of years. As is well-known, a major milestone in this development was set in 1961 by Gell-Mann and Ne'eman, dubbed as the Eightfold Way [6]. One may wonder how this scheme would have been set up if isospin would have been understood physically. Within the scope of this article, it is my aim to show that a physical interpretation allows a less heuristic alternative for the Eightfold Way.

To show that such a novel view might be a useful complement to present theory, some problems will be addressed that are difficult to solve with present-state theory. Examples of such problems are mass related, because present theory shows a weakness in that respect. In particular, it will be shown how the mass spectrum of hadrons can be calculated to a rather high precision, how the masses of the weak interaction bosons and the Higgs boson can be calculated and why, for instance, the mass of a neutral pion is 4.6 MeV larger than that of a charged pion.

After the introduction in paragraph 2 of an unconventional non-electron format of a Dirac particle, in the next two paragraphs, the quark and the archetype meson (pion) will be profiled in terms of this particle. In paragraph 5 a description is given of the quark-scaling theorem as a basic quark property next to its polarisable dipole moment under a scalar potential. Paragraph 6 contains an assessment by theory of the Higgs boson mass, followed in paragraph 7 by an an assessment by theory of the $Z$ boson. In paragraph 8 the structural view as developed in this article will be compared and related with the mathematical view of the Standard Model. Paragraph 9 deals with the electroweak unification. In paragraph 10, a short description of baryons is given on the basis of the developed theory. Paragraph 11 contains a discussion and the conclusion. In these texts, quite some results are invoked from previously documented works in publications and preprints. The highlight on the quark-type Dirac particle and the quark-scaling theorem as two unrecognized theoretical principles will place those in a better context.

## 2. Dirac particles with a polarisable dipole moment in a scalar potential field

The canonical set of gamma matrices is given by,

$$\gamma_0=\begin{bmatrix} I & 0 \\ 0 & -I\end{bmatrix};\gamma_1=\begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0\end{bmatrix};\gamma_2=\begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0\end{bmatrix};\gamma_3=\begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0\end{bmatrix};\beta=\begin{bmatrix} I & 0 \\ 0 & I\end{bmatrix}. \tag{2}$$

The calculation of the excess energy of an electron in motion subject to a vector potential $\boldsymbol{A}(A_0, A_x, A_y, A_z)$, gives [2,7],

$$\Delta E=\frac{e}{2m_0}\begin{bmatrix} \overline{\sigma} & 0 \\ 0 & \overline{\sigma}\end{bmatrix}\hbar.\mathbf{B}+\mathrm{i}\frac{e}{2m_0}\begin{bmatrix} 0 & \overline{\sigma} \\ \overline{\sigma} & 0\end{bmatrix}\hbar/c.\mathbf{E}, \tag{3}$$

in which $\overline{\sigma}$ is the Pauli vector, defined by

$$\overline{\sigma}=\sigma_1\mathbf{i}+\sigma_2\mathbf{j}+\sigma_3\mathbf{k}, \tag{4}$$

in which ($\mathbf{i},\mathbf{j},\mathbf{k}$) are the spatial unit vectors and in which $\mathbf{B}$ and $\mathbf{E}$ are generic field vectors derived from the vector potential. The matrices are state variables with a real eigenvalue

$|\overline{\sigma}| = 1$, such that the angular momentum (associated with $\mathbf{B}$) can be conceived as a spin vector with eigenvalue $|\boldsymbol{\hbar}|$. Next to the angular momentum a second momentum $\boldsymbol{\hbar/c}$ (associated with $\mathbf{E}$) can be identified with eigenvalue $|\boldsymbol{\hbar/c}|$. In terms of these two dipole moments, eq. (3) can be written as,

$$\Delta E = \frac{e}{2m_0}\,|\boldsymbol{\hbar}|.\mathbf{B} + \mathrm{i}\frac{e}{2m_0}\,|\boldsymbol{\hbar/c}|.\mathbf{E} \tag{5}$$

The electron has a real first dipole moment ($e\hbar/2m_0$), known as the magnetic dipole moment, and an imaginary second dipole moment ($\mathrm{i}e\hbar/2m_0c$), known as the anomalous electric dipole moment. The latter is one of the two anomalies of his theory pointed out by Dirac. He noticed a negative energy solution next to a positive energy solution. And he noticed a real magnetic moment next to an imaginary electrical dipole moment. About the first thing he remarked that that the problem would disappear if the electron would change his polarity, but that “this is a phenomenon not yet observed”. About the second thing he remarked that he doubted about the physical meaning of an imaginary electrical dipole moment. Whereas he welcomed the real magnetic dipole moment as a confirmation of a known physical phenomenon (“the spinning electron”), he suggested that the imaginary electrical dipole moment would be a mathematical artefact as a result of “the artificial multiplication of his wave function to create an Hamiltonian that resembles the one of previous theories”. In fact, however, the negative energy solution is a result of an artificial multiplication as well, because by deriving a set of linear wave equations from the Einsteinean energy relationship $E$,

$$E = \sqrt{(c|\mathbf{p}|)^2 + (m_0c^2)^2}\,,$$

in which $\mathbf{p}$ is the three-vector momentum. Dirac allowed the heuristic manipulation,

$$W = \pm E\,, \tag{6}$$

while he could equally have chosen,

$$W = \pm \mathrm{i}E\,. \tag{7}$$

Whereas, as a consequence from discovery of the positron, a negative value of $W$ has been accepted as physically viable, it is not the case for an imaginary value of $W$, while a negative value is not necessarily less strange than an imaginary one. Hence, supposing that an imaginary $W$ would turn the imaginary electric dipole moment into a real one, a discovery of a Dirac particle with a polarisable dipole moment in a scalar potential field, would make an imaginary $W$ as physically viable as a negative $W$. Although a negative $W$ is commonly denoted as “negative energy”, it is not more than an artificial manipulation on the true physical entity “energy”. Having this in mind, let us consider how the imaginary second dipole moment might be turned into a real one. Before doing so, let us rewrite Dirac's expression for the electron in terms of a generic Dirac particle, i.e., as,

$$\Delta E = \frac{g}{2m_0 c}\begin{bmatrix} \overline{\sigma} & 0 \\ 0 & \overline{\sigma} \end{bmatrix} \boldsymbol{\hbar}.(\nabla \times \mathbf{A}') + \mathrm{i}\frac{g}{2m_0 c}\begin{bmatrix} 0 & \overline{\sigma} \\ \overline{\sigma} & 0 \end{bmatrix} \boldsymbol{\hbar/c}.(\nabla A_0'), \tag{8}$$

in which $g$ is a generic coupling factor and in which $A'(A_0', A_x', A_y', A_z') = A'(A_0', \mathbf{A}')$ is a dimensionless generic vector potential. Due to the imaginary sign in front of the second term, such a particle is not polarisable in a scalar potential field. This will change if the $\beta$ matrix of the canonical Dirac set is modified into the "fifth gamma matrix", such that

$$\gamma_0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}; \gamma_1 = \begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix}; \gamma_2 = \begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{bmatrix}; \gamma_3 = \begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{bmatrix}; \beta = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \tag{9}$$

thereby, like an elementary algebraic evaluation shows, modifying the Pauli property (1) into

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 0 \text{ if } \mu \neq \nu; \text{ and } \gamma_0^2 = -1; \gamma_i^2 = -1; \ \beta^2 = -1. \tag{10}$$

This removes the imaginary sign from the second dipole, because an analytical elaboration shows that the excess energy expression is modified into,

$$\Delta E = \frac{g}{2m_0 c}\begin{bmatrix} \overline{\sigma} & 0 \\ 0 & \overline{\sigma} \end{bmatrix} \boldsymbol{\hbar}.(\nabla \times \mathbf{A}') + \frac{g}{2m_0 c}\begin{bmatrix} 0 & \overline{\sigma} \\ \overline{\sigma} & 0 \end{bmatrix} \boldsymbol{\hbar/c}.(\nabla A_0'). \tag{11}$$

However, whereas the canonical formulation (2) is compatible with

$$W = \pm\sqrt{(c|\mathbf{p}|)^2 + (m_0 c^2)^2}\ , \tag{12}$$

in which $\mathbf{p}$ is the three-vector momentum, the modified formulation (9) is compatible with

$$W = \pm \mathrm{i}\sqrt{(c|\mathbf{p}|)^2 + (m_0 c^2)^2}\ , \tag{13}$$

The proof of the sequence (9-13) is left to the reader, who may use the analysis shown in the appendix of Ref.[7]. Particles with an imaginary $W$ as in (13) , can be viewed as particles with imaginary mass. If such particles move at superluminal speed $v > c$, their (Einsteinean) energy is real. This will be clear from

$$E = \sqrt{\{\frac{\mathrm{i}m_0 v}{1-(v/c)^2}\}^2 + (\mathrm{i}m_0 c^2)^2} > 0 \text{ for } v > c. \tag{14}$$

Dirac particles in this state are known as "tachyons". Because of their superluminal speed, it are particles of high interest if they would exist. Within the scope of this article, hypothetical particles with imaginary (Einsteinean) energy are of interest at subluminal speed $v < c$ as well, because of the real value of the second dipole moment. This raises the question if physics laws allow the existence of such particles or not. If they do, they might have been remained hidden because of lack of physical evidence. To estimate the relevancy of this question it is instructive considering a somewhat different modification of the canonical Dirac set, in which the the $\beta$ matrix is multiplied with $\mathrm{i}$, such that, rather than (2) or (9), we have

$$\gamma_0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}; \gamma_1 = \begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix}; \gamma_2 = \begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{bmatrix}; \gamma_3 = \begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{bmatrix}; \beta = \mathrm{i}\begin{bmatrix} \mathrm{I} & 0 \\ 0 & \mathrm{I} \end{bmatrix}, \tag{15}$$

which represents the so-called imaginary-mass Dirac equation [8],

$$(\mathrm{i}\hbar\gamma^{\mu}\partial_{\mu}\psi - \mathrm{i}m_0 c\psi) = 0,$$

and which meets the Pauli-property,

$$\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 0 \text{ if } \mu \neq \nu; \text{ and } \gamma_0^2 = 1; \gamma_i^2 = -1; \ \beta^2 = -1. \tag{16}$$

Note the correspondences and the differences between the Pauli properties of the three Dirac modes. Curiously, while the “tachyonic modification” returns two real dipole moments, the “imaginary mass modification” still returns an imaginary second moment. Apparently, there is a fundamental difference between the two modifications. As known from literature [8], the tachyonic format is Lorentz invariant, while the imaginary mass format is not. It is therefore fair to conclude that the possible existence of subluminal tachyonic Dirac particles is physically viable.

Note: The Dirac format (9) is a replacement for the erroneous format reported in [7, eq. (27)]. The author is indebted to prof. D. Zeppenfeld, who pointed to the error.

Under the hypothesis that the quark is a Dirac particle with an imaginary $W$, it possesses a polarisable dipole moment in a generic scalar nuclear field $\nabla A_0$ with magnitude $p$ in given by,

$$p = \frac{g}{m_0}\frac{\hbar}{2c}, \tag{17}$$

in which $g$ is a generic unknown nuclear coupling factor. Similarly as a negative $W$ allows the existence of energetic positrons, an imaginary $W$ may allow the existence of energetic quarks. Within this perspective, the “$W$ status” can be viewed as “absolute energy” $|W| = E$, which might be positive, negative or imaginary, depending on whether the energetic domain is gravitational, electric or nuclear.

Similarly as in the case of particle/antiparticle state, quarks in the two states of the polarisable dipole moments can be regarded as two different quarks, which by convention can be indicated as an “up isospin” $u$ quark and a “down isospin” $d$ quark.

Note: Within the scope of this article a symbolic notation will be adopted for the state of (the common angular) nuclear spin. The up-state nuclear spin of an $u$ quark (or a $d$ quark) and the down-state of an $u$ quark (or or a $d$ quark) will be indicated as, respectively $u$ and $\underline{u}$ (or $d$ and $\underline{d}$ for $d$ quarks). The particle state and the antiparticle state of an $u$ quark will be indicated as, respectively $u$ and $\bar{u}$. Analogously $d$ and $\bar{d}$ for $d$ quarks.

Summarizing: accepting the existence of Dirac particles with imaginary $W$, allows considering the quark as a particle that, under influence of its dipole moment, may "spin" (i.e., can be polarized) in a scalar nuclear potential field. The associated quantity $\bar{\sigma}$ is a state variable that can be conceived as isospin next to the nuclear spin associated with the elementary angular momentum $\hbar$. This quark has two different modes, allowing to make a distinction between an $u$ quark and a $d$ quark.

## 3. Profiling a quark

Let us proceed by profiling a quark as a field. One of the issues to cope with in the context of this article are the different semantics of the field concept in classical physics, in quantum mechanical physics and in particle physics. In classical physics the field is the static solution of an energetic wave equation. The field has an energetic interpretation. In quantum mechanics the field is a solution of Schrödinger's equation, which in fact is the non-relativistic limit of Dirac's equation. This field has an probabilistic interpretation, because its integrated squared value is considered as the probability that a particle is at some moment in some spatial position. In particle physics theory the two fields are unified in a single concept: the quantum field. This is done on the basis of (second) quantization. This allows the description of processes that are subject to an interchange between matter and energy, such as occur in decay processes and scattering processes, including, for instance, recoil. This Quantum Field Theory (QFT) is one of the pillars of the Standard Model of particle physics. Because interchange between matter and energy is beyond the scope, the field view within the scope of this article is either classical, formalized as $\Phi$, or quantum mechanical, formalized as a multi-component spinor $\Psi(\psi_\mu)$ eventually reduced to a single component $\psi$.

Modeling the quark's field as a classical scalar $\Phi$, it can be characterized by a Lagrangian density with the format

$$\mathrm{L} = -\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + U(\Phi) + \rho\Phi, \tag{18}$$

in which $U(\Phi)$ is the potential energy of a symmetric energetic background field and in which $\rho\Phi$ is the source term. If the background field would have the format,

$$U(\Phi) = U_{DB} = \lambda_{DB}^2 \frac{\Phi^2}{2}, \tag{19}$$

the stationary format of the wave equation, obtained after application of the Euler-Lagrange equation from the Lagrangian, is the inhomogeneous Helmholtz equation [9], which for a pointlike source $\rho$ shows the solution

$$\Phi_{DB} = \Phi_0 \frac{\exp(-\lambda_{DB} r)}{\lambda_{DB} r}. \tag{20}$$

Such a field (20) is of a type as described by Debije [10] for a charged particle in an ionic plasma The wave equation associated with this Lagrangian (18,19) has the same format as the Klein-Gordon equation, which originally has been erroneously conceived as a relativistic extension of the Schrödinger one. In present quanum field theory, this equation is considered as the field equation for a massive spin-zero particle. In a formal way, one might say that a background field of energy has given mass to a mass less boson. A more physical interpretation is the parallel with the scalar field expression for a charged particle in an ionic plasma, In the Debije's theory, it is the influence of the polarized ionic plasma composed by elementary dipoles that shields the mass less boson field from the electric charged pointlike source.

Similarly as an electron, the quark has an energetic monopole, represented by the source term $\rho\Phi$ in the Lagrangian (18). For an electron, the monopole is an electric point charge. For the quark it is the nuclear equivalent of the electric point charge. Next to the monopole, the electron and the quark have two dipole moments [2,7]. These dipole moments are the results from the elementary angular momentum ***ħ*** and the elementary mass dipole moment ***ħ/c***. In the case of an electron, these dipole moments give rise to, respectively, a real magnetic dipole and an *imaginary* electric dipole. In the case of a quark, these dipole moments give rise to, respectively, a real equivalent of the magnetic dipole and a *real* nuclear equivalent of the electric dipole. While, due to its imaginary value, the electric dipole moment of the electron cannot be polarized in a scalar potential field, the nuclear equivalent can, because of its real value.

Similarly as the monopole of the electron, the monopole of the quark, spreads a scalar potential field. This field is able to polarize the electric dipole equivalent of another quark. Such a dipole spreads an en energetic potential with $x^{-2}$ dependency along the orientation axis of the dipole. As a consequence, an equilibrium of forces can arise between a repelling force from the $r^{-1}$ monopole field dependency and the attractive force with $x^{-2}$ dipole field dependency from suitably aligned dipoles of two quarks. Because nuclear forces have a short range, these potential fields must experience a shielding effect akin to the shielding of the field of an electric point charge in an ionized plasma. This shielding is known as the Debije effect. It occurs under influence of an omni-present fluidal field of energy. In particle physics such a background field is known as the Higgs field, in gravity known as the cosmological background field due to the Cosmological Constant. Hence, in qualitative terms, the potential field of a quark along the axis of the polarisable dipole, can be expressed as,

$$\Phi(\lambda x) = \Phi_0 \exp(-\lambda x)\{\frac{1}{(\lambda x)^2} - w\frac{1}{\lambda x}\}, \tag{21}$$

in which $\lambda$ (with dimension m$^{-1}$) is a measure for the range of the nuclear potential, in which $\Phi_0$ (in units of energy, i.e. joule) is a measure for the quark's "charge", and in which $w$ is a dimensionless weigh factor that relates the strength of the monopole field to the dipole field. The far field, decaying as $\exp(-\lambda x)/\lambda x$ is due to the monopole. As will be shown later, it can be seen as the weak interaction between the quarks. The near field, decaying as $\exp(-\lambda x)/(\lambda x)^2$ can be seen as the strong interaction between the quarks. The strong interaction is due the polarisable dipole..

While the spatial Debije format (10) of the field has been straightforwardly calculated from the functional expression of the background field (9), the spatial field expression of the quark's field has only been derived indirectly. One may ask if it would be possible to arrive at the format (11) analytically from a functional field expression as well. Obviously, the simple unbiased symmetric background field expression (9), in which $U(\Phi)=0$ for $\Phi=0$, has to be modified for the purpose. The most simple approach is modifying (19) into

$$U(\Phi)=-\frac{\mu_H^2}{2}\Phi^2+\frac{\lambda_H^2}{4}\Phi^4. \tag{22}$$

For positive values of $\lambda_H^2$ and $\mu_H^2$, it is a broken field that is zero for

$$\Phi_0=(\mu_H/\lambda_H)\sqrt{2},$$

known as the vacuum expectation value.

(Note: this field format, originally conceived by Nambu [11] from quite a different perspective, has been dubbed later as the Higgs field because of its particular property to give mass to spin-1 particles, shown by Higgs [12] and Englert and Brout [13].)

Unfortunately the high non-linearity of this field prevents deriving an analytical solution $\Phi(r)$ from (22) and (18). However, a numerical procedure allows deriving a two-parameter expression for $\Phi(r)$ that closely approximates a true analytical solution. In this approach a generic Ansatz format is adopted for $\Phi(r)$ from which an expression is retrieved of $U(\Phi)$. Subsequently, a fit of is searched on (22). In this approach, first of all, the Euler-Lagrange equation is applied on the static Lagrangian density (18). Hence, from

$$\frac{\partial \mathrm{L}}{\partial \Phi}-\partial_i\left(\frac{\partial \mathrm{L}}{\partial(\partial_i \Phi)}\right)=0, \tag{23}$$

we have from (18),

$$\partial_i\Phi\partial^i\Phi=\frac{d}{d\Phi}U(\Phi)+\rho. \tag{24}$$

The Ansatz format of the field $\Phi(r)$ is chosen as,

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$$\Phi(r) = \Phi_0' \frac{\exp(-\lambda r)}{\lambda r}\{\frac{\exp(-\lambda r)}{\lambda r} - 1\}. \tag{25}$$

The rationale behind the choice (25) is the assumption that the inter-quark potential will behave similarly as the inter-nucleon potential [14].

Substitution of (25) into (23) and subsequent calculation of $U(\Phi)$ gives a fit with (22) for $\mu_H^2$ and $\lambda_H^2$, such that

$$\frac{1}{2}\mu_H^2 = 1.06\lambda^2 \text{ and } \frac{1}{4}\lambda_H^2 = 32.3\frac{\lambda^2}{\Phi_0'^2}. \tag{26a}$$

A numerical calculation and a proof for this fit has been documented in [15].

Without loss of generality $\Phi_0'$ can be rescaled to the vacuum expectation value $\Phi_0$ (22) by modifying (26a) into

$$\frac{1}{2}\mu_H^2 = 1.06\lambda^2 \text{ and } \frac{1}{4}\lambda_H^2 = \frac{(1.06\lambda)^2}{\Phi_0^2}. \tag{26b}$$

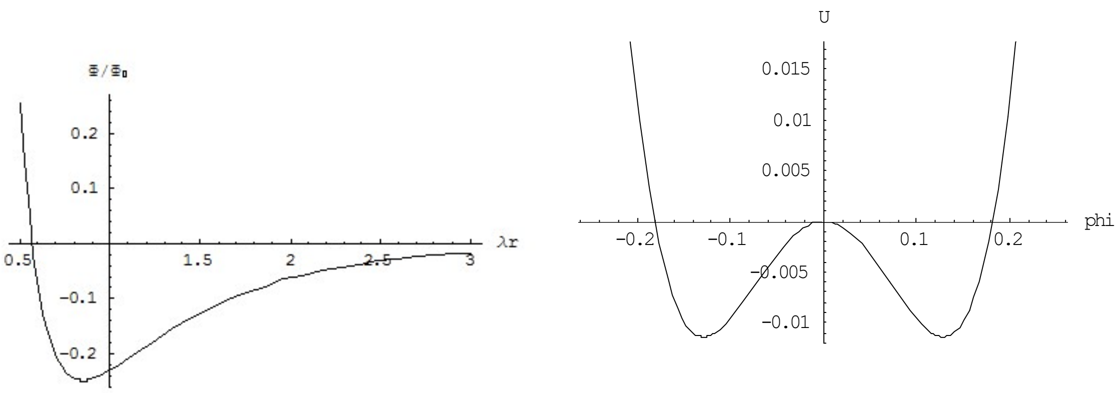


**Fig. 1.** (Left) The quark's scalar field $\Phi / \Phi_0$ as a function of the normalized radius $\lambda x$ ; (Right) The background field $U_H(\Phi) = -U(\Phi)$ retrieved from the spatial expression.

The two-parameter field is indistinguishable from the three-parameter field,

$$\Phi(r) = \Phi_0 \exp(-\lambda r)\{\frac{1}{(\lambda r)^2} - w\frac{1}{\lambda r}\} \text{ for } w = 1/0.555. \tag{27}$$

The quark's field would show the characteristics as shown in Figure 1. It would imply that a quark would be repelled by any other quark under influence of the far field, but attracted by the near field, thereby giving rise to mesons as stable two-quark junctions and baryons as three-quark junctions.

Unfortunately, this radial symmetric format is not viable, because it violates the renormalization constraint. However, comparing (27) with (21) reveals a striking correspondence. Nevertheless, there is a major difference as well. While the derivation of (27) has been based upon a presupposed energetic monopole model for the quark, (21) is the result of a dipole moment next to a monopole. Hence, by restricting the validity of (27) to the dipole axis $x$, the renormalization problem is removed by rewriting (27) as a sum of a far field and a near field, such that,

$$\Phi(x)=\Phi_F(x)+\Phi_N(x) \text{ with } \Phi_F(x)=-w\Phi_0\frac{\exp(-\lambda x)}{\lambda x} \text{ and } \Phi_N(x)=\Phi_0\frac{\exp(-\lambda x)}{(\lambda x)^2} \quad (28)$$

The conclusion therefore is that the Higgs field has to be interpreted as the shielded radial symmetric field of an energetic monopole in conjunction with a one-dimensional dipole field. The quark, conceived as a subluminal tachyon-type Dirac particle, in this article futher denoted as a *pseudo-tachyon*, is compatible with this model. The near field is due to the dipole and gives an interpretation for the near field that glues the quarks together in hadronic structures. The break in the field that spoils the symmetry of the Debije field, thereby modifying it into the Higgs field, can be ascribed to the quark's polarisable dipole moment.

## 4. Profiling mesons and baryons

Let us suppose that we wish to build the archetype meson (quark plus antiquark) and the archetype baryons, i.e. the proton and the neutron (three quarks) by a single archetype quark. Would that be possible, and if not, what kind of theoretical instruments (read axioms) would be needed to do so? Figure 2 shows a schematic configuration between two elementary quarks. The meson consists of a quark and an antiquark. The nuclear spins would be either anti-parallel or parallel, thereby making, respectively, a pseudo-scalar meson (pion) or a vector meson (rho meson).

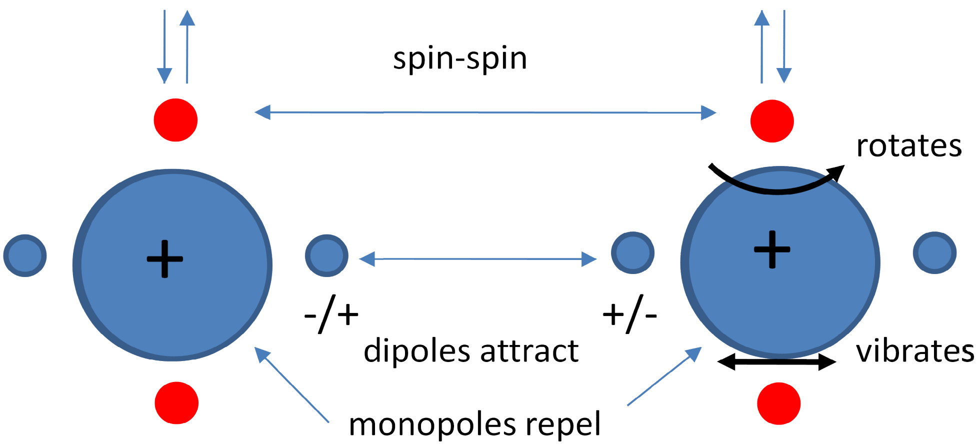


**Fig. 2.** A quark has two real dipole moments, hence two dipoles. One of these (horizontally visualized) is polarisable in a scalar potential field. The other one (vertically visualized) is not. The dipole moments are subject to spin statistics. However, the polarity of the horizontal one is restrained by the bond: the horizontal dipoles are only oriented in the same direction: either inward to the centre or outward from the centre.

As noted before, the far field, decaying as $\exp(-\lambda x)/\lambda x$, due to the monopole, can be seen as the weak interaction between the quarks. The near field, decaying as $\exp(-\lambda x)/(\lambda x)^2$ is due the polarisable dipole. Unlike quark bonds, such lepton bonds don't exist, because of the lack of such dipole. The possible antiparallel states of the two different quark states $u$ and $d$ are shown in the upper part of Table II. Note the allowance of a mixed state, while identical isospin states are not allowed. The possible parallel states are shown in the lower part. These states show an additional mode, because of a possible triplet state for parallel spins. While in the Standard Model two different archetypes quarks $u$ and $d$ have been defined by an axiom, these two quark modes are a theoretical consequence from the quark conceived as a pseudo-tachyon.

What about the archetype baryon? Its structure, made possible by the nature of the polarisable spins, being either inward or outward oriented, may either show up as nuclear spin 1/2 (two spins parallel, one anti parallel) or as nuclear spin 3/2 (three spins parallel) provided that Pauli-statistics would not prevent their existence. As shown in the upper part of Table II, the ground state configuration with nuclear spin 1/2 shows two different possible modes. In the context of the Standard model made possible by the isospin axiom, in our structural model as a theoretical consequence from the quark model as a pseudo tachyon. In the Standard Model, the Pauli conflict in the 3/2 spin $uuu$ and $ddd$ configurations is solved by an additional theorem next to the isospin one. This additional axiomatic theorem is known as color charge. Considering that a triplet state of two electrons in the 2s orbital of an hydrogen atom shows up as a consequence of spin-spin interaction between electron and nucleus, one might argue that any of two quarks in a triangular structure of three quarks, like shown in figure 3, are allowed being in the same state of nuclear spin. From that perspective, three quarks in a symmetrical triangular structure are allowed being in the same state of nuclear spin, such a shown in the lower part of Table II. Comay [16] has given a detailed analysis for this phenomenon.

In a sense, giving a structural physical explanation for isospin and color force is irrelevant for the task finding covariant descriptions for the hadron behavior under nuclear forces. In that respect a mathematical axiomatic approach is not better or worse than a structural physical approach.

**Table II**

| pseudoscalar, spin antiparallel | | | | | | |
|---|---|---|---|---|---|---|
| meson | scalar spin inward/outward | spin (nuclear) | isospin sum | recode | bias | symb |
| $(q\overline{q})$ | $(\underline{u}\overline{d})$ | antipar | +1 | | 0 | $\pi^+$ |
| | $(\underline{d}\overline{u})$ | antipar | -1 | | 0 | $\pi^-$ |
| | $(\underline{u}\overline{d}+\underline{d}\overline{u})/2$ | antipar | 0 | | 0 | $\pi^0$ |

(mixed mode allowed, two isospins not allowed, isospin flip allowed))

| vector, spin parallel | | | | | | | |
|---|---|---|---|---|---|---|---|
| meson | scalar spin inward/outward | triplet | spin (nuclear) | Isospin sum | bias | recode | symb |
| $(q\bar{q})$ | $(u\bar{d})$ | | par | +1 | 0 | | $\rho^{+}$ |
| | $(d\bar{u})$ | | par | -1 | 0 | | $\rho^{-}$ |
| | $(u\bar{d}+\bar{u}d)/2$ | | par | 0 | 0 | | $\rho^{0}$ |
| | | $(u\bar{d}+\underline{\bar{u}}\,\underline{d})/2$ | par | 0 | 0 | | $\omega$ |

(mixed mode allowed, isospin flip allowed, nuclear spin flip allowed. The antiparticle has opposite electric charge w.r.t. to the charge of the particle)

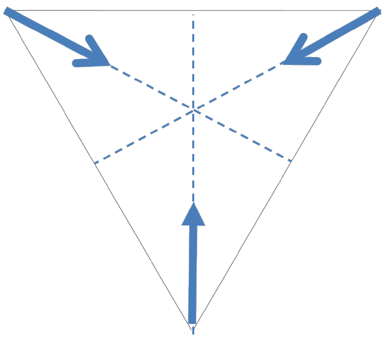

**Fig.3:** The baryon as a three-quark (an) harmonic oscillator

**Table III**

| baryon | scalar spin inward/outward | spin (nuclear | isospin sum | bias | charge | symb |
|---|---|---|---|---|---|---|
| | $(ud)\underline{u}$ | $\pm 1/2$ | +1 | +1/2 | 1 | p |
| | $(du)\underline{d}$ | $\pm 1/2$ | -1 | +1/2 | 0 | n |

(two isospins in different nuclear spin are allowed against a third particle)

| baryon | scalar spin inward/outward | triplet | spin (nuclear) | isospin sum | bias | charge | symb |
|---|---|---|---|---|---|---|---|
| $(qqq)$ | $(ud)u$ | | $\pm 3/2$ | +1/2 | +1/2 | 1 | $\Delta^{+}$ |
| | $(du)d$ | | $\pm 3/2$ | -1/2 | +1/2 | 0 | $\Delta^{0}$ |
| | | $(uu)u$ | $\pm 3/2$ | +3/2 | +1/2 | 2 | $\Delta^{++}$ |
| | | $(dd)d$ | $\pm 3/2$ | -3/2 | +1/2 | -1 | $\Delta^{-}$ |

(two isospins in the same nuclear spin are allowed against a third particle)

Inspection of the meson and baryon tables made up by the two basic quarks $u$ and $d$ show a clear impact of the isospin states on the electric charge of the observable hadrons. A particular concern is the charge integrity of the hadron. Full charge parity for the $u$ quark and the $d$ quark seems to violate it. The problem can be avoided if asymmetrical charge splits $+2/3e$ and $-1/3e$ are assigned to, respectively, the $u$ quark and the $d$ quark. Another way-out is the assignment of a bias for the baryon state. This allows to maintain full symmetry.

Unfortunately, the simple adoption of a single archetype quark in two different modes, is, by far, not adequate for the explanation of observations on mesons and baryons. A first extension to the considerations just given have led to the inevitable conclusion that next to the $u$ quark and the $d$ quark at least a third quark had to exist. This has led to the adoption to yet another axiom. There seemed being no escape than defining a new elementary particle that became known as the $s(\text{trange})$ quark. In this article it will be shown that its existence can be explained as a further consequence from the quark conceived as a polarisable Dirac particle. Before doing so, let us first extend the meson and baryon tables by including the $s$ quark. It can be done systematically similar as in the case of the basic quarks by distinguishing between the nuclear spin antiparallel configuration (spin +/- 1/2 for baryons) and the spin parallel configuration (spin +/- 3/2 for baryons). The $s$ quarks appears being a particle with negative electric charge. The $c(\text{harmed})$ quark, identified later in 1974) appeared being just positively charged. Because the electric charge of the hadron is an holistic attribute, one may still explain the charge of the hadrons as a consequence from asymmetrical charge splits or as symmetrical contributions on top of a bias.

**Table IV**

| Mesons (light sector) | | | | | | | |
|---|---|---|---|---|---|---|---|
| meson | pseudo scalar | symb | vector | | isospin sum | Q | mass (MeV/c$^2$ |
| | $(u\bar{d})$ | $\pi^+$ | $(u\bar{d})$ | $\rho^+$ | 1 | 1 | 139/775 |
| | $(d\bar{u})$ | $\pi^-$ | $(d\bar{u})$ | $\rho^-$ | -1 | -1 | |
| | $(u\bar{d}+\underline{d}\bar{u})/2$ | $\pi^0$ | $(u\bar{d}+\bar{u}d)/2$ | $\rho^0$ | 0 | 0 | |
| $(q\bar{q})$ | | | $(u\bar{d}+\underline{\bar{u}}\underline{d})/2$ | $\omega$ | 0 | 0 | n.a/782 |
| | $(u\bar{s})$ | $\mathrm{K}^+$ | $(u\bar{s})$ | $\mathrm{K}^{*+}$ | 1 | 1 | 489/892 |
| | $(s\bar{u})$ | $\mathrm{K}^0$ | $(s\bar{u})$ | $\mathrm{K}^{*0}$ | 0 | 0 | |
| | $(d\bar{s})$ | $\bar{\mathrm{K}}^0$ | $(d\bar{s})$ | $\bar{\mathrm{K}}^{*0}$ | 0 | 0 | |
| | $(s\bar{d})$ | $\bar{\mathrm{K}}^-$ | $(s\bar{d})$ | $\bar{\mathrm{K}}^{*-}$ | -1 | -1 | |
| | $(s\bar{s})$ | x | $(s\bar{s})$ | $\varphi$ | 0 | 0 | n.a/1020 |

From the rest masses of the hadrons, shown in the tables IV and V, it is obvious that the parallel nuclear spin configurations show significant higher values than the antiparallel configurations. The nuclear spin flip between parallel and antiparallel is known as strong interaction. The spin +/- 1/2 table of the eight light ($u,d,s$) baryons is known as an octet, and the spin +/- 3/2 table of the ten light ($u,d,s$) baryons is known as a decuplet. Note the mass difference between the $\Lambda^0$ baryon and the $\Sigma^0$ baryon. It makes a difference whether quarks with equal mass are in parallel or whether quarks with unequal mass are in parallel. In the +/- 3/2 configuration the difference has disappeared and the two configurations coincide. Hence, it is obvious that de nuclear spin orientations have a major impact on the rest masses of the hadrons. More about this in quantitative terms will be discussed later in this article.

**Table V**

| Baryons (light sector) | | | | | | |
|---|---|---|---|---|---|---|
| baryon | spin $\pm$1/2 | symb | spin $\pm$3/2 | symb | Q | mass (MeV/c2) |
| | $(ud)\underline{u}$ | p | $(ud)u$ | $\Delta^{+}$ | 1 | 938/1232 |
| | $(du)\underline{d}$ | n | $(du)d$ | $\Delta^{0}$ | 0 | |
| | | | $(uu)u$ | $\Delta^{++}$ | 2 | |
| | | | $(dd)d$ | $\Delta^{-}$ | 1 | |
| | $(ud)\underline{s}$ | $\Lambda^{0}$ | | | 0 | 1115/ |
| $qqq$ | $(us)\underline{u}$ | $\Sigma^{+}$ | $(us)u$ | $\Sigma^{*+}$ | 1 | 1170/1385 |
| | $(ds)\underline{d}$ | $\Sigma^{-}$ | $(ds)d$ | $\Sigma^{*-}$ | -1 | |
| | $(us)\underline{d}$ | $\Sigma^{0}$ | $(us)d$ | $\Sigma^{*0}$ | 0 | |
| | $(s\underline{s})u$ | $\Xi^{+}$ | $(ss)u$ | $\Xi^{*+}$ | 1 | 1314/1535 |
| | $(s\underline{s})d$ | $\Xi^{-}$ | $(ss)d$ | $\Xi^{*-}$ | -1 | |
| | $(s\underline{s})s$ | x | $(ss)s$ | $\Omega^{-1}$ | -1 | n.a. /1672 |

As noted before, whereas in the Standard Model the emergence of quarks heavier than the archetype $u/d$ has been accepted by defining new elementary particles, it will be demonstrated now that such particles are a theoretical consequence of the Dirac quark as a polarisable particle. If so, the heavier quarks will no longer be elementary. From inspection of the basic meson and the basic baryon in figure 2, respectively figure 3, it will be clear that those stable structures show a two-quark (an)harmonic oscillator, respectively a three-quark (an)harmonic oscillator. Both structures can be analyzed by a one-body equivalent. Obviously, the meson is easier to handle than the baryon, albeit that the meson has to be analyzed in its center-of-mass frame and subsequent relativistic correction. In that respect the baryon is different. However, a three-body problem is notoriously difficult. One might oppose that these (an)harmonic oscillator structures are oversimplifications of the actual problem, because they suggest that the behavior of a quark can be captured in a non-relativistic Schrödinger-type wave function, while actually a relativistic Dirac-type wave function is required. But let us see where this road takes us.

## 5. The meson

Conceiving the pion as a structure in which a quark couples to the field of the antiquark with the generic quantum mechanical coupling factor $g$ (it will turn out later that its value can be exchanged with $\Phi_0$ under invariance of the product $g\Phi_0$), the pion can be modeled as the one-body equivalent of a two-body oscillator, described by the equation for its wave function $\psi$ ,

$$-\frac{\hbar^2}{2m_m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+\{U(d+x)+U(d-x)\}\psi = E\psi\,;\;\; U(x)=g\Phi(x)\,, \tag{28}$$

in which $\Phi(x)$ is the quark's scalar field as derived before and eventually expressed by (18), $2d$ the quark spacing, $m_m$ is the reduced mass that embodies the two massive contributions from the constituting quarks, $V(x)=U(d+x)+U(d-x)$ its potential energy, and $E$ the generic energy constant, which is subject to quantization.

It will be clear from (28) that the potential energy $V(x)$ can be expanded as,

$$V(x)=U(d+x)+U(d-x)=g\Phi_0(k_0+k_2\lambda^2x^2+....)\,, \tag{29}$$

in which $k_0$ and $k_2$ are dimensionless coefficients that depend on the spacing $2d$ between the quarks.

Note that the effective mass $m_m$ of the two quarks is not necessarily the same as the constituent mass that results from an a-posteriori assignment from the non-observable rest mass of the pion calculated from the observable decay products. The constituent mass is mainly a result of the ground state energy of the oscillator, which is taken up from the field. Furthermore, it has to be kept in mind that this model holds in the center of mass frame, so that a lab frame interpretation will need a relativistic correction. To facilitate the analysis, (28) is normalized as,

$$-\alpha_0\frac{\mathrm{d}^2\psi}{\mathrm{d}x'^2}+V'(x')\psi = E'\psi\,, \tag{30}$$

in which $\alpha_0=\dfrac{\lambda^2\hbar^2}{2m_m g\Phi_0}$, $x'=x\lambda$, $d'=d\lambda$, $E'=\dfrac{E}{g\Phi_0}$, $U'(x')=\dfrac{U(\lambda x)}{g\Phi_0}$ and

$$V'(x')=U'(d'+x')+U'(d'-x')=k_0+k_2x'^2+.......$$

In previous work [[17, eq. (C2)] it has been proven that,

$$\alpha_0=\frac{k_0^2}{2k_2}\,. \tag{31}$$

Normalized quantities in this text will be indicated by a "prime" (').The coefficients $k_0(d')$ and $k_2(d')$ can be straightforwardly calculated from (30) and (25) as,

$$k_0=2\Big(\frac{\exp(-2d')}{d'^2}-\frac{\exp(-d')}{d'}\Big)$$

$$k_2=\frac{\exp(-2d')}{d'^4}(6+4d'^2+8d')-\frac{\exp(-d')}{d'^2}\Big(2+d'+\frac{2}{d'}\Big)\,. \tag{32}$$

The two quarks in the meson settle in a state of minimum energy, at a spacing $2\lambda d = 2d'_{\min}$, such that [17,18],

$$d'_{\min} = \lambda d = 0.853;\ k_0 = -1/2 \text{ and } k_2 = 2.36. \tag{33}$$

Note: the field format (27) has been preferred above the indistinguishable field format (17,18) because (27) is a two-parameter format, while (25) is a three-parameter one.

The archetype, the pion, is the two-quark oscillator in its ground state. The first excitation state transforms a pion into a kaon. The mass ratio between the two is the same as the mass ratio of the normalized energy constants $E' - k_0$. This is not trivial and it reflects the basic theorem of the theory. This theorem states that the energy wells of the two quarks are not massive. Instead, the mass attribute of two-quark junctions (mesons) and three-quark junctions (baryons) is made up by the vibration energy as expressed by the energy state of the quantum mechanical oscillator that they build. The distribution of this mass over constituent quarks is a consequence of this mechanism. Unfortunately, an analytical calculation of the $E' - k_0$ ratio of kaons over pions, is only possible for the quadratic approximation of the series expansion of the potential energy $V'(z')$. A more accurate calculation requires a numerical approach. A procedure to do so has been documented in [17, Appendix C]. It shows that some simple lines of code in Wolfram's *Mathematica* [19] may do the job. The numerically calculated ratio of the energy constants appears to be 3.57 instead of 3 as it would have been in the harmonic case. The result explains the excitation of the 137 MeV/$c^2$ pion mass to the 490 MeV/$c^2$ mass of the pseudoscalar kaon. This result gives a substantial support for the viability of the theory as will be further developed in this article. This result also gives rise to the question if other mesons can be regarded as a result from enhanced excitation. Table VI gives a survey of the calculated ratios for higher excitation ratios. It gives the pseudoscalar $\eta'$ meson as a candidate from second level excitation. The table gives no candidate for third level excitation. As shown in [20], the corresponding level of energy would imply a meson state with a positive value for the binding energy (as is reflected in the value of $k_0$), which prevents a sustainable quasi-stable configuration.

**Table VI: meson excitations**

| | | | |
|---|---|---|---|
| Bottom level | $E'_{bind}$ = - 1/2 | mass ratio | mass in MeV/$c^2$ |
| Ground state | $E'_0 - E'_{bind}$ = 0.84 | 1 | 137<br>(pion = 135-140) |
| First excitation | $E'_1 - E'_{bind}$ = 3.00 | 3.57 | 489<br>(kaon = 494-498) |
| Second excitation | $E'_2 - E'_{bind}$ = 6.06 | 7.21 | 988<br>($\eta'$= 958) |
| Third excitation | $E'_3 - E'_{bind}$ = 9.94 | 11.83 | ??? |

## 6. The quark-scaling theorem and its impact

The wave function of the simple pion model as shown in (30) is Schrodinger's wave equation, which in fact is the non-relativistic approximation of Dirac's covariant wave equation. As is well known, Dirac adopted the Einsteinean energy formula as a starting point. He might have chosen Einstein's geodesic equation instead. There is no reason why the momenta in the geodesic equation would not allow the same momentum-wave function transformation as in the energy equation. But why doing so? The consideration is that the geodesic equation may give additional results on top of those from the energy equation. The reason is that it contains an additional symmetry: apart from energy conservation, it complies momentum conservation. As will be shown in this paragraph, exploitation of this symmetry will reveal an interesting theorem.

Let us consider the quantum mechanical wave equation (30) once more in its denormalized format,

$$-\frac{\hbar^2}{2m_m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + g\Phi_0\{k_0 + k_2\lambda^2 x^2 + \ldots.\}\psi = E\psi. \tag{34}$$

This represents an anharmonic quantum mechanical oscillator characterized by quantum steps $\hbar\omega$ related with the effective mass $m_m$, such that

$$\frac{1}{2}m_m\omega^2 = g\Phi_0 k_2\lambda^2 \rightarrow \frac{m'_m(\hbar\omega)^2}{(\hbar c)^2} = 2g\Phi_0 k_2\lambda^2. \tag{35}$$

Conventionally, $m_m$ represents is the central mass of the oscillator. In the relativistic model described in chapter 4, it does not represent the individual masses of the two bodies, but it is an equivalent mass that captures the energy of the field. As usual, $\omega$ is related with the vibration energy $E_n = (n+1/2)\hbar\omega$. Considering that the pion decays into a fermion via the weak interaction boson, it makes sense to equate the boson $\hbar\omega$ with the weak interaction boson. Hence,

$$\hbar\omega = \hbar\omega_W. \tag{36}$$

It also implies that the actual bond between the quark and the antiquark in a meson is sustained by the weak interaction boson. Hence, the spacing $2d\lambda = 2d'_{\min}$ is expected about equal to a half wave length of the weak interaction boson $\hbar\omega_W$. Hence,

$$\lambda = \frac{2(\hbar\omega_W)d'_{\min}}{\alpha\pi(\hbar c)}, \tag{37}$$

in which $\alpha$ is a dimensionless correction factor of order 1. From (35)-(37), we have

$$\Phi_0 = \frac{m'_m}{8gk_2}\frac{(\alpha\pi)^2}{d'^2_{\min}}. \tag{38}$$

At this point, I would like to invoke a particular relationship from previously documented work [18]. In this work it has been shown that the 2D quantum mechanical wave equation as shown in (34) can equally well be derived from Dirac's equation as commonly derived from the Einsteinean energy relationship, as well as derived from Einstein's geodesic equation. The equivalence of the two approaches applied to the anharmonic quantum mechanical oscillator has revealed the relationship (see eq.(24) of [18]),

$$g\Phi_0 = \frac{k_2\lambda^2}{2k_0^2}\frac{(\hbar c)^2}{m'_m}. \tag{39}$$

From (35) and (39), we have,

$$\hbar\omega_W = 2|k_0|g\Phi_0 = g\Phi_0. \tag{40}$$

Hence, from (37) and (40),

$$\frac{g\Phi_0}{\lambda} = \frac{\alpha\pi(\hbar c)}{2d'_{\min}}. \tag{41}$$

in which $k_0 = 1/2$ and $d'_{\min} = 0.853$ as shown in (33). This ratio holds for all quarks. It means that the strength $\Phi_0$ as well as the range $\lambda^{-1}$ of the potential field may be different for different quark flavors under invariance of the of the $\Phi_0/\lambda$ ratio. From (38 and (41), we have,

$$m'_m = \frac{8k_2 d'^2_{\min}}{(\alpha\pi)^2} g\Phi_0 = \frac{4k_2(\hbar\omega_W)}{k_0^2(\alpha\pi)^2} d'^2_{\min}, \tag{42}$$

It is tempting to suppose that $m'_m$ is the center-of-mass equivalent of the pion's rest mass. Unfortunately, as will be shown later, this rest mass is highly influenced by the nuclear spin interaction. In an elaboration of this result toward an expression of the Gravitational Constant $G$ in terms of quantum mechanical quantities, the unknown dimensionless constant $\alpha$ of order 1 has been assessed as $\alpha \approx 0.69$, [18]. In the next paragraph, its value will be discussed in a different context, which nevertheless will give the same result.

Note that $\Phi_0$ is a quantity that expresses the "strength" of the constituting quark next to the quantity $\lambda^{-1}$ that expresses the spatial range of its strength. Their product is constrained by the invariant (41). This identifies the scaling theorem of quarks. Different quark flavors may have different values for their strength and their spatial range, but always under the constraint of (41). Its relevance goes beyond pions. As to be discussed later in more detail, other mesons have a different spacing between the two quarks. Hence, (41) expresses the mass of these mesons as a function of this spacing $2d'_0$. Figure 4 shows the mass of the mesons as a function of the spacing parameter $d'_0$ between the quarks. The upper curve representing the mass is determined by $k_2(d'_0)$. The lower curve is determined by $k_0(d'_0)$ and represents the binding energy between the quarks.

In this calculation, the electromagnetic interactions have been ignored, because their influence is considered to be of second order as compared to the nuclear interaction. More on the electromagnetic interaction will be subject of chapter 10. Interestingly, the kaon energy, shown in Table VI does not only correspond with the energy of a pion in its state of first excitation, but also with the ground state energy of a quark junction at smaller spacing, thereby composing the kaon as a $u\bar{s}$ or a $\bar{u}s$ bond, composed by the $u$ quark next to the heavier $s$ quark, such as illustrated in figure 4. The lower curve in the figure, representing the binding energy, should remain negative to allow a stable bond. Hence, the light sector $(u,d,s)$ stops after second excitation from the ground state. However, the $\varphi$ state $(s,\bar{s})$ is a new symmetrical state, from which new excitations may arise, and so on.

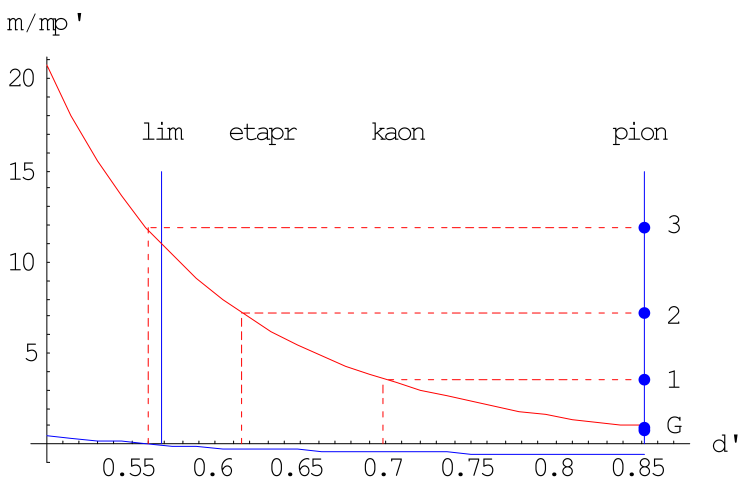


**Fig. 4.** The light sector limit. The graph shows the increase of the massive energy of a quark/antiquark pair relative to the pion state as a function of the quark spacing. Two excitation levels beyond the pion's ground state are converted into the ground state of, respectively, the kaon and the $\eta'$, thereby producing the $(u,s)-$ quark family. Third level excitation is prevented by the loss of binding energy (lower curve).

As noted before, the scaling theorem and the quark description as a pseudo tachyon are not part of canonic particle physics theory. In my previous studies it has been shown that the scaling theorem can be successfully applied for calculating the mass spectrum of mesons and baryons [20].

## 7. The Higgs boson

In the preceding chapters it has been demonstrated that the meson's mass spectrum can be explained from the quark's far field as defined in (28), supplemented by a near field from a dipole moment. The far field is a scalar field obtained from the steady state solution of a Proca-type wave equation with the format

$$\frac{1}{c^2}\frac{\partial^2 r\Phi}{\partial t^2}-\frac{\partial^2}{\partial r^2}r\Phi+\lambda^2 r\Phi=\rho_H(r,t)\,, \tag{43}$$

in which $\rho_H(r,t)$ is a Dirac-type pointlike source that can be expressed as,

$$\rho_H(r,t)=4\pi\lambda\Phi_0\delta(r)H(t)\,, \tag{44}$$

in which $H(t)$ is Heaviside's step function. Its solution is given by [21],

$$r\Phi(r,t)\leftrightarrow\frac{\Phi_0}{\lambda}\frac{1}{s}\exp[-(\lambda r\sqrt{s^2/(\lambda c)^2+1})]\,. \tag{45}$$

If, under violence of particle collisions, the equilibrium between the quarks is broken, the far field bosons will show up in decay channels of pairs of gamma photons, W-bosons or Z-bosons, which will manifest themselves into a decay path of fermions. Momenta and energies of these fermions can be measured and can be traced back to numerical values for the energy of a nuclear boson pair. So, ultimately, the Higgs field will show up as two quantum fields, instead of the single one that is expected by the Standard Model. The massive energy of the far field part, if interpreted as a single boson, would therefore be assigned as,

$$m'_H\approx 2\lambda(\hbar c)\,. \tag{46}$$

Subsequent application of (37) on this gives,

$$m'_H=\frac{4d'_{\min}m'_W}{\alpha\pi}\rightarrow\alpha=\frac{4d'_{\min}}{\pi}\frac{m'_W}{m'_H}\,, \tag{47}$$

which, under consideration of $m'_W$ = 80.4 GeV and $m'_H$ = 127 GeV for the Higgs boson just gives $\alpha\approx$ 0.69 quoted before. One might wonder about the factor 2. The Standard Model value $m'_H$ of the Higgs boson in natural units documented in [1] gives, under consideration of (26),

$$m'_H=\mu_H\sqrt{2}\approx 2\lambda\rightarrow m'_H=2\lambda(\hbar c)\,. \tag{48}$$

This is just the same as (46). The result gives a strong support to the viability as developed in this article, including its compatibility with the Standard Model. However, unlike as in the Standard Model, the numerical value is now established by theory.

## 8. The Z boson

While the meson model shown in figure 2 shows two sets of dipole moments, we have only considered so far the role of the polarisable dipoles. As a consequence of the nuclear spins, one may expect a nuclear equivalent of the magnetic moment of an electron as well. Such a dipole moment shows up in Dirac's four-component wave equation. Let us try to calculate its influence on the model so far developed.

Similarly as the magnetic moment of the electron in its orbit around a proton or positron interacts with the magnetic moment of the proton or positron, the nuclear moments of the quark interact. For its analysis, I'll assume Griffith's model as a starting point. In Griffith's model, the interaction energy $U_{12}$ between an electron and the proton nucleus of the hydrogen atom amounts to [22],

$$U_{12} = \mu_0 \frac{\gamma_{g1}\gamma_{g2}e^2}{6m_1m_2}\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{\pi d_0^3} = \frac{4}{6c^2}\frac{e^2}{4\pi\varepsilon_0}\frac{\gamma_{g1}\gamma_{g2}}{m_1m_2}\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{d_0^3}, \tag{49}$$

in which $\mu_0$ and $\varepsilon_0$ are the Maxwellian permeability constants, $\gamma$'s the gyromagnetic ratios, $m$'s the rest masses and in which $d_0$ is radius of the orbit. To establish the nuclear equivalent of the electromagnetic potential, the far field force $F_F$ evoked by a quark is compared with the electromagnetic force $F_e$. Generally, under consideration of (27),

$$F_e = -e\frac{\partial}{\partial r}\frac{e}{4\pi\varepsilon_0 r} \quad \text{and} \quad F_F = -g\frac{\partial}{\partial r}\Phi_0\frac{\exp(-\lambda r)}{\lambda r}. \tag{50}$$

There is no reason why these forces would be the same. What is clear, however, is, that $g\Phi_0/\lambda$ plays a similar role as $e^2/(4\pi\varepsilon_0)$, i.e.,

$$\frac{e^2}{4\pi\varepsilon_0} \leftrightarrow \frac{g\Phi_0}{\lambda}. \tag{51}$$

Hence, from (49) and (41) we have for two identical particles $m_{qu}$,

$$U_{12} = \frac{1}{c^2}\frac{4}{6}\frac{q_e^2}{4\pi\varepsilon_0}\frac{\gamma^2}{m_{qu}^2}\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{d_0^3} \to \frac{1}{c^2}\frac{2}{3}\frac{g\Phi_0}{\lambda}\frac{\gamma^2}{m_{qu}^2}\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{d_0^3} = \frac{1}{c^2}\frac{2}{3}\left(\frac{\pi\hbar c}{\sqrt{2}d'_{\min}}\right)\frac{\gamma^2}{m_{qu}^2}\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{d_0^3} \tag{52}$$

The spins will align themselves in parallel or in anti-parallel, which gives, respectively,

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = \frac{\hbar^2}{4} \quad \text{and} \quad \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = -\frac{3\hbar^2}{4} \tag{53}$$

Hence, the energy difference between the parallel spin condition and the antiparallel condition is given by

$$\Delta E = \frac{1}{3c^2}\left(\frac{\pi\hbar c}{\sqrt{2}d'_{\min}}\right)\frac{\gamma^2}{m_{qu}^2}\frac{\hbar^2}{d_0^3}\,. \tag{54}$$

For the positronium, we have from (52)

$$\Delta E = \frac{1}{c^2}\frac{4}{6}\frac{q_e^2}{4\pi\varepsilon_0}\frac{\gamma^2}{m^2}\frac{1}{d_0^3} = \frac{2\gamma^2}{3m^2c^2}\frac{\alpha_{em}\hbar c}{d_0^3}, \tag{55}$$

in which $\alpha_{em}$ is the electromagnetic fine structure constant given in the relationship $q_e^2 = 4\pi\varepsilon_0\alpha_{em}$, and in which the Bohr radius $d_0$ is given by,

$$d_0 = \frac{\hbar}{\alpha_{em}(m/2)c}\,. \tag{56}$$

Hence, from (55) and (56),

$$\Delta E = \frac{1}{3}\alpha_{em}^4 mc^2\,. \tag{57}$$

Actually, the difference is 7/4 times larger. This is due to an additional amount of energy as a consequence of the recoil of the bond in the higher state of energy. The correction factor can be found from the positronium case. Including the recoiling influence (beyond the scope of this article), the actual amount is [23,24],

$$\Delta E = \frac{1}{3}\alpha_{em}^4 mc^2 + \frac{1}{4}\alpha_{em}^4 mc^2 = \frac{7}{4}\left(\frac{1}{3}\alpha_{em}^4 mc^2\right). \tag{58}$$

Let us proceed by taking the recoil correction into account. Hence, from (54) and (58),

$$\Delta E = \frac{7}{12c^2}\left(\frac{\pi\hbar c}{\sqrt{2}d'_{\min}}\right)\frac{\gamma^2}{m_{qu}^2}\frac{\hbar^2}{d_0^3}\,. \tag{59}$$

Using the center-of-mass frame value of $\lambda$ as shown in (37), gives, under consideration that $\gamma = 2$ and, as before, $\alpha \approx 0.69$,

$$\Delta E = \frac{7}{12c^2}\left(\frac{\alpha\pi\hbar c}{d'_{\min}}\right)\frac{4\gamma^2}{m_{qu}^2}\frac{\hbar^2}{d'^3_{\min}}\left(\frac{2d'_{\min}}{\alpha\pi(\hbar c)}\hbar\omega_W\right)^3\frac{d'^3_{\min}}{d'^3_0} = \frac{7}{12}\frac{32\gamma^2}{(\alpha\pi)^2}\frac{(\hbar\omega_W)^3}{m'^2_{qu}} = 15.9\frac{(\hbar\omega_W)^3}{m'^2_{qu}}\,. \tag{60}$$

The bond between the two quarks is somewhat different from the bond between the electron and the positron in the positronium. All we know about the quarks is that the collapse of the lowest state of energy of the quark-antiquark bond results in a lab frame rest mass $m_\pi$ with massive energy $m'_\pi = m_\pi c^2$. Supposing that $m'_\pi$ is the lab frame value of the weak interaction boson $\hbar\omega_W$ and taking into consideration that the mass $m$ of a two-body oscillator is the reduced mass from two contributions $2m$, we have from (60),

$$\Delta E = 15.9\frac{m_\pi'^3}{(2m_\pi')^2} = 3.97m_\pi'. \tag{61}$$

This energy is the energy difference between the pions $m_\pi'$ and the rho mesons $m_\rho'$. These energies can be expressed in terms of the mass $m_u$ of a constituting quark , respectively, as,

$$m_\pi' = 2m_u' - 3Am_u' \quad \text{and} \quad m_\rho' = 2m_u' + Am_u'. \tag{62}$$

Hence,

$$\Delta E = m_\rho' - m_\pi' = 4Am_u' \to \Delta E = \frac{4A}{2-3A}m_\pi'. \tag{63}$$

Equating (61) and (63) gives, $A \approx 0.5$. We may go a step further by invoking the mass relationships due to the nuclear spin interactions as derived in Griffith's textbook in conjunction with the theoretically established ratio $3.57$ of the kaon mass over the pion mass, listed in TableIV . Taking the rest mass energy of the pion $m_\pi' = 140$ MeV as a reference, we may calculate the constituent masses $m_u$ and $m_s$ of the $u/d$ quarks, repectively, the strange quark $s$ from the following set of equations,

$$2m_u' - 3A_m' = m_\pi' = 140 \text{ MeV}. \tag{64a}$$

$$m_K' = m_{qu}' + m_s' - 3\frac{m_u'^2}{m_u' m_s'} A_m' = 3.57\, m_\pi' = 489 \text{ MeV}. \tag{64b}$$

$$A_m' = 0.51 m_\pi' \tag{64c}$$

Solving this set for $m_u'$ and $m_s'$ reveals $m_u'$ = 300 MeV and $m_s'$ = 478 MeV.

This means that the constituent masses of the quarks can be theoretically derived from a single reference for which we have adopted the constituent mass of the archetype meson. This mass of the constituent meson is the lab frame value of the weak boson interaction boson that binds the quark in the archetype meson. This implies that there is no reason to consider the quark flavors as elementary. Note the difference with the calculation in Griffiths' textbook, which is purely empirically based and denoted as "shaky", but nevertheless rather accurate. The quark model based upon the polarisable dipole moment of Dirac's third particle, has given it now a clear theoretical basis.

There is somewhat more. Because the spin spin interaction is a boson phenomenon next to to the weak interaction, one may expect that, while the weak interaction manifests itself as the weak interaction boson, the spin spin interaction will manifest itself as a boson as well. Apparently, whereas the weak interaction boson has a lab frame equivalent in terms of the pion's rest mass, the center-of-mass equivalent of the spin-spin mass difference $Am_u'$ is a boson with an energy equivalent to the amount of,

$$\hbar\omega_A = \frac{775-140}{4}.\frac{80.4}{140} = 91.16 \text{ GeV}. \tag{65}$$

This is just the value of the Z-boson. It is fair to conclude that the Z-boson is the manifestation of the spin spin interaction. Note that, while in the Standard Model the value of the Z boson is empirically established, the Z-boson identified as spin spin interaction boson is actually determined by theory.

## 9. Relationship with the Standard Model

The given description of the bond between the $u$ quark and the $d$ quark has revealed the existence of two different binding forces that each can be modeled in terms of force interacting particles. The interacting force due to the polarisable dipole moment of a quark in a scalar potential field has been identified as the $W$ boson and the interaction force due to the nuclear spin interaction has been identified as the $Z$ boson. Let us now take a more abstract point of view, in which no knowledge is available about the actual physical mechanism of the nuclear force. This brings us to the basic question in particle physics, which, in words, is the simple one: how to describe the field of a quark in an ambient field of nuclear forces? The recipe is trying to find a covariant equivalent for the quark's field in an ambient field from the quark's field in free space. Because in the context of the Standard Model the quark has not been recognized as polarisable in a scalar field , physical knowledge of the nuclear forces could not been taken into account. While for electromagnetic interactions the gauge needed for covariance could be understood from a physical mechanism, the gauge for nuclear interactions had to be conceived from an abstract point of view. Its description might reveal the relationship between the structural view developed in this article with the gauge based view adopted in the Standard Model. This is the issue to be discussed in this paragraph.

The interaction model between two quarks shown in the preceding paragraph is based upon spatial field descriptions. Such spatial descriptions are uncommon in the canonic field descriptions of the Standard model, in which fields are exclusively described in functional field parameters. It is instructive viewing the field of a nuclear particle as the mapping of its momenta $p_\mu$ on the amplitudes $\Psi_\mu$ of the four components of the solution of Dirac's equation, i.e., as

$$\{p_0, p_1, p_2, p_3\} \rightarrow \{\Psi_0, \Psi_1 \Psi_2, \Psi_3\}. \tag{66}$$

This mapping is visualized in figure 5 for 1+2 dimensionality.

The left-hand part is a geometric interpretation of Einstein's energy law in Hawking metric $(+,+,+,+)$ for $(\mathrm{i}ct, x, y, z)$, $\mathrm{i} = \sqrt{-1}$. The well-known Einsteinean energy expression of a generic free moving particle with rest mass $m_0$ is given as,

$$E_W = \sqrt{(m_0 c^2)^2 + (c|\mathbf{p}|)^2}\,, \tag{67}$$

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In which $\mathbf{p}$ is the three-vector momentum ($\mathrm{d}s/\mathrm{d}t$, not be confused with the fourvector momentum $p$). Under adoption of the Hawking metric (like, for instance, adopted by Perkins [25]),

$$E_W^2 = -p_{00}^2 c^2 = (m_0 c^2)^2 + c^2 p_1^2 + c^2 p_2^2 + c^2 p_3^2 , \tag{68}$$

which can be normalized as,

$$p_{00}'^2 + p_1'^2 + p_2'^2 + p_3'^2 + 1 = 0 ; \quad p_\mu' = \frac{p_\mu}{m_0 c} . \tag{69}$$

This allows representing the momentum space of a moving particle in free space as a sphere with unit radius.

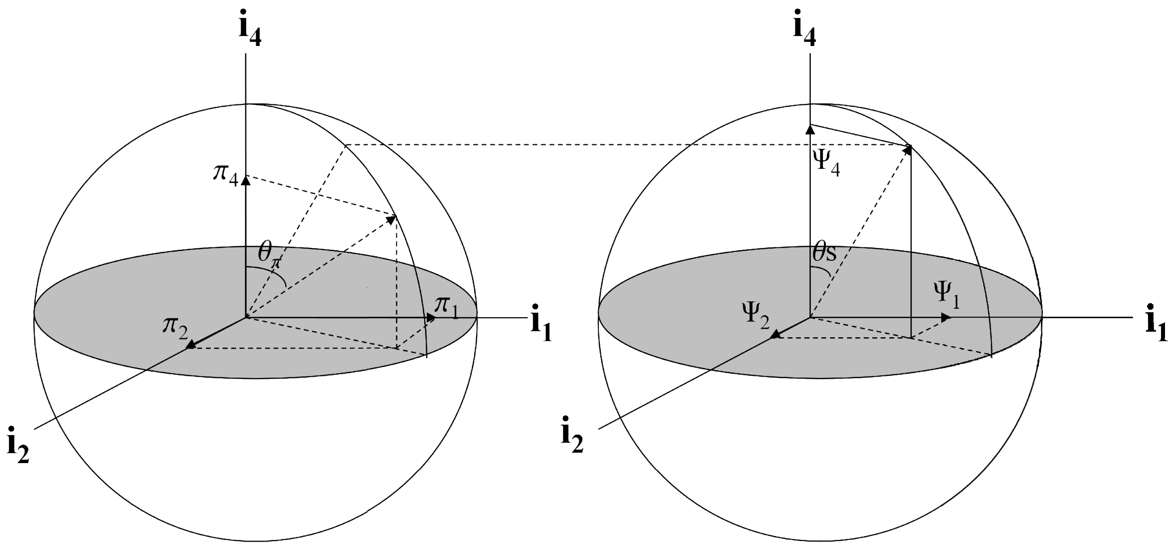


**Fig. 5.** A visual interpretation of the mapping of the particle's momenta into amplitudes of Dirac's wave function solution. Note that this mapping is not 1 to 1.

The right-hand part is a geometric interpretation of the absolute values of the amplitudes $\Psi_\mu$ of the four components of the solution of Dirac's equation. The amplitudes themselves are complex quantities. As a consequence of the semantics of the particle's wave function, these amplitudes can be represented as orthogonal vectors in a unit sphere. Note that this mapping is not one-to-one. In momentum space, the angle $\vartheta_\pi$ between the temporal momentum $p_0$ and the vector sum of the spatial momenta $p_i$ is a global invariant. As long as the particle's energy is not changed by a field of force, the angle remains the same. In spinor space, there is a characteristic angle $\vartheta_s$ between the component $\Psi_0$ associated with the temporal momentum and the vector sum of the components $\Psi_i$ associated with the spatial momenta. Although the mapping is not one-to one, the angle $\vartheta_s$ is globally invariant. Under influence of forces on the momenta, the angle $\vartheta_\pi$ will change, while the radius of the momentum space will remain the same owing to the normalization. The angle in the spinor space will change as well and the radius of the spinor space will remain the same because of the wave function semantics. These angles play a role in the modification of Dirac's free space equation into a covariant one. By definition, the covariant equation, valid for particles moving in a field of forces, has the same format as the free space equation after redefining the normal differential operators into covariant ones, i.e. $\partial_\mu \to D_\mu$. The prescription how to do it, is the modification of a global invariant quantity into a local invariant one. By modifying the global invariance of the Lorentz transform into a local invariant one, Einstein has been able to derive the transformation rule for the covariant derivatives that modified his equations of Special Relativity in free space into covariant equivalents for his equations of General Relativity. Paul Dirac's prescription for making his equation (1) covariant in a conservative field of forces $\boldsymbol{A}(A_0,\mathbf{A})$,

$$p'_\mu \to p'_\mu + gA'_\mu \quad \text{and} \quad p'_\mu \to \hat{p}_\mu\psi + gA'_\mu\psi\,;\;\; \hat{p}'_\mu = \frac{1}{m_0 c}\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x_\mu}, \tag{70}$$

can be interpreted as the modification of the global invariance of $\vartheta_\pi$ and $\vartheta_s$ into local invariant ones, because (70) represents, as we shall see below, just infinitesimal rotations in, respectively, momentum space and spinor space. Effectively, these rotations takes place in 2D space, as long as a single particle is involved.

It is instructive to consider the particle's antiparticle in this picture. The momentum amplitude of the antiparticle has the same value as the amplitude of the vector sum of the spatial momenta of the particle. And, in spite of the fact that the mapping from momenta to the values of the spinor components is non one-to-one, the absolute value of the amplitude of the spinor component associated with the temporal momentum is equal to the value of the vector sum in spinor space of the amplitudes of the spinor components associated with the spatial momenta of the antiparticle.

This picture allows to represent the particle-antiparticle bond as a 2 x 2 matrix $\Psi_{pa}$,

$$\Psi_{pa} = \begin{bmatrix} \Psi_{1t} & \Psi_{1s} \\ \Psi_{2t} & \Psi_{2s} \end{bmatrix}, \tag{71}$$

in which $\Psi_{it}$ represent the components associated with the temporal momenta and in which $\Psi_{is}$ represent those associated with the spatial momenta. Considering that the solution of Dirac's equation in free space, like, for instance, shown on page 220 of Griffith's textbook [1], implies that the absolute value of the amplitude of the temporal part and the absolute value of the sum of the amplitudes of the spatial part of the particle are antisymmetric with respect to those of the antiparticle, allows to conceive the particle-antiparticle bond of an $u$ quark with the antiparticle of a $d$ quark as an SU(2) Lie group. Its matrix has the following properties,

$$\Psi_{1t}\Psi_{1t}^{*}+\Psi_{1s}\Psi_{1s}^{*}=1;\ \Psi_{2t}\Psi_{2t}^{*}+\Psi_{2s}\Psi_{2s}^{*}=1;\ \Psi_{2s}=\Psi_{1t}^{*}\ \text{and}\ \Psi_{2t}=\Psi_{1s}^{*}. \tag{72}$$

Because of this relationship, the matrix (71) is unitary, i.e.

$$\Psi_{pa}\Psi_{pa}^{T}=1, \tag{73}$$

in which $\Psi_{pa}^{T}$ is the transpose conjugate of $\Psi_{pa}$.

Note that the elements of $\Psi_{pa}$ are complex numbers.
A complex *n x n* matrix has $2n^2$ real parameters. The unitary condition on the rows removes $n^2$ of these and an additional one is removed by the constraint of unit determinant. That leaves 3 degrees of freedom for the SU(2) operator. The matrix (71) can then be generically represented as,

$$\begin{bmatrix} e^{i\beta}\cos\alpha & e^{i\gamma}\sin\alpha \\ -e^{-i\gamma}\sin\alpha & e^{-i\beta}\cos\alpha \end{bmatrix}. \tag{74}$$

Obviously, this matrix is unitary, thereby meeting the constraints as imposed by (72). Lie-group theory states that any matrix multiplication with the generic SU(2) format as defined in (74) leaves the object in the group. Hence, the transformation that maintains the desired property of Lagrangian equivalence is given by

$$\Psi\to\Psi\exp(\mathrm{i}\overline{\sigma}\overline{\vartheta})\ \text{with}\ \overline{\vartheta}=\overline{\vartheta}(\vartheta_1,\vartheta_2,\vartheta_3)\ \text{and}\ \overline{\sigma}=\overline{\sigma}(\sigma_1,\sigma_2,\sigma_3), \tag{75}$$

as long as the matrices $\overline{\sigma}=\overline{\sigma}(\sigma_1,\sigma_2,\sigma_3)$ match with (74). The most simple ones are the three Pauli matrices,

$$\sigma_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix};\ \sigma_2=\begin{bmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{bmatrix};\ \sigma_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \tag{76}$$

From (75) should follow $D_\mu\Psi\to D_\mu\Psi\exp(\mathrm{i}\overline{\sigma}\overline{\vartheta})$. This is true if

$$D_\mu\Psi=\partial_\mu\Psi-\mathrm{i}\Psi\partial_\mu(\overline{\sigma}\overline{\vartheta}). \tag{77}$$

By identifying

$$\overline{\sigma}\overline{\vartheta}=\sigma_k\vartheta^k=g_W\sigma_k W^k, \tag{78}$$

in which $g_W$ is a generic dimensionless coupling factor,

we get,

$$D_\mu\Psi=(\partial_\mu-\mathrm{i}g_W\sigma_k W_\mu^k)\Psi;\quad k=1,2,3. \tag{79}$$

Note: the subscript in $g_W$ has been added for distinction from the coupling factor $g$ introduced in (8) in a somewhat different context.

Because $\sigma_k W_k$ are operations in the field domain with a complex number type, $W_k$ cannot be identified as mappings of real valued momenta. Hence, it makes sense to redefine,

$$W_1^+ = W_1 + \mathrm{i}W_2\,;\;\; W_1^- = W_1 - \mathrm{i}W_2\,;\;\; W^0 = W_3\,. \tag{80}$$

From (79) and (80),

$$D_\mu \Psi = \{\partial_\mu - \mathrm{i}g_W(\tau_1 W_\mu^+ + \tau_2 W_\mu^- + \tau_0 W_\mu^0)\}\Psi\,, \tag{81}$$

in which $\tau_k$ now are real valued matrices,

$$\tau_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix};\; \tau_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix};\; \tau_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{82}$$

Note that as yet the $+$ sign and the $-$sign have no electrical meaning here. Note that this covariant derivative derived from SU(2) perspective contains the Pauli elements of Dirac's equation as represented by (1). It includes the influence of the angular momentum. However, particular properties of the field of forces that necessitates a covariant derivative are not yet taken into account. The relevancy within the context of this article is that the recognition of the full-dimensionality of Dirac's equation (which includes the spin phenomenon) in a bond of SU(2) particles, reveals the existence of three interaction forces or, equivalently, the existence of interaction bosons in three modalities. It will be clear that apart from this conclusion, more is needed for proving the viability of a stable bond of SU(2) constituents. So, the next question to be addressed is: "what are the characteristics of the energetic background field that guarantees such a stable bond?". The answer is a simple one: its Lagrangian density should remain locally invariant under substitution of a covariant derivative as specified by (79). One may expect that this condition will require certain properties of the interaction bosons ($W_\mu^+, W_\mu^-, W_\mu^0$). This gives the recipe for defining a covariant derivative, formally dubbed as gauge, in Dirac-type wave equations of particle bonds. The gauge for particle bonds with a wave function (= field) that is subject to the unitary constraint, has been originally generically formulated by Yang and Mills [26].

Whereas the recipe for finding the answer to the problem is clear, finding the solution itself of is not a piece of cake. The GSW (Glashow, Salam, Weinberg) electroweak theory [27,28,29] has given the answer. Not surprisingly, the nuclear background energy field has the format of the Higgs field, such as introduced by (22). Under the influence of such a field, the bosons shown in (74), which are mass less in free space, gain mass. The proof for this can be found in many textbooks and tutorials. Whereas the mass of the bosons $W_\mu^+$ and $W_\mu^-$ is the same, the mass of the third boson is different, because it evolves from a mix of contributions from the nuclear field and the electromagnetic field. It makes the neutral $Z$ boson. Omitting the influence of the electromagnetic field in the GSW theory would make the mass of the $Z$ boson the same as those of $W_\mu^+$ and $W_\mu^-$, which is in conflict with experimental evidence. Inspection of (74) however, makes clear that there is no reason why the coupling factor $g_W$ should be necessarily equal for all three bosons. Accepting this theoretical option would identify the $W_\mu^0$ as a $Z$ boson without being mixed up with electromagnetic energy, while still having a mass different from the $W$ bosons. More on this will be discussed in the next paragraph.

It is fair to conclude that this overview brings the origin of the bosons, as developed in the preceding paragraphs from a structural point of view, in agreement with the origin of the bosons conceived from a pure abstract point of view of gauging and the adoption of a heuristic format for the background field. Importantly and in addition to this, it has to be noted that, whereas the mass ratio of the $W$ bosons over the $Z$ boson in the structural view could be assessed by theory, such as shown in the preceding paragraph, the very same ratio has remained in the GSW theory a value for which an empirical assessment is required. However, by omitting the electromagnetic field in the GSW theory, the unification of the nuclear forces with electromagnetism is now missing. How to include it, will be the subject of the next paragraph.

### 10. Electroweak unification

In the analysis so far presented in this article, the electromagnetic interaction has been regarded as a second order effect. In this chapter, it will be shown that the scaling theorem applied on quarks conceived as pseudo-tachyons is a powerful instrument for calculating the mass difference between the charged pion and the neutral pion. The task to be done is including the electromagnetic interaction into account as an additional force on the nuclear force between the quark and the antiquark. We may combine this additional force with the weak interaction force, implying that each quark feels a repulsive force $F(r)$ from the other quark, such that

$$F(r) = -g\frac{\mathrm{d}\Phi}{\mathrm{d}r} \pm p\frac{e^2}{4\pi\varepsilon_0 r^2}, \tag{83}$$

in which $p$ is a dimensionless factor, which depends on the composition of the pion.

Conventionally, it has been taken for granted that $u$ quarks and $d$ quarks show a different electrical behavior. From empirical evidence it has been concluded that $u$ quarks are charged as $2/3e$ and that $d$ quarks are charged as $-1/3e$. Within the view so far developed in this text, this asymmetry is an unexpected parity violation. As shown in Table III, the asymmetry disappears by hypothesizing a certain charge bias in the baryon configuration. It can be understood from the following two observations. The first one is that in Dirac's theory of the electron theory the origin of the electric charge polarity is supposed being due to the particle/antiparticle state of the electron. The second one is that in the theory of quarks the origin of electric charge polarity is supposed being due to state of isospin (like in the Standard Model) or, equivalently due to the state of the second dipole moment (like in the scope of this article). From these observations, it is fair to suppose that both these states contribute in a similar way to the charge of the particle under consideration. Hence, in the case of mesons, the net contribution from the particle/antiparticle state is zero, whereas it gives a pedestal (bias) in the case of baryons. Accepting that electric charge is a holistic property of the hadron, rather than an individual property of a quark, the isospin/second dipole moment contributions are symmetrical. Considering that the neutral pion does not show electromagnetic interaction, because the spin states in the mixed condition just cancel, and concluding that the electromagnetic interaction increases the strength of the far field, we may write for the latter,

$$\Phi_F(r) = \frac{\Phi_0}{\lambda r}\exp(-\lambda r) + p\frac{e^2}{4\pi\varepsilon_0 r^2}, \tag{84}$$

in which $p = 0$ for the neutral pion and $p = 1$ for the charged pion. Let us rewrite (84) in terms of the electromagnetic fine structure relationship,

$$e^2 = 4\pi\varepsilon_0\hbar c g_e^2, \tag{85}$$

in which $g_e^2$ is the well-known fine structure constant $g_e^2 = \alpha_{em} \approx 1/137$. Hence, from (85) and (84), the interaction force $F(r)$ between the quark and the antiquark, under consideration of equal contributions of charge can be written as,

$$F(r) = -g\frac{\mathrm{d}\Phi}{\mathrm{d}r} + \frac{p}{4}\frac{g_e^2\hbar c}{r^2}. \tag{86}$$

The far field potential $\Phi_F(r)$ can now be written after including the influence of the electric interaction as,

$$\Phi_F(r) = \frac{\Phi_0}{\lambda r}\exp(-\lambda r) + \frac{p}{4}\frac{g_e^2\lambda\hbar c}{\lambda r}. \tag{87}$$

Note that denying a charge difference between $u$ quarks and $d$ quarks implies that mesons don't show an electric dipole moment.

Now, the potential $\Phi(x')$ of the field built up by the quarks felt by the center of mass, expanded along the dipole axis, is built up by the near field $\Phi_N(x')$ from the dipole moment, the far field $\Phi_F(x')$ component of the weak interaction, and the electromagnetic potential $\Phi_{em}(x')$, such that

$$\Phi(x') = \Phi_N(x') - \Phi_F(x') - \Phi_{em}(x') \quad \text{with } x' = \lambda x,$$

$$\Phi_N(x') - \Phi_F(x') = \Phi_0(k_0 + k_2 x'^2 + .....),$$

$$\Phi_{em}(x') = pw\frac{g_e^2}{4g}\frac{(\hbar c)\lambda}{d'}(2+\frac{2}{d'^2}x'^2+.....);\ d'=d\lambda\ . \tag{88}$$

( $w$ is the weighting factor $w \approx 1/0.55$ as introduced in (27) for balancing the far field against the near field)

Note: $\Phi'_{em}$ holds under the assumption of equal charge distribution over the two quarks. The dimensionless factor $w$ is the one introduced in (49) for weighing the far field relative to the near field. As discussed, this factor is close to 2. This factor has not been taken into account in my previous study [17] on the electromagnetic interaction. Eq. (88) can now be written as,

$\Phi(x') = \Phi_0(k'_0 + k'_2)x'^2 + .....)$ , with

$$k'_0 = k_0 + \frac{1}{2}pw\frac{g_e^2}{g}\frac{(\hbar c)\lambda}{d'\Phi_0};\quad k'_2 = k_2 + \frac{1}{2}pw\frac{g_e^2}{g}\frac{(\hbar c)\lambda}{d'^3\Phi_0}\ . \tag{89}$$

Hence, after invoking the invariance $\Phi_0/\lambda$ as expressed by (41),

$$k'_0(d') = k_0(d') + pw\frac{g_e^2 d'_{\min}}{\alpha\pi d'};\ k'_2(d') = k_2(d') + pw\frac{g_e^2 d'_{\min}}{\alpha\pi d'^3}\ . \tag{90}$$

Whereas In the case of neutral pions ( $p=0$ ), the constants $k_0$ and $k_2$ are not affected by electromagnetic interaction, implying $k'_0(d') = k_0(d'_{\min})$ and $k'_2(d') = k_2(d'_{\min})$ , it is not the case for charged pions. As a consequence of the electric charge the spacing between the two quarks in equilibrium is shifted apart by an amount of say $2\delta$, such that $d'_0 = d'_{\min} + \delta$. The potential is minimum if $k'_0$ is minimum. Hence, the minimum values for $k'_0$ and the spacing values for minimum potential are slightly different from $k_0(d'_{\min}) = 1/2$ and $d'_{\min} = 0.852$ as they were calculated without the electromagnetic interaction. As a consequence of the shift, the mass formula (42) shows,

$$\frac{m'_{\pi\pm}}{m'_{\pi 0}} = \frac{k'_2}{k_2}\frac{k_0^2}{k'^2_0}\ . \tag{91}$$

Hence, from (91),

$$\Delta m'_\pi = m'_{\pi\pm} - m'_{\pi 0} = m'_{\pi 0}(\frac{m'_{\pi\pm}}{m'_{\pi 0}} - 1) = m'_{\pi 0}(\frac{k'_2}{k_2}\frac{k_0^2}{k'^2_0} - 1)\ . \tag{92}$$

As long as $k_2(d'_{\min} + \delta) \approx k_2(d'_{\min})$ and $k_0(d'_{\min} + \delta) \approx k_0(d'_{\min})$ , we have from (90) and (92),

$$\frac{m'_{\pi\pm}}{m'_{\pi 0}} = \frac{k'_2}{k_2}\frac{k_0^2}{k'^2_0} \approx (1+\frac{1}{k_2}\frac{wg_e^2}{\alpha\pi d'^2_{\min}})(1-\frac{2}{k_0}\frac{wg_e^2}{\alpha\pi}) \to \Delta m'_\pi \approx \frac{wg_e^2}{\alpha\pi}(\frac{1}{k_2 d'^2_{\min}} - \frac{2}{k_0})m'_\pi\ . \tag{93}$$

It shows that both the curving parameter $k_2$ of the field potential and the binding force parameter $k_0$ have an influence on the mass difference between a charged pion and a neutral pion. With $w = 1/0.55$, $g_e^2 = 1/137$, $\alpha = 0.69$, $k_2 = 2.36$ and $k_0 = -1/2$ and $m'_\pi = 140$ MeV, it gives $\Delta m'_\pi = 2.92$ MeV. The difference with the empirical value $\Delta m'_\pi = 4.6$ MeV is due to the assumption $k_2(d'_{\min} + \delta) \approx k_2(d'_{\min})$. This assumption is somewhat inaccurate, because the more accurate calculation, documented in [17], gives,

$$k_2(d'_{\min} + \delta) = k_2(d'_{\min}) + \delta \frac{\mathrm{d}}{\mathrm{d}d'} k_2(d'); \quad \delta = \frac{g_e^2}{\alpha\pi} \frac{d'_{\min}}{(d'_{\min}+1)^2}; \quad \frac{\mathrm{d}}{\mathrm{d}d'} k_2(d') = -19.87. \tag{94}$$

Taking this into consideration, we have for (93),

$$\Delta m'_\pi \approx \frac{w g_e^2}{\alpha\pi} \left(\frac{1}{k_2 d'^2_{\min}} + \mathrm{corr} - \frac{2}{k_0}\right) m'_\pi; \ \mathrm{corr} = \frac{d'_{\min}}{k_2 (d'_{\min}+1)^2}. \tag{95}$$

This gives $\Delta m'_\pi = 4.7$ MeV, which is pretty close to experimental evidence. As proven in [17], this sensitivity for the spacing shift on the curving parameter $k_2$ also explains that, whereas charged pions heve a larger mass value than neutral pions, the opposite is true for kaons.

Rather than considering the electromagnetic interaction separately from the nuclear $W/Z$ interaction, one might prefer unifying the interactions in a single covariant derivative. This requires a modification of (81), which had to be modified anyway because of a different coupling factor of $W^0$ bosons to the Higgs field as compared with $W^\pm$ bosons. Moreover the covariant derivative should allow for electromagnetic bosons that are not affected by the Higgs field. In the GSW theory this requirement is met by hypothesising the existence of an additional (mass less) interaction boson $B_\mu$, next to the $W$s, which mixes up with the $W_\mu^0$ boson, such that the interaction of the $W$ bosons and the $B$ boson with the Higgs field produces massive charged bosons $W_\mu^\pm$, a massive neutral boson $Z_\mu$ and a mass less electromagnetic boson $A_\mu$. The components of this theory are, (a) a proper field format for the quark-antiquark junction conceived as an SU(2) group, (b) a proper format for the Higgs field operating on the quark junction and (c) a proper format for the covariant derivative.

The pion spinor as shown in (71) can be conceived as an SU(2) doublet $\Phi$ of two complex fields, symbolically represented as,

$$\Phi = \begin{bmatrix} \Phi^+ \\ \Phi^0 \end{bmatrix}; \quad \Phi^+ = \frac{\varphi_1 + \mathrm{i}\varphi_2}{\sqrt{2}}; \ \Phi^0 = \frac{\varphi_3 + \mathrm{i}\varphi_4}{\sqrt{2}}, \tag{96}$$

The (field) Lagrangian of the Higgs field can be represented as,

$$\mathcal{L} = (\partial_\mu \Phi)^{\mathrm{T}} (\partial^\mu \Phi) + V(\Phi), \text{ with } V(\Phi) = \frac{\mu_H^2}{2} \Phi^{\mathrm{T}}\Phi - \frac{\lambda_H^2}{4} (\Phi^{\mathrm{T}}\Phi)^2,$$

in which,

$$\Phi^{T}\Phi = \begin{bmatrix}\Phi^{+} * & \Phi^{0} *\end{bmatrix}\begin{bmatrix}\Phi^{+} \\ \Phi^{0}\end{bmatrix} = \Phi^{+} * \Phi^{+} + \Phi^{0} * \Phi^{0} = \frac{\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2}{2}. \quad (97)$$

The covariant derivative gets the format as defined before in (88).

$$D_\mu \varphi = \{\partial_\mu - \mathrm{i}g_W(\tau_1 W_\mu^+ + \tau_2 W_\mu^- + \tau_0 W_\mu^0) - \mathrm{i}g_e B_\mu)\}\varphi, \quad (98)$$

Note: whereas at this point a physical motivation for the conception of the SU(2) doublet of two complex fields is given from the pion spinor, such a motivation is lacking in most, if not all, texts on the GSW theory.

Replacing the normal derivatives in the field Lagrangian (90) by the covariant ones as defined in (98) and subsequent elaboration of the fields as variations around the field minimum of the Higgs field (known as the vacuum expectation value), produces new fields in which the bosons $W_\mu^\pm$, have gained mass and in which a massive neutral boson $Z_\mu$ shows up next to a mass less electromagnetic boson $A_\mu$. Whereas in the GSW theory the $Z$ boson is related with electromagnetism, in this article the $Z$ boson has been related with the nuclear spin interaction between quarks. The correspondence and the difference shows up by comparing the covariant derivative of the GSW theory by an alternative one in which electromagnetism is not included, such that

$$D_\mu \varphi = \{\partial_\mu - \mathrm{i}g_W(\tau_1 W_\mu^+ + \tau_2 W_\mu^-) + \mathrm{i}g_Z \tau_0 W_\mu^0)\}\varphi. \quad (99)$$

Note: in most texts on GSW the (unknown) coupling factors $g_W$ and $g_Z$ are written as $g/2$ and $g'/2$. As noted by Klauber [30], this is just by convention, because the factor $1/2$ is sometimes omitted as well.

The potential $V(\Phi)$ is minimum at $\Phi_0$ found by differentiation as

$$\Phi^T \Phi = \frac{\mu_H^2}{\lambda_H^2} = \frac{\Phi_0^2}{2} \to \Phi_0 = \frac{\mu_H}{\lambda_H}\sqrt{2}, \quad (100)$$

corresponding to the vacuum expectation value indentified in (22). By substituting the covariant derivative (99) into the field Lagrangian (97). In principle, one may try to obtain field expressions in terms of $\varphi$, but, like shown in the GSW theory, it is more effective trying to obtain field expressions in terms of variations around the vacuum expectation value (100). How to do so, can be found in many texts [1,30]. Doing so in terms of the extended format (98) or in terms of the "naked" format (99) doesn't make an essential difference. Elaboration shows that the mass less bosons gain mass, such that

$$m_W' = g_W \Phi_0; \; m_Z' = g_Z \Phi_0. \quad (101)$$

Next to these boson masses an additional boson is produced, known as the Higgs boson, which is considered as being the carrier of the background field, to the amount of,

$$m'_H = \mu_H \sqrt{2}. \tag{102}$$

Note that $\lambda_H$ is dimensionless. For $m'_W$ and $m'_H$ it are the same expressions as shown in (40), respectively in (48). While in the context of the GSW theory the numerical value of the mass ratio $m_Z / m_W (= g_W / g_Z)$ has remained a value to be assessed empirically, the numerical value mass ratio in this article has been derived by theory as shown by (65). The same holds for the Higgs boson. Eqs. (46-47) shows a numerical relationship between $m'_W$ and $m'_H$. Note that these results do not reveal numerical values for the coupling constants $g_W$ and $g_Z$, nor for the vacuum expectation value $\Phi_0$. The numerical values are related to $m'_W$ as the reference. However, whereas electromagnetism has been considered in this text so far as a second order add-on effect, it is an integral part of the GSW theory, in which electromagnetism is unified with the weak interaction on a theoretical fundament. Let us return to (98) to show how.

As compared to the "naked" format (99) it contains an additional boson field to account for the electromagnetism that shows up in the SU(2) doublet field. Without an additional mechanism there is no reason why this additional boson field would not gain mass as a result of the same mechanism why the $W$ bosons do. The idea that such a mechanism could exist became a corner stone in the GSW theory. From the straightforward covariant derivative (98) Weinberg constructed a modified one by hypothesizing a mechanism that mixes the (mass less) boson fields $W_\mu^0$ and $B_\mu$ into two other (mass less) bosons fields $A_\mu$ and $Z_\mu$ such that,

$$\begin{bmatrix} A_\mu \\ Z_\mu \end{bmatrix} = \begin{bmatrix} \cos\vartheta_W & \sin\vartheta_W \\ -\sin\vartheta_W & \cos\vartheta_W \end{bmatrix} \begin{bmatrix} B_\mu \\ W_\mu^0 \end{bmatrix}, \tag{103}$$

which is equivalent with

$$\begin{aligned} A_\mu &= B_\mu \cos\vartheta_W + W_\mu^0 \sin\vartheta_W \\ Z_\mu &= -B_\mu \sin\vartheta_W + W_\mu^0 \cos\vartheta_W \ . \end{aligned} \tag{104}$$

Multiplying the upper equation with $\sin\vartheta_W$, the lower one with $\cos\vartheta_W$ and addition gives $W_\mu$. Multiplying the upper one $\cos\vartheta_W$ etc., gives $B_\mu$. Hence,

$$W_\mu^0 = Z_\mu \cos\vartheta_W + A_\mu \sin\vartheta_W \quad \text{and} \quad B_\mu = -Z_\mu \sin\vartheta_W + A_\mu \cos\vartheta_W\,. \tag{105}$$

If the Weinberg angle $\vartheta_W$ has a suitable value, the mass less boson field $Z_\mu$ gains mass, while the mass less boson field $A_\mu$ remains mass less. In this condition the mass ratio $m'_W / m'_Z$ happens to be,

$$\frac{m'_W}{m'_Z} = \cos\vartheta_W \,. \tag{106}$$

The discovery of neutral bosons $m'_Z$ ($\approx$80.4 GeV) with a different mass value from charged bosons $m'_Z$ ($\approx$91.2 GeV) is considered as the viability proof of the GSW theory, in spite of the inability to assess a numerical value for $\vartheta_W$ by theory. It confirms Weinberg's hypothesis that an SU(2) field, like the pion's one, contains a particular physical mechanism that creates electromagnetism. Let us proceed by considering how the covariant derivative (98) is influenced by the created $A_\mu$ field. Let us evaluate (98) under consideration of (105), thereby omitting non- $A_\mu$ containing terms for simplicity reason. Doing so, we have from (98) and (105),

$$\begin{aligned}
&D_\mu\varphi = \{\partial_\mu - \mathrm{i}g_W\tau_0 W^0_\mu - \mathrm{i}g_e B_\mu) + .........\}\varphi = \\
&\{\partial_\mu - \mathrm{i}g_W\tau_0(Z_\mu\cos\vartheta_W + A_\mu\sin\vartheta_W) - \mathrm{i}g_e(-Z_\mu\sin\vartheta_W + A_\mu\cos\vartheta_W) + .......\}\varphi \rightarrow \\
&D_\mu\varphi = \{\partial_\mu - \mathrm{i}(g_w\tau_0 + g_e\frac{\cos\vartheta_W}{\sin\vartheta_w})A_\mu\sin\theta_W + .....\}\varphi.
\end{aligned} \tag{107}$$

This can be written as,

$$D_\mu\varphi = (\partial_\mu - \mathrm{i}q_{\mathrm{u,d}}A_\mu + ....)\varphi\,;, \tag{108a}$$

in which

$$q_{u,d} = (\pm I_z + \frac{Y}{2})(2g_w\sin\vartheta_W) \quad \text{with} \quad I_z = \frac{\tau_0}{2}\,; Y = \frac{g_e}{g_W}\frac{\cos\vartheta_w}{2\sin\vartheta_W}\,. \tag{108b}$$

Because of the eigen value $\tau = 1$, we have conveniently,

$$g_W = \frac{e}{2\sin\vartheta_W}; \quad Y = 1 = \frac{g_e}{g_W}\frac{\cos\vartheta_w}{2\sin\vartheta_W} \rightarrow \frac{g_W}{g_e} = \frac{2\sin\vartheta_W}{\cos\vartheta_W}\,, \tag{109}$$

This relationship is graphically illustrated in figure 6. The expression (108a) for the charge of the $SU(2)$ bond has been shown in Table I as an expression for the charge of the composing quarks with dedicated values for $I_z$ and $Y$. It has to be noted, though, that the GSW theory produces a holistic result for the quark-antiquark bond. That means that the assignment $Y = 1/3$ to $u$ and $d$ quarks is arbitrary, because its influence disappears in the junction of a quark and an antiquark. Considering that the bias $Y$ is a holistic property of the composite particle, there is no particular need to assign asymmetrical charges to the composing quarks. This view is reflected Table VII.

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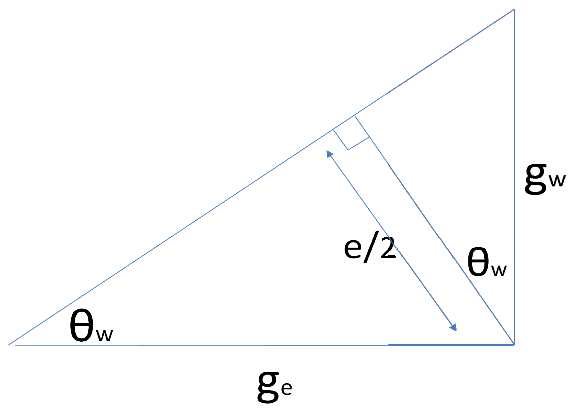


**Fig.6.** Graphical picture of the Weinberg angle.

By substitution of the covariant derivative (107) into (97), the masses of the Higgs boson, the masses of the $W^{+/-}$ bosons and the $Z$ boson are found, under consideration of the relationships (109) respectively as,

$$m'_H = \mu_H \sqrt{2}\,;\; m'_W = (c_0 g_W)\Phi_0\,;\; m'_Z = m'_W / \cos\vartheta_W\,,$$

$$\text{with } \Phi_0 = \frac{\mu_H}{\lambda_H}\sqrt{2} \text{ and } g_W = \frac{e}{\sin\vartheta_{W.}}\,, \tag{110}$$

in which $c_0$ is a dimensionless normalization constant. As noted before, $\lambda_H$ is a dimensionless quantity. The derivation of these relationships can be found in many texts on the electroweak theory.

**Table VII**

| Fermions | $u$ | $\bar{d}$ |
|---|---|---|
| S- spin | 1/2 | 1/2 |
| $m_0$-mass | ? | ? |
| $I_Z$ -weak isospin charge | 1/2 | 1/2 |
| Y-hypercharge | $Y$ | $-Y$ |
| $Q$ | $(\pm I_z + Y)q_e$ | $(\pm I_z - Y)q_e$ |
| $Q_{u\bar{d}} = Q_u + Q_{\bar{d}} = \pm 2I_z$ | $\pm q_e,0$ | |

It is fair to conclude that Weinberg's hypothetical axiomatic mechanism to explain the origin of the $Z$ boson is the same as the spin-spin interaction mechanism as developed in this article from a structural point of view. However, the GSW theory is unable to relate the masses of numerical values of this $Z$ boson and the Higgs boson with the mass value of the $W$ bosons, while the structural theory can. On the other hand, whereas in the structural theory the electric charge of the mesons is just accepted as a physical attribute related to isospin and unified with the weak force in a physical model that enables to calculate its influence on the mass of the quark-antiquark junction, the electroweak theory explains the origin of the electrical charge in a model that unifies the weak interaction with electromagnetism by a mathematical theory.

## 11. Strong interaction

One of the problems left is the unification of the GSW theory with strong interaction. This problem has given rise to the development of the QCD (Quantum Color Dynamics) theory, in which an additional boson is introduced, dubbed as gluon, as the carrier of an additional force next to the electromagnetic force and the weak interaction. This is done by extending the $SU(2)$ model for the quark junction between the $u$ quark and the $\bar{d}$ quark toward the $SU(3)$ model for three-quark junctions (baryons). Before discussing that model, let us consider the decay of the vector-type rho-meson into the pseudo-scalar type pion. As a rule of thumb, it is usually said that strong interaction decay comes first before weak interaction decay takes place. For that reason, the rho-pion decay is considered as a strong interaction mechanism. However, the structural view as developed in this article has revealed that this decay is nothing else but the nuclear spin flip in the spin-spin interaction process. Because we have identified the spin-spin interaction as the physical interpretation of Weinberg's axiomatic mixing process, it is rather questionable if the rho-pion decay needs a description in terms of an additional bosonic carrier. It has been shown in this article so far, that a simplified structural physical model of two-quark junctions has given a comprehensible view on the rather abstract highly-sophisticated mathematical $SU(2)$ electroweak theory. It might well be that such a simple physical model for three-quark junctions may do as well to give a comprehensible view on QCD.

## 12. Baryons

Whereas a meson can be conceived as the one-body equivalent of a two-body harmonic oscillator, a baryon can be conceived as the one-body equivalent of a three-body harmonic oscillator. The one-body equivalent of the three-body quantum mechanical oscillator can be analyzed in terms of pseudo-spherical Smith Whitten coordinates [31]. The Smith-Whitten system of coordinates is six-dimensional. Next to a (hyper)radius $\rho$, the square of which is the sum of the squared spacings between three bodies, there are five angles $\varphi, \vartheta, \alpha, \beta, \gamma$, in which $\varphi$ and $\vartheta$ model the changes of shape of the triangular structure and in which $\alpha, \beta$ and $\gamma$ are the Euler angles. The latter ones define the orientation of the body plane in 3D-space. The planar forces between three identical interacting bodies are not only the cause of dynamic deformations of the equilateral structure, but are also the cause of a Coriolis effect that result in vibra-rotations around the principal axes of inertia of the three-body structure [32]. The application of this approach for baryons has been documented by the author in [33], showing that the wave equation of the quasi-equilateral baryon structure can be formulated as

$$-\alpha_0\{\frac{d^2\psi}{d\rho'^2}+\frac{5}{\rho'}\frac{d\psi}{d\rho'}+\frac{R(m,v,k)}{\rho'^2}\psi\}+V'(\rho')=E'\psi\,,$$

in which $\alpha_0 = \frac{\hbar^2 \lambda^2}{6mg\Phi_0}$; $E' = \frac{E}{3g\Phi_0}$; $V' = \frac{V}{3g\Phi_0}$; $\rho' = \rho\lambda$, and

$$V(\rho') = 3g\Phi_0(k_0 + k_2\rho'^2 + .......) \tag{109}$$

$$R(m, v, k) = 4m + |v-k|(4m + |v-k| + 4)$$

This wave function is the three-body equivalent of the pion's two-body wave equation shown in (57). In the ground state $m = 0$. Hence,

$$R = R(0, v, k) = l(l+4)\,;\; l = |v-k|\,. \tag{110}$$

The radial variable $\rho$ is the already mentioned hyper radius. The potential field is just the threefold of the potential field in the wave equation of the pion. There are three quantum numbers involved. Two of those are left in the ground state, effectively bundled to a single one. The quantum number $k$ allows a visual interpretation, while $v$ is difficult to visualize. The impact of $k$ is shown in figure 7. It illustrates the motion of the center of mass under influence of $k$. Note that this rotation is quite different from a rotation of the triangular frame around the center of mass. It is the center of mass itself that rotates, while the frame does not. Actually, the small motions of the individual quarks are responsible for this motion. As shown in [29], this relatively simple wave function expression allows a pretty accurate calculation of the mass spectrum of baryons. The octet states in the baryon classification are the counter part of the meson pseudoscalar states, the decuplet states are the counter part of the vector mesons. A single integer step in the quantum number $l$, brings the p,n/$\Delta$level to the $\Sigma/\Sigma^*$-level, etc. The results of the mass calculations are shown in the right-hand part of the tables VIII and IX.

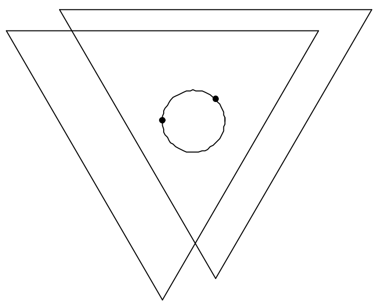

**Fig. 7:** . Physical interpretation of the motion associated with the angular quantum number $k$ .

As before, these tables are constructed by conceiving the $u$ quark and the $d$ quark as the archetype quark in a different state of isospin. The up-state nuclear spin of an $u$ quark (or a $d$ quark) and the down-state of an $u$ quark (or or a $d$ quark) are indicated as, respectively $u$ and $\underline{u}$ (or $d$ and $\underline{d}$ for $d$ quarks).

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**Table VIII: Re-interpretation of the light baryon octet**

| nuclear spin 1/2 (octet) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| baryon | Isospin modes | code | total isospin | bias | charge | symb | mass calc. | mass act. |
| $(uu)\underline{u}$ | $(\uparrow\downarrow)\uparrow$ | $ud\,\underline{u}$ | +1/2 | +1/2 | 1 | p | 934 | 938 |
| | $(\uparrow\downarrow)\downarrow$ | $ud\,\underline{d}$ | -1/2 | +1/2 | 0 | n | 934 | 939 |
| $(uu)\underline{s}$ | $(\uparrow\downarrow)\uparrow$ | $ud\,\underline{s}$ | -1/2 | +1/2 | 0 | $\Lambda^0$ | 1105 | 1115 |
| $(u\underline{u})s$ | $(\uparrow\uparrow)\downarrow$ | $u\underline{u}s$ | +1/2 | +1/2 | 1 | $\Sigma^+$ | 1170 | 1189 |
| | $(\downarrow\downarrow)\downarrow$ | $d\underline{d}s$ | -3/2 | +1/2 | -1 | $\Sigma^-$ | 1170 | 1197 |
| | $(\uparrow\downarrow)\downarrow$ | $u\underline{d}s$ | -1/2 | +1/2 | 0 | $\Sigma^0$ | 1170 | 1192 |
| $(ss)\underline{u}$ | spin conflict | | | | | | | |
| $(s\underline{s})u$ | $(\downarrow\uparrow)\uparrow$ | $s\underline{s}u$ | -1/2 | +1/2 | 0 | $\Xi^0$ | 1314 | 1321 |
| | $(\downarrow\uparrow)\downarrow$ | $s\underline{s}d$ | -3/2 | +1/2 | -1 | $\Xi^-$ | 1314 | 1314 |
| $(ss)\underline{s}$ | spin conflict | | | | | | | |

**Table IX: Re-interpretation of the light baryon decuplet**

| | spin 3/2 (decuplet) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| baryon | isospin modes | code | | total isospin | bias | charge | symb | mass calc. | mass act. |
| $(uu)u$ | $(\uparrow\downarrow)\uparrow$ | $(ud)u$ | | +1/2 | +1/2 | 1 | $\Delta^+$ | 1237 | 1232 |
| | $(\uparrow\downarrow)\downarrow$ | $(ud)d$ | | -1/2 | +1/2 | 0 | $\Delta^0$ | 1237 | 1232 |
| | $(\uparrow\uparrow)\uparrow$ | $(uu)u$ | | +3/2 | +1/2 | 2 | $\Delta^{++}$ | 1237 | 1232 |
| | $(\downarrow\downarrow)\downarrow$ | $(dd)d$ | | -3/2 | +1/2 | -1 | $\Delta^-$ | 1237 | 1232 |
| | | | | | | | | | |
| $(uu)s$ | $(\uparrow\downarrow)\downarrow$ | $(ud)s$ | | +1/2 | +1/2 | 1 | $\Sigma^{*+}$ | 1377 | 1382 |
| | $(\downarrow\uparrow)\downarrow$ | $(du)s$ | | -1/2 | +1/2 | 0 | $\Sigma^{*0}$ | 1377 | 1387 |
| | $(\downarrow\downarrow)\downarrow$ | $(dd)s$ | | -3/2 | +1/2 | -1 | $\Sigma^{*-}$ | 1377 | 1383 |
| | | | | | | | | | |
| $(ss)u$ | $(\downarrow\downarrow)\uparrow$ | $(ss)u$ | | -1/2 | +1/2 | 0 | $\Xi^{*0}$ | 1521 | 1531 |
| | $(\downarrow\downarrow)\downarrow$ | $(ss)d$ | | -3/2 | +1/2 | -1 | $\Xi^{*-}$ | 1521 | 1536 |
| $(ss)s$ | $(\downarrow\downarrow)\downarrow$ | $(ss)s$ | | -3/2 | +1/2 | -1 | $\Omega^-$ | 1671 | 1672 |

While the mesons in this article are considered as members of an SU(2) group, it makes sense considering the baryons as members of an SU(3) group. This is most obvious if one the three quarks would be in opposite isospin state. This is reflected in the wave function representation shown in (108). Because the archetype quarks are supposed to be identical, they hold each other in equilibrium by spatial momenta $(p_x, p_y)$ with relative values of, respectively, $(\sqrt{3}/2, 1/2), (-\sqrt{3}/2, 1/2)$ and $(0,1)$. These values are reflected in the spatial components of the wave function. While two of the quarks can be in up isospin state with, the third one has to be in down isospin state.

$$\Psi_{pa} = \begin{bmatrix} \Psi_{1t} & \Psi_{1x} & \Psi_{1y} \\ \Psi_{2t} & \Psi_{2x} & \Psi_{2y} \\ \Psi_{3t} & \Psi_{3x} & \Psi_{3y} \end{bmatrix} \rightarrow \begin{bmatrix} -ib & a\sqrt{3}/2 & a/2 \\ -ib & -a\sqrt{3}/2 & -a/2 \\ -ib & 0 & -a \end{bmatrix}. \tag{111}$$

It is not difficult to prove that, under proper scaling of the amplitudes, this matrix is unitary (i.e. $\Psi\Psi^+ = 1$, in which $\Psi^+$ is the transpose conjugate of $\Psi$) and that its determinant is equal to 1 for any value of the ratio $a/b$. In a conservative field of forces, like it is the case of interaction between the quarks as a consequence of their nuclear potential fields, the ratio $a/b$ is subject to change. This implies that the nine-component spinor $\Psi_{amp}$ may rotate over eight spatial angles $\overline{\vartheta}(\vartheta_i)$ in a nine-dimensional spinor space. This rotation is the equivalence of the weak interaction in the meson case. The bosons involved in SU(3) are known as gluons. Because the wave function shown in (109) is a single dimensional center of frame Schrodinger approximation of the generic nine-dimensional wave function (111), the fine nuances have disappeared. But, in fact, there is no conflict here with the Standard Model.

## 13. Discussion and conclusion

In this article it has been shown how two unrecognized theoretical consequences from Dirac's electron theory may influence the view on the Standard Model of particle physics without substantially affecting its basics of SU(2) and SU(3) gauging, electroweak unification and the mass generation mechanism from the Higgs field. Most of the presented results have been documented in literature in more detail by me before, some in journals, others in prepublications that met opposition because of a seeming conflict with common views that are considered as proven in the wealth of studies and experiments in the high standard of present theory. The highlight on two additional principles that are not yet covered in the Standard Model, may help showing that results obtained before might be useful complements to the present status. This holds in particular for the mass calculations, because, while mass, next to charge, is the main attribute of physical particles, its assessment in the Standard Model is mainly empirically based and has not yet reached the same high standard as many other attributes. The two basic principles highlighted in this article that can be added to the Standard Model are,

1. The quark is an unrecognized Dirac particle that has, next to the well-known real dipole moment associated with the elementary angular momentum ***h***, a second real dipole moment associated with an elementary linear dipole ***h/c***, which, unlike as in the case of electrons, is polarisable in a scalar potential field.
2. Deriving Dirac's fermionic wave equation from Einstein's geodesic equation rather than from Einstein's energy expression reveals a complementary property to the quark conceived as a Dirac particle described in the first highlight. This property is the invariance of the frame-independent ratio $\Phi_0/\lambda$, in which $\Phi_0$, expressed in units of

energy, is a measure of the quark's potential and in which $\lambda$, expressed in $m^{-1}$, is a measure for the range of the quark's potential. In the article, this property is dubbed as the quark-scaling theorem.

The details of the derivation of these two principles can be found in, respectively, [7] and [18]. Some disclaimers on these two references have to memorized, though. As explained in paragraph 2, the Dirac mode of the quark that shows a polarisable dipole moment under a scalar potential, is a pseudo tachyon, which is different from the erroneous one in [7]. Second to this, it is worthwhile to mention that the near field description of the quark in [18] has been conceived by hypothesis, because the awareness of the second polarisable dipole moment was lacking at the time.

In the theory reviewed in this article, the quark is an energetic pointlike particle that emits energy into the vacuum filled with background energy. The format of this nuclear energy is seen as a particular mode of absolute energy $|W| = E$ , in the sense that absolute energy not only embodies negative Einsteinean energy next to positive one, but also imaginary Einsteinean energy. The symmetry of the quark's potential due to the background energy is broken. In the view developed in this article, the break is due to the polarisable dipole moment. Apart from giving a physical interpretation to the Higgs field, it interprets isospin as a physical state of polarization of this dipole moment. By conceiving the bond between the quark and the antiquark as a result of an equilibrium of forces evoked by the far field of the nuclear potential and the near field from the polarisable dipole moment, the origin of gravitational (= baryonic) mass is explained as the vibration energy of the center-of-mass in this bond. The viability of this origin has been proven in [18] by a successful numerical calculation of the gravitational constant $G$ in terms of quantum mechanical quantities. It gives a structural view on the bond between quarks that enables the modeling of mesons and baryons as excitable anharmonic quantum mechanical oscillators that enables to calculate the mass spectrum of hadrons quite accurately. The quark scaling mechanism reveals fundamental relationships between the quarks, such that, apart from the single archetype, they don't need no longer be considered as elementary particles.

The mass value of the Z-boson and the mass value of the Higgs boson, which in the present canonic theory both are empirically assessed, have been related by theory with the mass value of the weak interaction boson, which is shown to be the non-relativistic equivalent of the lab frame mass value of the pion. Although the view outlined in this article is shown being compatible with SU(2) and SU(3) gauging as well as with electroweak unification, the view on electric charge is slightly different. Whereas in the Standard Model electric charge is seen as an attribute of an individual quark, the adopted view in this article is considering electric charge as a holistic attribute of an hadron, evoked as a result from the spin characteristics of all dipole momenta involved in the hadron.

The theory described in this article is a structural view on particle physics with a physical interpretation on some of the axiomatic principles adopted in the mathematical formalism of the Standard Model. The model description of the mesons and the baryons is based upon a non-relativistic approximation of the multi-dimensional Dirac's fermionic wave equation to a single-dimensional Schrödinger one in the center of mass frame of hadrons, extended by separate additions of some second order effects not covered in the approximation and interpreted in the lab frame after relativistic correction. Hence, it lacks the rigidity of the conventional Standard Model description. On the other hand, the recognition and inclusion of the quark's polarisable dipole moment and the quark-scaling theorem reveal results that are not obtained so far in the present state of theory.

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