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Time, Equilibrium, and General Relativity

Harmen H. Hollestelle
Independent Researcher, CommonRoom, Bandung/Sukabumi, West-Java, Indonesia

ABSTRACT

Considered is "time as an interval" including time from the past and from the future, in contrast to time as a moment. Equilibrium as the basis for a description of changing properties in physics is understood to depend on the "mean velocity theorem", while a "time" of equilibrium resembles a center of weight. This turns out to be a good method to derive properties for any function of time t including space coordinates q(t) and expressions for the time dependent Hamiltonian. Introduced are derivatives depending on time intervals instead of time moments and with these a new relation between the Lagrangian L and the Hamiltonian H. As an application introduced is a step by step method to integrate stationary state "local" time interval measurements to beyond "locality" in General Relativity. Because of limits on the measures of the resulting time intervals and their asymmetry, this allows for a probabilistic interpretation of quantities that have these intervals as time domain in QM. Their asymmetry also questions the time reversal symmetry of GR. Another application of time intervals is the discussion of the measurement of starlight radiation energy and QM wave packet collapse as an example of a time dependent Hamiltonian. Finally a relation between starlight frequency, metric and space- and time intervals is found. Discussed is how finite and asymmetric time intervals correspond to time dependent H and symmetric infinite time intervals to a time independent H. From there, in cosmological perspective, finite time intervals can help to describe how entropy change could relate to dark energy.

Keywords: time interval, equilibrium, graphs, derivative, metric, general relativity, starlight radiation, qm wave packet collapse, cosmology

1. INTRODUCTION AND OVERVIEW OF RESULTS

In this article three new ideas are introduced to support the concept of time as a time interval. These are a new definition of time coordinates, the introduction of time intervals for derivatives and the application of the "mean velocity theorem" to describe equilibrium. The time coordinates, asymmetric to the past and future, agree with a asymmetric time experience and from there the introduction of time intervals is natural. The concept of time as a time moment is basic to many theories in physics. However with time moments one cannot easily understand change or continuity. Hamilton's principle of least action depends on a time interval, however it involves virtual, not real, path variations. The resulting Lagrangian equations, that do describe equilibrium effectively, depend on derivatives to time moments only. Newtonian equilibrium as well only applies derivatives to time moments. There the problem of time moments related to change already emerges. The new description of equilibrium, based on time intervals, is proposed to complete the time interval description.

The "mean velocity theorem" (paragraph 3) includes a graphical way to describe a "time" of equilibrium in the sense of a center of weight, and naturally provides the possibility to introduce time intervals and derivatives to time intervals. Also it provides an intuitively clear understanding of symmetries and asymmetries during equilibrium. From it follow derivatives and commutation properties related to time intervals for any function of time moments t (paragraph 4). The properties of space coordinates q(t) thus derived are applied throughout the further parts of this article. In paragraph 8) introduced is the specific time interval necessary for derivatives to time intervals.

Time coordinates and their properties are defined in paragraph 6) and paragraph 8). Time is assumed to depend on two elements that added together result in a one-dimensional time coordinate. One of these elements is anti-symmetric for past and future, and it counts time with positive numbers. The other one is symmetric and decisive for from when time is counted. With these definitions time coordinates do not commute and the value of a product of time dependent quantities does depend on their writing order.

The derivative to time intervals and the "mean velocity theorem" are applied to derive expressions for the time dependent Hamiltonian (paragraph 5 and 7) and its time interval derivative (paragraph 8). The commutation properties for q(t) derived in paragraph 4) are the basis for these results, however this Hamiltonian can be derived independently also from the equilibrium definition in terms of the generator of time transformations. A step by step transformation for time intervals prepares for how in General Relativity stationary state "local" time interval measures can be integrated, towards "non-local" time interval measures (paragraph 11). As a second application the QM description of the measurement of starlight radiation energy is expressed in terms of the time interval derivative of the time dependent Hamiltonian in paragraph 12). The final paragraph (13) concerns the relation between time interval, and space interval, starlight radiation frequency and metric tensor. Discussed is how time intervals being asymmetric and finite and Hamiltonian time dependence are related. Change in terms of energy transformation, following the description of wave collapse in paragraph 12), corresponds to time interval properties and could have implications also in cosmology.

2. EQUILIBRIUM WITH TIME INTERVALS AND A TIME DEPENDENT HAMILTONIAN

Newton's laws relate applied forces and the second derivatives to time moments of the space coordinates q, for a given mass m [Goldstein, 1]. Equilibrium is described as the applied forces being "equal" to the relative changes of the velocities x, that are the first derivatives dq/dt to time moments t of the space coordinates q, the differences being "equal" to zero.

For a conservative system that is described with a kinetic energy T quadratic in the derivatives dq/dt, the forces F = -∂V/∂q are derived from V, meaning all other energy. For a conservative system the kinetic energy is conserved for a closed actual path. Equilibrium based on Hamilton's principle of least action implies that the integral: I = ∫ L dt, from time t1 to t2, with L = T – V the Lagrangian, is an extremum for the actual path of motion compared to other possible paths. Otherwise said the δ variation of the integral I is zero: δ I = δ (∫ L dt|Δt1t2) = 0. This means that the integral I for the actual path is locally stationary, does not change for infinitesimal changes of the path, and thereby determines equilibrium: the total energy H0 = T + V is time and space independent and the change in T is the same as the change in –V, thus according to δ I = 0 the first order variation of both T and V with any varied path is zero. A δ variation means the considered time interval t1 to t2 remains actual and fixed while the considered, virtual or possible however not actual, path may vary from the actual path. From there one derives the Lagrangian equilibrium equations, for L = L(x = dq/dt, q), that are equivalent to those of a system in Newtonian equilibrium [Goldstein, 1], [Arnold, 2]. This is a description in terms of energy quantities like the Lagrangian and the Hamiltonian. Newtonian equilibrium is independent of δ variation considerations, however similarly applies time moment derivatives of q.

The total energy H0 = T + V remains time independent for any system. The Hamiltonian H is the Legendre transform of the Lagrangian L, and is a function of the parameter p, and H(p,q) = p.x(p) – L(x, q). With p.x is meant a scalar product of the vectors p and x. For scalar products like these in the following the relevant factor cos(p, x) is not applied however it will return in the discussion paragraph 13). The specific relation x = x(p) is defined with dL/dx = p for x(p). From the Lagrangian equilibrium equations for a conservative system it follows that H equals H0 = T + V and is time independent as well. The relation between H and L as each other's Legendre transform will remain valid for the new equilibrium description in paragraph 3) and 5) including a time dependent Hamiltonian H. The Hamiltonian H can be evaluated for a certain time interval from the difference of L and the asymptotic function p.x, with the "mean velocity theorem", reconsidering the relation dL/dx = p for x(p) which is a time moment derivative relation. For a Newtonian or conservative system H reduces again to the total energy H0 as required.

3. THE "MEAN VELOCITY THEOREM" AS A BASIS TO DESCRIBE VARIATION AND CHANGE AND (A-) SYMMETRIES

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The symmetry properties of a system tell which transformations do not change the value of the Hamiltonian. Similarly when the value of H changes with some transformation parameter this means an asymmetry exists for some property. This agrees with the essence of Curie's (i.e. Pierre Curie) principle [Curie, 3]. Discussion of Curie's principle in relation to the Higgs mechanism can be found in [Katzir, 4] and [Earman, 5]. In qm field theories group representations of symmetries are applied to derive particle properties, and the absence of symmetries gives clues to derive differences between properties and for transitions and changes [Veltman, 6]. In this article concentrated is on time intervals and time elements and the time dependent Hamiltonian.

Consider the "mean velocity theorem" [Hannam, 7] [Dijksterhuis, 8], that can be visualized with graphs. The theorem states that the area below a horizontal line is the same as the area below a sloped line, when the two lines meet and cross each other at that value at the parameter interval for which the sloped line reaches its average, "mean", value. The first line means constant velocity and the second one means varying velocity in the case of a time parameter. Because of where the two lines meet and cross each other the theorem is also called the "fixed point theorem". In Medieval age it was derived as the "mean speed theorem", with the help of graphs. The "mean velocity theorem" is in itself a way to imagine equilibrium, like the center of weight is an "average" place. The evaluation of mean velocity graphs was generalized from one dimension to higher dimensional spaces by Brouwer, who also introduced the term "fixed point theorem" [Hocking and Young, 9].

The mean velocity theorem is part of a tradition of thinking how changing properties can be described. Newton introduced derivatives, for instance to describe continuously in time the change of velocity in terms of applied forces. To relate the function L(x = dq/dt, q) = T – V to H(p, q) as a function of a new coordinate p with dL/dx = p at x(p) was a consequence when a description in terms of energies became an alternative to the description in terms of paths. A traditional derivative depends on a limiting process from a surrounding interval towards one moment in time or one space point. It remains to be interpreted what this limit means for the description of the continuity of variables that change with time or start to change with time. For a derivative to an interval instead of to one moment these difficulties do not exist. Within quantum mechanics, change is related to probability and discontinuity. Initially in qm reasoning the concept of space and time was to be disregarded in favor of abstract energy levels at least in the quantum domain. Any attempt to localize for instance with the help of paths is refuted [Beller, 10].

Energies relate to symmetries naturally: energies can remain invariant during variation of a property, while actual coordinates mostly vary in any case. This is a reason why energy quantities can be a basis for symmetry and equilibrium description. Especially when H is time dependent and describes change, or when it describes invariance as the absence of change, a time derivative depending on time intervals seems more appropriate then a time derivative depending on a time moment. Equilibrium, similarly, needs time intervals rather than a time moment to be defined properly, since it only exists where one is in equilibrium with another one. Indeed Hamilton's principle of least action is also defined for a time interval: the time interval [t1, t2]. Arnold mentions a criterion for a equilibrium x0 of a system dx/dt = f(x): x(t) = x0 for all t is a solution of this system, i.e. f(x0) = 0, [Arnold, 2]. One can say equilibrium means a quantity exists that expresses invariance and symmetry as being the change of several other quantities. The formulation of equilibrium from the mean velocity theorem is crucial because it describes the interdependence of one moment values of a function with a certain interval average of this same function.

4. INTERVAL DERIVATIVES AND INTERVALS

The definition of a comparative derivative of a quantity or function, say f(x), to an interval ΔX that includes the parameter x(t) for some specific t belonging to Δt = [t1, t2], using " " notation to emphasize the difference with a traditional derivative to the parameter x for the specific x = x(t), is:

1) "df/dx"|ΔX = < df/dx >|ΔX = ∫ (df/dx) dx (1/|ΔX|)

Equation 1) depends on the interpretation of the relevant "mean velocity" graph as a comparison between average and slope line. This comparison is similar to an equilibrium definition for the slope line and it liberates the derivative

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from a one value limit to an interval in equilibrium. With < … >|ΔX is meant the average for the interval ΔX = [x(t1), x(t2)] where t is a one dimensional parameter for simplicity. x(t) belongs to the interval ΔX and ΔX in turn should include x(t). For convenience also is defined the interval ΔY(y(t)) = [x(t1), y(t)] for any y(t) belonging to ΔX. For y=x(t2) there is ΔY(y) = ΔX = [x(t1), x(t2)]. Also |ΔX |= |x(t2) – x(t1)| = |x(t2)| because the value of x(t1) is quite arbitrary, and one may organize that x(t1) = 0. At least x(t2) > x(t) The interval ΔX is interpreted as the domain for the function f(x). The following approximation is valid for all y belonging to ΔX: "df/dx"|ΔY = "df/dx"|ΔX (y/x(t2)) and thus < df/dx >|ΔY = < df/dx >|ΔX (y/x(t2)). This means that any function f allows for a linear approximation for the complete interval ΔX. A linear approximation might be positive or negative of sign depending on f(x) being increasing or decreasing. For all x belonging to ΔX and for all increasing positive f(x), this approximation means the evaluation of f(x)/x ≈ "df(x)/dx"|ΔX or written as a linear equation f(x) ≈ "df/dx"|ΔX x, while assumed is f(x = 0) = 0. For decreasing positive functions f(x), "df(x)/dx"|ΔX ≈ - f(x)/x, and similarly for negative functions. For the space coordinate q(t) one finds "dq/dt"|Δt ≈ +/- q/t, for a positive, increasing respectively positive decreasing q and for Δt = [t1, t2]. From "dq/dt"|Δt ≈ - q/t follows the approximation [1/t, q] = -2q/t and [t, q] = -2qt and

2a) "dq/dt"|Δt = 1/2 [1/t, q(t)]

and this commutation bracket relation is inferred to be a valid equation for all functions and for all t belonging to Δt, not only for q(t), valued at "equilibrium" being the equilibrium from the "mean velocity theorem" for Δt. The following definition for a comparative derivative is inferred to be valid for any interval Δt = [t1, t2]:

2b) "df/dt"|Δt = 1/2 [1/t, f(t)]|Δt = 1/2 (1/t1 f(t1) – f(t2) 1/t2)

Writing "comparative" commutation brackets in this way suggests a similar definition with 1/2 [t, f(t)]|Δt = 1/2 (t1f(t1) – f(t2) t2), being the comparative integral of f(t). With equations 1) and 2a/b) derivatives to an interval ΔX or Δt are defined as an alternative to traditional time moment derivatives at x = x(t) at time moment t. Equation 2b) can also be evaluated for t1 = 0 due to the linear approximation above. On the right side, still, time moment functions remain. These definitions are independent of the traditional derivative and finding a function f(t) by traditional integration does not provide a solution for a comparative derivative equation immediately. However from the above it can be argued that a positive, decreasing, function q(t) is proportional with 1/t. With the comparative derivative, and the above approximation as a comparative method, the following equation is directly derived for the Legendre transforms f and g for which g = p.x(p) – f:

3) "df/dx"|ΔX = < df/dx >|ΔX = 3/2 p – 3 < g >|ΔX 1/x(t2)

Equation 3) does not replace the Legendre transform relation for f and g. On the contrary, it defines the comparative derivative for f(x) to an interval ΔX, while the Legendre relation g = p.x(p) – f remains intact. Thus equation 3) defines "df/dx"|Δt as a derivative to an interval while again the right side of the expression contains time moment dependent functions. This occurs because the interval ΔX and the specific time moment coordinate t are related. The progress with equation 3) is in the application of the derivative to an interval ΔX, which itself depends on the time interval Δt = [t1, t2]. To avoid infinite regress chosen is to keep p and x(t2) as time moment parameters included in equation 3). In this way an interval does not have an interval as border. The comparative derivative definition agrees with a theorem [Arnold, 2] concerning the equal value of averages of a function for a t interval and a q interval. Following the usual identification f = L and g = H the traditional derivative of the Lagrangian is dL/dx = p at x(p) while the comparative derivative "dL/dx"|ΔX for interval ΔX can differ from p, because of the liberation of the derivative from a one value limit to an interval equilibrium. With equation 3) the traditional Lagrangian equilibrium equations and equilibrium itself become time interval dependent.

5. TIME INTERVAL AVERAGES

Even for H time dependent, the Lagrangian L and the Hamiltonian H are assumed to remain the Legendre transform of each other. With f = L and g = H and x the comparative time derivative of q, and writing H = H0 + ΔH(t), to accompany equation 3) one finds:

4) L(x(p)) + H(p) = 2T + ΔH = p.x = p."dq/dt"|Δt

Both p and x are functions of t and related to the time interval Δt = [t1, t2] as in paragraph 4). Just as equation 3) also equation 4) contains both time interval and time moment parts. For now the time moment t and time interval Δt remain unspecified. Assumed is that the mass m is a constant in time and that T is quadratic in p. T can also be understood to be quadratic in dq/dt in some cases and for a Newtonian system these definitions are the same. Consider the function G* = p.q, [Goldstein, 1]. Following Goldstein's description with a generalized force k, and when applying comparative derivatives like in par. 4), then H time dependent implies "dG*/dt"|Δt = p.q/t and < k >|Δt = (p(t2) – p(t1)) 1/|Δt| with k = "dp/dt", and there is:

5) < 2T >|Δt = k.q - < k.q >|Δt

For H = H0 + ΔH time dependent one can write also the following relations:

6a) < ΔH >|Δt = - < d(p.q)/dt >|Δt

6b) < 2T >|Δt + < k.q >|Δt = - < ΔH >|Δt

Equations 6) can be compared to the usual virial equation < 2T >|Δt + < k.q >|Δt = 0. For a system in Newtonian equilibrium with time independent H both - d(p.q)/dt and < ΔH >|Δt are zero, for other systems in equilibrium from the "mean velocity theorem" with time dependent H these expressions, including - "d(p.q)/dt" = - p.q/t, turn out to be non zero however equal following equation 6a). Equation 6b) provides an addition to existing virial equilibrium equations. Parameter p relates to H and L through the Legendre transformation equation 4). For a time dependent H the traditional definition of V with k = - ∂V/∂q might have to be changed. The definition of T quadratic in p will be followed in the remaining.

6. TIME COORDINATES AND TIME ELEMENTS

Equation 1) that defines comparative derivatives to an interval asks for a specification of what is an interval, especially for derivatives to time. One assumes that a) time is measured with counting, b) there is a present moment now, without knowing what that means yet, c) for the future one counts time further into the future from some moment in the future, however for the past one counts differently: one counts rather from some moment in the past. Whereas the future goes further from us now, away from us now, the past is coming towards us now, nearer to us. d) time is linear and there is only one time coordinate that does not allow for higher dimensional properties like turning. Traditionally time description with time moments is 0-dimensional: the time moment now is the same everywhere all the time, even when measured or counted differently at different places and it is not possible to change, to "go", to another time moment independent of others like is possible in space. Time intervals discussed here are 1-dimensional closed intervals that can overlap. To make this more precise: think of the moment now as a yet undefined time belonging to a time interval comprising parts of both the future and the past. Considering the future one counts time with element (i) positively from some time, say: (n) + (i = 0), to a time ta in the future: ta = (n) + (i) > 0. When considering the past, one counts time with element (i) positively from some time in the past, say: (-n) + (i = 0), to a time tb in the past: tb = (-n) + (i) < 0. These definitions specify time for the future and the past respectively, by counting both with the same +(i), with the i included in (i) only a positive real number or zero. For both two time elements is used the ( ) notation and the sum of these (n) and (i) elements added together is by definition the time coordinate t, which remains however 1-dimensional. A past time similar to the future time with +(n) includes +(-n), with the minus sign contained in +( ) to clarify it is forward oriented, even for negative n, and it is combined with forward counting time with +(i). A time interval [tb, ta] emerges with parts of the past and the future both. There is with these definitions a symmetry and a anti-symmetry between past and future. A time interval could also be defined with the symmetric choice ta = +(n) + (i) and tb = -(n) - (i): an interval [tb, ta] would then become [-ta, ta] and the past is then counted backwards with -(i). (-n) and –(n) are not the same, the latter one being backwards oriented, and to be combined with –(i) while (-i) is not possible. The above assumption c) means the element (n) is symmetric and the counting element (i) anti-symmetric for past and future: the interval [tb, ta] equals [(-n) +(i), (n)+(i)]. A past time and a future time can be defined independent of each other with different (n) for past and future or counting with different (i) for past and future. In this article the interval [tb, ta] is defined such that ta and tb are interdependent through (n), (-n) and (i). Assuming the counting element to be the same (i) for both past and future agrees with the anti-symmetric part of time experience. From the discussion of time element properties in paragraph 8) it follows that time coordinates do not commute.

7. THE TIME DEPENDENT HAMILTONIAN

The time dependent part ΔH of the Hamiltonian H = H0 + ΔH is, for convenience, written in the form:

7) ΔH = exp (- (c.q)F) G exp (+ (c.q)F)

F and G are functions independent of the space coordinates q. The vector c is added with the dimension of q-inverse to make (c.q) a scalar product. In this description not yet q as a function of t, meaning an equilibrium solution q = q(t), is determined. Eventually when specific equilibrium equations for when H time dependent are applied equilibrium solutions ΔH are found from these. At the end of this paragraph with the equilibrium solution q(t) from the “mean velocity theorem” (paragraph 4) these equilibrium equations and the solutions for ΔH are found confirmed. The usual operator writing convention is: to the left includes to the right. In this case, rather time dependent functions are present, however still the writing order has to be cared for, because q and t do not always commute. All function parts relate to the same time moment t and ΔH is still completely time moment dependent. H and ΔH are energy quantities just like H0 and this means both should have a real value.
ΔH seems similar to the standard way to describe functions when for instance calculating exponents of matrices. Such expressions are applied extensively when representations are studied and also for gauge transformations. However ΔH and equation 7) describe an energy quantity as a function of time, not considered are field theories or operators.

The unspecified equation 7) has meaning as a trial expression, chosen for its simplicity: below, from the definition of the equilibrium equations for a time dependent H with equations 10) and 11), found are solutions for F and G and thus for ΔH. Some considerations for clarity are:
The exponent function on the right is accompanied by its inverse on the left to achieve linear space coordinate system transformation invariance, i.e. $\partial \Delta H/\partial q = 0$ when q changes accordingly and t remains constant, at least when q and t commute and ΔH is time independent. For ΔH not time independent this is achieved when F remains independent of both t and q and commutes with t.
The writing order of equation 7) resembles equivalence transformation writing order for (matrix) functions which is the reverse of unitary transformation writing order for operators. This suggests the interpretation of the q part of equation 7) for ΔH to be a coordinate transformation, along q and –q, of the t part.
The t part of ΔH, that is: G, is being found further on to be the comparative derivative of a time dependent function that is similar to a “time dependent” constant of Planck h. In paragraph 9) such a function, h+, is introduced that differs from h only when H ≠ H0. Indeed for a starlight radiation measurement event with initially a constant energy E = hν there is derived E = h+/Δt during time dependent wave packet collapse.

Because the q and t parts of ΔH can be separated due to the above F and G properties, specific free infinitesimal transformations for q or t independent of each other are possible. Then [Arnold, 2] from equation 7) and applying the solutions q(t) from paragraph 4), it follows, when ΔH = ΔH(q(t), t), ΔH can also, like q(t), be written as equal to G(t + D) = ΔH(q = 0.q0, t + D), that is: ΔH evaluated at a time t + D different from t while q = 0.q0 remains invariant. A possible singularity for G at t = ts within Δt is avoided when D is chosen such that Δ(t + D) does not includes this singularity. q(t) can be evaluated with the results of paragraph 4) and D follows from q(t). A ts = 0 singularity exists for the solution for G introduced below, however this is not easily considered as a possible moment now since time is not reversal symmetric following the definition of time coordinates with elements (n) and (i) in paragraph 6). When writing ts = 0 meant is ts = 0.t0, while the t0 is left out. With "df/dt"|Δt = 1/2 [1/t, f]|Δt equal to a comparative derivative (paragraph 4), this comparative commutation bracket result is the same as the Poisson brackets [f, G_] that defines derivatives with the time transformation generating function G_, however now including the partial derivative.

When one considers a function v as a generator of infinitesimal contact transformations and applies Poisson brackets, one can write for any function u, [Goldstein, 1], [Arnold, 2]:

8) $\delta u = \varepsilon\, [u, v] + \varepsilon\, \partial u/\partial t^*$

δ means the δ variation and ε means the variation dt* of the parameter t* corresponding to v. One may choose v equal to the Hamiltonian H, when H is time independent, meaning the system is Newtonian with H = H0. Then ε, the time parameter variation dt*, is equal to dt and v = H equals the generator of time transformations G_. Also like before (equations 6) < ΔH(t) >|Δt = (p.q) 1/t = 0 in this case. This is not new. Time dependence of H can be included in ΔH writing H = H0 + ΔH. In this case v = H still equals the generator of time transformations G_, and still ε = dt* = dt, however v is not equal to H0 anymore. The above transformation equation 8) with u = H, when applying the comparative derivative introduced with equation 1), reduces to:

9) $\text{“}d\Delta H/dt\text{”}|\Delta t = [\Delta H, v] + \partial \Delta H/\partial t$

Compared with transformation equation 8) there is a change of the placing of the parameter variation ε = dt: δΔH = [ΔH, v] ε + ∂ΔH/∂t* ε. The placing of time parameters is not trivial since they are assumed to not necessarily commute, also with other parameters. Equation 8) applies the traditional formulation with ε to the left, and for traditional commuting time moment variables this is the same as with ε to the right. The right side placing of ε is in agreement with the definition of averages with equation 1) where 1/|ΔX| = 1/(x(t2) – x(t1)) is placed on the right side as well.
Since v = G_ and ε = dt for both time dependent or time independent H, the following equations 10) and 11), being just those for comparative equilibrium when H = H0, that are the same as the Lagrangian equilibrium equations for H = H0, are assumed to remain valid for comparative equilibrium when H = H0 + ΔH and time dependent and with v still identified with H even while H ≠ H0: even for H = H0 + ΔH and differing from H0 the canonical equations 10/11) are saved while applying comparative derivatives with time intervals instead of the usual derivatives with time moments. For H ≠ H0 and time dependent the generator v = G_ itself by definition does not remain canonical in the sense of dH/dt = 0 from equation 8). At the end of this paragraph it is confirmed starting from the assumptions 10/11) that indeed when H ≠ H0, H is not time independent, and the generator v = G_ does not remain canonical, in the sense of "dH/dt" = 0 from equation 9) either. Thus saving the canonical equations 10/11) does not interfere with the consistency of the time dependence of H. For H = H0 and time independent the time interval derivative, comparative derivative, description reduces to the traditional derivative description (paragraph 2 and 4).

This means a transition or change for say a real object, described assuming equilibrium and with a time dependent energy quantity H as its property, is recognizable from H in both descriptions or senses and change or no change occurring itself is independent of descriptions and senses, and thus the (occurrence of the) change is real and not relative of description or sense. This is not at all trivial. Also one can choose the description most clear to describe change. Time intervals in relation to the transition or change when it occurs in the above two descriptions are further discussed in paragraphs 8) and 13). With the comparative description, at least, most clearly one recognizes and describes change by including the possibility of asymmetrical and finite time intervals to exist. Time being not translational invariant when change is occurring and vice versa is a basic addition to include time explicitly within the symmetry principle referred to at the beginning of paragraph 3). It also is a basis for discussing the origin of GR.

With the assumptions equations 10/11) one finds comparative equilibrium equations 12/13) for ΔH:

10) δqi = qi(t+dt) – qi(t) = dqi = ∂v/∂pi dt , δpi = pi(t+dt) – pi (t) = dpi = - ∂v/∂qi dt

11) "dq/dt"|Δt = ∂v/∂p , "dp/dt"|Δt = - ∂v/∂q

12) "dΔH/dt"|Δt = ∂ΔH/∂q "dq/dt"|Δt + ∂ΔH/∂p "dp/dt"|Δt + ∂ΔH/∂t

13) "dΔH/dt"|Δt = - ["dp/dt"|Δt, "dq/dt"|Δt]|Δt + ∂ΔH/∂t

The brackets in equation 13) are commutation brackets. Equation 13) can be derived from the results of paragraph 4), independent of the assumptions above. A solution of equation 12) or 13) is found with the functions F = i and G = h/t. Here i is just the imaginary number unit. The solution ΔH can be written in two ways:

14a) ΔH = exp (- (c.q)i) h/t exp(+ (c.q)i)

14b) ΔH = h/t (1 + (c.q)^2 + …)

Chosen is to keep intact the order of the different parts of ΔH since time parameters do not always commute as argued before in paragraph 6). Therefore the exponent version expression 14a) makes sense, being not simply equal to h/t. For F and G matrices this can be different. One can write the exponents within ΔH as Taylor series and one finds the series version expression 14b) with (c.q)^2 being the lowest order term in (c.q) assuming ΔH and the vector c are space orientation invariant. The series version for ΔH gives real values as required. This can be proven for the exponent version too, considering that when q and t do commute there is ΔH = G and both versions are trivially the same.

Because (c.q) can be equal to a multiple n of 2π for some choice of c(t) for q = q(t), for this q(t) the series version 14b) for ΔH is valid and exactly the same as the exponent version because then all exponents and their Taylor series are equal to 1. Now assume the relevant time interval Δt = [t1, t2] = [tb, ta] includes borders with t = ta and t = tb and c is chosen such that (c(t).q(t)) = n(t) 2π for these q(t) and t. For any t belonging to the interior of this time interval, the series version is still correct. One applies the mean velocity theorem to assert that the transformation from the t domain, say the interval Δt, to ΔH(t) in the above approximation, is continuously connected along the whole t domain interval. The theorem confirms that there is at least one x in the domain of any function f, such that for < f > the average of f, there is < f > = x. When there is only one such x, necessarily this x belongs to the interior and not to the border of the domain of f. When there are two such x at least one of these two belongs to the interior of the domain of f, two such x in the border would contradict < f > = x. When three or more such x exist, then at the most two belong to the border, and at least one belongs to the interior of the domain of f. Thus in any case at least one such x belongs to the interior of the domain of f. This means the transformation from the interior of Δt to ΔH(t) is continuously connected to this transformation from the border of Δt to ΔH(t). For this reason the series and the exponent version of ΔH are assumed to be equivalent following standard topology.

When G_ is the generator of time transformations, for equations 11) propose the following solutions ∂G_/∂p = "dq/dt"|Δt = - q/t, and thus G_ = - (p.q) 1/t = < ΔH >. There is ∂G_/∂q = - "dp/dt"|Δt = + p/t. These solutions mean q is positive and decreasing and p is negative and increasing following the description in paragraph 4) with "dq/dt"|Δt = - q/t and "dp/dt"|Δt = - p/t. Following paragraph 5) there is ∫ΔHdt 1/Δt = - (p.q) 1/t and thus ΔH = - "d(p.q)/dt"|Δt = - (p.q) 1/t = G_. With the above comparative derivatives of p and q, and taking care of the proper commutation bracket relations with t, it follows:

15a) "dΔH/dt"|Δt = [ΔH, G_] + ∂ΔH/∂t = - ∂G_/∂t = G_ 1/t ≠ 0

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The brackets are Poisson brackets in this equation. It is possible to write ΔH and its comparative derivative in terms of p, q, and t, applying commutation brackets, without reference to any solution ΔH from F and G. From "d(ΔH)/dt"|Δt = ΔH/t = - (p.q)/t^2, and "d(ΔH)/dt" |Δt = (p.q)/t^2 + p[1/t,q]|Δt 1/t, it follows that is required [1/t, q] |Δt = -2 q/t. This commutation relation and similar ones were derived in paragraph 4). This is an independent confirmation starting from the results of paragraph 4) and with the inference "df/dt" |Δt = 1/2 [1/t, f]|Δt, defined with equations 2), for the time dependent H equilibrium assumptions equations 10) and 11). It follows:

15b) "d(ΔH)/dt"|Δt = "d(1/2 p[1/t, q]|Δt)/dt"|Δt = 1/2 p[1/t, q]|Δt 1/t

Always ∂G_t/∂t = 0, however ∂G_/∂t ≠ 0. "dΔH/dt"|Δt is non zero depending on [1/t, q(t)]|Δt ≠ 0 while these both are time interval Δt dependent. Notice in relation to equation 13) that always Δ*p.Δ*q ≥ h for Δ* variances, following the qm uncertainty relations, however this will be discussed in paragraph 9). In conclusion, the generating function G_ does not leave H = H0 + ΔH, or ΔH itself, invariant, meaning the following:

16) The time transformation is canonical for a time independent Hamiltonian, however non canonical for a time dependent Hamiltonian, in both the time moment description sense and the comparative time interval description sense.

8. TIME COORDINATES, ONCE MORE, AND TIME INTERVALS AND THE TIME INTERVAL DEPENDENT HAMILTONIAN

Consider the following transformation of t, applying the exponent version of ΔH = exp (- (c.q)i) G exp(+ (c.q)i): ΔH(q = 0.q0, t) = G(t) equals ΔH(q(t), t') for t' the transformed of t. This type of free transformation was discussed before in relation to equation 7). The series version of ΔH from equation 14) supports this transformation with: 1/t' = 1/t (1 - (c.q)^2) and, by including a minus sign and with the positive and decreasing equilibrium solution q = q(t) derived in paragraph 4), defined is transformation A:

17) A: tb = - (1 - (c.q(ta))^2)^(-1) ta = - (1 – (c.q0)^2 (t0/ta)^2)^(-1) ta

Just this transformation A: tb = tb(ta) is applied to define the interval [tb, ta] = [tb = tb(ta), ta]. tb is part of the past when ta is part of the future due to the minus sign. The meaning of this definition in terms of time elements (n) and (i) is discussed in alinea b) below. It is not meant that ΔH(tb) = ΔH(ta) for all ta and that H remains time independent. With this definition the comparative derivative with equation 1) acquires the specific time domain Δt = [t1, t2] = [tb, ta] for ΔX. The time moment now is not considered. To derive comparative derivatives with this interval Δt is assumed to be approximately justified with regard to the original interval ΔY = [0, y] encompassing x(t). Recalling equation 13) one finds for H = H0 + ΔH, applying comparative derivatives to Δt = [tb, ta] = Δtbta and commutation brackets:

18) "dΔH/dt"|Δtbta = Δ(ΔH)/Δt|Δtbta = - ["dp/dt"|Δtbta, "dq/dt"|Δtbta]|Δtbta + ∂ΔH/∂t

This is the basis for defining a new function ΔH2, with the dimension of energy like ΔH:

19a) ΔH2(tb, ta) = - Δ(ΔH)/Δt|Δtbta Δtbta = - exp (- (c.q(ta))i) h/tbta Δtbta exp(+ (c.q(tb))i)

ΔH2 depends only on Δtbta and its borders tb and ta. This is possible because the q dependent part and the t dependent part appear separated in ΔH. This suggests the following definition for comparative derivatives for any function h1(q(t), t) with separated parts for q and t like for ΔH:

19b) h2(Δtbta) = - "dh1/dt"|Δtbta = - 1/2 [1/t, g]|Δtbta

Higher order comparative derivatives can be considered as well. The commutation bracket result from equation 19b) is the same as the exponent result of equation 19a) for h1 = ΔH and h2 = ΔH2/Δt, by application of equations 2) and

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of the results of Appendix A). The function h+ introduced later on in paragraph 9) can be inserted as well. With h1 = h+/Δt one finds again h1 = ΔH and h2 = ΔH2/Δt = - 2 h+/(Δt)^2 from equations 22) in paragraph 9). A function h0 emerges, that resembles h+, with 1/2 h1 = - 1/2 [1/t, h0]|Δt. This also means comparative derivatives of h0 can be meaningful and non zero even when h0 is a constant, while Δt has non zero measure, i.e. tb ≠ ta. This is a purely time interval dependent result. A similar result with traditional derivatives would be a contradiction. A constant function h0 leads to some difficulties related with the mean velocity theorem, and needs interpretation: the specific equilibrium solution q = q(t) relates tb and ta. For H = H0, q and t commute and there is ΔH = G and G(tb) = G(-ta) meaning tb = -ta and thus q(t) = 0.q0 for all t. From this value of equilibrium solution q it follows tb and ta are infinite with opposite sign (this is a reason for difficulties with the mean velocity theorem) and the comparative derivative to Δtbta of h0 indeed is zero, h1 = ΔH = 0, as expected from H = H0, while h0 equals a possibly finite function 2 h+/Δt ta = h+ that equals the constant of Planck for only H = H0. In this way encountered are time intervals [-ta, ta] that are symmetric and infinite for H time independent and time intervals [tb, ta] that are asymmetric and finite for H time dependent.

The description of time coordinates is continued with the following properties:

a) Time is regarded as part of reality: the value of (n) and (i) should be real numbers, however with dimension of time. In the following a difference between – (n) and (-n) is attended to. t+ is defined to be in the future with t+ = (n) + (i) > 0.t0, and t- in the past with t- = (-n) + (i) < 0.t0, such that:

20a) t+ - t- = 2(t+ - t0)

20b) t+ + t- = 2t0

20c) (-n) = - (n) – 2(i) + 2t0

Equation 20c) is the result of the other two definitions, equations 20a/b). The measure of the interval [t-, t+] is “twice” that of the interval [t0, t+] or the interval [t-, t0]. This defines the relation between the time interval [t-, t+] and the time t0, that is an indication for the time equilibrium of the interval. t0 however cannot be interpreted as the time moment now. From addition of the equations 20a) and 20b) one understands that 2t+ = 2t+ and t- - t- = 2t0 – 2t0 = (2-2)t0 = 0.t0, with 0.t0 interpreted as the time unit for addition and elements can be transported to the other side of the equal sign when multiplied with -1. Applied is that for 1.t0 exists the addition inverse - 1.t0. Still, addition of non equal time elements depends on the properties of their (n) and (i) parameters.

1.t0 = t0 is interpreted as the time unit for multiplication. Time variables do not commute in most cases. t0iv = 1/t0 is the multiplication inverse for t0 with t0.t0iv = 1. A time multiplication inverse however is itself not a time coordinate. The product of two or more time elements or variables left of the equal sign can only result in a product of a similar number of time elements or variables at the right side of the equal sign.

When writing equations often variables are transported from one side of the equal sign to the other side, and then inverses are necessarily occurring. This means one value has to be divided by another value within expressions. Special is the multiplication unit t0: t.t0 = t for time t and t0 the multiplication unit can be correct, regarding dimensionality, when the product is interpreted as vector product while time t, rather than being a vector in higher dimensional space, remains the sum of two elements (n) and (i) together being one coordinate in a 1-dimensional time space. Higher dimensional time coordinate spaces are imaginable, when taking care that time remains without unreal properties.

b) Following equation 17) with t+ in the future: t+ > 0.t0, t- is defined to be equal to t+’ = - (1 – (c.q0)^2 (t0/t+)^2)^(-1) t+, and thus t- < 0.t0 is valid for (t+)^2 > (c.q0)^2 t0^2. This definition means that t+ = (n) + (i) > 0.t0, with A transforms to t- = (-n) + (i) < 0.t0 and from t+ = ta it follows t- = tb. Together with equations 20) that specify the relation between t+ and t- and t0, transformation A defines t0: t0 = (n = 0) + (i = e) = (0) + (e) with e chosen any real

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positive number. t0 being the time unit for multiplication means: t.t0 = t0.t = t. There is t+.t0 = t+ = (n) + (i). For the special scale (e) = (i) and with t.(n = 0) = (n = 0).t = 0.t, this means t+.t0 = (n)(i) + (i)(i) = (n) + (i) and also t0.t+ = (i)(n) + (i)(i) = (n) + (i). Since t+.t0 = t0.t+ there is [(n), (i)] = 0 and (n)(i) = (i)(n) = (n) and (i)(i) = (i). In general in any product all (n) and (i) elements are present. However when accepting q(n) = (q.n) and q(i)= (q.i) for all non negative real numbers q the above multiplications remain valid within the t0 = (n = 0) + (i) scale. The following properties result as well: t-.t+ = t+.t- = 1/2 (t-^2 + t+^2) and (t-/t+) (t+/t-) = 1. For t coordinates other than t+ and t- = t+' and for their commutation properties one has to start from different (n) and (i) and derive commutation values and other properties for all t's independently.

9. THE ORDER OF TIME DEPENDENT QUANTITIES AND THE CONSTANT OF PLANCK

The expression for "d(ΔH)/dt"|Δt from equation 13) is rewritten with Δ variations, defining the variations equal to differentials with "dΔH/dt"|Δt = Δ(ΔH)/Δt, and with commutation brackets:

21) $\Delta(\Delta H)/\Delta t = - [\ \Delta p/\Delta t, \Delta q/\Delta t\ ] + \partial\Delta H/\partial t = (\Delta p.\Delta q - \Delta q.\Delta p)1/\Delta t^2 + \partial\Delta H/\partial t$

To derive this result one applies the commutation relations for q and t from paragraph 4) and equations 11). In agreement with the above interpretation that time coordinates do not commute, Δ variations, because they are rewritings of time interval derivatives, are considered to be non commuting just the same and their order should be taken care of: for their products introduced are the new quantities hpq = Δp.Δq and hqp = Δq.Δp. These quantities are comparable to and have the same dimension as h, the constant of Planck, as it appears in the standard uncertainty relation Δ*p.Δ*q ≥ h, [Sakurai, 11], where Δ* means a variance. hpq and hqp depend on the writing order of Δp and Δq and the scalar product value of these variations will change when this order is changed. The relation: hpq – hqp = 0 only when H = H0 and vice versa, can be derived directly, from equations 6). All Δ* variances should have the same value as Δ variations, for which will be given further arguments below. Apart from h_ = hpq – hqp one can define also h+ = 1/2 (hpq + hqp). These quantities seem quite arbitrary, however it is clear that h+ reduces to the constant of Planck h and h_ reduces to zero when H is time independent and equals H0.

A second uncertainty relation is: Δ*E.Δ*t ≥ h (often written as Δ*E.Δ*t ≈ h), with h again the constant of Planck, usually with Δ*E and Δ*t in this order [Merzbacher, 12]. When E = p.p/2m is just the kinetic energy T, and "dE/dt"|Δt = ΔE/Δt = 1/2 (Δp/Δt.Δq/Δt + Δq/Δt.Δp/Δt) for a Newtonian system with p/m = Δq/Δt, then ΔE = - h+/Δt = - h/Δ*t = - Δ*E. The relation ΔE = - h+/Δt is consistent with the de Broglie relation p = hk/2π for ΔE = - Δ*E and Δt = Δ*t. For a time dependent H ≠ H0, with h+ ≠ h, still ΔE = - h+/Δt is regarded valid.

In order to agree with the above qm relation Δ*p.Δ*q ≥ h for wave packets, variances and differentials should have equal value: this follows from including p/m = Δq/Δt in ΔE/Δt. Indeed for the quantities E, q, and t in the above description there is no mention of variances, instead Δ is interpreted as part of a derivative, i.e. as a Δ variation. While relating the measurement of Δt and Δq to Δ*p, one has to interpret also Δ*p as part of a derivative with Δ*p = Δp. All this follows from the narrative that a stationary wave packet can somehow be "observed" during passing, as is argued when deriving these uncertainty relations [Sakurai, 11].

Due to the Einstein relation E = hν, a stationary state wave packet allows for a "nearly" precise E for each natural frequency ν, and stationary means there is time "enough" (meaning Δt large) for the variance of E to be reduced "enough", [Merzbacher, 12]. However the event of wave packet collapse is not a stationary state event. The value of the variance Δ*E not necessarily has to be small compared to E. Below, applied is the simple equivalence E = - ΔE = - ΔT for the measurement of starlight radiation with frequency ν, arguing that the collapse of the wave function is complete with E(before) = hν = V(after) while no work-function is considered. Then the problem of the value of the variances disappears. All variances from now will be interpreted as variations. Whereas h+ resembles a variation on the "average" constant of Planck h, the reason to exist for h_ is commutation brackets [Δp, Δq] being different from zero if only to the slightest when H is time dependent and $\Delta H \neq 0$. One can indeed verify directly that h_= hpq – hqp is not equal to zero for a time dependent H from equations 15).

When one agrees that $E = T = - \Delta E = - \Delta T$, then $\Delta E = \Delta T < 0$ and $E = h+/\Delta t$ for a positive kinetic energy T while variances and variations have a different sign. Then hqp can be identified with $\Delta T.\Delta t$ for the kinetic energy T. The identification of hpq with $- \Delta V.\Delta t$ follows from the definition of ΔV from the action $– k.\Delta q$ for a generalized $k = \Delta p/\Delta t$.

22a) $\Delta H/\Delta t = \Delta(\Delta H)/\Delta t = - h\_/h+ \Delta E/\Delta t + \partial \Delta H/\partial t$

22b) $\Delta(T - V)/\Delta t = - 2 h+/(\Delta t)^2 = 2 \Delta T/\Delta t$

Notice that $T(tb) \neq 0$ and $V(ta) \neq 0$ while $T(ta) = V(tb) = 0$, and $H0 = T(tb) = V(ta)$, still $\Delta T = - \Delta V$. Leaving the total energy H0 invariant is maintained throughout the description with comparative derivatives to time intervals.

10. TIME INTERVALS AND THE METRIC TENSOR

The principle of least action is often applied to derive a relation between kinetic energy T and the metric path-length $\Delta\rho$ with $(\Delta\rho)^2 = \Sigma ij\ mij\ \Delta qi.\Delta qj$ for a metric tensor mij. One may follow this derivation to find how the metric tensor is related to starlight radiation energy. A Δ variation means that the end points q1(t1) and q2(t2) remain the same, however the total transit time t2 – t1 may vary, in contrast to a δ variation where the total transit time remains constant. At the i-th part of the path this does not necessarily involve a different time variation $|\Delta ti|$ for each i, and $|\Delta ti|$ can be assumed to be the same for all i. For a Δ variation with end points q invariant defined is $|\Delta qi|/|\Delta ti| = ci/|\Delta ti|$, with $|\Delta qi| = ci$ a constant. With this assumption and $mij = \delta ij$ for space symmetric in all directions one finds the standard relation [Goldstein, 1]:

23) $(\Delta\rho)^2 = \Sigma i\ mij\ (ci.cj) = Trace(mij)\ (ci.ci) = 2\ T/m\ (\Delta t)^2$

A metric tensor $mij = \delta ij$ is only valid for Cartesian space coordinates. To describe 4-space a different mij including possibly off diagonal terms and time coordinate parts is needed as is usual in GR. In paragraph 12) an energy change expressed in terms of ΔH and $\Delta H2$ is derived related to the starlight energy Elight = hν. Elight is interpreted as a kinetic energy following the de Broglie relation $p = h/\lambda$, where λ is the wave length of the starlight wave packet and p its “momentum”. Only for a Newtonian situation with p/m = “dq/dt”|Δt and T quadratic in p, equation 23) is directly valid, however it may be assumed to be valid in other situations. The Δt from equation 23) is the same as the Δt from p/m. What is new here is that this brings in direct relation starlight radiation energy hν and time intervals and the metric tensor mij for distances and paths.

24) $h\nu = 1/2\ m\ Trace(mij)\ (ci.ci)\ (1/\Delta t)^2 = 1/2\ m\ Trace(mij)\ c\text{-}light^2 = 3/2\ m\ (mii)\ c\text{-}light^2$

The constant c-light is the velocity of light. The appearance of the “mass” m in a wave description is resolved in the discussion paragraph 13). The time interval Δt in equation 23) and 24) refers to the stationary situation just before measurement and wave packet collapse, and is different from $\Delta t = [tb, ta]$ defined in paragraph 8) which is the same as the time interval Δt of the measurement event. Nevertheless these equations relate in principle time intervals with space intervals and are a basis for deriving a 4-space metric and a metric dependent energy like is usual in general relativity, now in a qm measurement context.

11. TIME INTERVALS AND GENERAL RELATIVITY

In General Relativity metric tensor and distances are related to gravitational energy. Einstein discussed local distances with the concept “standard measuring rod” for local measurements within GR [Einstein, 13]. In [Hollestelle, 14] the concept “dot” is introduced to describe local places and local distances for which step by step addition is possible towards distance measurements beyond locality in GR in a cosmological setting.

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Just like this a step by step method is proposed to measure time intervals beyond the time interval Δt = [tb, ta]. Consider transformation B: t' = (1 – (c.q(t))^2)^(-1) t, similar to transformation A without the overall minus sign. Where A (equation 17) defines the interval Δt = [tb, ta] with tb = - t'(ta), B defines steps from Δt to Δt': from [tb_0, ta_0] = [tb, ta] to [tb_1, ta_1] = [tb'_0, ta'_0] and continuing with [tb_n, ta_n] = [tb'_n-1, ta'_n-1] until the final time interval Δt(n = n2) while for all n interval Δt(n) includes time parts of the future and of the past, like Δt(n = 0) = [tb, ta]. The parameter n, varying from 0 to n2, is different from the n, related to a choice for c, introduced in paragraph 7).

For ΔH at time t the series version is assumed to be valid with only the lowest terms. From paragraph 4) applied is the solution q(t) = q0.t0/t. The result is that ΔH(t'_n) = ΔH(t_n) for all n, when transformation B is written in the following way, with c = c', while the sign of t' remains the same as the sign of t:

25a) B: (t'_n)^2 = 1/2 (t_n)^2 (1 – (c.q0)^2 (t0/t_n)^2)

The lowest terms series version for ΔH(t) is only valid when (c.q0)^2 (t0/t)^2 << 1. According to paragraph 7) however a second requirement is (c.q0) t0/t = n 2π = N with n a certain integer (different than the step defining parameter n) at t = ta_n, and likewise for t = tb_n, together the borders of Δt(n). When t and t' are related through B, it follows c = c' approximately for N >> 1. Both requirements can be achieved by introducing a scale transformation C for t0. C transforms t0 to t0* and this means t* = t t0/t0*, and (c*.q0*) t0*/t* = N (t0*/t0)^2 for (c.q0) invariant with C. When N >> 1 there should be (t0*/t0)^2 << 1/N for the series version in lowest terms to be valid at the t0* scale with (c*.q0*)^2 (t0*/t*)^2 << 1. For this scale ΔH(t*') = ΔH(t*) when t*' = t'(t*) is the transformed of t* with transformation B, and this relation can be rewritten in the following way:

25b) B: (t*')^2 = 1/2 (t*)^2 (1 – (c*.q0*)^2 (t0*/t*)^2)

Identification t* = t_n means ΔH(t*) at t0* scale is saved as an invariant for transformation B. This does not mean ΔH(tb_n) = ΔH(ta_n), since ΔH(t) = G(t) at t0 scale for t equal to ta_n and tb_n where the exponents become equal to 1 by definition of c. At t0 scale the proof that the series version is equal to the exponent version is valid. At t0 scale t'^2 /t^2 < 0, however this t' relates to the next step with transformation B from Δt to Δt' and is not relevant for the equal versions proof that depends on transformation A and ta_n and tb_n. Equation 25b) implies |t*'| < |t*| and by making steps with the reverse of B the requirement for |Δt*(n)| increasing with n i.e. |Δt*(n)| - |Δt*(n-1)| > 0 is fulfilled. The time interval Δt*(n) fulfills the requirements to include both past and future parts when (c*.q0*)^2 (t0*/t*)^2 << 1 which is secured by definition with transformation C. Reversing transformation B to B(-1) implies creating steps from Δt*(n) = [tb_n, ta_n] to Δt*(n-1) and further, and these intervals can be re-named and rearranged interchanging n and n-1 etc. From equation 25b), B(-1) is defined with:

26) B(-1): t*_n = t*'_n-1 = 2^(1/2) t*_n-1 (1 + 1/2 (c*.q0*)^2 (t0*/t*_n-1)^2)

Then |Δt*(n)| = [tb_n, ta_n] = |Δt*'(n-1)| > |Δt*(n-1)| for B(-1) for all n, and re-defined is Δt*(n = n2) for B to Δt*(n = 0) for B(-1) to be equal to [tb, ta] which is the original time interval Δt at step 0. Equation 26) can be approximated with t*' = 2^(1/2) t* and thus after each step from Δt* the next interval will encompass again times that always fulfill the requirement for B(-1), i.e. (c*.q0*)^2 (t0*/t*)^2 << 1. However t = t* t0*/t0 and the second requirement reads: (c .q0) t0/t = n 2π = N and after rewriting: N = (c.q0) t0/t* t0/t0*. When c does not change, N will be proportional to 1/t*. This means with N = n 2π the lower limit for N is n = 1 and for t* similarly (c.q0) t0 (t0/t0*) (n = 1)/2π and for this t* the maximal time interval after the last step is reached.

Started is from Δt*(n = 0) = [tb, ta] that is a "local" time interval that can be given a measure. With each step the interval borders tb and ta are further transformed with B(-1) to result in beyond local however measurable time intervals Δt*(n) with ΔH(ta/b*_n) = ΔH(ta/b*_n-1). Because of the requirements from paragraph 7) there are lower and upper limits for these intervals. These limits on the time interval measure |Δt*(n)| also indicate that time interval dependent functions or quantities, that depend on Δt*(n) as time domain, can be giving an interpretation related to qm probability. This is an interesting result in its own right. The term “local” is a three-space term, for time intervals the term “timely” is preferable. The transformation B(-1) completes the description of the step by step method for the integration of measurements of “timely” time intervals to beyond “timely-ness”, considered as a basis for all time interval measurements in General Relativity.

12. STARLIGHT RADIATION ENERGY IN A QM MEASUREMENT

To describe the collapse of a wave packet during a QM measurement of starlight radiation with a time dependent Hamiltonian H(t) = H0 + ΔH(t) started is from equation 14b), the series version: ΔH (t) = h/t (1 + (c.q)^2 + …) for t just before tb or just after ta. ΔH(tb) differs from ΔH(ta) when wave packet collapse, during a non-stationary state measurement event, re-emerges in the time dependence of H during Δt = Δtbta. For times t < tb and t > ta the Hamiltonian remains stationary and is equal to its value at tb and ta respectively:

27a) ΔH(tb) = h/tb (1 + (c.qb)^2 + …) with qb = star source space coordinate ≈ average distance to the starlight wave sphere measured from the zero space coordinate place qi ≈ starlight wave sphere radius rs at time tb”

27b) ΔH(ta) = h/ta (1 + (c.qa)^2 + …) with qa = measurement place space coordinate

At time tb the starlight wave has reached the space origin at qi. In the case of two measurement apparatus, measurement at one of these will exclude measurement at the other since the complete wave has collapsed to one place. The starlight radiation wave is regarded as one unity, and measuring the wave energy means counting its wave packets at a certain place qa. Somehow a light wave from a star source at a time t is related to a certain propagation sphere radius r(t), related to the velocity of light, and r(tb) = rs. During a measurement time interval Δt, wave occurrences can be measured or counted a number of times #n, depending on the initial energy E*, emitted during a similar time interval, that corresponds to the number of stationary state wave packets at t < tb: E* = #n hν. During measurement event Δt counted are not just one wave packet, rather the complete wave and all the #n wave packets, with the complete energy E* arriving at q(ta) = qa. This agrees with the traditional qm description of wave radiation measurements and wave packet collapse [Wichmann, 15], and the description of qm measurements in a cosmological context in [Hollestelle, 14]. Chosen is for a wave packet collapse description rather than a probability description to remain near to the above wave picture of light including light propagation. In the following #n and its relation to the star source energy E* will not be further specified, however it is possible that #n = 1, when the light wave just consists of one wave packet. #n = 0 does not easily agree with measurement of #n, it then seems no star light is detected during Δt.

The measurement event at qa near qi can be chosen with (c.qa) << 1. qi is, like qa, at a distance rs to the star itself and thus part of the starlight wave sphere surface at time tb. Starlight E* is assumed to originate from the star without preferred direction and appears at distances r(t) from the star simultaneously, where r(tb) = rs is approximately the same as the average distance of the starlight wave sphere surface to the origin qi, with rs = |qb|. Then E(t < tb) = E(complete) = E* = #n hν with ν the constant light wave frequency. In the following all H, L, p and q describe properties of one wave packet. For the complete wave is used a subscript c: E*= Ec = #nE, etc. One assumes that for one wave packet energy E equals hν = T(tb) = - ΔT, when light wave energy is considered to be kinetic. V(tb) = 0 and T(ta) = 0 and V(ta) = T(tb), when all energy after the collapse is included in V. The total energy H0 = T + V is conserved throughout the collapse event.

Evaluated are the difference between < H(tb) > for a time interval just before the collapse of the wave packet and < H(ta) > for a time interval just after the collapse of the wave packet, applying equation 3).

28) < H(ta) >|Δta - < Hc(tb) >|Δtb = 1/2 (pa.xa – pb.xb) – 1/3 ((“dL/dx”|Δxa).xa - (“dLc/dx”|Δxb).xb)

Again, subscript c means all #n wave packets together for t near tb, while at ta no subscript is used since the wave has collapsed at those t near ta. As before x = “dq/dt”Δt. There is Hc(tb) = #n hν + ΔHc(tb), with again ν the wave

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frequency. Hc(tb) just before the event is time independent, thus for its average one may write the value at tb: < Hc(t < tb) > = H(tb), and similarly < H(t > ta) > = H(ta): before tb and after ta a stationary state is assumed. From the uncertainty relations, recalling paragraph 9), |pb| equals h/|qb| = h/rs for Δp = - p and Δq = - q. Both p and q are independent of #n. ΔHc(tb) follows from the Legendre transform relation for L and H, evaluated for #n=1 and for unspecified #n, while pb.xb remains the same for both cases: pb.xb = #n 2T(tb) + ΔHc(tb) = 2T(tb) + ΔH(tb) with T(tb) = E = hν. Equation 28) then reads as:

29) #n hν/Δt = #n H0/Δt + Δ(ΔH(ta) - ΔHc(tb))/Δt - 1/3 Δ(L(ta) - #n L(tb))/Δt – 1/2 Δ(pa.qa – pb.qb)/Δt

Solving ΔHc(tb) as a function of #n, and with ΔH2 from equation 19), the following two sets of equations follow, each set for #n unspecified and for #n = 1:

30a) hν = - 3/2 (2 #n – 3)^(-1) ΔH2(tb,ta) / hν = + 3/2 ΔH2(tb,ta) (#n = 1)

30b) hν = + 3/2 (2 #n – 3)^(-1) (ΔH(ta) - ΔH(tb)) / hν = - 3/2 (ΔH(ta) - ΔH(tb)) (#n = 1)

These equations do not imply that the frequency ν depends on the right side quantities like #n, rather ν depends only on the properties of the star source and the variable is ΔH. Applying the relation between ta and tb from equation 17) one finds for #n = 1, in the series version:

30c) hν = 3/2 h (1/tbta) Δtbta (- 1 + 1 + (c.rs)^2 + …)

ΔH2(tb,ta) is a function of the interval Δtbta and its borders ta and tb while ΔH(t) is a function of time moments t. With ΔH2(tb,ta) the description of the time interval dependent H is complete. Comparing equations 30) with equations 22) it follows:

31) ΔH2(tb,ta) = - exp (- (c.qa)i) h/tbta Δtbta exp(+ (c.qb)i) = - 1/2 h_/Δtbta = -2/3 (2#n – 3) hν

A relation h_/Δtbta^2 and h/tbta exists, that corresponds with the relation of Δtbta with its borders tb and ta. The wave packet energy E = hν is a kinetic energy and is positive since ν is a counting parameter, counting occurrences per time unit. Then according to equation 30b) ΔH(tb) decreases to ΔH(ta) with ΔH(tb) > ΔH(ta) and ΔH2(tb,ta) > 0 for #n = 1. For this situation with only one wave packet ΔH2 = 2/3 hν. For all #n > 1 the relation is: ΔH(tb) < ΔH(ta) and ΔH2(tb,ta) < 0. Not considered are negative energies like for instance appear in Dirac's theory of relativistic quantum mechanics. The possible influence of a work-function is not considered either. A proof that with ΔH2 from equation 19) the above equations 30a) and 30b) correspond to the same frequency ν is given in appendix A). A specific choice for the constant c is needed for this: (c.rs) = 2π and this means that, depending on naïve quantization of the complete wave, at time tb: c = ν/c-light (2π^2) with c-light the velocity of light. The constant c can be measured from observation of E* or c-light and ν, apart from (c.rs) = 2π and an estimate for rs. A measurement for rs is an indirect test for the light wave measurement and wave collapse description in terms of measurement event time interval Δt.

13. DISCUSSION

Spatial distance measurements allow for translations of a local measurement place since space is translation invariant. Translation of measurement event time intervals that are "local" or "timely" is not possible because the time coordinate and time interval Δt are not translation invariant. The interval Δt changes from event to event while the relevant equilibrium changes with it. The intuition, based on time experience, is that a time interval should be asymmetrical. With translation invariance a symmetric time interval defined with Δt = [-ta, ta] like is possible for space intervals could have been possible. Then the equilibrium does not change when ta, and –ta with ta, changes. The symmetrical and anti-symmetrical properties of the time interval Δt = [tb, ta] depend on (n) and (i) and define the change of the equilibrium. The time equilibrium of the asymmetric "slope", with the "mean velocity theorem", is

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a liberation of one value time averages to a changing time interval. This description with finite time intervals seems to be justified at least for situations with time dependent events, events with change, and a time dependent Hamiltonian. Re-writing the result from paragraph 8): time intervals that are symmetric and infinite relate to H time independent and time intervals that are asymmetric and finite to H time dependent.

Curie's principle can be applied directly to state that finite(…) asymmetrical time intervals are real for events with a time dependent Hamiltonian. However this is not a statistical interpretation of time, like for instance the qm time interpretation of Campbell [Beller, 10] because the time coordinate is not defined to be probabilistic, rather with elements (n) and (i). It is questionable whether (n) and (i) that define [tb, ta] support time reversal symmetry for GR. The discussion in paragraph 7) by including time intervals to Curie's principle with emphasis on the realness of change in time needs interpretation in qm.

Frequency is a property of a wave phenomenon, while time is a coordinate of 4-space. With counting by frequency one means counting occurrences within a time interval, which is a finite event time interval. Counting by time rather means counting time itself till the (next) occurrence, which means matching to an in-definite event time interval. Counting by frequency can be meaningfully repeated giving finite results. Counting by time does not allow for a zero occurrence result. For the time interval infinite, Δt measures become non additive (paragraph11).

The time interval dependent energy quantity ΔH2 = -"dΔH/dt"|Δt Δt equals -2/3 (2#n – 3) hν, following equations 30) and thus ΔH2 is related to wave frequency ν, a counting parameter. The number of occurrences #n itself is expected to depend on ν in a complex way, and inferred is that #n is proportional to |Δt| at least for |Δt| >> 1: the measurement time interval Δt relates to the radiation time interval of the star source and thus to #n. This means the time interval description has a direct interpretation with counting and measurements including measurable properties like #n and ν. The interpretation of qm measurements and wave packet collapse is not conclusive or definitive, [Beneducio, 16], [Van Kampen, 17]. Van Kampen discusses entropy change in relation to qm measurements. This is interesting in relation to the result in paragraph 12) concerning the change of ΔH during wave packet collapse.

The three-space metric tensor mij has the well known property [Goldstein, 1]: Trace(mij) is proportional to kinetic energy T. Positive kinetic energy and positive metric distances are expected to occur together. The kinetic energy relation for starlight T = Elight derived in paragraph 12) is: Elight = hν (ΔH stationary when t < tb, earlier than Δt, or when ΔH remains time independent during Δt) and Elight = 3/4 (2#n – 3)^(-1) h_/Δt (ΔH time dependent during Δt) thus defines a metric in 4-space. Wave energy with constant value hν exists only when the light wave is stationary however the value itself of ν is a remaining constant. h_ is variable with #n and Δt and can only be non zero during time interval Δt, and only when there is interaction.
For a light wave with velocity c-light there is Trace(mij) = 2 Elight /mc-light^2 and this can be interpreted as a metric tensor mij for which Trace(mij) relates to both the wave "path" and to its frequency. The factor mc-light^2 including a certain mass m is just a way of writing remaining from the discussion in paragraph 10) starting from kinetic energy and is re-written below within the wave description. In 4-space the metric free path length including the time coordinate is (Δτ)^2 = Trace(mij) Δq^2 - mtt c-light^2 Δt^2. For a "local" 4-space distance and assuming that during the measurement time interval Δt, including wave packet collapse, the light velocity property does not alter, (Δτ)^2 remains zero and the time part of the 4-space metric tensor remains mtt = 2 Elight/mc-light^2 (Δq/Δt)^2 1/c-light^2. The value of (Δq/Δt)^2 resembles an apparent light velocity when assumed constant, however it is determined by and varies with the measurement event properties and is not a natural quantity.
The metric tensor part mtt can be evaluated when during the measurement event ΔH is time dependent. During Δt, following the description in paragraph 12), Elight depends like above on h_ only when ΔH time dependent and h_ is non zero, and it is found: h_ = p.q – q.p = - 2 Δp/Δt q0t0 and also h_ = 2 (ΔH(ta) - ΔH(tb)) Δt cos(p, q) where equilibrium equations 11) and equations 30) are applied. With these expressions for h_ the factor cos(p, q) re-emerges from the scalar product of the vectors Δp = - p and Δq = - q (paragraph 12). The energy mc-light^2 equals hν when the light wave is stationary however it can be assumed to be equal to h+/Δt during event time interval Δt. Thus during Δt the metric time span mtt Δt^2 is equal to 3/2 (2#n - 3)^(-1) h_ /h+ (Δq/Δt)^2 c-light^(-2) Δt^2. When

during Δt there is ΔH(ta) - ΔH(tb) = 0, and ΔH remains time independent, h_ = 0 and p.q = q.p, and the light wave does not interact, regardless of Δt. In this case the description of paragraph 12) when relating Elight and h_ does not apply.
The factor cos(p, q) depends on the angle by p and q and equals zero for an applied force perpendicular to the light wave "path", due to the relation Δp = - p. Thus again one finds h_ = 0 and indeed the relation for Elight and h_ does not apply. However in this case ΔH can still change with ΔH(ta) - ΔH(tb) ≠ 0 and thus can be assumed to be correlated with a asymmetric finite Δt, following the discussion in paragraph 8). Δt then is also a occurrence time interval, and its interpretation gains natural event value. Of course measurement events and natural events could somehow be related when one intends to measure a property relating to a natural event, i.e. perform an experiment in physics like for instance measuring the frequency of a light wave emitted by a star source. However this is not any further specified in this article. When Δt infinite, there is again ΔH(ta) - ΔH(tb) = 0, meaning there is no interaction and no wave collapse.
It is inferred that a light wave from a star source due to such a applied force can follow a complete finite circle path as path of propagation during a finite time interval Δt only when ΔH changes during Δt. The change of ΔH in this description is positive (#n > 1) or negative (#n = 1) all due to wave collapse. Assuming the reverse is possible just as well (like for a light wave originating from a star source), this would mean, in cosmology, a radiation energy collecting/losing universe where energy is unlocked/stored in a unknown way, indicating for instance a process or interaction with dark energy, this energy being non radiative. Alternatively a frequency shift could occur, thus altering the energy of a light wave packet, however this being extra ordinary. A dark energy increase and increasing V would mean an entropy increase for the radiation part T of the universe, due to wave collapse. A decrease of V can be imagined, for instance at a source where radiation emerges, say dark energy collectively being the origin of light waves propagating away from this source. Also this inference gives some support for ergodic theory that predicts that exact re-occurrence, in continuous space, does exist only within a infinite time interval.

The time interval of the measurement event depends on the measurement specifications and can be chosen |Δt| >> 1. The distance of the investigated star depends on the star source choice, and if this source can be identified it could be one for which its distance is determined very securely and fixed, and for which |rs| >> 1. Even with |Δt| >> 1 according to the result in paragraph 11) the measurement event time interval can be measurable when taking care of the limits mentioned there. This has cosmological implications and still is subject of further study. Largest time intervals and largest distances rs can thus be related through the metric tensor mij.

APPENDIX A): EQUATIONS 30) and 31)

The final result from paragraph 12) with equations 30a/b) and equation 31) is:

hν = 3/2 h/tatb Δt (- 1 + 1 + (c.rs)^2 + …) = 3/2 ΔH2(tatb) = - 3/2 exp(- (c.ra)i) h (1/tbta) Δtab exp(+ (c.rs)i)

The space coordinates qb and qa are represented by rs and ra. The series is derived from the product of Taylor series for the exponents exp (- (c.rs)i) and exp (+ (c.rs)i). This is defined in paragraph 7). The constant c is yet unspecified, however a choice is proposed later on to simplify the result for this case. When this choice is applied earlier the final result for any c cannot be derived. There is:

32) exp(-(c.rs)i) exp(+(c.rs)i) = (1 + (-c.rsi) + 1/2! (-c.rsi)^2 +…) (1 + (c.rsi) + 1/2! (c.rsi)^2 +…)

33) = 1 + Σ|n (1/n!) ((c.rsi)^n + (-c.rsi)^n)) + Σ|n (Σ|k (1/k!n!) (-c.rsi)^k (c.rsi)^n + (c.rsi)^k (-c.rsi)^n))

Both summations in equation 33) start from k, n = 1 to infinity. For k - n = even the terms in the second summation equal 2 (-1)^k (c.rsi)^(k + n). For k - n = not even the terms of this summation equal zero because (-1)^k = - (-1)^n. One gathers together all terms of order v in (c.rsi), while maintaining that for v = even: k = even and for v = not even: k = not even, for the non zero terms. Then the second summation in equation 33) can be written as:

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34) Σ|v=even Σ|k=even 2 (1/k!(v-k)!) (-1)^k (c.rsi)^v + Σ|v= not even Σ|k=not even 2 (1/k!(v - k)!) (-1)^k (c.rsi)^v

Summation is only from k = 1 till k = v. One can insert the binomium equality k! (v - k)! = v! (v/k)^(-1) where the symbol (v/k) means the usual "v over k" to derive the following complete expression:

35) 1 + Σ|v (i)^v (1/v!) ((c.rs)^v + (-c.rs)^v) + Σ|v 2 (i)^v (1/v!) (c.rs)^v (Σ|k (-1)^k (v/k))

The summation for k from k = 1 till k = v, at the right side, equals: - 1 + Σ|k (-1)^k (v/k) = - 1, and to this summation, the k = 0 term is added. One arrives at:

36) 1 + Σ|v (i)^v (1/v!) (-(c. rs)^v + (-c. rs)^v)

Every term with v = even is zero. The above derivation can easily be repeated with the first exponent including ra instead of rs. Recall (c.ra) << 1. One then remains with:

37a) exp(-(c.rai)) exp(+(c.rsi)) = 1 + Σ|v (i)^v (1/v!) ((-(c.rs)^v + (-c.ra)^v) + 2 (-1 + (-1)^v) (c.rs)^v)

= exp(-(c.rs)i) exp(+(c.rs)i) + Rest

37b) Rest = Σ|v (i)^v (1/v!) ((-c.ra)^v + (-c.rs)^v – 2 (c.rs)^v)

= exp(-(c.ra)i) + exp(-(c.rs)i) - 2 exp(+(c.rs)i) = - 2 (exp(+(c.rs)i) – exp(-(c.rs)i) - exp(-(c.rs)i) + exp(-(c.ra)i)

From this it follows that Rest = 0 is a good estimate for a specific choice for the constant c. The result is:

38) exp(-(c.rs)i) exp(+(c.rs)i) + Rest = exp(-(c.ra)i) exp(+(c.rs)i)

The constant c within (c.rs) can be chosen such that there is (c.rs) = 2π and exp (+/-(c.rs)i) = exp(+/-2πi) = 1. Including exp(-(c.ra)i) ≈ 1 for (c.ra) << 1, this means for this choice for c, Rest = 0 as expected. Notice that for (c.rs) = 2π the exponents in ΔH are equal to 1. Equation 38), for any choice for c, resulting in exponents in ΔH possibly differing from 1, and with a possibly non zero Rest function, is the main result of this appendix.

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