

Optimal Parking Occupancy with and without Differentiated Parking: A Macroscopic Analysis

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A very high parking occupancy can negatively influence the traffic performance of an area by causing very long cruising times. A very low parking occupancy, on the other hand, is inefficient from a space utilization perspective. Thus, this paper proposes a framework to compute the optimal parking occupancy rate over a given time horizon based on a macroscopic traffic and parking model. This rate is set high enough to ensure an efficient usage of the parking infrastructure. However, it should also guarantee a high likelihood of finding parking in order to eliminate the drivers' time wasted in cruising for parking and the added congestion it causes. The model outputs are based on small data collection efforts and low computational costs, and they can be generated without complex simulation software using a simple numerical solver. Multiple vehicle types are included into our methodology allowing us to generate insights about the optimal parking occupancy with or without differentiated parking (i.e., parking for specific vehicles, such as fuel and electric vehicles). In times of a modal shift towards electric vehicles, cities can use our model to evaluate how much parking supply (with battery charging opportunities) they would like to dedicate to electric vehicles in order to achieve optimal traffic and parking results, and whether a differentiated or semi-differentiated parking policy is desirable. We illustrate our framework in a case study of a central area within the city of Zurich, Switzerland, showing the traffic and parking impacts (e.g., average searching time for parking, total revenue created by parking fees, optimal parking occupancy rate) for different proportions of fuel and electric vehicles in the parking demand and/or supply. Our results confirm that optimal occupancy rates are between 80 % and 90 % for most realistic scenarios. We then discuss how these rates might change depending on various demand and supply relationships, and according to different parking policies. We show that equal proportions between electric vehicles in the demand and their parking spaces in the supply lead to the best traffic performance in the area. We also provide the tools for cities to analyze their loss in performance if they do not react, e.g., to an increasing demand for electric vehicles over time. Moreover, we illustrate how some of these risks can be mitigated by having more flexible parking policies, e.g., allowing electric vehicles to use parking spaces for fuel vehicles.

Keywords: optimal parking occupancy rate; macroscopic traffic and parking model; multiple vehicle types; electric vehicles; cruising-for-parking; parking policies.

1. Introduction

Traffic congestion and its associated costs (i.e., loss in time, loss of productivity of workers sitting in traffic, increase in cost of transporting goods through congested areas, waste of fuel) often conflict with an efficient usage of the parking infrastructure. A rise in on-street parking occupancy is not always positive. A very high parking occupancy rate can drastically reduce drivers' likelihood of finding parking, increasing cruising times and leading, in turn, to a worse traffic performance in the network. A low parking occupancy, on the other hand, is inefficient from a space utilization perspective. The problem becomes more complicated with differentiated parking (e.g., specific parking spaces for fuel and electric vehicles such that electric vehicles can charge their batteries while parking). Our study proposes a macroscopic model to determine the optimal parking occupancy rate to minimize the impacts on traffic and at the same time maximize the usage of the available parking spaces. Moreover, we analyze how the optimal parking occupancy might be affected by a modal shift towards a specific vehicle type such as electric vehicles, with and without differentiated parking.

The stochasticity of vacant on-street parking spaces and their impact on the traffic performance is often underestimated. Even if on-street parking spaces in an area might have low occupancy rates at specific times of the day, there might be times when drivers must spend a considerable amount of time cruising for parking ([Shoup \(1999, 2005, 2006\)](#)). [Vickrey \(1954\)](#) was the first to discuss the possibilities of achieving a specific on-street parking occupancy rate such that there is a parking space on each block available at almost all times. His proposal decreases congestion and the cruising-for-parking time on the network, and it could be achieved through smart on-street parking pricing policies. However, the required technology was beyond the means at that time. [Arnott et al. \(1991\)](#) proposed different parking meter rates across time at different locations during the morning rush-hour in a downtown area to be able to control the order in which on-street parking spaces are occupied. They used network equilibrium models to regulate traffic and parking usage with the help of their parking fee policy. This leads to low-income workers trying to avoid paying high parking fees and parking further away from their destination in the city center. Their model, however, assumes that traffic performance parameters (e.g., travel speed) are fixed for all model conditions, i.e., traffic performance dependencies are not included in their framework. [Arnott and Inci \(2006, 2010\)](#), [Arnott and Rowse \(2009, 2013\)](#) and [Arnott et al. \(2015\)](#) examined stationary on-street parking policies to determine the relationship between cruising-for-parking, the on-street parking occupancy, and the congestion in a downtown area. The studies focused on social optimum and user equilibrium methodologies. [Anderson and de Palma \(2004\)](#), [Zhang et al. \(2008, 2011\)](#) and [Qian et al. \(2012\)](#) investigated agent-based parking pricing models or alternative downtown parking policies incorporating on-street parking occupancy rates in order to improve the traffic performance. [Wang et al. \(2019\)](#) derived a bi-modal traffic equilibrium model to investigate the optimal parking supply considering the scale economy of transit. A real-time parking pricing approach focusing on space utilization and parking access was developed as a dynamic Stackelberg leader-follower game theory model in [Mackowski et al. \(2015\)](#). [Zakharenko \(2016\)](#) developed a uniform parking pricing scheme for all parking sites focusing on the parkers' arrival rates and the parking occupancy rate in a heterogeneous parking environment. He showed that the purpose of pricing methods affects more the parking departures than the arrivals. A real-time parking pricing methodology for a parking lot was modeled in [Qian and Rajagopal \(2013\)](#) using its occupancy rate and a system optimal parking flow minimization problem. [Qian and Rajagopal \(2014\)](#) developed a parking pricing model under stochastic demand minimizing the total travel time of the system according to real-time parking occupancy data collected by parking sensors. This model was later extended to include departure time and parking location choices ([Qian and Rajagopal \(2015\)](#)). [Tamrazian et al. \(2015\)](#) proposed efficient learning algorithms to predict parking occupancy rates using historical and real-time data. [Javale et al. \(2019\)](#) developed a smart parking pricing algorithm using electronic IoT-based sensors to determine the optimal parking fee depending on various factors including the current occupancy rate, the time of day and the parking space locations

within the network. This agent-based methodology can also be used to predict the future occupancy rate in the area. [Lei and Ouyang \(2017\)](#) modeled an intelligent parking system with a dynamic parking reservation strategy and a parking fee varying by the chosen location such that the drivers should make parking reservations for a future time period prior to their trips. Compared to these long-term demand management strategies taking into account user equilibrium and drivers' competition, [Jakob et al. \(2018\)](#) developed a dynamic macroscopic parking pricing scheme focusing on the short-term effects of congestion and traffic performance. The framework not only uses the parking occupancy but also the searching traffic to maximize the revenue for a city while simultaneously minimizing the total searching time in the area. [Cao and Menendez \(2015\)](#) presented a macroscopic methodology to estimate the effects of parking on urban traffic dynamics accounting for the competition between vehicles searching for an available parking space. They later validated the model using data from the city of Zurich, Switzerland, in [Cao et al. \(2019\)](#). [Cao and Menendez \(2018\)](#) extended the methodology to quantify the potential cruising time savings associated with intelligent parking services. [Leclercq et al. \(2017\)](#) used the macroscopic fundamental diagram (MFD) from [Geroliminis \(2009, 2015\)](#) and [Geroliminis and Daganzo \(2008\)](#) to analyze the relationship between the aggregated travel distance before parking and the on-street parking occupancy in an urban network. Based on a trip-based MFD formulation using experimental data from the city of Lyon, France, parking search laws were investigated to understand how the distance to park behaves when the parking occupancy rate changes dynamically. [Liu and Geroliminis \(2016\)](#) and [Zheng and Geroliminis \(2016\)](#) used an MFD approach to develop a dynamic parking pricing methodology in order to analyze the cruising-for-on-street-parking and the impact of parking capacity and duration focusing on the commuters' morning peak.

As real-time pricing schemes require a lot of information, they are harder and more expensive to implement compared to policies setting the parking meter rate *ex ante* by block and time of day to achieve a target on-street parking occupancy rate. [Shoup \(1999, 2005, 2006\)](#) developed an on-street parking scheme according to a target parking occupancy rate and suggested setting this rate to 85 %. We confirm these findings using our model and discuss how these rates might change depending on various demand and supply relationships following different parking policies. A modified version of his proposed scheme has been implemented in San Francisco using empirical data collected by parking meters for on-street parking spaces ([SFpark \(2009\)](#); [Millard-Ball et al. \(2014\)](#); [Pierce and Shoup \(2013\)](#)). [Arnott \(2014\)](#) developed a simple, structural model for steady-state on-street parking to determine the optimal on-street parking occupancy rate on a block in the network. The static results show that the optimal occupancy and parking meter rates are dependent on the parking demand, i.e., at busier blocks with a higher parking demand the target parking occupancy rate should be set to a higher value. [De Vos and van Ommeren \(2018\)](#) focused on the effects of parking occupancy rates on walking distances towards the drivers' destinations in a residential area in Amsterdam, Netherlands. The drivers' walking distances only increase when the parking occupancy rate exceeds 85 % in the area. [Zakharenko \(2019a\)](#) used the information from parking occupancy sensors to help drivers during their search for an available parking space. Increasing the parking price in congested areas with parking sensors can lead to a higher turnover rate for parking spaces. However, it is not necessary to install parking sensors for all parking spaces. By considering price discrimination and by pricing parking locations differently, it is possible to set the optimal parking fee for sensed parking lower compared to non-sensed parking spaces. [Zakharenko \(2019b\)](#) enhanced this work by investigating when it is more reasonable to steer heterogeneous drivers away from available parking spaces in order to reserve privileged parking spaces. This framework uses second-best pricing policies and is extended by studying parking for drivers with special needs.

In contrast to existing studies focusing on agent-based searching-for-parking traffic, our model is based on much less data requirements and has lower computational costs. A city can simulate their target parking occupancy rate over, e.g., the peak hours of the day, to guarantee an optimal trade-off between an efficient usage of the parking infrastructure

and a high likelihood of finding parking as to improve the traffic performance in a central area. The results can then be used to set the optimal parking occupancy rate *ex ante* and to establish measures (e.g., parking pricing policies ([Jakob and Menendez \(2020\)](#))) in some areas in order to achieve this target rate in the future. These measures, however, are considered out-of-scope in this paper. Our study defines the parking occupancy rate to be optimal by trying to minimize cruising time for all vehicles. At the same time, the parking occupancy rate is set to a high enough value to ensure an efficient usage of the parking infrastructure. The model outputs can be generated with a simple numerical solver and without complex simulation software.

The contributions of this paper are threefold. First, our research proposes a framework to compute the optimal parking occupancy rate based on a macroscopic traffic and parking model. We determine this single optimal rate over a given time horizon for an area within a city. Second, the extension of our macroscopic model to include multiple vehicle types provides us some insights about the parking occupancy's dependency on specific vehicle types (e.g., fuel and electric vehicles). We analyze a differentiated parking policy with vehicle type dependent parking spaces (e.g., fuel vehicles park at fuel vehicle parking spaces, and electric vehicles park at spaces with battery chargers), and a semi-differentiated parking policy, considering no parking space restrictions for some vehicle types (e.g., electric vehicles can park at both parking spaces for fuel and electric vehicles). We then compare these two policies to a parking scheme without any parking differentiation. In all cases, our framework allows us to analyze the traffic and parking impacts (e.g., average searching time for parking, total revenue from parking pricing, optimal parking occupancy rates) of a modal shift towards a specific vehicle type, such as electric vehicles. Cities have the option to evaluate how to react towards a constantly varying parking demand and how much parking supply to dedicate to electric vehicles in order to have the best balance between traffic performance, optimal parking occupancies, and a high revenue for the city. Third, our methodology offers quick evaluation possibilities for the impacts on the optimal parking occupancy rate caused by a change in parking demand, supply, or parking duration in the network. We illustrate our proposed model using real data from a central area within the city of Zurich, Switzerland.

This paper is organized as follows. Section 2 presents the strategy to determine the optimal parking occupancy rate based on a macroscopic traffic and parking model without and with parking differentiation using multiple vehicle types. Section 3 illustrates the use of the methodology to find the optimal parking occupancy for an area within the city of Zurich, and discusses the impact of different modeling inputs. Section 4 concludes this paper.

2. The optimal parking occupancy rate: A macroscopic model for multiple vehicle types

First, we explain our macroscopic traffic and parking framework differentiating multiple vehicle types (section 2.1). Second, we present our optimization strategy to find the optimal parking occupancy rate in an area (section 2.2). Third, we show how to determine the average cruising time for parking and the parking revenue for the city over a defined time horizon (section 2.3). The main variables and parameters used in this study are introduced in Table 1.

Table 1. List of main variables and parameters.

Notation	Definition
L	Size (length) of the network.
T	Length of the simulation's time horizon.
t	Length of a time slice.
E	Set of vehicle types indexed by e . This might include, e.g., fuel and electric vehicles.
K	Set of user groups for the network's demand indexed by k . Each user group has a different value of time (VOT).
A^e	Total number of public parking spaces for vehicles of type $e \in E$.
$A^{i,e}$	Number of available parking spaces for vehicles of type $e \in E$ at the beginning of time slice i .
VOT^k	VOT for user group $k \in K$.
p^e	Hourly parking fee for vehicles of type $e \in E$.
\bar{t}_d^e	Average parking duration of vehicles of type $e \in E$.
i_{cmax}	Time slice in which the number of vehicles having started to cruise for parking which is required to determine the maximum cruising time per vehicle for vehicles cruising at the beginning of time slice i .
ACT^i	Average cruising time for vehicles parking at the beginning of time slice i .
ACT_{all}	Average cruising time for parking across all vehicles over the whole time horizon T .
occ^i	Parking occupancy rate across all parking spaces at the beginning of time slice i .
$occ^{i,e}$	Parking occupancy rate of parking spaces for vehicles of type $e \in E$ at the beginning of time slice i .
I	Total revenue resulting from parking fees for all user groups K over T .
ε	Proportion of electric vehicles within the traffic demand entering the area for all time slices.
ζ	Proportion of parking spaces for electric vehicles with battery charging possibilities compared to the total parking supply.

2.1. A macroscopic traffic and parking framework for multiple vehicle types

This section extends the parking-state-based matrix in [Cao and Menendez \(2015\)](#) for multiple vehicle types as they not only vary within the demand, but also have different parking needs, e.g., an electric vehicle might be charged while parking. The matrix consists of different parking-related traffic states and the transition events between those states. All trips are assumed to be made by cars, i.e., the mode choice has been previously made. The same is true for the decision between different types of vehicles. Our framework evaluates a compact urban area with a relatively homogeneous network of length L . E denotes the set of vehicle types (e.g., electric vs. fuel), and K represents the set of user groups for the network's demand. Each user group has a different value of time (VOT), VOT^k . These user groups can be dependent on the drivers' residence location, income, career, working state, etc. The number of vehicles in each traffic state for vehicle type $e \in E$ and user group $k \in K$ are updated iteratively over time based on the number of vehicles in each transition event. These iterations finish when the whole time horizon is evaluated, or a defined criterion is reached (e.g., all the cars leave the area). The time horizon T (e.g., a day) is divided into small time slices t (e.g., 1 minute), such that the traffic and parking conditions are assumed to be steady within each time slice, but they can change over multiple time slices. The variables in Table 2 are restricted to the variables required to determine the optimal parking occupancy rate in section 2.2. Details about their mathematical formulations can be found in [Cao and Menendez \(2015\)](#), but a summary is given here for the readers' convenience. The number of vehicles searching for parking at the beginning of time slice $i + 1$, $N_s^{i+1,k,e}$, is updated in Eq. (1) for all $k \in K$ and $e \in E$. Vehicles starting to search for parking in the area (i.e., $n_{nsi/s}^{i,k,e}$) join this state, and vehicles accessing parking (i.e., $n_{s/p}^{i,k,e}$) leave this state.

$$N_s^{i+1,k,e} = N_s^{i,k,e} + n_{nsi/s}^{i,k,e} - n_{s/p}^{i,k,e} \quad (1)$$

We determine the number of vehicles parked at the beginning of time slice $i + 1$, $N_p^{i+1,k,e}$, in the area in Eq. (2) for all $k \in K$ and $e \in E$. Vehicles accessing an available parking space (i.e., $n_{s/p}^{i,k,e}$) join this traffic state, and vehicles departing from parking for an external destination (i.e., $n_{p/nse}^{i,k,e}$) leave this state.

$$N_p^{i+1,k,e} = N_p^{i,k,e} + n_{s/p}^{i,k,e} - n_{p/nse}^{i,k,e} \quad (2)$$

The transition events $n_{nsi/s}^{i,k,e}$, $n_{s/p}^{i,k,e}$, and $n_{p/nse}^{i,k,e}$ are estimated macroscopically based on the size of the network, the likelihood of finding parking, and the distribution of parking durations, respectively (for details see [Cao and Menendez \(2015, 2018\)](#)). Notice that we do not need to record the individual locations of each vehicle and parking space over time, i.e., only the average number of vehicles in each traffic state and transition event during each time slice is tracked. All traffic state variables need an initial condition as an input to the model that can be measured, assumed or simulated.

Table 2. The traffic state and transition event variables used to determine the optimal parking occupancy rate.

	Notation	Name	Definition
Traffic states	$N_s^{i,k,e}$	Searching for parking	Number of vehicles of type $e \in E$ for user group $k \in K$ searching for parking at the beginning of time slice i .
	$N_p^{i,k,e}$	Parking	Number of vehicles of type $e \in E$ for user group $k \in K$ parking at the beginning of time slice i .
Transition events	$n_{nsi/s}^{i,k,e}$	Start to search for parking	Number of vehicles of type $e \in E$ for user group $k \in K$ starting to search for parking during time slice i .
	$n_{s/p}^{i,k,e}$	Access parking	Number of vehicles of type $e \in E$ for user group $k \in K$ accessing parking during time slice i .
	$n_{p/nse}^{i,k,e}$	Depart parking	Number of vehicles of type $e \in E$ for user group $k \in K$ departing from parking and moving towards a destination outside the area during time slice i .

The traffic states and transition events are illustrated in Fig. 1 based on two different vehicle types ($E = \{1, 2\}$): fuel and electric vehicles. Note that some state and transition event variables are omitted in Fig. 1 for simplicity.

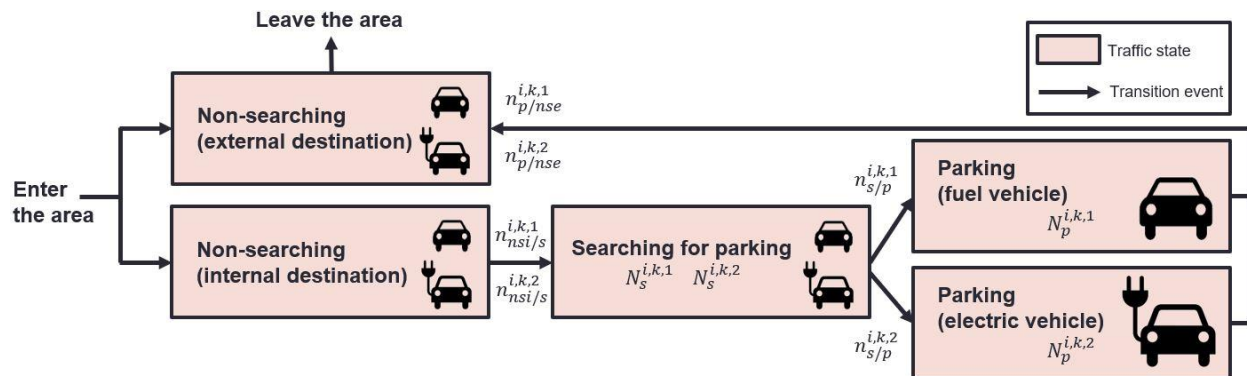


Fig. 1. Traffic and parking framework for two different vehicle types: Fuel and electric vehicles.

Vehicles enter the area with a destination outside (“non-searching (external destination)”) or inside the network (“non-searching (internal destination)”). The first group of vehicles represents the through-traffic. The latter group (fuel or electric vehicles) searches for available parking spaces (“searching for parking”) before parking (“parking”). The non-differentiated parking scheme is presented in section 2.1.1. In case of differentiated parking, we consider two different levels of flexibility: fully differentiated parking and semi-differentiated parking. They are introduced in sections 2.1.2 and 2.1.3, respectively. All parking searchers are homogeneously distributed within the overall traffic and no drivers cancel their trip towards their internal destinations. After having parked and paid the parking fee depending on their parking duration, the vehicles drive towards their next destination outside the network (“non-searching (external destination)”), and leave the area.

2.1.1 Non-differentiated parking

The non-differentiated parking scheme assumes all parking spaces A to be identical and uniformly distributed in the area. A differentiation between on-street and garage parking is not made here, but could potentially be added following the same rationale as in [Jakob and Menendez \(2019\)](#). The parking availability, A^i , is updated in Eq. (3) according to A and the number of vehicles parked, $N_p^{i,k}$, at the beginning of time slice i .

$$A^i = A - \sum_{k=1}^K N_p^{i,k} \quad \forall i \in \{1, \dots, T\} \quad (3)$$

As A^i is restricted by A , $0 \leq A^i \leq A$ is valid for all time slices i .

2.1.2 Differentiated parking policy

In our fully differentiated parking policy, we assume that vehicles are only allowed to park at their specific vehicle type parking spaces, i.e., fuel and electric vehicles are only allowed to access parking spaces for fuel and electric vehicles, respectively (Fig. 2a).

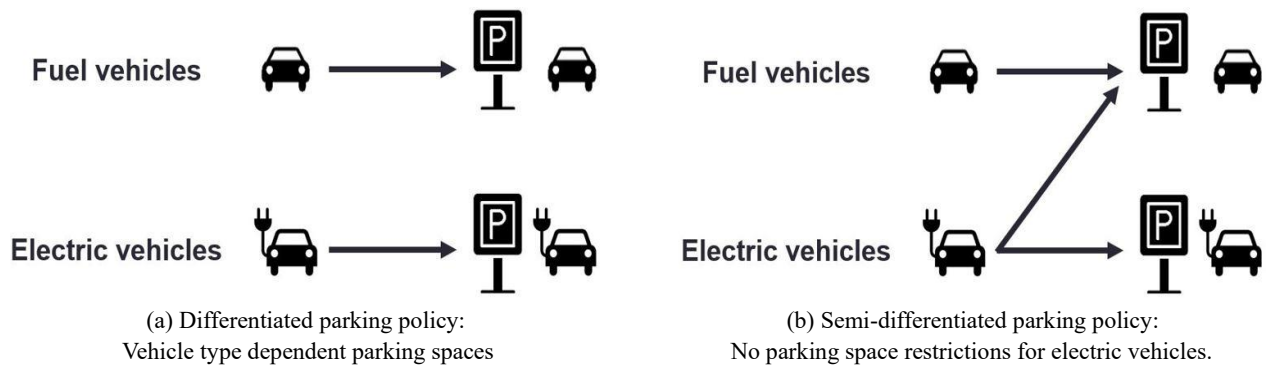


Fig. 2. Two differentiated parking policies for the access of parking spaces by fuel and electric vehicles.

The parking spaces A^e for each vehicle type $e \in E$ are assumed to be identical and uniformly distributed in the area. The parking availability, $A^{i,e}$, for $e \in E$ is updated in Eq. (4) according to A^e and the number of vehicles parked, $N_p^{i,k,e}$, at the beginning of time slice i .

$$A^{i,e} = A^e - \sum_{k=1}^K N_p^{i,k,e} \quad \forall i \in \{1, \dots, T\}, \quad \forall e \in \{1, \dots, E\} \quad (4)$$

As $A^{i,e}$ is restricted by A^e , $0 \leq A^{i,e} \leq A^e$ is valid for all time slices i .

2.1.3 Semi-differentiated parking policy

The semi-differentiated parking policy assumes that no parking space restrictions are in place for vehicles of type $e = 2$, $e \in E$. On the other hand, vehicles of type $e = 1$, $e \in E$ can only access their dedicated parking spaces. Assuming electric vehicles are part of $e = 2$ and fuel vehicles belong to $e = 1$, the electric vehicles can access parking spaces for fuel and electric vehicles (Fig. 2b). However, it is reasonable to assume that they generally prefer parking spaces with battery charging options. In case these spaces are not available anymore, electric vehicles access parking spaces for fuel vehicles. Fuel vehicles can only access their dedicated parking spaces and get fined when parking at spaces with battery charging opportunities. Both fuel and electric vehicles have the same chance to access an available fuel parking space in case all parking spaces with battery chargers are occupied.

The parking availabilities $A^{i,1}$ and $A^{i,2}$ are updated in Eq. (5) and Eq. (6) depending on whether there are enough parking spaces for vehicles of type $e = 2$ available in time slice i , i.e., we check whether the net number of electric vehicles moving to parking during time slice $i - 1$, $\sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2})$, can access the available parking spaces, $A^{i-1,2}$, at the beginning of time slice $i - 1$. If this is the case, all vehicles of type $e = 2$ access their preferred parking spaces, and $A^{i,2}$ is updated to $A^{i-1,2} - \sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2})$ (Eq. (6)). This leads to only vehicles of type $e = 1$ accessing their dedicated parking spaces, and $A^{i,1}$ is updated to $A^{i-1,1} - \sum_{k=1}^K (n_{s/p}^{i-1,k,1} - n_{p/nse}^{i-1,k,1})$ (Eq. (5)). If there are not enough preferred parking spaces for the vehicles of type $e = 2$ (i.e., $A^{i,2} = 0$ (Eq. (6))), all remaining vehicles move towards parking spaces for the vehicles of type $e = 1$, and $A^{i,1}$ is updated to $A^{i-1,1} + A^{i-1,2} - \sum_{k=1}^K (n_{s/p}^{i-1,k,1} + n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,1} - n_{p/nse}^{i-1,k,2})$ (Eq. (5)).

$$A^{i,1} = \begin{cases} A^{i-1,1} - \sum_{k=1}^K (n_{s/p}^{i-1,k,1} - n_{p/nse}^{i-1,k,1}), & \text{if } \sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2}) \leq A^{i-1,2} \\ A^{i-1,1} + A^{i-1,2} - \sum_{k=1}^K (n_{s/p}^{i-1,k,1} + n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,1} - n_{p/nse}^{i-1,k,2}), & \text{if } \sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2}) > A^{i-1,2} \end{cases} \quad (5)$$

$$A^{i,2} = \begin{cases} A^{i-1,2} - \sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2}), & \text{if } \sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2}) \leq A^{i-1,2} \\ 0, & \text{if } \sum_{k=1}^K (n_{s/p}^{i-1,k,2} - n_{p/nse}^{i-1,k,2}) > A^{i-1,2} \end{cases} \quad (6)$$

As $A^{i,1}$ and $A^{i,2}$ are restricted by A^1 and A^2 , respectively, $0 \leq A^{i,1} \leq A^1$ and $0 \leq A^{i,2} \leq A^2$ are valid for all time slices i .

2.2. Optimal parking occupancy rate

In this section, we determine the optimal parking occupancy rate based on our macroscopic traffic and parking model for different vehicle types $e \in E$. We formulate the parking occupancy rate $occ^{i,e}$ (section 2.2.1) for $e \in E$, and the average cruising time ACT^i (section 2.2.2) at the beginning of time slice i before we present the optimization framework (section 2.2.3).

2.2.1 Parking occupancy rate

We determine the parking occupancy rate, $occ^{i,e}$, for parking spaces for vehicles of type $e \in E$ (Eq. (7)) and, occ^i , for all parking spaces in the area independently of their vehicle type (Eq. (8)).

$$occ^{i,e} = 1 - \frac{A^{i,e}}{A^e} \quad (7)$$

$$occ^i = 1 - \frac{\sum_{e=1}^E A^{i,e}}{\sum_{e=1}^E A^e} \quad (8)$$

Both formulations depend on the relation between the parking availability $A^{i,e}$ in time slice i and the total parking supply A^e for $e \in E$. For the non-differentiated parking policy, Eq. (7) and Eq. (8) become equivalent as all vehicles behave similarly, so we drop the superscript e for vehicle type.

2.2.2 Average cruising time at the beginning of each time slice

The cumulative number of vehicles for all $k \in K$ and $e \in E$ going through each transition event in Fig. 1 are illustrated in Fig. 3 over time. This figure allows us not only to visualize the number of vehicles cruising for parking

at time slice i (i.e., $\sum_{e=1}^E \sum_{k=1}^K N_s^{i,k,e}$), but also the maximum cruising time $i - i_{cmax}$ across those vehicles (Jakob et al. (2018)).

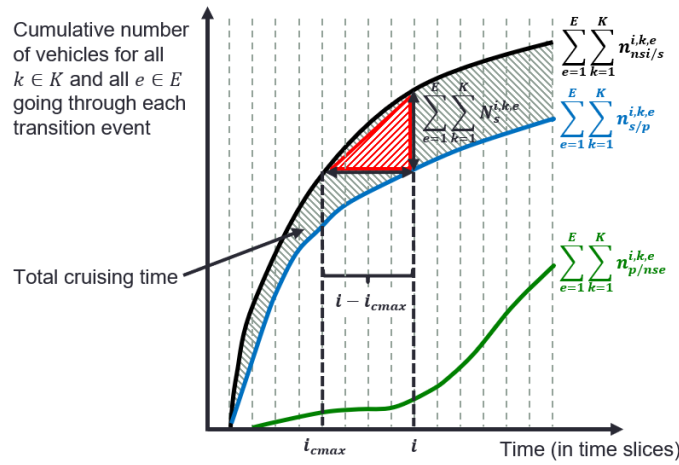


Fig. 3. The visualization of i_{cmax} at the beginning of time slice i within the queuing diagram.

We determine i_{cmax} in Eq. (9) based on the cumulative number of vehicles having accessed parking by time slice i (i.e., $\sum_{j=1}^i \sum_{e=1}^E \sum_{k=1}^K n_{s/p}^{j,k,e}$). Then, we estimate the time at which the cumulative number of vehicles having started to search for parking (i.e., $\sum_{j=1}^{i_{cmax}} \sum_{e=1}^E \sum_{k=1}^K n_{nsi/s}^{j,k,e}$) is the same.

$$\text{Find } i_{cmax}, \text{ s. t. } \sum_{j=1}^{i_{cmax}} \sum_{e=1}^E \sum_{k=1}^K n_{nsi/s}^{j,k,e} = \sum_{j=1}^i \sum_{e=1}^E \sum_{k=1}^K n_{s/p}^{j,k,e} \quad (9)$$

Approximating the area highlighted in Fig. 3 as a red triangle, we compute the average searching time across vehicles searching for parking during time slice i , ACT^i in Eq. (10). Recall that t is the length of a time slice.

$$ACT^i = \frac{i - i_{cmax}}{2} \cdot t \quad (10)$$

ACT^i is required for our optimization framework in section 2.2.3 to propose the optimal parking occupancy rate which minimizes cruising time for all vehicles at all times.

2.2.3 Optimization framework

Our framework combines ACT^i (Eq. (10)) with $occ^{i,e}$ (Eq. (7)) or occ^i (Eq. (8)), respectively. $m(ACT^i)$ denotes the moving average over the number of s values of ACT^i as a function of $occ^{i,e}$. The parameter s is considered as an input to the model. The optimization model to determine the optimal parking occupancy rate for parking spaces for vehicles of type $e \in E$ is formulated in Eq. (11). Note that we can replace $occ^{i,e}$ in Eq. (11) by occ^i to compute the rate for all parking spaces in the area.

$$\max \left\{ \arg \min (m(ACT^i)) \right\}_{0 \leq occ^{i,e} \leq 1} \quad (11)$$

Our optimization framework tries not only to minimize the average searching time for parking, but also to maximize the parking occupancy rate. This should lead to an efficient usage of the parking infrastructure and at the same time a high likelihood of finding parking in order to eliminate the drivers' time wasted in cruising for parking and the added congestion it causes. Fig. 4 visualizes this optimization strategy (Eq. (11)) for $m(ACT^i)$ as a function of $occ^{i,e}$

showing the valid solution set in green. We solve it using a simulation-based approach, such that $m(ACT^i)$ is determined depending on $occ^{i,e}$ or occ^i for all time slices i over T .

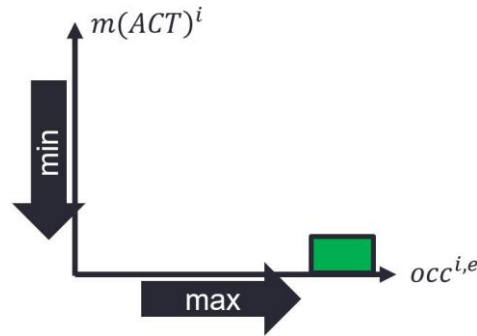


Fig. 4. Optimization strategy (Eq. (11)) for $m(ACT^i)$ as a function of $occ^{i,e}$.

2.3. Traffic performance and parking revenue

The vehicles' average cruising time for parking, ACT_{all} , over the whole time horizon T is represented in Eq. (12), reflecting the traffic performance in the area. We determine the total cruising time (gray shaded area in Fig. 3), $t \cdot \sum_{i=1}^T \sum_{e=1}^E \sum_{k=1}^K N_s^{i,k,e}$, and divide it by the total number of vehicles accessing an available parking space over T , $\sum_{i=1}^T \sum_{e=1}^E \sum_{k=1}^K n_{s/p}^{i,k,e}$. Less available parking spaces might lead to more vehicles searching for parking, $N_s^{i,k,e}$, and thus to traffic congestion in the network.

$$ACT_{all} = \frac{t \cdot \sum_{i=1}^T \sum_{e=1}^E \sum_{k=1}^K N_s^{i,k,e}}{\sum_{i=1}^T \sum_{e=1}^E \sum_{k=1}^K n_{s/p}^{i,k,e}} \quad (12)$$

Once the drivers depart from their parking spaces, $n_{p/nse}^{i,k,e}$, they pay their parking fee p^e subject to their individual parking duration. The average parking duration across drivers is \bar{t}_d^e . Note that both p^e and \bar{t}_d^e can vary by $e \in E$ as some vehicle types might have different parking requirements (e.g., electric vehicles might need to park longer in parking spaces with charging possibilities for their batteries). The total revenue from parking pricing for the city is determined in Eq. (13).

$$I = \sum_{i=1}^T \sum_{e=1}^E \sum_{k=1}^K n_{p/nse}^{i,k,e} \cdot p^e \cdot \bar{t}_d^e \quad (13)$$

Recall that for non-differentiated parking we can drop the superscript e in Eq. (12) and Eq. (13).

3. Applications

This section presents a case study of a central area within the city of Zurich, Switzerland, to determine the optimal parking occupancy rate for fuel and electric vehicles. As the interest in electric vehicles is continuously increasing, our findings evaluate the traffic performance and parking impacts of a modal shift towards electric vehicles with respect to the average searching time for parking, the total revenue from parking pricing, and the parking occupancy. Our methodology is implemented using a simple numerical solver such as Matlab based on real data obtained by [Cao et al. \(2019\)](#).

3.1. A case study for an area within the city of Zurich, Switzerland

The total length of all roads in our area, around the shopping district Jelmoli (0.28 km²) in the city center of Zurich, is $L = 7.7$ km. There are 539 parking spaces in the area (Cao et al. (2019)). As these parking spaces have no time limit, drivers can park there for the whole time horizon of 1 day. We divide the working day into time slices of 1 minute, i.e., $t = 1$ min, so that $T = 1440$ min. All parking spaces have an hourly fee of $p^e = 2.25$ CHF approximating the average value in the city center of Zurich (Cao et al. (2019); Jakob and Menendez (2019)). The parking demand, the distribution of parking durations, and initial conditions are determined using an agent-based model in MATSim that have been validated for the city of Zurich in Waraich and Axhausen (2012). We split the parking demand of 2687 trips into $K = 4$ different user groups (892/ 956/ 838/ 956 trips) associated with different VOTs ($VOT^1 = 29.9$ CHF/h; $VOT^2 = 25.4$ CHF/h; $VOT^3 = 25.8$ CHF/h; $VOT^4 = 17.2$ CHF/h) based on the estimated VOT mean values for car journeys in Switzerland (Axhausen et al. (2006)). However, only 77 % (2069 trips) of the daily traffic searches for parking and the remaining 23 % (618 trips) does not search for parking (through-traffic). The parking durations are described by a probability density function (pdf) following a gamma distribution with a shape parameter of 1.6 and a scale parameter of 142, and the average parking duration is 227 min (Cao et al. (2019)). We initially assume that the pdf of the parking durations is the same for all vehicle types. However, we relax this assumption in section 3.4. The traffic properties of this are based on those presented in Ambühl et al. (2017) and Loder et al. (2019). The initial conditions include $\sum_{e=1}^2 \sum_{k=1}^4 N_p^{0,k,e} = 183$ vehicles already parked in the area, and $\sum_{e=1}^2 \sum_{k=1}^4 N_s^{0,k,e} = 0$ vehicles searching for parking at the beginning of the working day.

3.2. Optimal parking occupancy rate, and traffic performance impacts

This section shows the optimal parking occupancy rate, and the resulting traffic performance when considering the differentiated and semi-differentiated parking space policies in sections 2.1.2 and 2.1.3 in comparison with a non-differentiated parking scheme from section 2.1.1. We first assume that the proportion of parking spaces for electric vehicles with battery charging possibilities, ζ (Eq. (14)), matches the proportion of electric vehicles entering the area, ε (Eq. (15)). This assumption, $\zeta = \varepsilon$, will be relaxed in section 3.3 when we analyze the trade-offs between different demand and supply proportions of electric vehicles. Note that ε is assumed to be equal for all time slices over one working day.

$$\zeta = \frac{A^2}{A^1 + A^2} \quad (14)$$

$$\varepsilon = \frac{\sum_{i=1}^T \sum_{k=1}^K n_{/ns}^{i,k,2}}{\sum_{i=1}^T \sum_{k=1}^K (n_{/ns}^{i,k,1} + n_{/ns}^{i,k,2})} \quad (15)$$

First, we evaluate our reference scenario with non-differentiated parking (scenario (a)), i.e., we do not distinguish between vehicles or parking spaces (section 2.1.1). We compare this scenario to the assumed scenarios (b) differentiated parking policy, section 2.1.2, and (c) semi-differentiated parking policy, section 2.1.3, reflecting two vehicle types, fuel ($e = 1$) and electric ($e = 2$) vehicles, with $\zeta = \varepsilon = 10$ %. Table 3 presents the results for these scenarios.

Table 3. Comparison of different scenarios considering non-differentiated, differentiated and semi-differentiated parking policies with $\zeta = \varepsilon = 10\%$ focusing on traffic and parking impacts. Value within parenthesis represents the percentage change with respect to the reference scenario.

Scenarios	(a)	(b)	(c)
	Reference scenario: Non-differentiated parking	Differentiated parking policy	Semi-differentiated parking policy
Optimal parking occupancy rate (single value for all parking spaces)	93.43 %	87.37 % (-6.5 %)	93.43 % (+0 %)
Optimal parking occupancy rate (parking spaces for fuel vehicles)	-	85.62 %	93.43 %
Optimal parking occupancy rate (parking spaces for electric vehicles)	-	87.6 %	93.63 %
Average time for vehicles searching for parking (min/veh)	3.14	3.77 (+20.1 %)	3.27 (+4.1 %)

Not surprisingly, scenario (a) with a non-differentiated parking scheme leads to the lowest average searching time (3.14 min/veh) over one working day. It also shows a high optimal parking occupancy rate (93.4 %). Note that we assume an optimistic case with all drivers going through all parking spaces in order to find the next available parking space in the area. This leads to slightly higher optimal parking occupancy rates than the 85 % suggested by [Shoup \(1999, 2005, 2006\)](#). Fig. 5 shows the average searching time as a function of the parking occupancy rate. It can be used to visualize the computation of the optimal parking occupancy rate using our optimization framework from section 2.2.3.

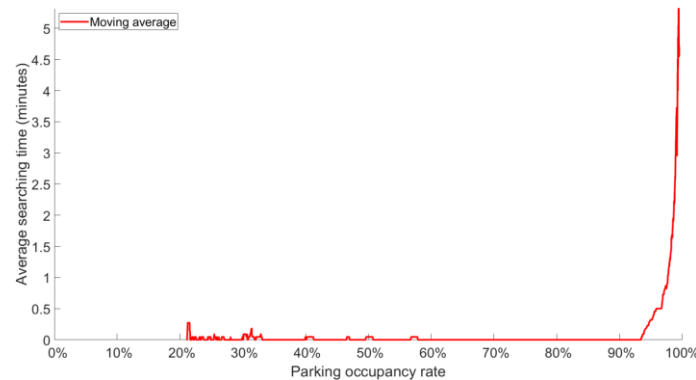


Fig. 5. Average searching time for different parking occupancy rates during a typical working day (scenario (a)).

The differentiated parking policy (scenario (b)) leads to an average searching time increase of 20.1 % compared to the reference scenario (a), and to an optimal parking occupancy rate reduction of 6.5 % on average. The optimal parking occupancy rate of parking spaces for fuel and electric vehicles are relatively similar to each other (85.6 % and 87.6 %, respectively). The semi-differentiated parking policy (scenario (c)) leads to an average searching time decrease of 13.3 % compared to scenario (b). This improvement comes with the same optimal parking occupancy rate as in scenario (a). The electric vehicles fill up their parking supply with battery charging options in the semi-differentiated parking policy before they decide to use fuel parking spaces in the area. Thus, they cause less cruising-for-parking traffic compared to the differentiated policy (scenario (b)), and this leads to a better traffic performance in the network. The average searching time for parking is only 4.1 % higher than in the reference scenario. This makes the semi-differentiated parking policy the preferred parking policy for most cities when facing a demand and supply change in terms of fuel and electric vehicles. Some drivers of electric vehicles might, however, require a parking space with battery chargers as their battery is almost empty. Depending on their planned activities, these drivers might decide to cruise for parking even when there are parking spaces for fuel vehicles available, or to drive to a different area instead. The latter might cause a change in the total parking demand, which is, however, out-of-scope in this paper.

3.3. Trade-offs between demand and supply for electric vehicles

Due to a modal shift towards electric vehicles, cities face the challenge of building new dedicated parking spaces or turning existing parking spaces for fuel vehicles into spaces with battery chargers. Then the question arises of how much parking supply shall be reserved for electric vehicles in order to react to a constantly varying parking demand over time. Compared to section 3.2 ($\zeta = \varepsilon$), we analyze in this section a mismatch between ζ and ε , and its effects on the traffic and parking model outputs. We run a simulation-based search algorithm to understand the impacts of all proportions ζ and ε on the traffic performance (Fig. 6a-b) analyzing the average searching time for the differentiated and semi-differentiated parking space policies in sections 2.1.2 and 2.1.3. The following figures are created using a cubic interpolation method of the results ([Hazewinkel \(1994\)](#)).

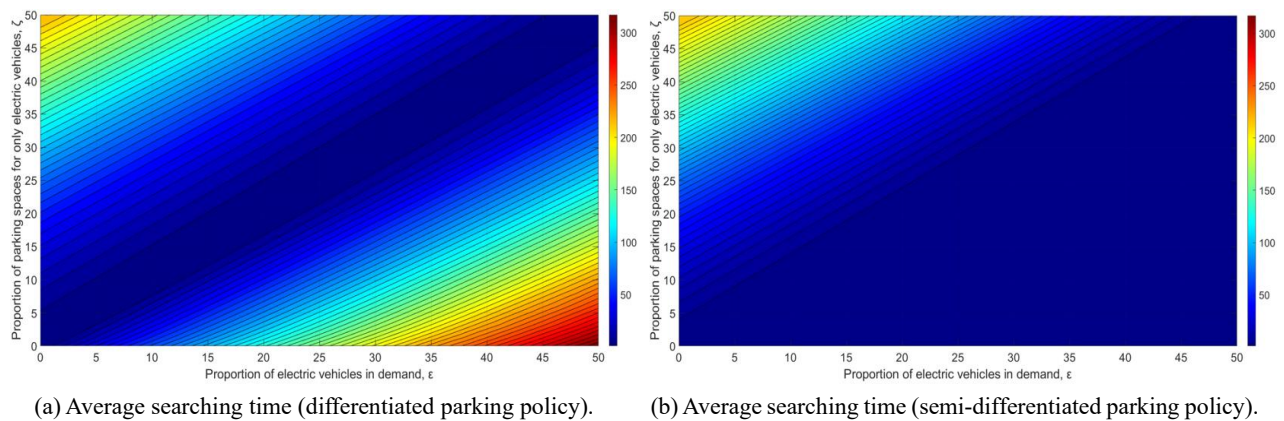


Fig. 6. Traffic performance impacts according to different demand and supply proportions for electric vehicles.

As one would expect, Fig. 6a shows that the average searching time for the differentiated parking policy is minimized for $\zeta \approx \varepsilon$ (i.e., along the diagonal). This is reasonable as the absolute size of the parking demand and the available supply are balanced for both electric and fuel vehicles, i.e., the proportion of vehicles searching for parking is similar to the proportion of available parking spaces over time. In other words, cities should aim to provide a proportion of parking spaces for electric vehicles, ζ , similar to the proportion of electric vehicles in demand, ε , to reduce cruising-for-parking in the area. The average searching time increases faster for $\zeta < \varepsilon$ compared to $\zeta > \varepsilon$. This is reasonable as both ζ and ε are below 50 %. The average searching time follows the opposite behavior when both ζ and ε are over 50 %. Either fuel or electric vehicles find it more difficult to find an available parking space when $\zeta \neq \varepsilon$. However, when a mismatch between ζ and ε cannot be avoided, it is safer to have a ζ higher than ε , i.e., an oversupply of parking spaces for electric vehicles rather than an undersupply, as long as $\varepsilon < 50$ %. Applying the semi-differentiated parking policy mitigates this problem (Fig. 6b) as electric vehicles can park everywhere. This leads to the same searching times for $\zeta \approx \varepsilon$ and $\zeta < \varepsilon$. In other words, as long as we do not oversupply parking spaces for electric vehicles, the traffic performance is acceptable in the area. Notice, however, that providing some parking with charging stations could potentially lead to a modal shift, and this could be desirable for the city. Such changes in the demand are considered out of the scope for this paper.

The total revenue from parking pricing (differentiated parking policy) over one working day (Fig. 7a) equals to 17,630 CHF for $0.7 \varepsilon \leq \zeta \leq 0.7 \varepsilon + 29$. An efficient usage of the available parking spaces leads to these high revenues. Beyond this ratio, $\zeta < 0.7 \varepsilon$ and $\zeta > 0.7 \varepsilon + 29$, the revenue decreases. The semi-differentiated parking policy (Fig. 7b) leads to a revenue of 17,628 CHF when $\zeta \leq 0.7 \varepsilon + 29$. The low average searching times (Fig. 6b) facilitate a more efficient usage of the available parking supply compared to the differentiated parking policy. An oversupply of parking spaces for electric vehicles and an undersupply of parking spaces for fuel vehicles reduce the parking revenue. Thus, if we want to increase revenues, we need to make sure that we do not undersupply parking

spaces for fuel vehicles, contrary to what we said when talking about traffic performance.

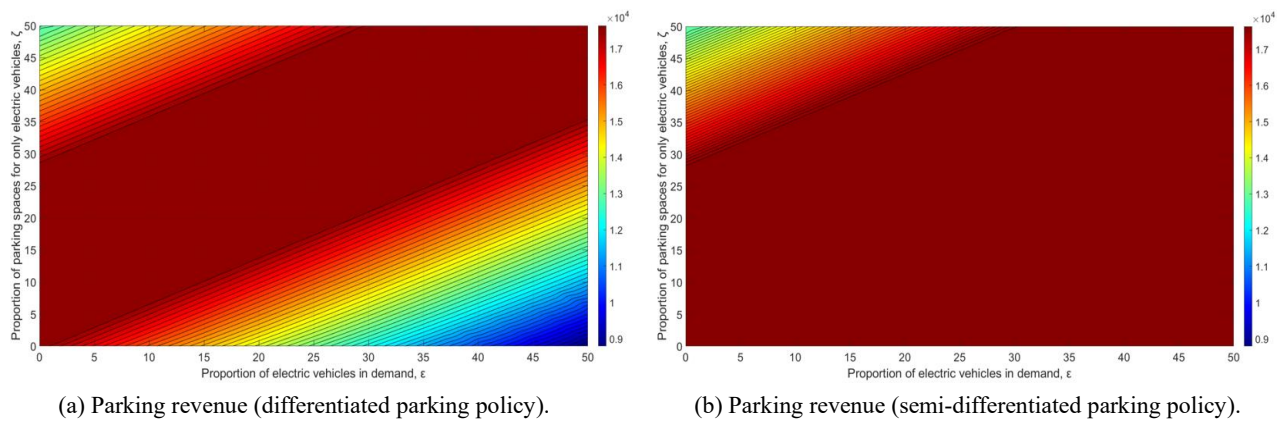


Fig. 7. Parking revenue impacts according to different demand and supply proportions for electric vehicles.

Fig. 8a-b analyze how the optimal parking occupancy rates of parking spaces for fuel and electric vehicles (differentiated parking policy) are affected by different proportions ζ and ε . The optimal occupancy rate is higher than 80 % when $\zeta > \varepsilon$ for the parking spaces for fuel vehicles (Fig. 8a), and when $0.9\zeta < \varepsilon < 3\zeta$ for the parking spaces for electric vehicles (Fig. 8b). The former can be explained by an undersupply of parking spaces for fuel vehicles which leads to high optimal parking occupancies reflecting the high parking demand of fuel vehicles. The latter can be explained by an efficient parking space usage due to low searching times in the network. It is still possible to achieve a high likelihood of finding parking for drivers of electric vehicles, even if the parking occupancy rates in the area are high. Beyond this, the optimal occupancy rates of parking spaces decrease in order to compensate for the mismatch between the demand and the supply for one of the vehicle types. Therefore, and given that guaranteeing different occupancy rates for different types of parking is rather complicated, it makes sense to set a single target value across all vehicle spaces. Such value should be around 80 % as long as there are similar proportions ζ and ε .

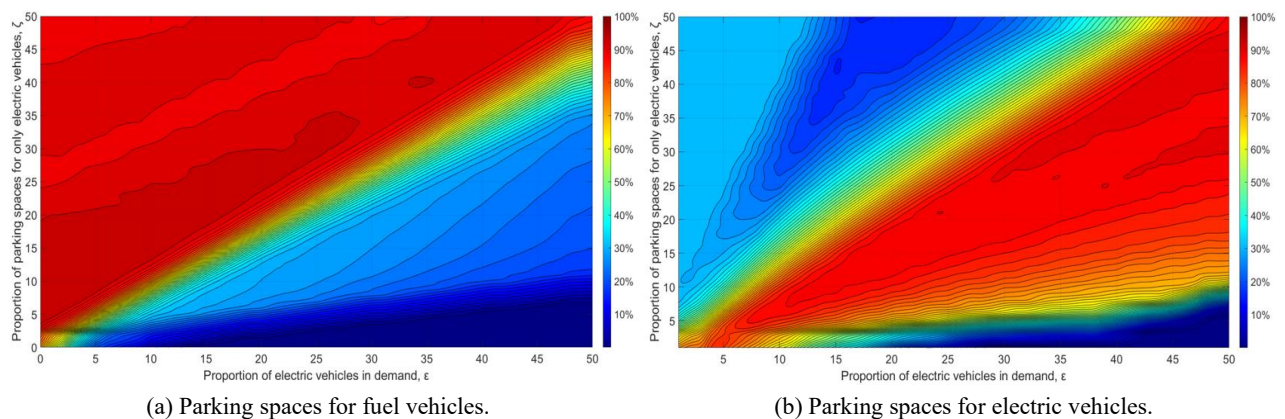


Fig. 8. Optimal parking occupancy rate for different demand and supply proportions for electric vehicles (differentiated parking policy).

This single target occupancy rate could be even higher, around 90 %, if we were to implement the semi-differentiated parking policy (see Fig. 9). Moreover, in this case, it would be applicable as long as $\zeta < \varepsilon$ which characterizes an undersupply of parking spaces for electric vehicles, as these vehicles can access both types of parking spaces. The semi-differentiated parking policy not only leads to a better traffic performance for $\zeta < \varepsilon$ (Fig. 6b) compared to the differentiated parking policy, but also to higher optimal parking occupancy rates for these proportions ζ and ε .

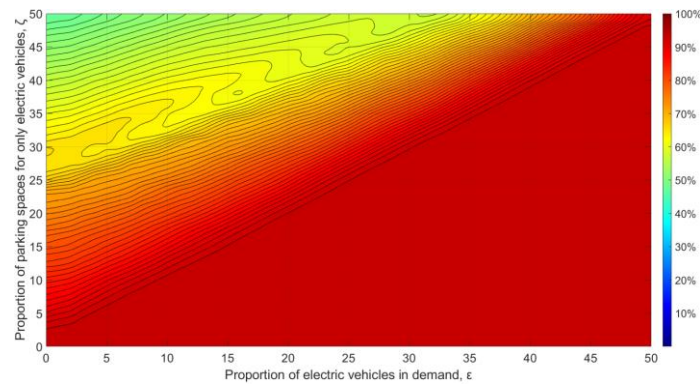


Fig. 9. Single optimal parking occupancy rate across all parking spaces in the area for different demand and supply proportions for electric vehicles (semi-differentiated parking policy).

In summary, achieving similar proportions, $\zeta \approx \varepsilon$, leads to the best traffic performance in the area for both the differentiated and the semi-differentiated parking policies. In the latter policy, the proportions $\zeta < \varepsilon$ can also lead to low searching times as drivers of electric vehicles can use fuel parking spaces instead of spaces with battery charging options. Equal proportions, $\zeta = \varepsilon$, come along with high revenues for the city and single optimal parking occupancy rates around 80 % (differentiated parking policy) and 90 % (semi-differentiated parking policy) across all parking spaces. Consequently, cities shall react towards a changing demand for electric vehicles over time by changing their supply accordingly. City councils can also use the results from this model to analyze the traffic performance loss when they do not react, e.g., to an increasing demand for electric vehicles over time. These risks can be evaluated as a function of the parking policy in place.

3.4. Sensitivity to changes in parking demand, supply, or parking duration

Here we present a sensitivity analysis for the differentiated (section 3.4.1) and the semi-differentiated parking policy (section 3.4.2) quantitatively evaluating the impacts of a change in parking demand, supply, or the distribution (pdf) of parking durations on the optimal parking occupancy rate across all parking spaces. We use as a reference the total demand of 2687 trips entering the area, the total supply of 539 parking spaces in the network, and the pdf of parking durations described in section 3.1. A more in-depth sensitivity analysis considering dependency between inputs ([Ge and Menendez \(2017\)](#)) is considered out of the scope for this paper.

3.4.1 Sensitivity analysis for the differentiated parking policy

Fig. 10-12 show how a decrease or an increase in demand, supply, and the electric vehicles' average parking durations affect the single optimal parking occupancy rates across all parking spaces depending on different proportions of ε with $\zeta = 10\%$ (Fig. 10a-12a), and different proportions of ζ with $\varepsilon = 10\%$ (Fig. 10b-12b).

An increase in demand together with a low $\varepsilon \leq 10\%$ (Fig. 10a), or a high $\zeta \geq 10\%$ (Fig. 10b) leads to optimal occupancy rates above 80 %. In these cases, electric vehicles do not cause additional searching for parking traffic as $\varepsilon < \zeta$. High proportions of electric vehicles in the demand, $\varepsilon > 10\%$, and low proportions of electric parking in the supply, $\zeta < 10\%$, lead to a decrease in optimal parking occupancy rates as the mismatch between the demand and the supply becomes more relevant. The low occupancy rates are needed to guarantee that despite the strong competition among electric vehicles looking for parking, cruising times are still minimized. For cases where $\varepsilon < \zeta$, as the demand entering the area decreases, the optimal occupancy rates do so as well. This can be explained by a decreasing absolute number of electric vehicles searching for parking compared to a constant parking supply with battery chargers, resulting in an increasing oversupply of parking spaces for electric vehicles as the total demand decreases.

Changes in parking supply (Fig. 11a-b) have a smaller impact on the optimal parking occupancy rates compared to changes in demand, mostly because the changes in absolute values are also much smaller. Recall that the total value of the supply is much smaller than the total value of the demand. However, the overall trends remain the same.

Last, changes in the electric vehicles' average parking durations have almost no impact on the optimal parking occupancy rates (Fig. 12a-b). Low $\varepsilon \leq 10\%$ (Fig. 12a), or high $\zeta \geq 10\%$ (Fig. 12b) lead to high optimal parking occupancy rates.

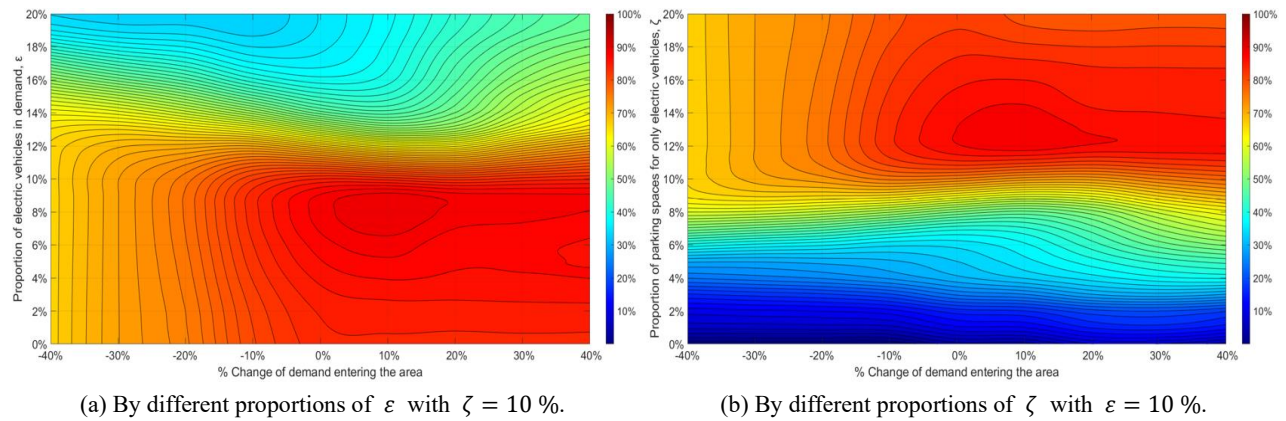


Fig. 10. Sensitivity analysis of the optimal parking occupancy rate across all parking spaces with respect to changes in the demand entering the area (differentiated parking policy).

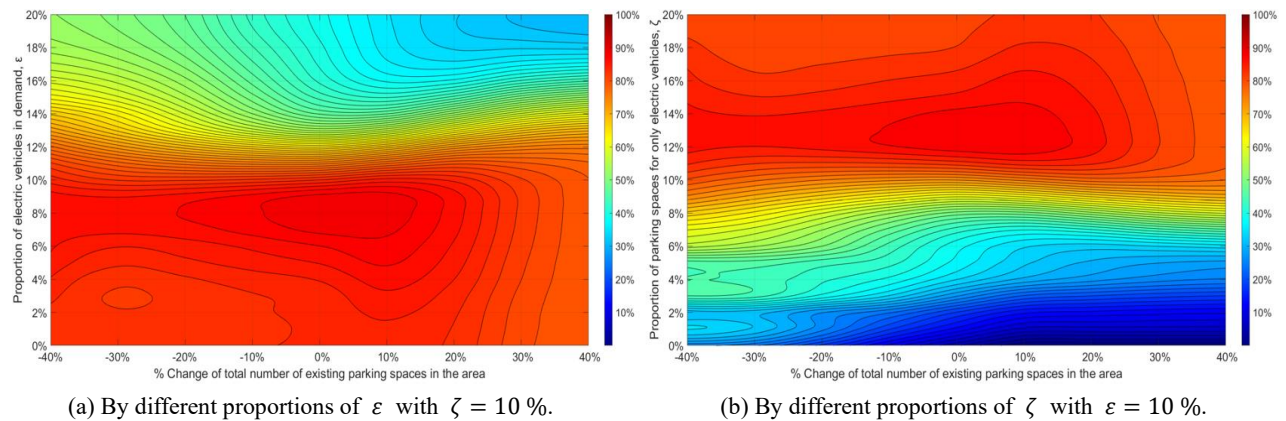


Fig. 11. Sensitivity analysis of the optimal parking occupancy rate across all parking spaces with respect to changes in the total parking supply in the area (differentiated parking policy).

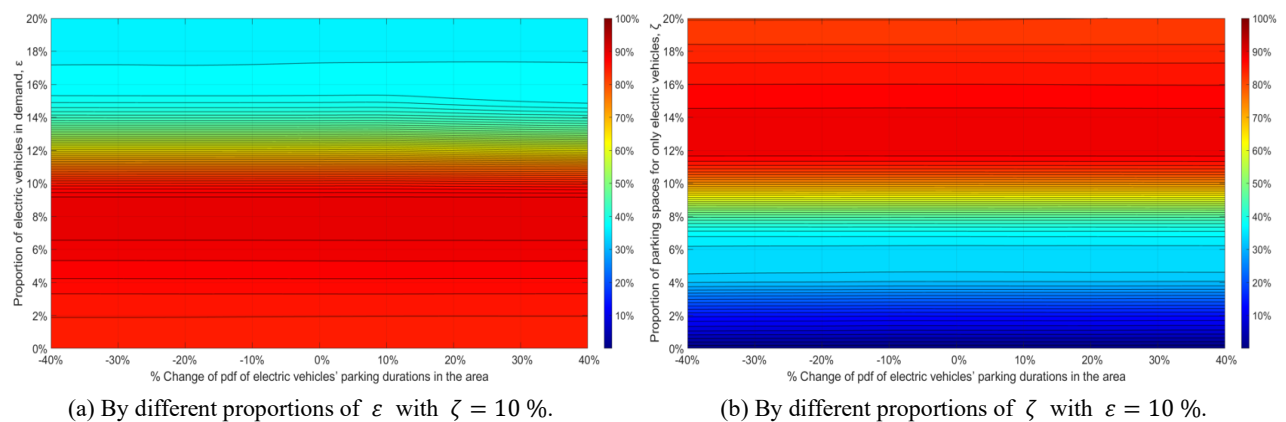


Fig. 12. Sensitivity analysis of the optimal parking occupancy rate across all parking spaces with respect to changes in the pdf of electric vehicles' parking durations in the area (differentiated parking policy).

3.4.2 Sensitivity analysis for the semi-differentiated parking policy

The semi-differentiated parking policy leads to generally higher optimal parking occupancy rates across all parking spaces compared to the differentiated parking policy in section 3.4.1. In this case, an increase in demand leads to the highest optimal parking occupancy rate of approximately 94 % for high proportions $\varepsilon \geq 10\%$ (Fig. 13a), or low proportions $\zeta \leq 10\%$ (Fig. 13b).

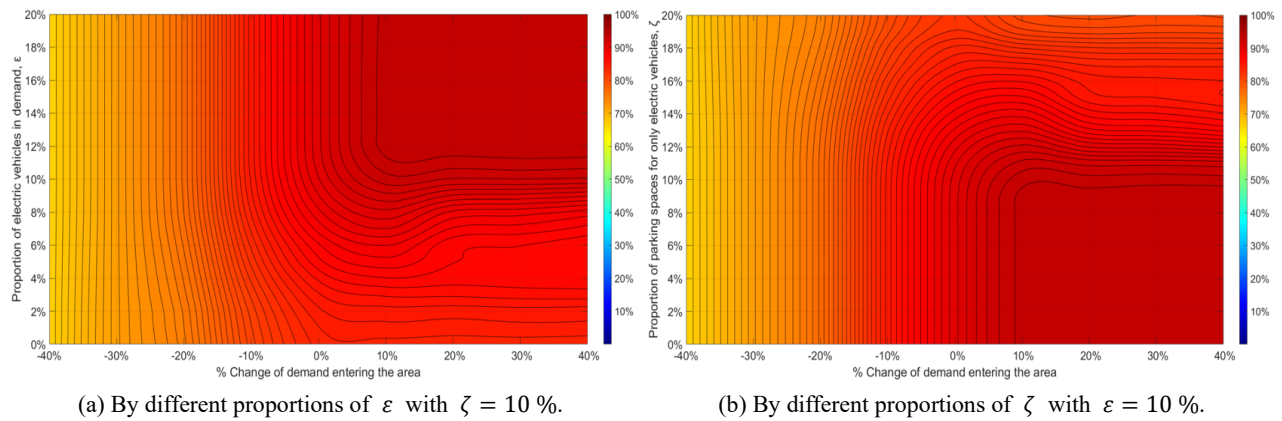


Fig. 13. Sensitivity analysis of the optimal parking occupancy rate across all parking spaces with respect to changes in the demand entering the area (semi-differentiated parking policy).

This is different than Fig. 10, as a high proportion of electric vehicles in the demand, ε , or a low proportion of parking spaces for only electric vehicles, ζ , do not necessarily increase the search times. Recall that in such cases electric vehicles will use a fuel parking space instead. Therefore, for any increases in demand, it is worth maintaining $\zeta \leq \varepsilon$. Note that we omit the figures showing the changes in supply and parking duration for the semi-differentiated parking policy as the variations in the optimal parking occupancy rates are hard to read due to their high values. As a matter of fact, there are no remarkable dependencies between changes in the total parking supply or the electric vehicles' average parking durations and the optimal parking occupancy rates. In the extreme case where the total supply were to be very limited with a semi-differentiated parking scheme, it is recommended to limit the supply for the non-differentiated vehicle class (here electric vehicles) compared to the differentiated vehicle class (here fuel vehicles) in order to achieve the best traffic performance in the network.

4. Conclusions

In this paper, we propose a model to determine the optimal parking occupancy rate for multiple vehicle types based on a macroscopic traffic and parking model over a given time horizon for an urban area. We demonstrate our methodology using real data from an area within the city of Zurich, Switzerland. Our findings confirm the optimal parking occupancy rates proposed by [Shoup \(1999, 2005, 2006\)](#), but we also discuss how these rates might change depending on various demand and supply relationships following different parking policies.

The usage of the proposed framework is far beyond the illustration presented here. Parking and/or congestion pricing measures ([Jakob and Menendez \(2020\)](#)) could be analyzed to achieve the optimal parking occupancy rate for some parking spaces in the network. A further consideration is to enhance the parking fees using responsive pricing schemes in order to optimize the total revenue from parking pricing ([Jakob et al. \(2018\)](#)). We could also enhance the model by including public transport and studying the multimodal demand effects on the parking occupancy ([Dacic et al. \(2020\)](#); [Loder et al. \(2017\)](#); [Paipuri and Leclercq \(2020\)](#); [Zheng et al. \(2014\)](#)). Future research could incorporate as well a differentiation between on- and off-street parking ([Jakob and Menendez \(2019\)](#)) and evaluate their impacts on the optimal occupancy rate. Our model does not explicitly account for delivery parking and assumes double parking does

not cause any issues in the area. [Roca-Riu et al. \(2017\)](#) investigated the development of dynamic delivery parking spaces that could be integrated into the proposed framework in future studies. Additionally, vehicles could prefer parking possibilities in a central street or area of the network compared to parking spaces elsewhere. This non-homogeneous environment could lead to different optimal parking occupancy rates by modeling adjacent subnetworks that are connected to each other.

Below, we summarize the main contributions of this paper and discuss their policy implications.

First, we propose a macroscopic model to determine the optimal parking occupancy rate in a central area that is based on small data collection efforts and has low computational costs. The model outputs can be generated with a simple numerical solver and without complex simulation software. Our study defines the optimal parking occupancy rate to minimize cruising time. The results help cities setting the optimal parking occupancy rate in order to guarantee an optimal trade-off between an efficient usage of the parking infrastructure and a high likelihood of finding parking such that the traffic performance is improved in the area. Multiple parking measures (e.g., parking pricing policies ([Jakob and Menendez \(2020\)](#))) could then be used to obtain this target rate over time. However, they are considered out-of-scope in this study.

Second, a modal shift towards a specific vehicle type (e.g., electric vehicles) will lead to new challenges for cities as they try to establish the required parking supply (e.g., parking spaces with battery charging opportunities for electric vehicles). Our framework not only allows us to evaluate the impacts on traffic performance, but also on optimal parking occupancy rates for different proportions of fuel and electric vehicles both in the demand and the supply. We investigate a non-differentiated parking policy, a differentiated parking policy with vehicle type dependent parking spaces (e.g., fuel vehicles park at fuel vehicle parking spaces, and electric vehicles park at their dedicated parking spaces), and a semi-differentiated parking policy, considering no parking space restrictions for some vehicle types (e.g., electric vehicles can park at any parking space). Our results for the city center of Zurich not only show that equal proportions between electric vehicles in the demand and their parking spaces in the supply lead to the best traffic performance in the area, but they also allow city councils to analyze their loss in performance if they do not react, e.g., to an increasing demand for electric vehicles over time. These risks can be evaluated for both the differentiated and the semi-differentiated parking policies. Overall, the semi-differentiated parking policy leads to better results in terms of traffic performance and parking revenues.

Third, we can analyze the dependency of the optimal parking occupancy rate (for the differentiated and semi-differentiated policies) on changes in parking demand, supply, or parking duration. Cities can use the sensitivity analyses to react to these changes by modifying the supply of parking spaces with battery chargers, or by adapting the target occupancy rates.

In summary, our model combines the advantages of an easy to implement methodology to determine the optimal parking occupancy rate for different vehicle types with the opportunities of evaluating traffic and parking impacts (e.g., average searching time for parking, total revenue from parking pricing, optimal parking occupancy rates) of a modal shift towards a specific vehicle type, such as electric vehicles with differentiated and semi-differentiated parking policies.

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The data used to support the findings of this study are available from the corresponding author upon request.

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