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The Distribution of Prime Numbers by the Symmetry of the Smallest Sphenic Number

M. Bousder[1]

[1]Lab of High Energy Physics, Modeling and Simulations,

Faculty of Science,University Mohammed V-Agdal, Rabat, Morocco

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Abstract

In this paper, we study the symmetry between the supersingular prime number according to smallest sphenic number. With this symmetry, we show that the elements of sporadic group generates all prime numbers with the order by a simple application.

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1 The symmetry of the smallest sphenic number

We use moonshine theory [1], we see that the first prime numbers are a supersingular prime [2]. We suggest, that from these numbers we can determine the distribution of prime numbers [3].

Def.1 We define the number **1** is a mirror prime, since $\mathbf{1} = 1 \times 1 \times \cdots$

We consider the set of prime between 1 and 30 of the supersingular prime

$$A_2^{29} = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

We delete $2, 3$,and 5 from A_2^{29}, it is to describe the set without the prime of smallest sphenic number [4]. Let A_7^{29} be a subset of A_2^{29},

$$\{1\} \cup A_7^{29} = \{\mathbf{1}, 7, 11, 13, 17, 19, 23, 29\} \tag{1.1}$$

Sphenic number S_c [5] is a product of tree prime numbers (p, q, r): $S_c = p \times q \times r$. The *smallest* sphenic number 30 is the product of the *smallest* three primes number

$$30 = 2 \times 3 \times 5 \tag{1.2}$$

Def.2 *We define the special prime numbers by the smallest three primes number*

$$\{2, 3, 5\} \tag{1.3}$$

Applying supersingular prime with to the smallest sphenic number, we find the first symmetry between the prime numbers.

We take p_+ a prime number which $30 \prec p_i^+ \prec 60$, For another prime number $p_j^- \in \{1\} \cup A_7^{29}$ (1.1) we show that

$$p_i^+ - 30 = 30 - p_j^- \tag{1.4}$$

the last equation remained to a symmetry between the first prime numbers, with respect to smallest sphenic number (1.2)

$$\begin{array}{c} \mathbf{1} \longleftarrow 7 \leftarrow 13 \leftarrow 17 \leftarrow 19 \leftarrow 23 \leftarrow 29 \leftarrow \mathbf{30} \\ \mathbf{30} \rightarrow 31 \rightarrow 37 \rightarrow 41 \rightarrow 43 \rightarrow 47 \rightarrow 53 \rightarrow 59 \end{array} \tag{1.5}$$

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The symmetry of smallest sphenic number leads to

$$\begin{array}{ccc}
\mathbf{30} - 29 = 1 & \mathbf{30} \times 2 - 29 = 31 & \mathbf{30} \times 3 - 29 = 61 \\
\mathbf{30} - 23 = 7 & \mathbf{30} \times 2 - 23 = 37 & \mathbf{30} \times 3 - 23 = 67 \\
\mathbf{30} - 19 = 11 & \mathbf{30} \times 2 - 19 = 41 & \mathbf{30} \times 3 - 19 = 71 \\
\mathbf{30} - 17 = 13 & \mathbf{30} \times 2 - 17 = 43 & \mathbf{30} \times 3 - 17 = 73 \\
\mathbf{30} - 13 = 17 & \mathbf{30} \times 2 - 13 = 47 & \mathbf{30} \times 3 - 13 = *\mathbf{77} \\
\mathbf{30} - 11 = 19 & \mathbf{30} \times 2 - 11 = *\mathbf{49} & \mathbf{30} \times 3 - 11 = 79 \\
\mathbf{30} - 7 = 23 & \mathbf{30} \times 2 - 7 = 53 & \mathbf{30} \times 3 - 7 = 83 \\
\mathbf{30} - 1 = 29 & \mathbf{30} \times 2 - 1 = 59 & \mathbf{30} \times 3 - 1 = 89
\end{array} \tag{1.6}$$

In the chain (1.5), all the first prime numbers occur, also 2, 3,and 5 present on **30** (1.2) except 11.

To add the number 11 on the chain (1.5) we add a 49, is a semiprime [6]. If s is a semiprime, then is a product of two prime numbers (p, q): $s = p \times q$. We then get a new chain between prime numbers, mirror prime and semiprime

$$\begin{array}{c}
\mathbf{1} \longleftarrow 7 \longleftarrow \mathbf{11} \leftarrow 13 \leftarrow 17 \leftarrow 19 \leftarrow 23 \leftarrow 29 \leftarrow \mathbf{30} \\
\mathbf{30} \rightarrow 31 \rightarrow 37 \rightarrow 41 \rightarrow 43 \rightarrow 47 \rightarrow *\mathbf{49} \rightarrow 53 \rightarrow 59
\end{array} \tag{1.7}$$

Def.3 *We define* a chain of prime numbers by a symmetry between **8 numbers**=\{*prime numbers and semiprime*\}, related to **30**.

example. chain 1: $\mathbf{1} \longleftarrow 7 \longleftarrow 11 \leftarrow 13 \leftarrow 17 \leftarrow 19 \leftarrow 23 \leftarrow 29 \leftarrow \mathbf{30}$;

chain 2: $\mathbf{30} \rightarrow 31 \rightarrow 37 \rightarrow 41 \rightarrow 43 \rightarrow 47 \rightarrow *\mathbf{49} \rightarrow 53 \rightarrow 59$.

In the next section, we will use the chain (1.7) to find a general relation which connects all the prime numbers with the mirror prime 1 and with whatever semiprime.

2 Distribution of prime numbres

We start by adapting the concept of the chain *def.3*; (1.7) to describe all the prime numbres , except the special prime numbers (1.3). By determining the position of each prime number, according to two integer variable. The integer $n \in \mathbb{N}^*$ represents the chain number or the position of the chain, the second integer $1 \leq m \leq 8$, represents the position of a number between 1 and 8, on a single chain.

In this case, we note the prime numbers by p_n^m and semiprime by $*p_n^m$. We prove a general relation to obtain all prime numbers with order

$$\forall n \in \mathbb{N}^*, \forall m \in [1, 8] : p_n^m = 30n - p_1^{9-m} \tag{2.1}$$

we cannot always find for each n a prime number, but this expression (2.1) includes in a condensed way all the prime numbers.

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for prime numbers we can write

$$\forall p \in \mathbf{P}, \exists n \in \mathbb{N}^*, \exists m \in [1,8] : p_n^m = 30n - p_1^{9-m} \tag{2.2}$$

With **P** is the set of prime numbers.
We replace n by $n+l$, with $l \in \mathbb{N}^*$ in Eq.(2.1), we obtain

$$p_{n+l}^m = 30l + \left(30n - p_1^{9-m}\right)$$

Which simplifies to

$$\forall n \in \mathbb{N}^*, \forall l \in \mathbb{N}^*, \forall m \in [1,8] : p_{n+l}^m = 30l + p_n^m \tag{2.3}$$

The above relation leads to the equations, in our proofs we make use of the following

$$\forall n \geq l : p_n^m - p_l^m = 30\,(n-l) \tag{2.4}$$

We also note that

$$\forall k \in \mathbb{N}^* : p_n^m - p_l^m = p_{n-l+k}^m - p_k^m \tag{2.5}$$

In the next section, we will see the distribution of prime numbers from Eq.(2.1).o

3 Catalog of prime numbers and semiprime

In this section we see the distribution of prime numbers p_n^m and semiprime (noted by $*\mathbf{p}_n^m$), using Eq.(2.1), we obtain

p_n^m	p_1^m	p_2^m	p_3^m	p_4^m	p_5^m	p_6^m
p_n^1	**1**	31	61	$*$**91**	$*$**121**	151
p_n^2	7	37	67	97	127	157
p_n^3	11	41	71	101	131	$*$**161**
p_n^4	13	43	73	103	$*$**133**	163
p_n^5	17	47	$*$**77**	107	137	167
p_n^6	19	$*$**49**	79	109	139	$*$**169**
p_n^7	23	53	83	113	$*$**143**	173
p_n^8	29	59	89	$*$**119**	149	179

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p_n^m	p_7^m	p_8^m	p_9^m	p_{10}^m	p_{11}^m	p_{12}^m	
p_n^1	181	211	241	271	∗**301**	331	
p_n^2	187	∗**217**	∗**247**	277	307	337	
p_n^3	191	∗**221**	251	281	311	∗**341**	
p_n^4	193	223	∗**253**	283	313	∗**343**	$\cdots$
p_n^5	197	227	257	∗**287**	317	347	
p_n^6	199	229	∗**259**	∗**289**	∗**319**	349	
p_n^7	∗**203**	233	263	293	∗**323**	353	
p_n^8	∗**209**	239	269	∗**299**	∗**329**	359	

In this description we can have other number generated by semiprime, for example $*\mathbf{343} = \mathbf{7}^3$. For some n and m, we cannot always have a prime number. the appearance of this number, makes ask a question; can we describe the non-prime numbers in Eq.(2.1) by sporadic group? [7].

This equation groups all prime numbers with the special prime numbers (1.2) and semi-prime. But there are also non-prime numbers, and the number of these numbers increases in the near of infinity.

In this catalog we can find all the prime numbers with order. We have $p_n^1 = \{3n-3\}\,1$, for example $p_1^1 = \{0\}\,1 = 1$, $p_2^1 = \{3\}\,1 = 31$, $p_8^1 = \{21\}\,1 = 211$, generally we obtain

$$\begin{aligned} p_n^1 &= \{3n-3\}\,1 \\ p_n^2 &= \{3n-3\}\,7 \\ p_n^3 &= \{3n+1\}\,1 \\ p_n^4 &= \{3n+1\}\,3 \\ p_n^5 &= \{3n+1\}\,7 \\ p_n^6 &= \{3n+1\}\,9 \\ p_n^7 &= \{3n+2\}\,3 \\ p_n^8 &= \{3n+2\}\,9 \end{aligned} \tag{3.1}$$

Theorem.

There are infinitely many pime numbers p_n^m in the Catalog on the column $n, \forall n \in \mathbb{N}^*$. And that there are always prime numbers of the form $p_n^1 = \{3n-3\}\,1$. And we write

$$\exists n \in Catalog\ ; \exists p_n^m \in \mathbf{P} : p_n^m = p_n^1 = \{3n-3\}\,1 \tag{3.2}$$

Proof. Let us Euclid's theorem [8]: let $p_1, p_2, p_3 \cdots p_N, \forall N \in \mathbb{N}^*$ be prime numbers in the order,the number $q = 1 + \prod_{i=1}^{N} p_i$ is a new prime number, with $p_1 = 2$.

Which simplifies to

$$q = 1 + 30 \times \prod_{i=4}^{N} p_i \tag{3.3}$$

we can see the presence of 30; Let us now Eq.(2.3), we take $(n = 1, m = 1)$

$$\forall k \in \mathbb{N} : p^1_{k+1} = 1 + 30k \tag{3.4}$$

We compare Eq.(3.3) and Eq.(3.4) we obtain

$$\forall k = \prod_{i=4}^{N} p_i : p^1_{k+1} = 1 + 30k \tag{3.5}$$

According to these results, there are infinitely many pime numbers in the Catalog. We compare Eq.(3.5) and Eq.(3.1), we see that the numbers p^1_{k+1} (3.5) are written by this way $p^1_{k+1} = \{3k\}\,1$.

Conclusion

We used a symmetry of prime numbers with respect to the smallest sphenic number. We find a description that is a little bit general, but we discover that this symmetry has been broken by number 49. The important result in this work, is that we arrive to describe the prime numbers, by a condensed distribution. Which means, that we are very close to determine the positions of each prime number.

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