Universal Scaling Laws of Hydraulic Fracturing

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Hydraulic fracturing has been studied by using dimensional analysis. The universal scaling law of the problem has been formulated. From this investigation, it has found that the key control variable in the process is total damage number $J=\frac{p+\frac{1}{2}\rho U^2}{\sigma_Y}$. The crack length is satisfied with the geometric similarity law if no response time is concerned, otherwise, it would not satisfy the similarity.

Keywords: Hydraulic fracturing, scaling law, fracture

INTRODUCTION

Hydraulic fracturing is a well-stimulation technique in which rock is fractured by a pressurized liquid. The process involves the high-pressure injection of 'cracking fluid' (primarily water, containing sand or other propants suspended with the aid of thickening agents) into a wellbore to create cracks in the deep-rock formations through which natural gas, petroleum, and brine will flow more freely. When the hydraulic pressure is removed from the well, small grains of hydraulic fracturing proppants (either sand or aluminium oxide) hold the fractures open [1][2].

A hydraulic fracture is formed by pumping fracturing fluid into a wellbore at a rate sufficient to increase pressure at the target depth. The whole cracking process can be viewed as an interaction between high pressure flow and rock, the flow acts as a spear and rock as a shield, they fight each other. The flow make the rock instable and rock resist it to keep itself stable. Delicate dynamical balance or equivalence of between the flow and rock is vital for the hydraulic fracturing process, therefore, the process must be controlled by certain type scaling laws. There are lost of numerical and test have been investigated. However, there is no any general discussion on the scaling law of the process. The aim of this paper is to apply dimensional analysis to formulate an universal scaling law for the process of d=hydraulic fracturing[1][2].

SCALING LAW OF HYDRAULIC FRACTURING

In the process of hydraulic fracturing, if we omit temperature, the process is controlled by following variables. The density of cracking fluid ρ, dynamical viscosity of cracking fluid μ, static professor p, flow velocity U; Yield strength of rock σ_Y, initial characteristic length of crack d, rock fracture stiffness K. Fracturing rocks at great depth frequently becomes suppressed by pressure, this suppression process is particularly significant in "tensile" (Mode 1) fractures which require the walls of the fracture to move against this pressure, so K can be chosen as $K=K_I$ od mode 1.

The hydraulic fracturing problem is to find current crack characteristic length D.

It is clear that the current crack characteristic length D must be a function of all other variables[3]

$$D=f(\rho,\nu,p,U,\sigma_Y,K_I,d). \tag{1}$$

The dimensions of variables can be listed in the following table.

TABLE I: Hydraulic fracturing dimensions

ρ	ν	p	U	σ_Y	K_I	d	D
mL^{-3}	L^2t^{-1}	$L^{-1}mt^{-2}$	Lt^{-1}	$L^{-1}mt^{-2}$	$L^{-1/2}mt^{-2}$	L	L

There are eight variables in the problem. Since it is mechanics system, which has three basic unit such as time, length and mass. Then from π theorem, we know we can construct five dimensionless parameters $\Pi_k, \quad k=1,..,5$. To get the Pi_k, we can choose d, σ_y, U as repeating variables.

The Π can be universally expressed as

$$\Pi_i=(\Psi_i)l^a\sigma_Y^bU^c. \tag{2}$$

where a,b,c are to to determined constants. If we replace the Ψ_i with D, p, ρ, K_I, ν, respectively, then we obtain five Π as follows

$$\begin{gathered}\Pi_1=\frac{D}{d},\quad \Pi_2=\frac{p}{\sigma_Y},\quad \Pi_3=\frac{\rho U^2}{\sigma_Y},\\ \Pi_3=\frac{K_I}{\sigma_Y\sqrt{d}},\quad \Pi_4=\frac{\nu}{Ud}.\end{gathered} \tag{3}$$

Therefore, the dimensionless relation of the problem can be expressed as

$$\frac{D}{d}=f(\frac{p}{\sigma_Y},\frac{\rho U^2}{\sigma_Y},\frac{K_I}{\sigma_Y\sqrt{d}},\frac{\nu}{Ud}). \tag{4}$$

where Π_1 is ratio of characteristic length, Π_2 is ratio of static pressure and yield strength, called static damage number, Π_3 is ratio of dynamic pressure and yield

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strength, called dynamic damage; Π_3 is inverse of the Irwin number Ir, Π_4 is inverse of Reynolds number Re.

If we denote total pressure P as the sum of static and dynamics pressure $P = p + \frac{1}{2}\rho U^2$.

$$\begin{aligned} \frac{D}{d} &= f(\frac{p + 1/2\rho U^2}{\sigma_Y}, \frac{K_I}{\sigma_Y \sqrt{d}}, \frac{\nu}{Ud}) \\ &= f(\frac{P}{\sigma_Y}, \frac{K_I}{\sigma_Y \sqrt{d}}, \frac{\nu}{Ud}). \end{aligned} \tag{5}$$

Adopting the well-known defined dimensionless variable, then the current crack length scale D can be expressed as simpler format

$$D = d \cdot f(J, \frac{1}{Ir}, \frac{1}{Re}). \tag{6}$$

where the total damage number is defined as $J = \frac{P}{\sigma_Y}$.

Formula (6)is an universal scale law of hydraulic fracturing. It represents combined influence of fracking flow Re, rock fracture stiffness Ir and flow-rock interaction J onto the crack scale. Since for given a crack the Irwin nubmer Ir is constant, so as longer as the model test has same cracking liquid and flow speed, the geometric similarity is valid. This geometric geometric similarity will breaks down when the response time of the rock materials is taken into account.

EFFECT OF TIME

It must noted that rock fracture would be happening in some time. Impact under high pressure, the development from micro cracking, growth, interconnecting to a macro crack will needs a time process, in other words, the rock has an characteristic fracture time t_p, with time dimension t.

Having concern the speed of energy wave in the rock, defined by $\sqrt{\frac{\sigma_Y}{\rho}}$, then the problem will have 9 variables in the following table.

TABLE II: Hydraulic fracturing dimensions with time effect

ρ	ν	p	U	σ_Y	K_I	d	D	t_p
mL^{-3}	L^2t^{-1}	$L^{-1}mt^{-2}$	Lt^{-1}	$L^{-1}mt^{-2}$	$L^{-1/2}mt^{-2}$	L	L	t

$$\Pi_6 = (\frac{d}{t_p})/\sqrt{\frac{\sigma_Y}{\rho}} = \frac{d}{t_p}\sqrt{\frac{\rho}{\sigma_Y}}. \tag{7}$$

If set $d\sqrt{\frac{\rho}{\sigma_Y}} = t_c$ as fracturing time of the rock, then $\Pi_6 = t_c/t_p$ is Debrah number De. The rock is softer or stronger as Deborah number is getting smaller or bigger, respectively.

Therefore the formula (6) can be extended into

$$D = d \cdot f(J, \frac{1}{Ir}, \frac{1}{Re}, De). \tag{8}$$

Because the Debrah number De contains length scale d, so the geometry similarity will break downs if take into account the response time t_p.

KEY CONTROL VARIABLE OF HYDRAULIC FRACTURE IS TOTAL DAMAGE NUMBER

Universal scaling law formula (6) shows that the length scale law is function of Ir, Re, J. For different stage, they will paly a different role.

If crack is in developing stage, which means that the Irwin number Ir must reach its critical value Ir_c, the formula (6) can be simplified into

$$D = l \cdot f(J, \frac{1}{Re}). \tag{9}$$

If Renolds number Re is small, the the above formula can further simplified into

$$D = d \cdot f(J) = l \cdot f(\frac{P}{\sigma_Y}) = l \cdot f(\frac{p + \frac{1}{2}\rho U^2}{\sigma_Y}). \tag{10}$$

Formula(5)indicates the control variables of the problem can be reduced to four, ie., d, P, σ_Y, D, in this case, we have

$$\begin{aligned} D &= d \cdot f(J) = Cd \cdot (\frac{P}{\sigma_Y})^\alpha \\ &= Cd \cdot (\frac{p + \frac{1}{2}\rho U^2}{\sigma_Y})^\alpha = CdJ^\alpha. \end{aligned} \tag{11}$$

From physics pint of view, the C and $\alpha > 0$ are constants to be determined by test.

If we compare effect of Ir, Re, J on the hydraulic fracturing, we can find the total damage number J is key control variable of the process. Because the total damage number is sum of static and dynamic, static pressure is constant, so to have a better fracturing, we must apply a high variable dynamics pressure.

CONCLUSIONS

The key control variable is the total damager number $J = \frac{p + \frac{1}{2}\rho U^2}{\sigma_Y}$. The crack length is satisfied the geometric similarity law if no response time is concerned, otherwise, would not stratified the similarity.

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