Supplement

L_p Loss Functions in Invariance Alignment and Haberman Linking

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Enclosed in this document are the supplemental materials for our article entitled "L_p Loss Functions in Invariance Alignment and Haberman Linking".

Supplement A contains additional tables to the results of Study 1 reported in the article. Supplement B contains additional tables to the results of Study 2 reported in the article.

Supplement A: Additional Results for Study 1

The following pages contain additional results for Simulation Study 1. Tables A1 to A4 show the average absolute bias of group means as a function of the number of groups ($G = 3$ or $G = 6$) and in the case of no DIF and for DIF. Tables A5 and A6 show the average RMSE of group means for $G = 3$ groups and in the case of no DIF and for DIF. Tables A7 to A10 show the average absolute bias of group standard deviations as a function of the number of groups ($G = 3$ or $G = 6$) and in the case of no DIF and for DIF. Tables A11 to A14 show the average RMSE of group standard deviations as a function of the number of groups ($G = 3$ or $G = 6$) and in the case of no DIF and for DIF.

Table A1

Study 1: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.003	.003	.001	.004	.003	.003	.004	.004
0.1	.003	.003	.001	.004	.002	.003	.004	.004
0.25	.003	.002	.001	.004	.002	.003	.003	.004
0.5	.003	.002	.000	.002	.002	.002	.003	.003
1	.002	.002	.000	.002	.001	.002	.003	.003
2	.002	.002	.002	.004	.001	.002	.003	.004
	$N = 500$							
0.02	.001	.001	.001	.001	.001	.001	.001	.001
0.1	.001	.001	.001	.001	.001	.001	.001	.001
0.25	.001	.001	.001	.001	.001	.001	.001	.001
0.5	.001	.001	.000	.001	.001	.001	.001	.001
1	.001	.000	.000	.001	.001	.001	.001	.001
2	.001	.001	.001	.001	.001	.001	.001	.001
	$N = 1000$							
0.02	.003	.003	.004	.003	.002	.003	.003	.003
0.1	.003	.003	.004	.003	.002	.003	.003	.003
0.25	.003	.003	.004	.003	.002	.003	.003	.003
0.5	.003	.003	.004	.003	.002	.003	.002	.003
1	.003	.003	.004	.003	.003	.003	.003	.003
2	.003	.003	.003	.003	.003	.003	.003	.003
	$N = 5000$							
0.02	.002	.002	.002	.002	.002	.002	.002	.002
0.1	.002	.002	.002	.002	.002	.002	.002	.002
0.25	.002	.002	.002	.002	.002	.002	.002	.002
0.5	.002	.002	.002	.002	.002	.002	.002	.002
1	.002	.002	.002	.002	.002	.002	.002	.002
2	.002	.002	.002	.002	.002	.002	.002	.002

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A2

Study 1: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.007	.007	.005	.010	.008	.008	.007	.007
0.1	.007	.007	.005	.010	.008	.008	.007	.007
0.25	.007	.007	.006	.010	.007	.008	.007	.007
0.5	.007	.007	.006	.010	.007	.007	.007	.007
1	.007	.006	.007	.009	.007	.006	.006	.006
2	.006	.006	.007	.008	.006	.006	.005	.005
	$N = 500$							
0.02	.005	.005	.005	.004	.005	.005	.005	.005
0.1	.005	.005	.005	.004	.005	.005	.005	.005
0.25	.004	.005	.005	.004	.005	.005	.005	.005
0.5	.004	.005	.005	.004	.005	.005	.004	.005
1	.004	.004	.005	.004	.005	.005	.004	.004
2	.004	.004	.005	.004	.004	.004	.004	.004
	$N = 1000$							
0.02	.001	.001	.004	.005	.002	.002	.002	.002
0.1	.001	.001	.004	.005	.002	.002	.002	.002
0.25	.001	.001	.003	.005	.002	.002	.002	.002
0.5	.001	.001	.003	.005	.002	.002	.002	.002
1	.001	.001	.003	.004	.001	.001	.002	.002
2	.001	.001	.003	.004	.001	.001	.002	.002
	$N = 5000$							
0.02	.001	.001	.001	.001	.001	.001	.001	.001
0.1	.001	.001	.001	.001	.001	.001	.001	.001
0.25	.001	.001	.001	.001	.001	.001	.001	.001
0.5	.001	.001	.001	.001	.001	.001	.001	.001
1	.001	.001	.001	.001	.001	.001	.001	.001
2	.001	.001	.001	.001	.001	.001	.001	.001

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A3

Study 1: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.016	.014	.044	.050	.010	.011	.019	.020
0.1	.017	.014	.044	.051	.010	.011	.024	.024
0.25	.017	.015	.045	.051	.012	.013	.022	.023
0.5	.020	.018	.047	.053	.016	.017	.024	.025
1	.036	.033	.070	.081	.033	.034	.044	.045
2	.073	.057	.119	.095	.061	.057	.066	.070
	$N = 500$							
0.02	.003	.002	.037	.037	.001	.002	.014	.015
0.1	.004	.003	.037	.038	.002	.002	.014	.015
0.25	.005	.004	.039	.040	.003	.003	.014	.014
0.5	.010	.010	.045	.045	.006	.006	.016	.017
1	.026	.024	.073	.081	.025	.026	.034	.035
2	.071	.053	.113	.087	.059	.053	.060	.068
	$N = 1000$							
0.02	.004	.004	.038	.037	.004	.004	.004	.004
0.1	.005	.005	.038	.039	.004	.004	.004	.004
0.25	.007	.007	.039	.040	.005	.005	.005	.005
0.5	.010	.009	.040	.042	.008	.008	.008	.009
1	.022	.021	.080	.086	.022	.023	.030	.031
2	.073	.055	.115	.090	.062	.055	.064	.071
	$N = 5000$							
0.02	.001	.001	.007	.008	.000	.000	.000	.000
0.1	.001	.001	.008	.008	.000	.000	.000	.001
0.25	.001	.001	.008	.008	.001	.001	.001	.001
0.5	.003	.003	.015	.015	.003	.003	.003	.003
1	.013	.012	.089	.094	.015	.016	.018	.018
2	.072	.054	.113	.086	.061	.054	.062	.070

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A4

Study 1: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.011	.011	.028	.032	.009	.010	.014	.015
0.1	.011	.012	.029	.032	.010	.011	.014	.016
0.25	.011	.012	.032	.036	.012	.012	.016	.016
0.5	.015	.015	.037	.042	.016	.017	.020	.021
1	.033	.031	.065	.080	.032	.033	.037	.038
2	.071	.057	.114	.101	.060	.057	.065	.069
	$N = 500$							
0.02	.005	.004	.038	.040	.004	.004	.007	.007
0.1	.005	.004	.037	.040	.005	.005	.008	.008
0.25	.006	.006	.038	.041	.006	.006	.009	.009
0.5	.010	.010	.045	.048	.009	.010	.011	.012
1	.027	.025	.078	.091	.027	.027	.030	.031
2	.072	.056	.110	.102	.062	.056	.063	.070
	$N = 1000$							
0.02	.004	.004	.028	.033	.004	.004	.006	.006
0.1	.004	.004	.031	.033	.004	.004	.006	.006
0.25	.005	.005	.032	.039	.005	.005	.007	.007
0.5	.008	.007	.039	.048	.007	.008	.009	.009
1	.022	.020	.089	.099	.022	.023	.025	.026
2	.073	.057	.111	.102	.062	.057	.063	.070
	$N = 5000$							
0.02	.001	.001	.008	.015	.001	.001	.001	.001
0.1	.001	.001	.008	.021	.001	.001	.001	.001
0.25	.002	.002	.013	.024	.002	.002	.002	.002
0.5	.003	.003	.013	.016	.003	.004	.004	.004
1	.014	.013	.089	.096	.016	.016	.017	.018
2	.072	.055	.111	.098	.061	.055	.063	.070

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A5

Study 1: Average Root Mean Square Error (ARMSE) of Group Means as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.052	.052	.061	.061	.051	.051	.052	.052
0.1	.052	.051	.060	.061	.051	.051	.051	.051
0.25	.051	.051	.060	.060	.051	.050	.051	.050
0.5	.051	.050	.058	.058	.050	.049	.050	.050
1	.049	.049	.055	.055	.049	.049	.049	.049
2	.048	.048	.054	.054	.048	.048	.049	.049
	$N = 500$							
0.02	.040	.039	.044	.045	.040	.040	.040	.039
0.1	.039	.039	.044	.044	.040	.040	.040	.039
0.25	.039	.039	.043	.044	.039	.039	.039	.039
0.5	.039	.039	.043	.043	.039	.039	.039	.039
1	.038	.038	.041	.041	.038	.038	.038	.038
2	.038	.038	.040	.040	.038	.038	.038	.038
	$N = 1000$							
0.02	.026	.026	.029	.029	.026	.026	.026	.026
0.1	.026	.026	.029	.029	.026	.026	.026	.026
0.25	.026	.026	.028	.028	.025	.025	.026	.026
0.5	.025	.025	.028	.028	.025	.025	.026	.026
1	.025	.025	.027	.027	.025	.025	.025	.025
2	.025	.025	.026	.026	.025	.025	.025	.025
	$N = 5000$							
0.02	.011	.011	.011	.011	.011	.011	.011	.011
0.1	.011	.011	.011	.011	.011	.011	.011	.011
0.25	.011	.011	.011	.011	.011	.011	.011	.011
0.5	.011	.011	.011	.011	.011	.011	.011	.011
1	.011	.011	.011	.011	.011	.011	.011	.011
2	.011	.011	.011	.011	.011	.011	.011	.011

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A6

Study 1: Average Root Mean Square Error (ARMSE) of Group Means as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.072	.070	.111	.116	.068	.067	.078	.077
0.1	.071	.069	.111	.116	.068	.067	.079	.078
0.25	.069	.068	.110	.114	.067	.067	.076	.077
0.5	.069	.067	.107	.111	.066	.066	.074	.075
1	.071	.068	.102	.111	.067	.067	.075	.075
2	.089	.080	.131	.114	.081	.080	.094	.093
	$N = 500$							
0.02	.043	.043	.092	.093	.042	.041	.061	.060
0.1	.043	.043	.091	.093	.042	.041	.060	.060
0.25	.043	.042	.090	.093	.042	.042	.058	.058
0.5	.043	.043	.090	.093	.042	.041	.057	.057
1	.049	.048	.091	.097	.048	.048	.057	.057
2	.080	.069	.120	.098	.072	.069	.079	.080
	$N = 1000$							
0.02	.028	.027	.083	.082	.026	.026	.028	.028
0.1	.028	.028	.083	.084	.026	.026	.028	.028
0.25	.028	.027	.082	.084	.026	.026	.028	.028
0.5	.028	.028	.080	.082	.026	.026	.029	.029
1	.035	.033	.090	.096	.034	.035	.045	.046
2	.077	.065	.118	.094	.067	.065	.075	.078
	$N = 5000$							
0.02	.011	.011	.038	.038	.011	.011	.011	.011
0.1	.011	.011	.038	.038	.011	.011	.011	.011
0.25	.011	.011	.038	.038	.011	.011	.011	.011
0.5	.011	.011	.048	.049	.011	.011	.012	.012
1	.017	.017	.092	.096	.019	.019	.023	.023
2	.072	.057	.114	.087	.062	.057	.065	.071

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A7

Study 1: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.004	.006	.006	.005	.007	.004	.007	.005
0.1	.004	.006	.006	.005	.006	.004	.007	.004
0.25	.004	.006	.006	.005	.006	.005	.007	.005
0.5	.004	.005	.005	.004	.006	.005	.007	.005
1	.003	.005	.005	.004	.006	.005	.006	.005
2	.003	.005	.005	.004	.006	.005	.006	.005
	$N = 500$							
0.02	.003	.002	.002	.003	.002	.002	.002	.002
0.1	.003	.002	.002	.002	.002	.002	.002	.002
0.25	.003	.002	.002	.003	.002	.002	.002	.002
0.5	.003	.002	.002	.003	.002	.002	.002	.002
1	.002	.002	.002	.003	.002	.002	.002	.002
2	.002	.002	.002	.002	.002	.002	.002	.002
	$N = 1000$							
0.02	.004	.005	.004	.004	.004	.005	.005	.005
0.1	.004	.005	.004	.004	.004	.005	.004	.005
0.25	.004	.005	.004	.004	.004	.004	.004	.005
0.5	.004	.004	.004	.004	.004	.004	.004	.004
1	.003	.004	.004	.003	.004	.004	.004	.004
2	.003	.003	.003	.003	.003	.003	.004	.004
	$N = 5000$							
0.02	.000	.001	.001	.001	.001	.001	.001	.001
0.1	.000	.001	.001	.001	.001	.001	.001	.001
0.25	.000	.001	.001	.001	.001	.001	.001	.001
0.5	.000	.001	.001	.001	.001	.001	.001	.001
1	.001	.001	.001	.001	.001	.001	.001	.001
2	.001	.001	.001	.001	.001	.001	.001	.001

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A8

Study 1: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.005	.003	.005	.008	.005	.004	.005	.004
0.1	.005	.003	.005	.008	.005	.004	.005	.004
0.25	.005	.003	.005	.008	.005	.004	.005	.004
0.5	.005	.003	.005	.008	.004	.004	.005	.004
1	.005	.003	.004	.007	.004	.003	.004	.004
2	.005	.003	.004	.006	.003	.003	.004	.004
	$N = 500$							
0.02	.004	.003	.003	.004	.003	.003	.003	.003
0.1	.004	.003	.003	.004	.003	.003	.003	.003
0.25	.004	.003	.003	.004	.003	.003	.003	.003
0.5	.004	.003	.003	.004	.003	.003	.003	.003
1	.004	.003	.003	.004	.003	.003	.003	.003
2	.004	.003	.003	.004	.003	.003	.003	.003
	$N = 1000$							
0.02	.002	.002	.003	.003	.002	.002	.002	.002
0.1	.002	.002	.003	.003	.002	.002	.002	.002
0.25	.002	.002	.002	.003	.002	.002	.002	.002
0.5	.002	.002	.002	.003	.002	.002	.002	.002
1	.002	.002	.002	.003	.002	.002	.002	.002
2	.002	.002	.002	.003	.002	.002	.002	.002
	$N = 5000$							
0.02	.001	.001	.001	.001	.001	.001	.001	.001
0.1	.001	.001	.001	.001	.001	.001	.001	.001
0.25	.001	.001	.001	.001	.001	.001	.001	.001
0.5	.001	.001	.001	.001	.001	.001	.001	.001
1	.001	.001	.001	.001	.001	.001	.001	.001
2	.001	.001	.001	.001	.001	.001	.001	.001

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A9

Study 1: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.009	.014	.017	.022	.013	.012	.016	.014
0.1	.009	.014	.018	.022	.014	.013	.017	.015
0.25	.008	.015	.018	.020	.014	.013	.016	.016
0.5	.009	.018	.021	.019	.017	.016	.019	.018
1	.012	.035	.035	.019	.037	.037	.037	.037
2	.020	.101	.094	.019	.083	.101	.082	.100
	$N = 500$							
0.02	.007	.010	.013	.011	.009	.009	.011	.011
0.1	.008	.011	.014	.011	.009	.009	.011	.011
0.25	.008	.013	.015	.011	.010	.010	.012	.012
0.5	.010	.016	.018	.013	.013	.013	.014	.015
1	.014	.032	.033	.018	.032	.034	.033	.034
2	.027	.105	.098	.025	.088	.105	.086	.104
	$N = 1000$							
0.02	.002	.002	.009	.010	.001	.001	.001	.001
0.1	.002	.002	.009	.010	.001	.001	.001	.001
0.25	.002	.003	.008	.010	.002	.001	.002	.002
0.5	.003	.006	.008	.009	.004	.004	.004	.004
1	.006	.019	.021	.015	.021	.022	.021	.022
2	.023	.101	.094	.022	.083	.101	.081	.100
	$N = 5000$							
0.02	.000	.001	.001	.001	.001	.001	.001	.001
0.1	.000	.001	.001	.001	.001	.001	.001	.001
0.25	.001	.002	.002	.001	.001	.002	.001	.002
0.5	.001	.003	.004	.002	.003	.003	.003	.003
1	.005	.014	.017	.013	.016	.018	.016	.017
2	.025	.103	.096	.023	.085	.103	.083	.101

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A10

Study 1: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.008	.011	.014	.016	.013	.013	.014	.014
0.1	.008	.012	.014	.015	.013	.013	.014	.014
0.25	.009	.013	.015	.016	.015	.015	.015	.015
0.5	.011	.018	.019	.018	.019	.018	.020	.019
1	.022	.038	.037	.029	.041	.041	.041	.042
2	.048	.105	.099	.055	.088	.105	.088	.104
	$N = 500$							
0.02	.005	.003	.011	.014	.003	.003	.004	.004
0.1	.005	.003	.011	.014	.003	.003	.004	.004
0.25	.005	.003	.010	.014	.003	.003	.004	.004
0.5	.007	.007	.010	.015	.006	.006	.007	.006
1	.015	.023	.025	.025	.025	.027	.025	.026
2	.048	.099	.092	.056	.081	.099	.080	.098
	$N = 1000$							
0.02	.004	.003	.007	.010	.003	.003	.003	.003
0.1	.004	.003	.007	.009	.003	.003	.003	.003
0.25	.004	.003	.007	.010	.003	.003	.004	.004
0.5	.006	.006	.008	.011	.006	.005	.006	.006
1	.014	.021	.023	.023	.023	.024	.023	.024
2	.049	.102	.095	.056	.084	.102	.082	.100
	$N = 5000$							
0.02	.001	.001	.001	.003	.001	.001	.001	.001
0.1	.001	.001	.002	.004	.001	.001	.001	.001
0.25	.001	.002	.002	.004	.002	.002	.002	.002
0.5	.002	.003	.004	.003	.003	.004	.003	.004
1	.008	.014	.017	.016	.016	.018	.016	.018
2	.048	.103	.096	.054	.085	.103	.083	.101

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A11

Study 1: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.046	.046	.047	.047	.045	.047	.045	.047
0.1	.046	.046	.047	.047	.044	.046	.045	.046
0.25	.045	.045	.046	.046	.044	.045	.044	.045
0.5	.044	.044	.045	.045	.043	.044	.043	.044
1	.042	.042	.043	.043	.041	.042	.041	.042
2	.041	.041	.042	.042	.040	.041	.040	.041
	$N = 500$							
0.02	.035	.034	.035	.036	.033	.033	.033	.033
0.1	.035	.034	.035	.035	.033	.033	.033	.033
0.25	.034	.033	.034	.035	.032	.033	.032	.033
0.5	.033	.033	.033	.034	.031	.032	.031	.032
1	.032	.031	.032	.032	.030	.031	.030	.031
2	.030	.030	.031	.031	.029	.030	.029	.030
	$N = 1000$							
0.02	.023	.023	.024	.024	.023	.023	.023	.023
0.1	.023	.023	.024	.023	.022	.023	.023	.023
0.25	.023	.023	.023	.023	.022	.023	.022	.023
0.5	.022	.023	.023	.023	.022	.022	.022	.022
1	.022	.022	.022	.022	.021	.022	.021	.022
2	.022	.022	.022	.022	.021	.022	.021	.022
	$N = 5000$							
0.02	.009	.009	.009	.009	.008	.009	.008	.009
0.1	.009	.009	.009	.009	.008	.009	.008	.009
0.25	.009	.009	.009	.009	.008	.009	.008	.009
0.5	.009	.009	.009	.009	.008	.009	.008	.009
1	.009	.009	.009	.009	.008	.009	.008	.009
2	.009	.009	.009	.009	.008	.009	.008	.009

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A12

Study 1: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.050	.049	.050	.052	.048	.049	.049	.049
0.1	.050	.048	.049	.051	.048	.049	.048	.049
0.25	.049	.047	.049	.050	.048	.048	.048	.048
0.5	.047	.046	.048	.049	.047	.047	.047	.047
1	.045	.045	.046	.046	.044	.045	.044	.045
2	.044	.044	.045	.045	.043	.044	.043	.044
	$N = 500$							
0.02	.034	.034	.035	.034	.035	.034	.035	.034
0.1	.034	.034	.034	.034	.034	.034	.034	.034
0.25	.034	.034	.034	.034	.034	.034	.034	.034
0.5	.033	.033	.033	.033	.033	.034	.033	.033
1	.033	.033	.033	.033	.032	.033	.032	.033
2	.032	.032	.033	.033	.032	.032	.032	.032
	$N = 1000$							
0.02	.024	.024	.025	.025	.024	.025	.024	.025
0.1	.024	.024	.025	.025	.024	.024	.024	.024
0.25	.024	.024	.025	.025	.024	.024	.024	.024
0.5	.024	.024	.024	.024	.023	.024	.023	.024
1	.023	.023	.024	.024	.023	.024	.023	.024
2	.023	.023	.024	.024	.023	.023	.023	.023
	$N = 5000$							
0.02	.010	.010	.011	.011	.010	.010	.010	.010
0.1	.010	.010	.011	.010	.010	.010	.010	.010
0.25	.010	.010	.011	.010	.010	.010	.010	.010
0.5	.010	.010	.010	.010	.010	.010	.010	.010
1	.010	.010	.010	.010	.010	.010	.010	.010
2	.010	.010	.010	.010	.010	.010	.010	.010

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A13

Study 1: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.051	.055	.061	.059	.057	.057	.058	.058
0.1	.051	.055	.061	.058	.056	.056	.058	.057
0.25	.050	.054	.060	.058	.055	.055	.056	.056
0.5	.050	.054	.059	.056	.055	.054	.056	.055
1	.047	.060	.061	.052	.062	.060	.062	.061
2	.048	.116	.115	.049	.097	.116	.097	.115
	$N = 500$							
0.02	.035	.035	.041	.041	.034	.034	.038	.038
0.1	.034	.035	.040	.040	.034	.034	.038	.038
0.25	.034	.036	.040	.040	.035	.034	.038	.037
0.5	.034	.037	.041	.039	.035	.035	.038	.038
1	.034	.046	.047	.039	.047	.047	.048	.047
2	.040	.111	.108	.040	.094	.111	.093	.110
	$N = 1000$							
0.02	.024	.025	.029	.029	.023	.023	.023	.024
0.1	.024	.024	.029	.029	.023	.023	.023	.023
0.25	.023	.024	.028	.028	.023	.023	.023	.023
0.5	.023	.024	.028	.028	.023	.023	.023	.023
1	.023	.030	.032	.028	.031	.032	.032	.033
2	.030	.105	.101	.030	.086	.105	.085	.104
	$N = 5000$							
0.02	.010	.010	.012	.012	.009	.009	.009	.009
0.1	.010	.009	.012	.012	.009	.009	.009	.009
0.25	.009	.010	.012	.012	.009	.009	.009	.009
0.5	.009	.010	.013	.013	.009	.010	.009	.010
1	.010	.017	.019	.017	.018	.020	.018	.020
2	.026	.104	.098	.025	.085	.104	.084	.102

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table A14

Study 1: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.057	.059	.062	.061	.062	.059	.062	.060
0.1	.057	.059	.061	.060	.061	.059	.062	.060
0.25	.056	.058	.061	.060	.061	.059	.062	.060
0.5	.055	.058	.060	.060	.062	.059	.062	.060
1	.057	.066	.067	.060	.073	.069	.073	.070
2	.071	.123	.123	.075	.107	.123	.107	.123
	$N = 500$							
0.02	.036	.036	.041	.041	.037	.037	.037	.038
0.1	.036	.035	.040	.041	.037	.037	.037	.037
0.25	.035	.035	.040	.041	.036	.036	.037	.037
0.5	.035	.035	.039	.041	.036	.036	.036	.036
1	.037	.041	.043	.043	.045	.044	.045	.044
2	.060	.108	.106	.066	.089	.108	.089	.107
	$N = 1000$							
0.02	.026	.026	.029	.030	.025	.025	.025	.026
0.1	.026	.025	.029	.030	.025	.025	.025	.025
0.25	.025	.025	.029	.030	.025	.025	.025	.025
0.5	.025	.025	.028	.030	.025	.025	.026	.026
1	.028	.032	.034	.034	.035	.035	.035	.035
2	.056	.106	.103	.061	.088	.106	.087	.105
	$N = 5000$							
0.02	.010	.010	.013	.015	.010	.010	.010	.010
0.1	.010	.010	.013	.016	.010	.010	.010	.011
0.25	.010	.011	.014	.016	.010	.011	.011	.011
0.5	.011	.011	.013	.014	.011	.011	.011	.011
1	.013	.017	.020	.020	.020	.021	.019	.021
2	.050	.104	.098	.055	.086	.104	.084	.102

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Supplement B: Additional Results for Study 2

The following pages contain additional results for Simulation Study 2. Tables B1 to B4 show the average absolute bias as a function of the number of groups ($G = 3$ or 6) and no DIF and the presence of DIF. Table B5 shows the average absolute RMSE for $G = 3$ groups in the condition of no DIF.

The following pages contain additional results for Simulation Study 2. Tables B1 to B4 show the average absolute bias of group means as a function of the number of groups ($G = 3$ or $G = 6$) and in the case of no DIF and for DIF. Table B5 shows the average RMSE for $G = 3$ groups in the condition of no DIF. Tables B6 to B9 show the average absolute bias of group standard deviations as a function of the number of groups ($G = 3$ or $G = 6$) and in the case of no DIF and for DIF. Tables B10 to B13 show the average RMSE of group standard deviations as a function of the number of groups ($G = 3$ or $G = 6$) and in the case of no DIF and for DIF.

Table B1

Study 2: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.016	.012	.024	.003	.010	.011	.010	.008
0.1	.015	.013	.022	.004	.010	.011	.008	.006
0.25	.016	.013	.022	.002	.011	.012	.008	.006
0.5	.016	.013	.022	.002	.012	.013	.007	.006
1	.016	.014	.022	.005	.014	.014	.009	.010
2	.019	.017	.011	.020	.017	.017	.019	.019
	$N = 500$							
0.02	.003	.006	.017	.009	.006	.006	.005	.006
0.1	.003	.006	.018	.008	.006	.006	.005	.005
0.25	.003	.006	.017	.008	.006	.006	.003	.004
0.5	.003	.006	.017	.007	.006	.006	.003	.003
1	.003	.005	.013	.004	.005	.005	.003	.003
2	.003	.005	.004	.006	.006	.005	.007	.007
	$N = 1000$							
0.02	.002	.004	.020	.014	.003	.003	.006	.007
0.1	.002	.003	.020	.014	.003	.003	.006	.007
0.25	.002	.003	.019	.014	.003	.003	.005	.006
0.5	.001	.003	.019	.013	.003	.002	.005	.005
1	.002	.003	.015	.011	.003	.003	.004	.004
2	.002	.003	.009	.004	.003	.003	.001	.001
	$N = 5000$							
0.02	.001	.001	.018	.017	.001	.001	.000	.001
0.1	.001	.001	.018	.017	.001	.001	.000	.001
0.25	.001	.001	.017	.017	.001	.001	.000	.001
0.5	.001	.001	.016	.015	.001	.001	.000	.000
1	.001	.001	.012	.012	.001	.001	.000	.001
2	.000	.000	.005	.004	.000	.000	.000	.001

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B2

Study 2: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.004	.010	.047	.041	.010	.010	.012	.018
0.1	.007	.010	.047	.049	.009	.010	.011	.017
0.25	.008	.011	.049	.084	.010	.010	.011	.014
0.5	.006	.012	.046	.073	.010	.011	.009	.013
1	.007	.013	.043	.056	.013	.014	.008	.008
2	.009	.017	.043	.076	.015	.017	.034	.030
	$N = 500$							
0.02	.004	.008	.031	.019	.007	.009	.005	.004
0.1	.003	.008	.031	.018	.006	.009	.004	.004
0.25	.003	.008	.031	.017	.006	.008	.004	.004
0.5	.003	.008	.031	.017	.006	.008	.003	.003
1	.002	.008	.030	.015	.006	.007	.004	.003
2	.003	.008	.021	.028	.008	.008	.009	.007
	$N = 1000$							
0.02	.005	.006	.027	.004	.005	.006	.004	.005
0.1	.005	.007	.027	.004	.005	.006	.004	.005
0.25	.005	.007	.026	.004	.004	.006	.004	.004
0.5	.005	.007	.025	.004	.004	.006	.004	.004
1	.005	.007	.021	.004	.004	.006	.004	.004
2	.005	.007	.014	.014	.005	.007	.006	.006
	$N = 5000$							
0.02	.001	.002	.034	.030	.003	.002	.002	.002
0.1	.001	.002	.033	.029	.003	.002	.002	.002
0.25	.001	.002	.032	.028	.003	.002	.002	.002
0.5	.001	.002	.029	.025	.003	.002	.002	.002
1	.001	.002	.020	.016	.003	.002	.003	.002
2	.001	.002	.008	.004	.002	.002	.004	.002

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B3

Study 2: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.081	.079	.082	.090	.095	.095	.115	.112
0.1	.081	.079	.085	.091	.096	.096	.117	.112
0.25	.083	.081	.083	.093	.098	.099	.119	.116
0.5	.094	.093	.091	.102	.099	.099	.121	.122
1	.115	.112	.108	.112	.112	.112	.133	.133
2	.139	.136	.133	.140	.136	.136	.165	.165
	$N = 500$							
0.02	.037	.037	.066	.069	.048	.047	.076	.078
0.1	.039	.038	.067	.070	.050	.050	.077	.082
0.25	.044	.043	.072	.075	.054	.055	.078	.083
0.5	.052	.051	.082	.085	.061	.062	.090	.093
1	.095	.094	.107	.110	.094	.094	.119	.119
2	.142	.140	.144	.148	.140	.140	.164	.164
	$N = 1000$							
0.02	.018	.018	.046	.043	.025	.026	.054	.053
0.1	.019	.019	.047	.045	.026	.026	.053	.052
0.25	.024	.024	.050	.049	.029	.029	.059	.059
0.5	.033	.033	.059	.059	.036	.036	.069	.069
1	.076	.076	.100	.100	.077	.078	.110	.110
2	.144	.144	.150	.151	.144	.144	.167	.167
	$N = 5000$							
0.02	.002	.002	.029	.027	.002	.002	.003	.003
0.1	.002	.002	.029	.028	.002	.002	.004	.003
0.25	.004	.004	.029	.028	.003	.004	.006	.005
0.5	.008	.008	.030	.029	.008	.008	.011	.011
1	.038	.038	.068	.068	.041	.042	.063	.063
2	.141	.141	.147	.147	.140	.141	.160	.160

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B4

Study 2: Average Absolute Bias (ABIAS) of Group Means as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.106	.111	.118	.109	.070	.070	.094	.090
0.1	.104	.112	.119	.112	.069	.070	.095	.092
0.25	.106	.115	.123	.113	.071	.071	.098	.089
0.5	.112	.117	.125	.120	.078	.079	.096	.093
1	.119	.125	.137	.124	.092	.093	.106	.107
2	.133	.138	.155	.145	.137	.138	.146	.147
	$N = 500$							
0.02	.072	.075	.102	.089	.029	.030	.049	.051
0.1	.073	.076	.104	.088	.030	.031	.052	.056
0.25	.079	.081	.107	.092	.033	.034	.057	.060
0.5	.090	.093	.114	.096	.041	.042	.063	.062
1	.115	.118	.130	.111	.071	.072	.087	.088
2	.137	.140	.153	.135	.139	.140	.148	.149
	$N = 1000$							
0.02	.028	.031	.081	.080	.015	.016	.023	.019
0.1	.031	.033	.084	.081	.015	.016	.025	.023
0.25	.036	.038	.090	.080	.018	.019	.029	.028
0.5	.050	.050	.101	.094	.024	.025	.037	.036
1	.106	.107	.129	.119	.054	.055	.070	.071
2	.139	.141	.153	.143	.140	.141	.155	.156
	$N = 5000$							
0.02	.003	.004	.052	.048	.002	.003	.003	.003
0.1	.004	.004	.052	.049	.002	.003	.003	.003
0.25	.006	.006	.053	.049	.003	.004	.004	.005
0.5	.011	.011	.056	.054	.006	.007	.009	.009
1	.061	.062	.112	.110	.027	.028	.036	.037
2	.139	.139	.148	.146	.139	.139	.152	.153

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B5

Study 2: Average Root Mean Square Error (ARMSE) of Group Means as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.077	.075	.075	.075	.073	.074	.082	.073
0.1	.075	.072	.074	.075	.072	.073	.077	.071
0.25	.074	.072	.074	.073	.072	.073	.075	.071
0.5	.071	.069	.069	.071	.068	.069	.067	.065
1	.064	.062	.061	.061	.061	.062	.060	.060
2	.069	.065	.060	.067	.065	.065	.069	.069
	$N = 500$							
0.02	.044	.043	.054	.056	.043	.044	.056	.058
0.1	.044	.043	.055	.055	.043	.044	.055	.059
0.25	.043	.042	.053	.053	.042	.042	.054	.056
0.5	.041	.041	.049	.050	.041	.041	.052	.053
1	.040	.039	.046	.046	.039	.039	.048	.048
2	.041	.041	.047	.049	.041	.041	.053	.053
	$N = 1000$							
0.02	.037	.037	.044	.042	.037	.037	.040	.040
0.1	.036	.036	.044	.042	.037	.037	.039	.040
0.25	.036	.036	.043	.041	.035	.036	.037	.038
0.5	.034	.034	.040	.038	.034	.034	.036	.035
1	.033	.033	.036	.034	.033	.033	.034	.034
2	.032	.032	.035	.034	.032	.032	.036	.036
	$N = 5000$							
0.02	.015	.015	.027	.026	.015	.015	.015	.015
0.1	.015	.015	.026	.025	.014	.014	.014	.014
0.25	.015	.015	.025	.024	.014	.014	.014	.014
0.5	.014	.014	.023	.023	.014	.014	.014	.014
1	.014	.014	.020	.019	.014	.014	.014	.014
2	.014	.014	.016	.015	.014	.014	.014	.014

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B6

Study 2: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.043	.007	.006	.044	.006	.010	.007	.010
0.1	.043	.007	.005	.044	.006	.009	.006	.009
0.25	.044	.008	.007	.045	.006	.010	.006	.010
0.5	.044	.008	.006	.045	.005	.008	.005	.008
1	.043	.007	.006	.043	.004	.007	.004	.008
2	.041	.006	.005	.040	.005	.006	.007	.007
	$N = 500$							
0.02	.019	.006	.006	.019	.009	.010	.009	.010
0.1	.019	.006	.005	.019	.009	.009	.009	.010
0.25	.021	.005	.004	.021	.009	.008	.009	.009
0.5	.021	.004	.004	.021	.008	.006	.007	.007
1	.020	.002	.002	.020	.005	.002	.005	.003
2	.018	.001	.001	.017	.004	.001	.004	.004
	$N = 1000$							
0.02	.009	.002	.003	.010	.007	.002	.007	.001
0.1	.009	.002	.003	.009	.006	.002	.007	.001
0.25	.009	.001	.003	.010	.006	.002	.006	.001
0.5	.009	.001	.003	.010	.005	.001	.006	.002
1	.009	.001	.003	.010	.004	.001	.004	.001
2	.009	.000	.001	.009	.002	.000	.002	.001
	$N = 5000$							
0.02	.001	.001	.004	.004	.004	.001	.004	.001
0.1	.001	.001	.004	.004	.004	.000	.004	.001
0.25	.001	.001	.004	.003	.003	.000	.003	.000
0.5	.001	.000	.003	.003	.003	.000	.003	.000
1	.002	.000	.002	.002	.002	.000	.002	.000
2	.002	.001	.001	.002	.001	.001	.001	.001

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B7

Study 2: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.096	.006	.020	.079	.007	.009	.007	.009
0.1	.094	.005	.020	.077	.006	.008	.007	.008
0.25	.084	.005	.021	.118	.006	.007	.006	.008
0.5	.075	.004	.021	.091	.006	.006	.006	.007
1	.063	.003	.020	.051	.005	.006	.007	.006
2	.053	.007	.007	.064	.006	.007	.037	.034
	$N = 500$							
0.02	.034	.005	.015	.023	.003	.004	.004	.004
0.1	.034	.004	.015	.023	.003	.004	.005	.004
0.25	.034	.005	.015	.024	.004	.004	.005	.004
0.5	.034	.004	.015	.024	.003	.004	.005	.003
1	.032	.004	.015	.024	.003	.004	.005	.003
2	.031	.005	.009	.024	.003	.005	.008	.007
	$N = 1000$							
0.02	.016	.003	.014	.017	.005	.002	.005	.002
0.1	.016	.003	.014	.017	.005	.003	.005	.003
0.25	.016	.003	.014	.017	.005	.003	.005	.003
0.5	.016	.002	.013	.017	.006	.002	.006	.002
1	.015	.002	.011	.015	.005	.001	.006	.002
2	.014	.002	.006	.011	.003	.002	.005	.004
	$N = 5000$							
0.02	.003	.001	.020	.018	.005	.001	.005	.001
0.1	.003	.001	.020	.017	.005	.001	.005	.001
0.25	.003	.001	.019	.017	.005	.001	.005	.001
0.5	.004	.001	.018	.015	.005	.001	.005	.001
1	.004	.001	.013	.010	.004	.001	.004	.001
2	.004	.001	.005	.005	.002	.001	.003	.001

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B8

Study 2: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.040	.017	.015	.040	.022	.019	.022	.019
0.1	.040	.017	.015	.040	.023	.019	.024	.019
0.25	.040	.017	.014	.041	.019	.018	.019	.018
0.5	.038	.013	.010	.038	.015	.013	.015	.013
1	.032	.012	.008	.033	.012	.012	.012	.011
2	.027	.018	.010	.028	.017	.018	.017	.017
	$N = 500$							
0.02	.019	.006	.003	.019	.008	.006	.008	.007
0.1	.019	.006	.002	.019	.007	.006	.008	.006
0.25	.019	.005	.002	.020	.006	.006	.007	.006
0.5	.018	.005	.002	.018	.006	.006	.006	.007
1	.017	.009	.005	.018	.008	.009	.008	.008
2	.018	.017	.013	.016	.017	.017	.015	.015
	$N = 1000$							
0.02	.008	.001	.004	.010	.007	.002	.006	.003
0.1	.008	.002	.004	.010	.007	.002	.006	.003
0.25	.008	.002	.004	.010	.008	.002	.007	.003
0.5	.008	.003	.004	.009	.007	.002	.007	.003
1	.009	.004	.003	.008	.006	.004	.006	.003
2	.014	.014	.010	.011	.013	.014	.011	.012
	$N = 5000$							
0.02	.003	.001	.006	.005	.003	.001	.003	.001
0.1	.003	.001	.006	.005	.003	.001	.003	.001
0.25	.003	.001	.005	.005	.003	.001	.003	.001
0.5	.003	.001	.005	.005	.002	.001	.002	.001
1	.003	.001	.002	.004	.002	.001	.002	.001
2	.013	.013	.011	.011	.013	.013	.010	.011

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B9

Study 2: Average Absolute Bias (ABIAS) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.105	.017	.031	.082	.017	.014	.017	.015
0.1	.098	.018	.031	.066	.018	.015	.017	.016
0.25	.094	.019	.033	.091	.017	.015	.017	.017
0.5	.084	.020	.035	.052	.018	.017	.018	.017
1	.078	.022	.036	.053	.021	.021	.018	.018
2	.069	.022	.030	.045	.022	.022	.009	.006
	$N = 500$							
0.02	.042	.012	.026	.037	.010	.008	.011	.012
0.1	.043	.013	.028	.037	.010	.008	.012	.013
0.25	.043	.014	.029	.037	.010	.009	.014	.014
0.5	.044	.016	.029	.037	.011	.011	.014	.013
1	.045	.020	.030	.037	.015	.015	.017	.017
2	.043	.021	.029	.034	.021	.021	.017	.018
	$N = 1000$							
0.02	.022	.008	.027	.029	.006	.006	.008	.006
0.1	.022	.008	.027	.028	.006	.007	.008	.007
0.25	.023	.009	.027	.029	.007	.007	.009	.009
0.5	.024	.012	.028	.029	.008	.009	.011	.011
1	.030	.019	.029	.030	.013	.014	.017	.017
2	.032	.022	.029	.030	.022	.022	.023	.023
	$N = 5000$							
0.02	.004	.002	.028	.026	.004	.002	.004	.002
0.1	.004	.002	.028	.026	.004	.002	.004	.002
0.25	.004	.002	.029	.027	.004	.002	.004	.002
0.5	.005	.003	.029	.026	.004	.003	.004	.003
1	.012	.012	.026	.024	.006	.007	.009	.009
2	.020	.020	.026	.024	.020	.020	.024	.025

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B10

Study 2: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.088	.076	.076	.088	.071	.076	.071	.076
0.1	.088	.075	.075	.087	.071	.075	.071	.075
0.25	.086	.072	.072	.086	.066	.072	.066	.072
0.5	.081	.066	.066	.081	.061	.068	.060	.068
1	.072	.057	.057	.072	.051	.057	.051	.057
2	.071	.054	.053	.071	.049	.054	.049	.054
	$N = 500$							
0.02	.059	.058	.059	.059	.052	.059	.052	.059
0.1	.058	.057	.058	.058	.051	.058	.051	.058
0.25	.057	.056	.056	.057	.050	.055	.050	.055
0.5	.054	.051	.052	.054	.045	.051	.046	.051
1	.047	.043	.043	.047	.038	.043	.038	.043
2	.046	.039	.040	.046	.037	.039	.037	.040
	$N = 1000$							
0.02	.038	.034	.035	.038	.032	.034	.032	.034
0.1	.037	.033	.034	.038	.032	.033	.032	.033
0.25	.036	.032	.032	.036	.030	.032	.030	.032
0.5	.034	.030	.030	.034	.028	.030	.028	.030
1	.029	.027	.027	.029	.025	.026	.025	.026
2	.027	.026	.026	.027	.024	.026	.024	.026
	$N = 5000$							
0.02	.015	.016	.016	.016	.014	.016	.014	.016
0.1	.015	.015	.016	.016	.014	.015	.014	.015
0.25	.015	.015	.015	.015	.014	.015	.014	.015
0.5	.014	.014	.015	.014	.013	.014	.013	.014
1	.013	.014	.014	.014	.013	.013	.012	.013
2	.013	.013	.013	.013	.012	.013	.012	.013

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B11

Study 2: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of No Differential Item Functioning (No DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.173	.081	.087	.177	.076	.081	.077	.082
0.1	.156	.079	.085	.188	.075	.080	.076	.080
0.25	.133	.076	.082	.262	.073	.076	.073	.076
0.5	.117	.069	.075	.215	.068	.073	.068	.073
1	.096	.062	.067	.098	.058	.064	.059	.064
2	.088	.063	.082	.126	.057	.063	.107	.102
	$N = 500$							
0.02	.068	.050	.055	.073	.052	.058	.051	.057
0.1	.067	.049	.055	.071	.051	.058	.050	.057
0.25	.065	.048	.053	.069	.049	.054	.049	.053
0.5	.062	.044	.050	.064	.045	.049	.045	.049
1	.056	.040	.045	.056	.039	.042	.040	.042
2	.053	.040	.043	.055	.038	.040	.039	.041
	$N = 1000$							
0.02	.042	.037	.042	.045	.037	.040	.037	.039
0.1	.041	.036	.041	.044	.036	.039	.036	.038
0.25	.040	.034	.039	.042	.035	.037	.035	.037
0.5	.038	.032	.036	.039	.032	.035	.032	.034
1	.035	.029	.033	.036	.028	.030	.028	.030
2	.035	.028	.030	.036	.028	.028	.028	.029
	$N = 5000$							
0.02	.014	.013	.026	.024	.015	.014	.015	.014
0.1	.014	.013	.026	.024	.015	.014	.015	.014
0.25	.014	.013	.025	.023	.014	.014	.014	.014
0.5	.013	.013	.023	.022	.014	.013	.014	.013
1	.013	.012	.018	.017	.013	.012	.013	.012
2	.013	.012	.014	.013	.012	.012	.012	.012

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B12

Study 2: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 3 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.095	.085	.085	.095	.083	.082	.083	.082
0.1	.095	.083	.083	.095	.081	.081	.082	.081
0.25	.093	.079	.079	.093	.074	.079	.075	.080
0.5	.084	.069	.068	.083	.065	.072	.065	.073
1	.070	.059	.059	.070	.054	.059	.054	.059
2	.065	.058	.057	.064	.053	.058	.053	.058
	$N = 500$							
0.02	.058	.052	.052	.058	.050	.051	.050	.052
0.1	.057	.050	.051	.057	.049	.051	.049	.051
0.25	.054	.048	.049	.055	.047	.048	.047	.049
0.5	.050	.044	.044	.051	.042	.045	.042	.046
1	.044	.039	.039	.044	.035	.039	.035	.039
2	.042	.040	.039	.041	.036	.040	.036	.040
	$N = 1000$							
0.02	.036	.035	.036	.037	.032	.036	.033	.037
0.1	.036	.034	.035	.036	.032	.035	.032	.036
0.25	.034	.033	.033	.035	.031	.034	.031	.035
0.5	.033	.031	.031	.033	.029	.031	.029	.031
1	.030	.028	.028	.030	.026	.028	.026	.028
2	.032	.029	.028	.031	.029	.029	.028	.029
	$N = 5000$							
0.02	.014	.015	.016	.015	.014	.015	.014	.015
0.1	.014	.014	.015	.015	.014	.014	.014	.014
0.25	.014	.014	.015	.015	.013	.014	.013	.014
0.5	.014	.013	.014	.014	.012	.013	.012	.013
1	.013	.013	.013	.013	.011	.012	.011	.012
2	.018	.018	.016	.016	.017	.018	.015	.016

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

Table B13

Study 2: Average Root Mean Square Error (ARMSE) of Group Standard Deviations as a Function of Sample Size in the Condition of Differential Item Functioning (DIF) and 6 Groups

p	IA1	IA2	IA3	IA4	HL1	HL2	HL3	HL4
	$N = 250$							
0.02	.170	.076	.083	.181	.077	.082	.078	.083
0.1	.140	.075	.082	.196	.076	.082	.076	.083
0.25	.133	.072	.080	.252	.072	.079	.073	.080
0.5	.114	.067	.075	.146	.067	.073	.067	.073
1	.100	.060	.069	.091	.059	.063	.058	.062
2	.093	.064	.069	.087	.057	.064	.056	.062
	$N = 500$							
0.02	.071	.057	.064	.071	.055	.057	.057	.059
0.1	.070	.056	.064	.070	.054	.057	.056	.060
0.25	.068	.054	.062	.068	.051	.055	.053	.057
0.5	.065	.051	.059	.064	.048	.052	.050	.053
1	.062	.048	.054	.059	.044	.047	.046	.049
2	.060	.050	.054	.057	.045	.050	.045	.049
	$N = 1000$							
0.02	.044	.041	.050	.050	.040	.042	.041	.042
0.1	.044	.039	.049	.049	.039	.041	.040	.042
0.25	.043	.038	.048	.048	.038	.040	.039	.041
0.5	.042	.037	.046	.046	.036	.038	.037	.039
1	.043	.037	.044	.044	.033	.035	.035	.037
2	.045	.037	.042	.043	.036	.037	.037	.039
	$N = 5000$							
0.02	.015	.014	.032	.031	.015	.015	.015	.015
0.1	.015	.014	.032	.031	.015	.015	.015	.015
0.25	.015	.014	.032	.031	.014	.015	.014	.015
0.5	.014	.014	.032	.030	.014	.014	.014	.014
1	.018	.018	.030	.028	.014	.015	.016	.016
2	.024	.024	.029	.028	.024	.024	.027	.028

Note. p = power used in invariance alignment (IA) or Haberman linking (HL) approaches; N = sample size; IA1 corresponds to the IA specification proposed by Asparouhov and Muthén [1], and HL3 is the original specification of Haberman [2].

References

[1] Tihomir Asparouhov and Bengt Muthén. Multiple-group factor analysis alignment. *Struct. Equ. Modeling*, 21(4):495–508, 2014.

[2] Shelby J. Haberman. *Linking parameter estimates derived from an item response model through separate calibrations* (Research Report No. RR-09-40). Educational Testing Service, 2009.