ON THE THEORY AND DESIGN OF COLD RESISTORS

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The concept of using special electrical circuit design realize a "cold resistors", that is, an active resistor circuitry with lowered effective noise temperature, was first introduced about 80 years ago. Later on, various kinds of artificial resistors were applied in different research areas, such as gravitational wave detection, photo-amplifiers and quartz oscillators. Their proofs of concepts were experimentally proved. Unfortunately, the complete theory was not found even though several attempts had been published, sometimes with errors. In this paper, we describe a correct and complete circuit theoretical model of a cold resistor system. The results are confirmed by computer simulations. A design tools for this circuit is also shown.

Keywords: Thermal noise; negative feedback; low-noise resistors; theory; design.

1. Introduction

The exploration of *cold resistors (CRs)* has a long history but no flawless theory. The goal of this paper to present and demonstrate a correct and complete theory for related circuit designs. In this section we briefly survey the foundation and some of the history of these efforts.

1.1. The Second Law of Thermodynamics

The thermal noise voltage (Johnson noise) in two parallel resistors R_h and R_c with temperatures $T_h > T_c$ generate a power flow P_{hc} (Fig. 1).

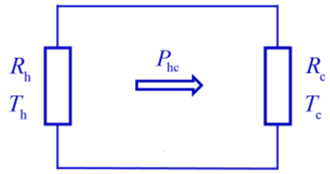

Fig. 1. T_h and T_c are the temperatures of resistors R_h and R_c respectively, where $T_h > T_c$. P_{hc} is the thermal power flow between these two parallel resistors.

The power flow P_{hc}, from R_h to R_c, in Δf frequency bandwidth can be expressed as [1]:

$$P_{hc} = 4k\left(T_h - T_c\right)\frac{R_h R_c}{\left(R_h + R_c\right)^2}\Delta f \ , \tag{1}$$

where k is the Boltzmann constant.

In the case of homogeneous temperature, $T_h = T_c$, the power flow is zero in accordance with the Second Law of Thermodynamics. Conversely, if we want to reduce the thermal noise of a resistor below the theoretical level, we need an active electronic device.

1.2. Definition of cold resistors

A *cold resistor (CR)* is a device emulating a resistor which has lower effective noise temperature T_{eff} than the temperature of its environment. Here T_{eff} is defined by the Johnson-Nyquist noise formula [2]:

$$S_u(f) = 4kT_{eff}R \ , \tag{2}$$

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where $S_u(f)$ is the power density spectrum of the thermal noise voltage, k is the Boltzmann constant, R is the resistance of the resistor.

The CR system can also be seen as a black box, see Fig. 2. The ambient temperature is T. At proper conditions, the CR behaves as a resistance with R_{eff} effective resistance and T_{eff} effective noise temperature, where $T_{eff} < T$.

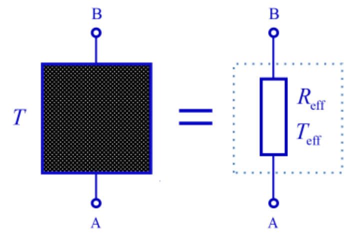

Fig. 2. The CR system as a black box. T is the ambient temperature. R_{eff} is the effective (dynamical) resistance and T_{eff} is the effective noise temperature of the CR, where $T_{eff} < T$.

1.3. Approaches for the realization of cold resistors

Cold resistors have a long history beginning in the 1940s [3-11]. Yet, this research is quite isolated and relatively unknown in the field of electronic noise. This is the probable reason that the possibility of producing an active device with Ohmic behavior and reduced noise temperature, that is, a cold resistor is not mentioned in textbooks, and this fact was a surprise to most noise researchers even at the 2018 international conference on Unsolved Problems of Noise [12].

1.3.1. The cold input resistance approach for cold resistor design

One type of CRs [3,4,5,6] is the effective resistance of the input (virtual ground) during a specific type of negative feedback, see Fig. 3. The dynamical resistance between the inverting input B and the ground and the related noise temperature are reduced. The amplifier can be as simple as a transistor.

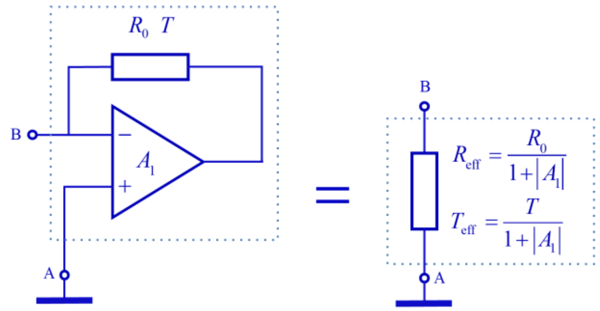

Fig. 3. CR system with cold input resistance approach. In the system, A_1 is the closed-loop gain of the amplifier and R_0 is the resistance of the feedback resistor.

The effective noise temperature T_{eff} is given as:

$$T_{eff} = \frac{T}{1+|A_1|}, \tag{3}$$

where A_1 is the closed-loop gain of the amplifier.

The equivalent circuit, see Fig. 3, is a grounded resistor with reduced resistance R_{eff}:

$$R_{eff} = \frac{R_0}{1+|A_1|}, \tag{4}$$

where R_0 is the resistance of the feedback resistor.

1.3.2. The cold output resistance approach for cold resistor design

Another type of CRs [7,8] is designed by the cold output resistance approach. This approach is based on the fact that the output resistance of an amplifier and a spurious voltage at its output are reduced by different scaling during negative feedback.

The proposed circuit realization first increases that output resistance of an amplifier of gain A_1 by a serial resistor placed at its output. Then the negative feedback is established by a second amplifier of gain A_2, see Fig. 4.

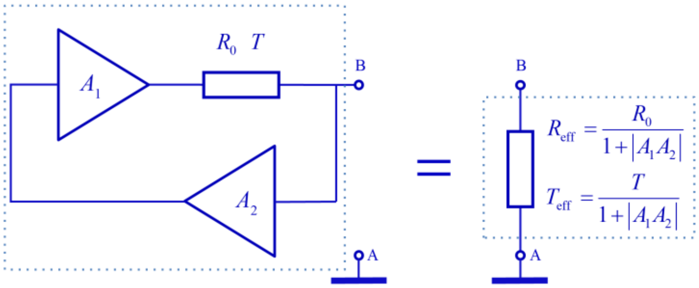

Fig. 4. CR system with cold output resistance approach. In the system, A_1 and A_2 are the closed-loop gains of the first amplifier and second amplifier respectively. R_0 is the resistance of the resistor at the output.

The effective resistance R_{eff} of the CR is [8]:

$$R_{eff} = \frac{R_0}{1+|A_1A_2|}, \quad (5)$$

where R_0 is the resistance of the resistor at the output, and A_1 and A_2 are the closed-loop gains of the amplifiers.

The effective noise temperature T_{eff} is [8]:

$$T_{eff} = \frac{T}{1+|A_1A_2|}. \quad (6)$$

The cold resistor appears between the ground and the output of the first amplifier - resistor system.

1.3.3. The AC-coupled diode approach for cold resistor design

A recent patent [9] by Linear Technology Corporation proposes a completely different method to create a CR. In the circuit, a diode is forward-biased by a low-noise DC current source, and it is connected to the external circuit by a coupling capacitor, see Fig. 5. The circuit performs as a nonlinear dynamical resistor with reduced noise temperature.

If the AC voltage between point A and point B is much less than kT/q, where q is the elementary charge, the dynamical resistance is fairly linear and the effective noise temperature T_{eff} is approximately half of the environment temperature T, provided the diode's current-voltage characteristic is near to ideal.

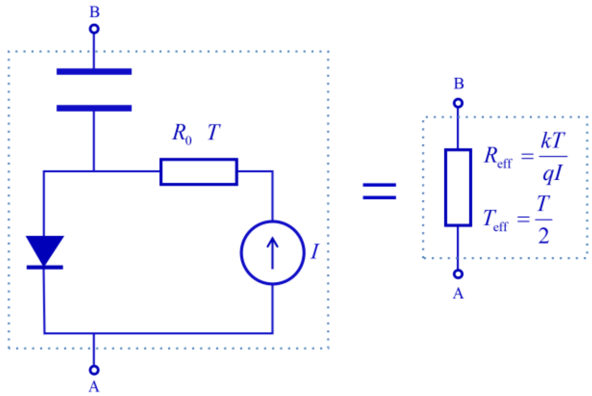

Fig. 5. Forward-biased, AC-coupled diode as a CR [9]. I is the current of the DC source and R_0 is the resistance of the resistor in the circuit.

Specifically [9], the effective resistance of the equivalent resistor R_{eff} is:

$$R_{eff} = \frac{kT}{qI}, \tag{7}$$

where k is the Boltzmann constant, I is the DC bias current, and the effective noise temperature T_{eff} is:

$$T_{eff} = \frac{T}{2}. \tag{8}$$

1.4. Former cold resistor theories and claims

In this section we briefly survey some of the most important former theoretical claims about CRs via negative feedback.

1.4.1. The cold resistor design proposed for transductors

CR was proposed for transductors in gravitational wave detection [4]. The generic scheme is the same as the one outlined in subsec. 1.3.1. The CR was created by connecting a resistor R_0 between the output and the inverting input of a low-noise operational amplifier, see Fig. 6, where I_n and U_n are the input current and voltage noise sources of the amplifier, respectively, and U_0 is the thermal noise source of the resistor.

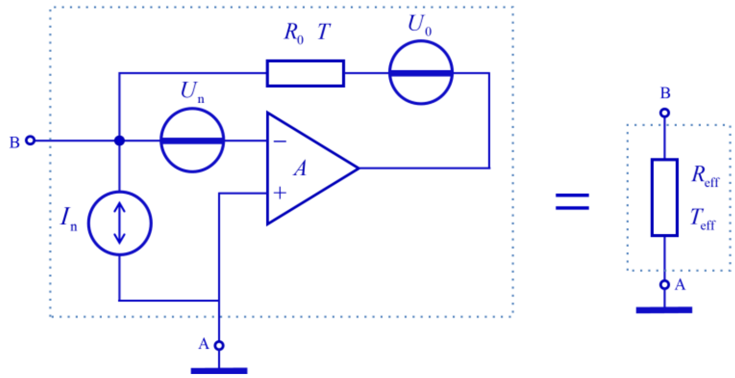

Fig. 6. Noise analysis circuit of the CR for gravitational wave detection. I_n and U_n are the input current and voltage noise sources of the amplifier, respectively, and U_0 is the thermal noise source of the resistor R_0 with temperature T.

The CR's effective resistance is given as:

$$R_{eff} = \frac{R_0}{1 + A_{opt}}, \tag{9}$$

where A_{opt} is the so-called optimum voltage gain of the operational amplifier [4]:

$$A_{opt} = \sqrt{\frac{4kTR_0 + R_0^2 I_n}{U_n}}. \tag{10}$$

The resulting minimum effective noise temperature between point A and point B is [4]:

$$T_{eff,\min} = \frac{\sqrt{\left(4kTR_0 + R_0^2 I_n\right)U_n}}{2kR_0}. \tag{11}$$

This theory [4] reaches the minimum effective noise temperature at the so-called optimum voltage gain. The derivation is straightforward and correct.

However, the theory is incomplete. Particularly, the variable R_0 contributes to not only to the value of the effective resistance in Eq. (9) but also to the minimum effective noise temperature in Eq. (11). But there are open questions that we will answer in this paper:

- Will there also be an optimum effective resistance?
- What is the lowest effective temperature for a given effective resistance?
- Is there an absolute lowest effective temperature that can be reached?

1.4.2. The cold resistor design proposed for quartz oscillators

Thesis [5] proposed a CR approach to achieve low-temperature mode in mechanical resonators. Their theoretical and experimental studies indicated that a CR scheme can be used to cool the mechanical mode of resonators as effectively as by liquid nitrogen.

In their system, the negative feedback between the output and inverting input takes place via a PID RC filter for a frequency-compensation purpose, see Fig. 7. U_t is the total thermal noise source of the filter.

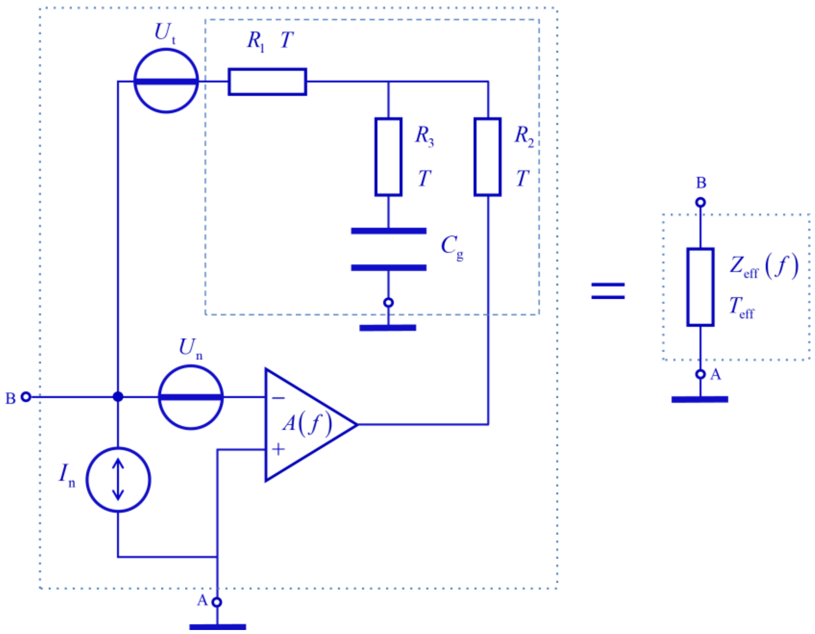

Fig. 7. Noise analysis circuit of the CR applied to quartz oscillators for cooling the loss component of the mechanical resonance. R_1, R_2, R_3 and C_g form a PID filter. U_t is the total thermal noise this filter. I_n and U_n are the input current and voltage noise sources of the amplifier, respectively, and $Z_{eff}(f)$ is the effective (cold) impedance between point A and B.

The effective impedance between point A and B is [5]:

$$Z_{eff}(f) = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2 + [1 + A(f)]R_3}, \qquad (12)$$

where R_1, R_2 and R_3 are the resistances of the resistors in the circuit respectively and $A(f)$ is the closed-loop gain of the operational amplifier.

The effective temperature of $Z_{eff}(f)$ is [5]:

$$T_{eff} = \frac{4kT\Re\left[Z_f(f)\right] + U_n\left|A(f)\right|^2 + I_n\left|Z_f(f)\right|^2}{4k\Re\left\{Z_f(f)[1 + A(f)]\right\}}, \qquad (13)$$

where $Z_f(f)$ is the generalized feedback impedance of the CR system:

$$Z_f(f) = [1 + A(f)]Z_{eff}(f) = \frac{[1 + A(f)](R_1R_2 + R_2R_3 + R_1R_3)}{R_2 + [1 + A(f)]R_3}. \qquad (14)$$

Note, there is a fundamental error (perhaps typo) in the above result [5] – missing T in the numerator in the equation giving the effective temperature. In Eq. (13) we already made this necessary correction.

Finally, similarly to the incompleteness of the theory, which we mentioned in subsection 1.4.1, this CR theory [5] also misses specifying the lowest possible effective temperature versus the effective resistance, for a given device.

In the next section, we deduce the correct and complete theory of CR systems with negative feedback.

2. Cold resistor system analysis

To deduce the correct theory of cold resistors, our formerly published [7,8] system will be used for the analysis. It is important to note that the difference between the output resistance based and input resistance based schemes is only a question of viewing the role of the resistor R_0 because point B in Fig. 4 can be viewed not only as output (of A_1) but also as input (of A_2).

2.1. Circuit analysis of the CR system

The practical CR circuitry is shown in Fig. 8. A passive resistor with resistance R_0 is connected to the output of a linear amplifier LA_1 with effective amplification $A_1 > 0$. The "new" output is point B. Another two linear amplifiers, LA_2 (in follower arrangement) and LA_3 with total effective amplification $A_2 < 0$ provide the negative feedback for LA_1 [7,8]. This arrangement serves a high input impedance for LA_2 at point B. Thus the current load at point B by LA_2 is negligible.

In the high open-loop amplification limit:

$$A_1 = \frac{R_1 + R_2}{R_2}, \tag{15}$$

$$A_2 = -\frac{R_4}{R_3}. \tag{16}$$

The (absolute) loop-amplification for this system is:

$$A_L = |A_1 A_2| = -A_1 A_2 = \frac{(R_1 + R_2) R_4}{R_2 R_3}. \tag{17}$$

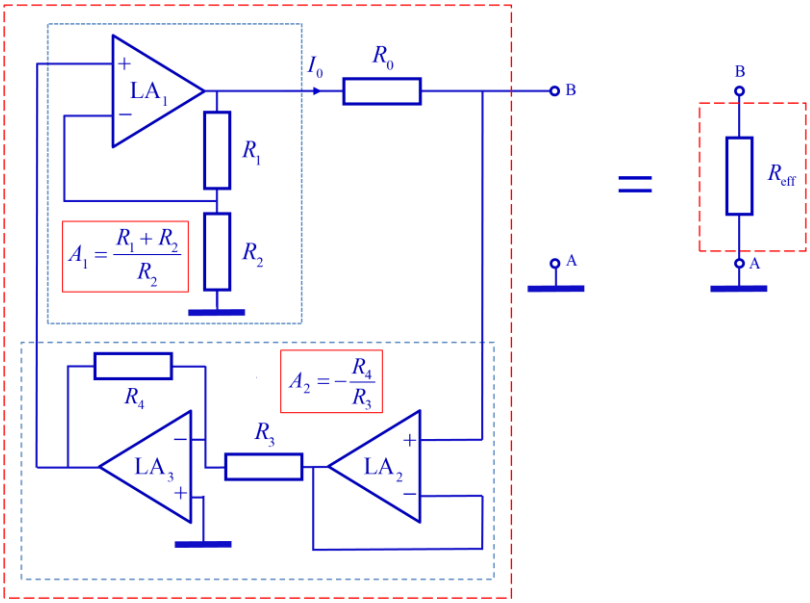

Fig. 8. In the CR system, LA_1, LA_2, LA_3, are linear operational amplifiers. A_1 is the effective amplification of LA_1. A_2 is the total effective amplification of LA_2 and LA_3. Resistors R_1, R_2, R_3 and R_4 are chosen so small that their thermal noise contributions are negligible. R_0 is the resistance that is getting transformed to become a dynamical resistor with reduced resistance and temperature. I_0 is the current through R_0.

If the external measurement current I_0 is injected into point B, the voltage U_B there satisfies:

$$U_B = A_1 A_2 U_B + I_0 R_0. \tag{18}$$

Thus:

$$U_B = \frac{I_0 R_0}{1 - A_1 A_2}. \tag{19}$$

Therefore, the effective (output) resistance of the CR is reduced by the loop-amplification as:

$$R_{eff} \equiv \frac{U_B}{I_0} = \frac{R_0}{1 + |A_1 A_2|}. \tag{20}$$

2.2. Noise analysis of the CR system

For the noise analysis, see Fig. 9, the thermal noise voltage source $U_0(t)$ of the resistor R_0; the voltage noise source $U_{n1}(t)$ and current noise source $I_{n1}(t)$ of LA_1; the noise voltage source $U_{n2}(t)$ and current noise source $I_{n2}(t)$ of LA_2; and the voltage noise source $U_{n3}(t)$ and current noise source $I_{n3}(t)$ of LA_3 are contributing to the total noise. We assume that the other resistance values are chosen so small that their thermal noise contributions are negligible. So we explore only their current-induce voltage noise contributions for the proper choice of the amplifiers, see Eq. (21).

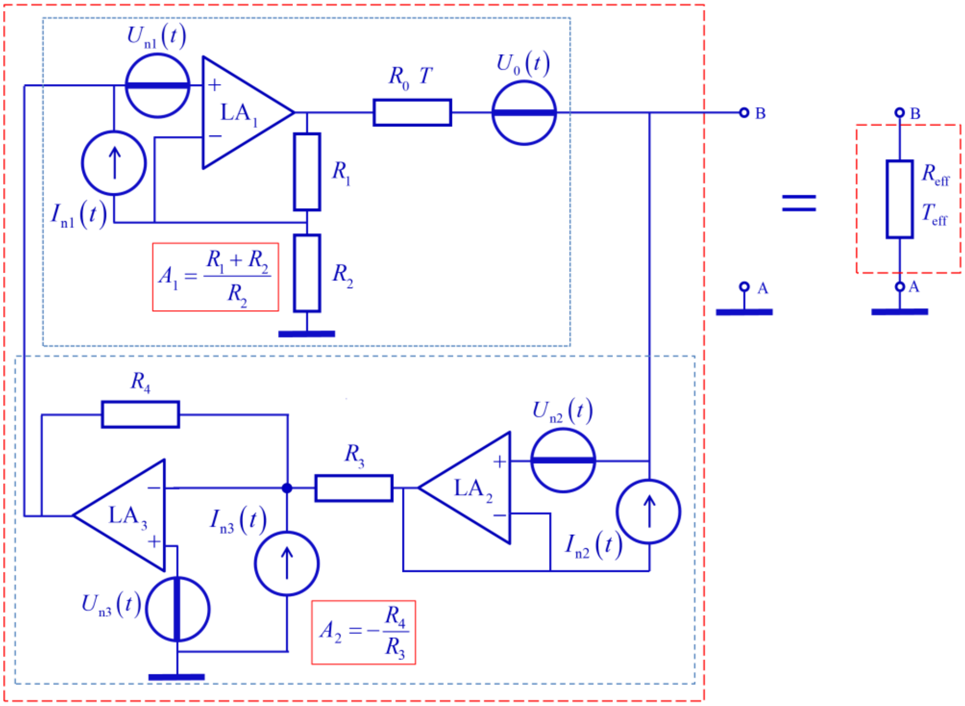

Fig. 9. Noise analysis of the CR system. $U_0(t)$ is the thermal noise voltage source of the resistor R_0. $U_{n1}(t)$, $U_{n2}(t)$ and $U_{n3}(t)$ are the voltage noise sources of LA_1, LA_2 and LA_3 respectively. $I_{n1}(t)$, $I_{n2}(t)$ and $I_{n3}(t)$ are the current noise sources of LA_1, LA_2 and LA_3 respectively.

R_0 is chosen much larger than the other resistors thus only its thermal noise is the observable Johnson noise contribution $U_0(t)$. The amplifier noise voltages and currents result in:

$$A_1\left\{A_2\left[U_B(t) + U_{n2}(t) + U_{n3}(t) + I_{n3}(t)R_4\right] + U_{n1}(t) - I_{n1}(t)R_1\right\} + \\ + I_{n2}(t)R_0 + U_0(t) = U_B(t) \quad , \tag{21}$$

where $U_B(t)$ is the instantaneous noise voltage between point A and B.

At this point we assume that R_1, R_2, R_3 and R_4, and the amplifier input current noise values are selected so that also the current-noise-induced-voltage-noise terms are negligible in Eq. (21). Thus we get a system with practical relevance:

$$A_1\left\{A_2\left[U_B(t)+U_{n2}(t)+U_{n3}(t)\right]+U_{n1}(t)\right\}+I_{n2}(t)R_0+U_0(t)=U_B(t) \quad . \tag{22}$$

From Eq. (22) we get the noise voltage at the output:

$$U_B(t)=\frac{A_1A_2}{1+A_L}\left[U_{n2}(t)+U_{n3}(t)\right]+\frac{A_1}{1+A_L}U_{n1}(t)+\frac{1}{1+A_L}R_0I_{n2}(t)+\frac{1}{1+A_L}U_0(t) \quad . \tag{23}$$

The total noise in Eq. (23) originates from three fundamental components:

(i) The voltage noise sources of the linear amplifiers (first and second term);

(ii) The current noise source of the linear amplifier LA_2

(iii) The thermal noise source of the resistor R_0.

These noise sources are independent thus the noise spectra of these terms are additive. The spectrum of the resultant noise voltage at point B is:

$$\begin{aligned} S_{eff}(f)=&\frac{A_L^{\,2}}{\left(1+A_L\right)^2}\left[S_{u2}(f)+S_{u3}(f)\right]+\frac{A_1^{\,2}}{\left(1+A_L\right)^2}S_{u1}(f)+\\ &+\frac{1}{\left(1+A_L\right)^2}R_0^{\,2}S_{i2}(f)+\frac{1}{\left(1+A_L\right)^2}S_0(f) \ , \end{aligned} \tag{24}$$

where $S_{i2}(f)$ is the noise current spectrum of LA_2, $S_{u1}(f)$, $S_{u2}(f)$ and $S_{u3}(f)$ are the input noise voltage spectra of the amplifiers, and $S_0(f)$ is the thermal noise spectrum of the resistor R_0.

The effective thermal noise spectrum of the CR is:

$$S_{eff}(f)=4kT_{eff}R_{eff} \ , \tag{25}$$

where R_{eff} and $S_{eff}(f)$ are given by Eqs. (20) and (24), respectively, and T_{eff} is the effective noise temperature of the CR. From Eqs. (20), (24) and (25) we obtain:

$$T_{eff}=\frac{A_L^{\,2}\left[S_{u2}(f)+S_{u3}(f)\right]}{(1+A_L)4kR_0}+\frac{A_1^{\,2}S_{u1}(f)}{(1+A_L)4kR_0}+\frac{R_0S_{i2}(f)}{(1+A_L)4k}+\frac{T}{1+A_L} \ , \tag{26}$$

where T is the temperature of the resistor R_0.

In the large loop-amplification limit, $A_L >> 1$, the expression of T_{eff} can be simplified as:

$$T_{eff} \approx \frac{A_L\left[S_{u2}(f)+S_{u3}(f)\right]}{4kR_0}+\frac{R_0 S_{i2}(f)}{4kA_L}+\frac{T}{A_L}= \\ =\frac{A_L\left[S_{u2}(f)+S_{u3}(f)\right]}{4kR_0}+\frac{R_0\left[S_{i2}(f)+S_i(f)\right]}{4kA_L} \quad , \tag{27}$$

where $S_i(f)$ is the thermal noise current spectrum of the resistor R_0. From Eq. (27):

$$\frac{dT_{eff}}{dR_0}=\frac{-A_L\left[S_{u2}(f)+S_{u3}(f)\right]}{4kR_0^{\,2}}+\frac{S_{i2}(f)+S_i(f)}{4kA_L}=0\,. \tag{28}$$

By solving Eq. (28), we can obtain the optimal resistance that makes T_{eff} minimal:

$$R_{0,\mathrm{opt}}=A_L\sqrt{\frac{S_{u2}(f)+S_{u3}(f)}{S_{i2}(f)+S_i(f)}}\,. \tag{29}$$

Then the minimal effective noise temperature $T_{eff,\min}$ can be calculated from $R_{0,\mathrm{opt}}$:

$$T_{eff,\min}=\frac{\sqrt{\left[S_{i2}(f)+S_i(f)\right]\left[S_{u2}(f)+S_{u3}(f)\right]}}{2k}\,. \tag{30}$$

The effective resistance of the CR is:

$$R_{eff,\mathrm{opt}}=\sqrt{\frac{S_{u2}(f)+S_{u3}(f)}{S_{i2}(f)+S_i(f)}}\,. \tag{31}$$

Note, the same results can be obtained by optimizing versus the A_L. From Eq. (27) we get:

$$\frac{dT_{eff}}{dA_L}=\frac{S_{u2}(f)+S_{u3}(f)}{4kR_0}-\frac{R_0\left[S_{i2}(f)+S_i(f)\right]}{4kA_L^{\,2}}=0\,. \tag{32}$$

From Eq. (32), the optimal loop-amplifiction $A_{L,\mathrm{opt}}$ minimizing T_{eff} is:

$$A_{L,\mathrm{opt}}=\sqrt{\frac{\left[S_{i2}(f)+S_i(f)\right]R_0^{\,2}}{S_{u2}(f)+S_{u3}(f)}}\,. \tag{33}$$

When the loop amplifier gain is set to $A_{L,\mathrm{opt}}$, the minimal effective noise temperature is:

$$T_{eff,\min}=\frac{\sqrt{\left[S_{i2}(f)+S_i(f)\right]\left[S_{u2}(f)+S_{u3}(f)\right]}}{2k}\,, \tag{34}$$

which is the same result as in Eq. (30) and the optimal effective resistance of the CR system is:

$$R_{eff,\mathrm{opt}} = \sqrt{\frac{S_{u2}(f)+S_{u3}(f)}{S_{i2}(f)+S_{i}(f)}}\,, \tag{35}$$

which is the same result as in Eq. (31),

From Eqs. (30) and (31), or Eqs. (34) and (35), we can deduce a practically useful design rule between $T_{eff,\mathrm{min}}$ and $R_{eff,\mathrm{opt}}$:

$$T_{eff,\mathrm{min}} = \frac{S_{u2}(f)+S_{u3}(f)}{2kR_{eff,\mathrm{opt}}}\,. \tag{36}$$

With Eq. (36), the designer can determine the lowest possible effective temperature for the aimed effective temperature, in the high loop-amplification limit with given amplifiers, provided A_L is >> 1.

3. Test results of the cold resistor system

For the CR circuitry in Fig. 8, we chose R_1=1 kOhm, R_2=34 Ohm, R_3=100 Ohm and R_4=1 kOhm. LA_1, LA_2 and LA_3 are ADA4625-1 operational amplifiers from Analog Devices Corporation. R_0 was varied. The simulations were carried out by the *LTspice XVII* industrial circuit tester/simulator system.

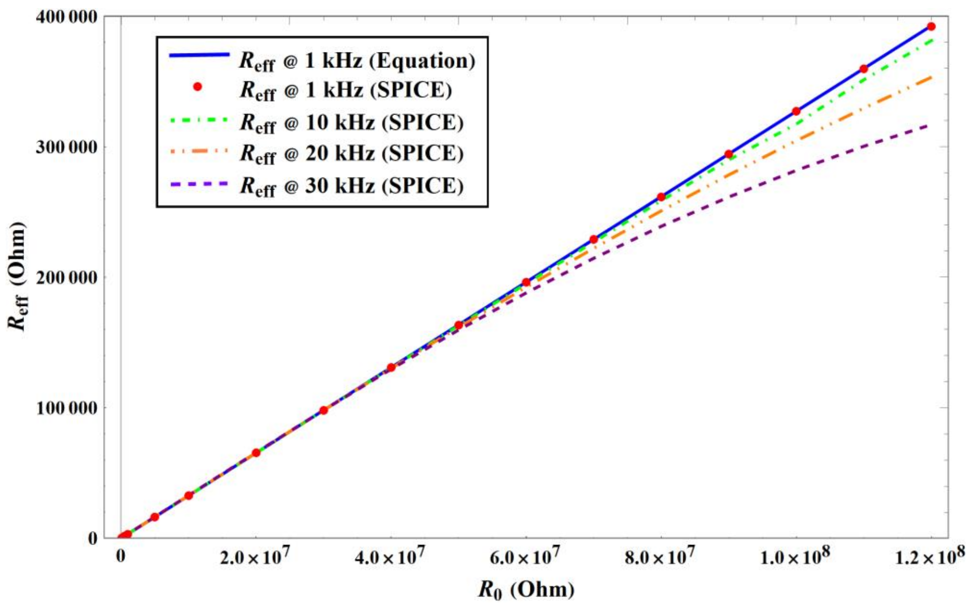

Fig. 10. The blue solid line shows the effective resistance R_{eff} versus R_0 at the ideal loop amplification by Eq. (33). Red dots, green, orange and purple dashed lines are the SPICE simulation results under 1 kHz, 10 kHz, 20 kHz, and 30 kHz, respectively.

In Fig. 10, the blue solid line shows the effective resistance R_{eff} versus R_0 at the ideal loop amplification by Eq. (33). Red dots, green, orange and purple dashed lines are the SPICE simulation results under 1 kHz, 10 kHz, 20 kHz, and 30 kHz, respectively. The practical results are limited by non-ideal features such as parasitic capacitances, phase shift, frequency characteristics, etc.

We also verified Eq. (36), the relationship between the CR's lowest effective noise temperature and its effective resistance. For the circuitry in Fig. 9, we chose R_1=1 kOhm, R_3=100 Ohm and R_4=1 kOhm. Both R_0 and R_2 were varied to fin the optimum. Fig. 11 shows the practical relationship between the lowest effective noise temperature $T_{eff,\mathrm{min}}$ and its effective noise temperature $R_{eff,\mathrm{opt}}$ for different frequency domains.

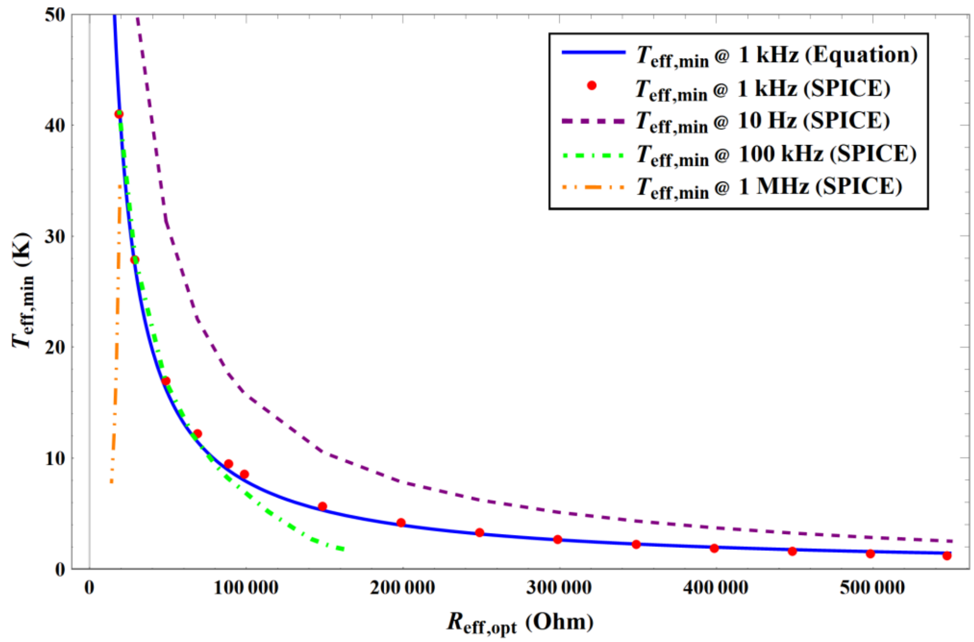

Fig. 11. The blue solid curve shows the theoretical relationship (Eq. (36)) between lowest effective noise temperature $T_{eff,min}$ and the effective resistance $R_{eff,opt}$. The red dots, and the purple, green and orange dashed curves represent the SPICE simulation results under 1 kHz, 10 Hz, 100 kHz and 1 MHz respectively.

The results in Fig. 11 show that at 1 kHz, the simulation results match the theory well. There is some derivation at 100 kHz for $R_{eff,opt}$ > 100 kOhm which can be predicted from the conclusion of Fig. 10. In the much lower frequency domain, the optimal temperature increases because of the 1/f noise which we could not take into the account in our equations due to the lack of information.

For the higher frequency domain (1 MHz), we see that the valid range of the theory vs $R_{eff,opt}$ is sharply limited by non-ideal features in the circuitry – large phase shift and reduced open-loop amplification of the operational amplifiers.

4. Conclusions

We gave the correct/complete circuit theoretical model of CR systems. The results are confirmed by computer simulations. We also deduced a practical "design tool" that shows the lowest effective temperature versus the effective resistance. With our test amplifiers, the lowest effective noise temperature that we could reach was 1.16 K.

References

[1] L.B. Kish, T. Horvath, Notes on Recent Approaches Concerning the Kirchhoff-Law-Johnson-Noise-based Secure Key Exchange, *Phys. Lett. A* **373** (2009) 2858-2868.

[2] C.D. Motchenbacher and J.A. Connelly, Low-Noise Electronic System Design (Wiley-Interscience, New York, 1993).

[3] M. J. O. Strutt and A. van der Ziel, Suppression of spontaneous fluctuations in amplifiers and receivers for electrical communication and for measuring devices, *Physica* **9** (1942) 513–527.

[4] K. Oide, Y. Ogawa and H. Hirakawa, Artificial cold resistors, *Jpn. J. Appl. Phys.* **17** (1978) 429–432.

[5] M. Huang, Cooling a Mechanical Resonator with an Artificial Cold Resistor, Master's Thesis, Leiden University (2015).

[6] R. L. Forward, Reducing noise in photoamplifier circuits with low noise artificial resistors, *J. Appl. Phys.* **53** (1982) 3365-3371.

[7] J. Song, L.B. Kish, Linear resistor plugin with ultra-low noise temperature without cooling, Provisional Patent Application, Texas A&M University, 62/720, 786, 8/2018

[8] J. Song and L. B. Kish, Does a Standalone, "Cold" (Low-Thermal-Noise), Linear Resistor Exist Without Cooling?, *Fluct. Noise. Lett.* **17** (2018) 1850030.

[9] M.T. Engelhardt (Linear Technology Corporation), Active differential resistors with reduced noise, *US patent*, # US9866245B2, January 9, 2018.

[10] R. L. Forward and G. D. Thurmond (Hughes Aircraft Company), Network for simulating low-noise temperature resistors, *US patent*, # 4156859, May 29, 1979.
[11] R. L. Forward and R. W. Peterson (Hughes Aircraft Company), Cold damping of mechanical structures, *US patent*, # 4199989, April 29, 1980.
[12] L.B. Kish, "Standalone cold virtual resistor? Can we cool by a linear amplifier without non-linearity, switches, or phase transition?", 8th International Conference on Unsolved Problems on Noise (UPoN2018), July 9-13, Gdansk, Poland.