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The mass-gap in Quantum Chromodynamics and a restriction on gluon masses

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Abstract

We prove that it is necessary to introduce the non-zero gluon masses into the fundamental Lagrangian of Quantum Chromodynamis in order to describe the mass-gap in the reaction of electron-positron annihilation into hadrons. A mew restriction on the gluon masses is obtained. The renormalized theory with non-zero Lagrangian gluon masses is constructed.

1 Introduction

Quantum Chromodynamics (QCD) is considered as the real theory of strong interactions since the famous discovery [1] of asymptotic freedom. The colour gauge symmetry of the Lagrangian ensures renormalizability and the possibility to use perturbaton theory in calculations. The gauge bosons of QCD, the gluons, are made massless to have gauge invariance. It is not possible to make gluons massive via the famous Englert-Brout-Higgs mechanism of the spontanious symmetry breaking [2] since colored Higgs particles are forbidden by experiments.

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In the present paper we prove that it is necessary to introduce the non-zero gluon masses into the fundamental Lagrangian of QCD to describe the mass-gap in the reaction of electron-positron annihilation into hadrons. Hence we get the theory with massive gluons which is the theory of the massive vector bosoms. This non-abelian Yang-Mills theory [3] with masses of the Proca type is known to be non-renormalizable, see [4].

Recently it was foumd that such a theory is in fact on mass-shell renormalizable [5]-[8], i.e. although the Green functions are non-renormalizable the physical S-marix elements are renormalizable. Thus one can have the consistent theory with massive gluons.

2 A restriction on the gluon masses and the mass-gap.

The standard QCD Lagrangian is

$$L_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\ \mu\nu} + i\overline{\psi}_f\gamma^\mu(\partial_\mu - igA^a_\mu T^a)\psi_f - m_f\overline{\psi}_f\psi_f \tag{1}$$

$$-\frac{1}{\xi}(\partial^\mu A^a_\mu)^2 + \partial^\mu\overline{c}^a(\partial_\mu c^a - gf^{abc}c^bA^c_\mu) + counterterms.$$

Notations are usual. The summation over the quark (flavour) index $f = u,..,t$ is assumed. The general covariant gauge with the parameter ξ is chosen. m_f is the renormalized quark mass, g is the strong coupling constant, $g^2/(16\pi^2) \equiv a_s$.

It is known [9], that the covariant gauge in (1) does not fix the gauge ambiguity uniquely. Hence QCD wuth the Lagrangian (1) is not the complete theory of strong interaction. It should be considered only as the theory which correctly reproduces the perturbative expansion of the complete theory [4]. One can choose e.g. the Hamilton gauge which fixes the gauge ambiguity uniquely [4] to get the complete theory. But this gauge is not convenient for perturbative calculations.

To show the necessity of the non-zero Lagrangian gluon masses for the mass-gap and to obtain a new restriction on the gluon masses let us consider electron-positron annihilation into hadrons via the virtual Z boson.

The quantity to be considered is the squared matrix element of the Z boson decay into hadrons summed over all final hadronic states. It is expressed in the usual way as the imaginary part of a correlator of quark currents

$$\sum_h < 0|J^\mu|h >< h|J^\nu|0 >= 2Im\Pi^{\mu\nu}, \tag{2}$$

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$$\Pi^{\mu\nu} = i \int e^{iqx} < 0|T(J^\mu(x)J^\nu(0))|0 > .$$

Here J^μ is the neutral quark current coupled to the Z boson in the Standard Model.

One type of Feynman diagrams among the diagrams contributing to the correlator (2) is of spesial interest. Theese are the three-loop diagrams consisting of two quark loops connected by two gluon propagators, each quark loop has a Z boson vertex. The diagram of this type was first calculated in [10]. The imaginary parts of these diagrams contain in particular Cutcosky cuts going only through two gluon propagators. These cuts give the contributions to the cross section of electron-positron annihilation which start from zero energy if gluons are massless. The contributions are of the order a_s^2 in the strong coupling constant. Of cours there is an infinite series of such contributions (starting from zero energy) coming from diagrams of higher orders in a_s, see e.g. [11].

One considers for convenience the famous R-ratio instead of the total cross section of electron-positron annihilation into hadrons. This ratio is the total cross-section itself divided by the tree level cross section of electron-positron annihilation into the muon-antimuon pare. One gets the perturbative QCD (pQCD) contribution to the R-ratio

$$R(s)_{pQCD} = \rho_{gluon}(s) + \rho_{quark}(s). \tag{3}$$

Here s is the squared momentum transfer of the process. $\rho_{gluon}(s)$ is the gluon contribution starting from zero s. It is produced by the Cutcosky cuts going via the gluon propagators only. $\rho_{quark}(s)$ is the quark contribution starting from the u-quark threshould $s = 4m_u^2$, where m_u is the mass of the lightest u-quark. It comes from the Cutcosky cuts going through quark propagators.

Thus pQCD gives non-zero contributions to $R(s)$ starting from zero s. This contradicts to experiments which dictate that non-zero contributions to $R(s)$ start only from the first physical threshould, i.e. the two-pion threshould $s = 4m_\pi^2$, where m_π is the pion mass. Hence one must somehow nullify the pQCD contributions in the energy interval $0 < s < 4m_\pi^2$.

The first suggestion is that one should not trust the perturbation theory below the two-pion threshould since the perturbative series heavily diverges at low energies. But the perturbative series is well defined at any energy since its coefficients are rigorously calculable in renormalizable theory at any energy.

One could also suggest that perturbative contributions are cancelled by non-perturbative terms, i.e. by contributions of the type

$$e^{-1/a_s} = 0 \cdot a_s + 0 \cdot a_s^2 + ..., \tag{4}$$

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which are invisible in the perturbative expansion at the poinr $a_s = 0_+$. But the non-perturbative terms have completely different analytical structure as compared to the perturbative terms. Hence non-perturbative contributions can not exactly cancel perturbative series in the continious interval $0 < s < 4m_\pi^2$.

We adopt here a constructive approach that the perturbation theory adequately reproduces the pertutrbative expansion of the exact solution of the complete theory. We could obtain this exact solution if we can do enough mathematics. If the perturbative expansion is non-zero below the two-pion threshould then the exact soilution is also non-zero there.

Thus the only way to move the glion contributions $\rho_{gluon}(s)$ above the two-pion threshould is to introduce the non-zero Lagrangian gluon masses. Since the corresponing Cutcosky cuts (crossing only gluon propagators) in the lowest order in a_s go only via two gluon propagators, we get the restriction for the lightest gluon mass $(2M_{gl})^2 > (2m_\pi)^2$ or

$$M_{gl} > m_\pi. \tag{5}$$

The Lagrangian gluon masses depend on the renormalization point. So the question arises what kind of the gluon mass M_{gl} should we choose in (5).

The natural choice is to take the perturbative pole mass. It arises as a pole of the complete perturbative gluon propagator which is odtained after summation of all loop propagator insertions. The perturbative pole mass is known to be a renormalization group invariant. It is the perturbative pole gluon mass which is taken in (5).

We would like to mention that in the previous works [6]-[8] we considered the process of electron-positron annihilation into hadrons via the photon. In that case the corresponding Cutcosky cuts (crossing only gluon propagators) in the lowest order in a_s go via three gluon propagators. Hence we got there the restriction $M_{gl} > \frac{2}{3}m_\pi$ which can be compared with the new result (5).

To produce the mass-gap in the energy interval $0 < s < m_\pi^2$ one should move also quark threshoulds above the two-pion threshould. This can be done by imposing the restriction $M_u > m_\pi$ on the perturbative pole mass of the lightest u-quark.

Thus we should modify the QCD Lagrangian by adding the gluon masses.

3 Adding the gluon mases to the QCD Lagrangian.

To construct QCD with massive gluons we will follow the approach of [5]. It is presently the only known way to obtain renormalizable (or on mass-

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shell renormalizable) theory with massive gluons without coloured scalars. One should start with a renormalizable theory using the Englert-Brout-Higgs mechanism of the spontaneous symmetry breaking [2]. Hence one adds to the standard QCD Lagrangian (1) the scalar part:

$$L_{QCD+scalars} = -\frac{1}{4}F^a_{\mu\nu}F^{a\ \mu\nu} + i\overline{\psi}_f\gamma^\mu(\partial_\mu - igA^a_\mu T^a)\psi_f - m_f\overline{\psi}_f\psi_f + \tag{6}$$
$$(D_\mu\Phi)^+ D^\mu\Phi + (D_\mu\Sigma)^+ D^\mu\Sigma - \lambda_1\left(\Phi^+\Phi - v^2\right)^2 - \lambda_1\left(\Sigma^+\Sigma - v^2\right)^2$$
$$-\lambda_2\left(\Phi^+\Phi + \Sigma^+\Sigma - 2v^2\right)^2 - \lambda_3\left(\Phi^+\Sigma\right)\left(\Sigma^+\Phi\right) + L_{gf} + L_{gc}$$
$$+counterterms,$$

where $\Phi(x)$ and $\Sigma(x)$ are the triplets of scalar fields in the fundamental representation of the $SU(3)$ colour group. We add two triplets to generate masses for all eight gluons. If one adds only one triplet of scalar fields then only five gluons get masses. We take for simplicity two different but otherwise identical scalar triplets. λ_i are coupling constants of scalar selfinteractions. v is the vacuum parameter. L_{gf} is the gauge fixing part in some renormalizable gauge and L_{gc} is the gauge compensating part with the Faddeev-Popov ghost fields.

One makes shifts of scalar fields to generate masses of gluons:

$$\Phi(x) = \begin{pmatrix} \phi_1(x) + i\phi_2(x) + v \\ \phi_3(x) + i\phi_4(x) \\ \phi_5(x) + i\phi_6(x) \end{pmatrix}, \quad \Sigma(x) = \begin{pmatrix} \sigma_1(x) + i\sigma_2(x) \\ \sigma_3(x) + i\sigma_4(x) + v \\ \sigma_5(x) + i\sigma_6(x) \end{pmatrix}. \tag{7}$$

This variant of the shifts is chosen to avoid non-diagonal in A^a_μ terms in the quadratic form of the gluon fields.

We obtain the following masss terms for gluons

$$L_M = m^2_{gl}\Big[(A^1)^2 + (A^2)^2 + (A^3)^2 + \frac{1}{2}(A^4)^2 + \tag{8}$$
$$\frac{1}{2}(A^5)^2 + \frac{1}{2}(A^6)^2 + \frac{1}{2}(A^7)^2 + \frac{1}{3}(A^8)^2\Big],$$

here $m^2_{gl} \equiv g^2v^2$ is the gluon mass parameter.

The scalar fields as usual are devided into unphysical Goldstone bosons and 'physical' Higgs fields. Here are four Higgs particles and eight Goldstone bosons after the spontaneous symmetry breaking.

For renormalization of ultraviolet divergences it is convenient to use the Bogoliubov-Parasiuk-Hepp subtraction scheme [12]. This scheme has a convenient property that counterterms of primitively divergent Feynman diagrams are truncated Taylor expansions of diagrams themselves at fixed values of external momenta. Thus counterterms of diagrams depending on some masses are also mass dependent. These subtractions should of course obey the Slavnov-Taylor identities [13],[14].

Let us consider only Green functions without external Higgs particles since we want to get rid of the Higgs fields from the Lagrangian.

In a renormalizable gauge one can remove all dagrams containing the Higgs propagators together with subtractions corresponding to these diagrams. These diagrams and subtractions are distinguishable due to their specific dependence on the Higgs masses. The remaining diagrams without the Higgs propagators stay renormalizable since they are not unfluenced by the removed subtractions depending on the Higgs masses.

It means that one can remove from the Lagrangian all terms with the Higgs fields and the corresponding counterterms with the specific dependendence on the Higgs masses. As a result one gets renormalizable theory of massive gluons without Higgs fields. This theory is renormalizable since it is obtained from the original renormalizable theory (6). The adding of the quark fields is straightforward.

Unitarity of the new theory can be established in the standard way [4] by the transition to the unitary gauge in the generating functional of Green functions.

The above derivation with the removal of the Higgs fields from the Lagrangian can be compared with the following case. Let us consider standard QCD with six quark flavours. One can remove all diagrams with the top quark propagators and their subtractions from all Green functions. At the Lagrangian level it means the removal of all terms with the top quark field and corresponding counterterms from the Lagrangian. Then one is left with the standard QCD Lagrangian with five quark flavours which is also renormalizable and unitary.

The Lagrangian of the obtained theory of massive gluons is quite involved since it contains eight gluons and eight Goldstone bosons and ghosts. To make it simpler one can consider the unitary gauge instead of a renormalizable gauge. Then Goldstone bosons and ghosts disappear and one has only physical degrees of freedom in the lagrangian.

Renormalizability is violated in the unitary gauge but on mass-shell renormalizability is known to survive. It means that only the S-matrix elements are renormalizable. As a result one obtains the on mass-shell renormalizable massive Yang-Mills theory of the Proca type. Hence the Englert-Brout-Higgs mechanism is an efficient mathematical tool to derive on mass-shell renormalizability of the massive Yang-Mills theory of the Proca type. This is far from to be obvious in advance.

The resulting Lagrangian of QCD with massive gluons in the unitary gauge is

$$L_{massive\ QCD} = L_M - \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + i\overline{\psi}_f\gamma_\mu D_\mu\psi_f - m_f\overline{\psi}_f\psi_f \qquad (9)$$
$$+counterterms,$$

where the expression for L_M with gluon mass terms is given in (8).

Let us note that on mass-shell renormalizability does not mean that quarks and gluons appear as external particles of S-matrix elements. It means that the S-matrix elements with the physical external particles are finite.

The one-loop β-function in the massive QCD with the Lagrangian (9) was calculated in [5]. It turns out that asymptotic freedom remains valid in QCD with massive gluons.

4 Conclusions

We have demonstrated that it is impossible to produce the mass-gap for the process of electron-positron annihilation into hudrons in QCD with zero Lgrangian gluon masses. One should introduce the non-zero Lagrangian gluon masses into the fundamental Lagrangian of QCD to produce the necessary mass-gap. We have obtained the new restriction on the lowesr perturbative pole gluon mass $M_{gl} > m_\pi$.

We have also demonstrated that it is possible to construct QCD with non-zero Lagrangian gluon masses retaining renormalizability and unitarity of the theory.

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