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Linear, Bidirectional and Circular Controlled Quantum Teleportation and Quantum State Sharing using a Seven Qubit Genuinely Entangled State

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Abstract

Multipartite entanglement is a resource for application in disparate protocols, of computing, communication and cryptography. In this paper, generation, characterisation and application of a genuine genuinely entangled seven-qubit resource state is studied. Theoretical schemes for quantum teleportation of arbitrary one, two and three qubits states, bidirectional teleportation of arbitrary two qubit states and probabilistic circular controlled teleportation as well as three schemes for undertaking tripartite quantum state sharing are presented.

Keywords: Quantum Computation, Multipartite Entanglement, Quantum State Sharing

I. INTRODUCTION

Quantum Entanglement, along with other general non-local quantum correlations, has sparked a revolution in Physics that has led to the emergence of Quantum Information (processing) [1–10]. It has been used in applications such as teleportation and super-dense coding [11–13].

Bennet et al first proposed a scheme for quantum teleportation, wherein a genuinely entangled Bell state was used to transmit an arbitrary single qubit [14]. Many different kinds of entangled quantum states have been used to teleport arbitrary quantum states since then, including W states [15, 16], Bell states [17, 18], GHZ states [19, 20] and multiqubit states [21–23]. There have been hop-by-hop and multi-hop quantum teleportation schemes proposed since then, as well as schemes to teleport GHZ-like states using two types of four-qubit states [24, 25]. Teleportation has been proposed in two-copy quantum teleportation scheme [26], in higher dimensions [27] and also shown to be possible over atmosphere channels [28]. More recently, various derivatives of the standard teleportation scheme have been proposed, including those used for controlled teleportation [29, 30], bidirectional teleportation [20, 31, 32], quantum secret sharing [33–35], quantum operation sharing [36, 37] and arbitrated quantum teleportation [38, 39]. What has also been of interest has been controlled teleportation of an entangled state in a noisy environment [39, 40].

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This broad area of research around entanglement and teleportation in noisy environments is related to another important area of work that centres around decoherence, decoherence-free subspaces and Markovian as well as Non-Markovian quantum noises [41–49]. Entanglement has also been key in quantum communication protocols [50–52], with Quantum Repeaters and Entanglement Purification being the subject of interest lately [53–55]. For information transfer using entanglement between parties, one needs an established entangled channel-state and means of classical communication. For a large number of parties, multipartite entanglement and entangled multiqubit states play the preeminent role, with states varying from GHZ- and W- states to clusters states [56, 57]. Teleportation of an arbitrary single qubit state using a channel comprising of an EPR pair was first demonstrated by Bennett et al [14]. Lately, W-GHZ composite states have been used for Teleportation as well as Superdense Coding of arbitrary quantum states.

Xin-Wei Zha et al [58] discovered a genuinely entangled seven-qubit state through a numerical optimization process, following the path taken by Brown et al [59] and Borras et al [60] to find genuinely entangled five-qubit and six-qubit states:

$$
\begin{aligned}
|\psi\rangle = \frac{1}{4\sqrt{2}}(&|0000000\rangle + |0000011\rangle + |0001101\rangle + |0001110\rangle + |0010001\rangle - |0010010\rangle \\
&+ |0011100\rangle - |0011111\rangle - |0100101\rangle + |0100110\rangle + |0101000\rangle + |0101011\rangle \\
&+ |0110100\rangle - |0110111\rangle - |0111001\rangle + |0111010\rangle - |1000100\rangle - |1000111\rangle \\
&+ |1001001\rangle + |1001010\rangle + |1010101\rangle - |1010110\rangle - |1011000\rangle + |1011011\rangle \\
&+ |1100001\rangle + |1100010\rangle + |1101100\rangle + |1101111\rangle + |1110000\rangle - |1110011\rangle \\
&+ |1111101\rangle - |1111110\rangle) \qquad (1)
\end{aligned}
$$

In this paper, the generation, characterisation and application of this seven-qubit (XWZ) state is explored. It is seen that this state can be used for teleportation of arbitrary single, double and triple qubit states, besides quantum state sharing. The form of the resource state is generalised to define a class of (2N+3) qubit multi-level entangled states of the form

$$
|\psi\rangle = \sum_i |GHZ_i\rangle|B_i^{(1)}\rangle|B_i^{(2)}\rangle...|B_i^{(N)}\rangle \qquad (2)
$$

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where $|GHZ\rangle$ is the GHZ state, $|B\rangle$ are Bell states, with the condition that

$$\langle GHZ_j|GHZ_k\rangle = 0 \;\; \forall \;\; j,k \;\; \text{for} \;\; j \neq k \tag{3}$$

$$\langle B_i^{(j)}|B_i^{(k)}\rangle = 0 \;\; \forall \;\; j,k \;\; \text{for} \;\; j \neq k \tag{4}$$

$$\langle B_i^{(j)}|B_k^{(j)}\rangle = 0 \;\; \forall \;\; i,k \;\; \text{for} \;\; i \neq k \tag{5}$$

A point to note here is that the three-qubit subsystems can also be formed with the GHZ-like and W states.

II. GENERATION AND CHARACTERISATION OF THE STATE

It has been found that this has a very interesting 3-2-2 structure, as follows:

$$\begin{aligned}|\psi\rangle = \frac{1}{2\sqrt{2}}(&|000\rangle_{135}|\psi_+\rangle_{24}|\psi_+\rangle_{67} + |001\rangle_{135}|\phi_-\rangle_{24}|\phi_+\rangle_{67} + |010\rangle_{135}|\psi_-\rangle_{24}|\phi_-\rangle_{67} \\ &+ |011\rangle_{135}|\phi_+\rangle_{24}|\psi_-\rangle_{67} + |100\rangle_{135}|\phi_+\rangle_{24}|\phi_+\rangle_{67} - |101\rangle_{135}|\psi_-\rangle_{24}|\psi_+\rangle_{67} \\ &- |110\rangle_{135}|\phi_-\rangle_{24}|\psi_-\rangle_{67} + |111\rangle_{135}|\psi_+\rangle_{24}|\phi_-\rangle_{67})\end{aligned} \tag{6}$$

This form helps us in devising a quantum circuit to generate the state, as shown in Figure 1 and realised on *IBM Quantum Experience*, giving us the state

$$\begin{aligned}|\psi\rangle = \frac{1}{2\sqrt{2}}(&|000\rangle_{135}|\psi_+\rangle_{24}|\psi_+\rangle_{67} + |001\rangle_{135}|\phi_-\rangle_{24}|\phi_+\rangle_{67} + |010\rangle_{135}|\psi_-\rangle_{24}|\phi_-\rangle_{67} \\ &+ |011\rangle_{135}|\phi_+\rangle_{24}|\psi_-\rangle_{67} + |100\rangle_{135}|\phi_+\rangle_{24}|\phi_+\rangle_{67} + |101\rangle_{135}|\psi_-\rangle_{24}|\psi_+\rangle_{67} \\ &+ |110\rangle_{135}|\phi_-\rangle_{24}|\psi_-\rangle_{67} + |111\rangle_{135}|\psi_+\rangle_{24}|\phi_-\rangle_{67})\end{aligned} \tag{7}$$

To retrieve the state in equation (2), we apply a unitary operator on qubits 1, 3 and 5: $U = I_{4\times4} \oplus (\sigma_z \otimes \sigma_z)$. This state has marginal density matrices for subsystems over one or two qubits that are completely mixed, with

$$\pi_{ij} = Tr_{ij}\rho_{ij}^2 = \frac{1}{4} \;\; \forall \;\; i,j \in \{1,2,3,4,5,6,7\}, i<j \tag{8}$$

$$\pi_i = Tr_i\rho_i^2 = \frac{1}{2} \;\; \forall \;\; i \in \{1,2,3,4,5,6,7\} \tag{9}$$

For three-qubit subsystems, some of the partitions have mixed marginal density matrices:

$$\pi_{ijk} = Tr_{ijk}\rho_{ijk}^2 = \frac{1}{8} \;\; \forall \;\; i,j \in \{1,2,3,4,5,6,7\}, i<j<k \wedge (ijk) \neq (127),(367),(457) \tag{10}$$

$$\pi_{127} = \pi_{367} = \pi_{457} = \frac{1}{4} \tag{11}$$

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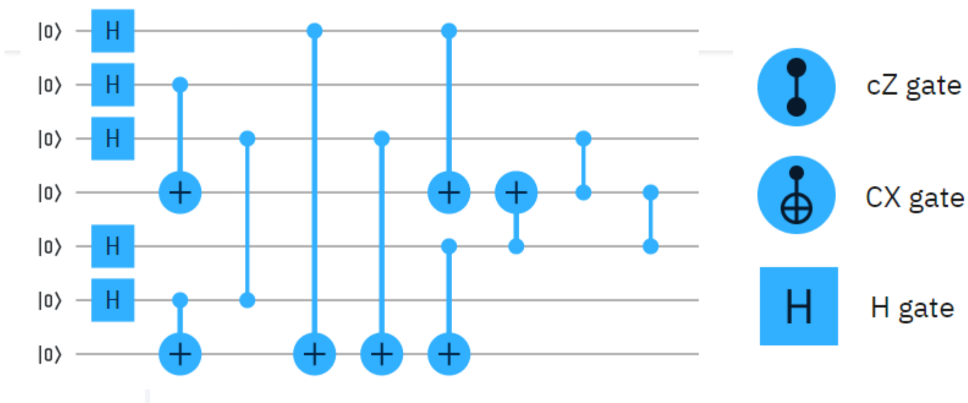

FIG. 1. Quantum Circuit for the generation of the seven-qubit genuinely entangled state, on *IBM Quantum Experience*. Here *CX gate* is the CNOT gate, *cZ gate* is the CPHASE gate and *H gate* is the Hadamard gate

III. QUANTUM TELEPORTATION

The seven-qubit genuinely entangled resource state $|\Gamma_7\rangle$ can be used for a number of applications. To begin with, schemes for teleportation of arbitrary one, two and three qubit states will be proposed.

A. Quantum Teleportation of an Arbitrary Single Qubit State

A scheme for quantum teleportation of an arbitrary single qubit state using the resource state $|\Gamma_7\rangle$ will be considered. In this case Alice possesses qubits 1, 2, 3, 4, 5, 6 and the 7th particle belongs to Bob. Alice wants to transport an arbitrary state $|\psi^{(1)}\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob. The combined state of the system is then,

$$|\Gamma_7^{(1)}\rangle = |\psi^{(1)}\rangle \otimes |\Gamma_7\rangle \tag{12}$$

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Alice then measures the seven qubits in her possession via the seven qubit orthonormal states as described below:

$$\begin{aligned}|\xi^{\pm}\rangle = |0000\rangle|\Psi^0_{GHZ}\rangle - |0001\rangle|\Psi^3_{GHZ}\rangle + |0010\rangle|\Psi^7_{GHZ}\rangle + |0011\rangle|\Psi^4_{GHZ}\rangle \\ - |0100\rangle|\Psi^5_{GHZ}\rangle - |0101\rangle|\Psi^6_{GHZ}\rangle + |0110\rangle|\Psi^2_{GHZ}\rangle + |0111\rangle|\Psi^1_{GHZ}\rangle \\ \pm (|1000\rangle|\Psi^2_{GHZ}\rangle - |1001\rangle|\Psi^1_{GHZ}\rangle - |1010\rangle|\Psi^5_{GHZ}\rangle - |1011\rangle|\Psi^6_{GHZ}\rangle \\ + |1100\rangle|\Psi^7_{GHZ}\rangle + |1101\rangle|\Psi^4_{GHZ}\rangle|1110\rangle|\Psi^0_{GHZ}\rangle - |1111\rangle|\Psi^3_{GHZ}\rangle)\end{aligned} \quad (13)$$

$$\begin{aligned}|\nu^{\pm}\rangle = |1000\rangle|\Psi^0_{GHZ}\rangle - |1001\rangle|\Psi^3_{GHZ}\rangle + |1010\rangle|\Psi^7_{GHZ}\rangle + |1011\rangle|\Psi^4_{GHZ}\rangle \\ - |1100\rangle|\Psi^5_{GHZ}\rangle - |1101\rangle|\Psi^6_{GHZ}\rangle + |1110\rangle|\Psi^2_{GHZ}\rangle + |1111\rangle|\Psi^1_{GHZ}\rangle \\ \pm (|0000\rangle|\Psi^2_{GHZ}\rangle - |0001\rangle|\Psi^1_{GHZ}\rangle - |0010\rangle|\Psi^5_{GHZ}\rangle - |0011\rangle|\Psi^6_{GHZ}\rangle \\ + |0100\rangle|\Psi^7_{GHZ}\rangle + |0101\rangle|\Psi^4_{GHZ}\rangle + |0110\rangle|\Psi^0_{GHZ}\rangle - |0111\rangle|\Psi^3_{GHZ}\rangle)\end{aligned} \quad (14)$$

where

$$|\Psi^{0,1}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|000\rangle \pm |111\rangle] \quad (15)$$

$$|\Psi^{2,3}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|001\rangle \pm |110\rangle] \quad (16)$$

$$|\Psi^{4,5}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|010\rangle \pm |101\rangle] \quad (17)$$

$$|\Psi^{6,7}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|100\rangle \pm |011\rangle] \quad (18)$$

Alice then conveys the outcome of the measurement results to Bob via two classical bits. Bob then applies a suitable unitary operation from the set $I, \sigma_x, i\sigma_y, \sigma_z$ to recover the original state, sent by Alice. In this way, one can teleport an arbitrary single-qubit state using the state $|\Gamma_7\rangle$.

B. Quantum Teleportation of an Arbitrary Two Qubit state

A scheme for the teleportation of an arbitrary two qubit quantum state will be considered. In this case Alice possesses qubits 1, 2, 3, 4 and 5, and the 6th and 7th particles belong to Bob. Alice wants to transport an arbitrary state $|\psi^{(2)}\rangle = \alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle$ to Bob. The combined state of the system is then,

$$|\Gamma^{(2)}_7\rangle = |\psi^{(2)}\rangle \otimes |\Gamma_7\rangle \quad (19)$$

So, Alice prepares the combined state

$$|\Gamma_7^{(2)}\rangle = \alpha(A_{00}|00\rangle + A_{01}|01\rangle + A_{10}|10\rangle + A_{11}|11\rangle) + \mu(B_{00}|00\rangle + B_{01}|01\rangle + B_{10}|10\rangle + B_{11}|11\rangle)$$
$$+ \gamma(C_{00}|00\rangle + C_{01}|01\rangle + C_{10}|10\rangle + C_{11}|11\rangle) + \beta(D_{00}|00\rangle + D_{01}|01\rangle + D_{10}|10\rangle + D_{11}|11\rangle) \quad (20)$$

where

$A_{00} = |0000\rangle|\Psi^0_{GHZ}\rangle + |0001\rangle|\Psi^4_{GHZ}\rangle - |0010\rangle|\Psi^5_{GHZ}\rangle - |0011\rangle|\Psi^7_{GHZ}\rangle$
$A_{11} = |0000\rangle|\Psi^1_{GHZ}\rangle + |0001\rangle|\Psi^5_{GHZ}\rangle - |0010\rangle|\Psi^2_{GHZ}\rangle - |0011\rangle|\Psi^7_{GHZ}\rangle$
$A_{01} = |0000\rangle|\Psi^6_{GHZ}\rangle - |0001\rangle|\Psi^2_{GHZ}\rangle + |0010\rangle|\Psi^4_{GHZ}\rangle + |0011\rangle|\Psi^0_{GHZ}\rangle$
$A_{10} = -|0000\rangle|\Psi^7_{GHZ}\rangle - |0001\rangle|\Psi^3_{GHZ}\rangle + |0010\rangle|\Psi^5_{GHZ}\rangle + |0011\rangle|\Psi^1_{GHZ}\rangle$
$B_{00} = |1000\rangle|\Psi^0_{GHZ}\rangle + |1001\rangle|\Psi^4_{GHZ}\rangle - |1010\rangle|\Psi^2_{GHZ}\rangle + |1011\rangle|\Psi^6_{GHZ}\rangle$
$B_{11} = |1000\rangle|\Psi^0_{GHZ}\rangle + |1001\rangle|\Psi^5_{GHZ}\rangle - |1010\rangle|\Psi^3_{GHZ}\rangle - |1011\rangle|\Psi^7_{GHZ}\rangle$
$B_{01} = |1000\rangle|\Psi^6_{GHZ}\rangle - |1001\rangle|\Psi^2_{GHZ}\rangle + |1010\rangle|\Psi^4_{GHZ}\rangle + |1011\rangle|\Psi^0_{GHZ}\rangle$
$B_{10} = -|1000\rangle|\Psi^7_{GHZ}\rangle - |1001\rangle|\Psi^3_{GHZ}\rangle + |1010\rangle|\Psi^5_{GHZ}\rangle + |1011\rangle|\Psi^1_{GHZ}\rangle$
$C_{00} = |0100\rangle|\Psi^0_{GHZ}\rangle + |0101\rangle|\Psi^4_{GHZ}\rangle - |0110\rangle|\Psi^3_{GHZ}\rangle + |0111\rangle|\Psi^6_{GHZ}\rangle$
$C_{11} = |0100\rangle|\Psi^1_{GHZ}\rangle + |0101\rangle|\Psi^5_{GHZ}\rangle - |0110\rangle|\Psi^3_{GHZ}\rangle - |0111\rangle|\Psi^7_{GHZ}\rangle$
$C_{01} = |0100\rangle|\Psi^6_{GHZ}\rangle - |0101\rangle|\Psi^2_{GHZ}\rangle - |0110\rangle|\Psi^4_{GHZ}\rangle + |0111\rangle|\Psi^0_{GHZ}\rangle$
$C_{10} = |0100\rangle|\Psi^7_{GHZ}\rangle - |0101\rangle|\Psi^3_{GHZ}\rangle + |0110\rangle|\Psi^5_{GHZ}\rangle + |0111\rangle|\Psi^1_{GHZ}\rangle$
$D_{00} = |1100\rangle|\Psi^0_{GHZ}\rangle + |1101\rangle|\Psi^4_{GHZ}\rangle - |1110\rangle|\Psi^2_{GHZ}\rangle + |1111\rangle|\Psi^6_{GHZ}\rangle$
$D_{11} = |1100\rangle|\Psi^1_{GHZ}\rangle + |1101\rangle|\Psi^5_{GHZ}\rangle - |1110\rangle|\Psi^3_{GHZ}\rangle - |1111\rangle|\Psi^7_{GHZ}\rangle$
$D_{01} = |1100\rangle|\Psi^6_{GHZ}\rangle - |1101\rangle|\Psi^2_{GHZ}\rangle + |1110\rangle|\Psi^4_{GHZ}\rangle + |1111\rangle|\Psi^0_{GHZ}\rangle$
$D_{10} = |1100\rangle|\Psi^6_{GHZ}\rangle - |1101\rangle|\Psi^3_{GHZ}\rangle + |1110\rangle|\Psi^5_{GHZ}\rangle + |1111\rangle|\Psi^1_{GHZ}\rangle$

where

$$|\Psi^{0,1}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|000\rangle \pm |111\rangle] \quad (21)$$

$$|\Psi^{2,3}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|001\rangle \pm |110\rangle] \quad (22)$$

$$|\Psi^{4,5}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|010\rangle \pm |101\rangle] \quad (23)$$

$$|\Psi^{6,7}_{GHZ}\rangle = \frac{1}{\sqrt{2}}[|100\rangle \pm |011\rangle] \quad (24)$$

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$$
\begin{aligned}
&(\alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle)|\Gamma_7\rangle = \\
&\quad \frac{1}{4}\{(A_{01} + B_{11} + C_{00} + D_{01})(\alpha|01\rangle + \mu|11\rangle + \gamma|00\rangle + \beta|01\rangle) \\
&\quad + (A_{01} + B_{11} - C_{00} - D_{01})(\alpha|01\rangle + \mu|11\rangle - \gamma|00\rangle - \beta|01\rangle) \\
&\quad + (A_{01} - B_{11} + C_{00} - D_{01})(\alpha|01\rangle - \mu|11\rangle + \gamma|00\rangle - \beta|01\rangle) \\
&\quad + (A_{01} - B_{11} - C_{00} + D_{01})(\alpha|01\rangle - \mu|11\rangle - \gamma|00\rangle + \beta|01\rangle) \\
&\quad + (A_{11} + B_{01} + C_{10} + D_{00})(\alpha|11\rangle + \mu|01\rangle + \gamma|10\rangle + \beta|00\rangle) \\
&\quad + (A_{11} - B_{01} + C_{10} - D_{00})(\alpha|11\rangle - \mu|01\rangle + \gamma|10\rangle - \beta|00\rangle) \\
&\quad + (A_{11} + B_{01} - C_{10} - D_{00})(\alpha|11\rangle + \mu|01\rangle - \gamma|10\rangle - \beta|00\rangle) \\
&\quad + (A_{11} - B_{01} - C_{10} + D_{00})(\alpha|11\rangle - \mu|01\rangle - \gamma|10\rangle + \beta|00\rangle) \\
&\quad + (A_{00} + B_{10} + C_{01} + D_{11})(\alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle) \\
&\quad + (A_{00} - B_{10} + C_{01} - D_{11})(\alpha|00\rangle - \mu|10\rangle + \gamma|01\rangle - \beta|11\rangle) \\
&\quad + (A_{00} + B_{10} - C_{01} - D_{11})(\alpha|00\rangle + \mu|10\rangle - \gamma|01\rangle - \beta|11\rangle) \\
&\quad + (A_{00} - B_{10} - C_{01} + D_{11})(\alpha|00\rangle - \mu|10\rangle - \gamma|01\rangle + \beta|11\rangle) \\
&\quad + (A_{00} + B_{10} + C_{01} + D_{11})(\alpha|10\rangle + \mu|11\rangle + \gamma|00\rangle + \beta|01\rangle) \\
&\quad + (A_{00} - B_{10} + C_{01} - D_{11})(\alpha|10\rangle - \mu|11\rangle + \gamma|00\rangle - \beta|01\rangle) \\
&\quad + (A_{00} + B_{10} - C_{01} - D_{11})(\alpha|10\rangle + \mu|11\rangle - \gamma|00\rangle - \beta|01\rangle) \\
&\qquad + (A_{00} - B_{10} - C_{01} + D_{11})(\alpha|10\rangle - \mu|11\rangle - \gamma|00\rangle + \beta|01\rangle)\} \quad (25)
\end{aligned}
$$

Now, Bob can carry out a combination of unitary operations, according to the given table, to obtain the original state teleported by Alice.

State Obtained by Bob	Unitary Operation
$\alpha\|01\rangle + \mu\|11\rangle + \gamma\|00\rangle + \beta\|01\rangle$	$I \otimes \sigma_x$
$\alpha\|01\rangle + \mu\|11\rangle - \gamma\|00\rangle - \beta\|01\rangle$	$\sigma_z \otimes \sigma_x$
$\alpha\|01\rangle - \mu\|11\rangle + \gamma\|00\rangle - \beta\|01\rangle$	$I \otimes i\sigma_y$
$\alpha\|01\rangle - \mu\|11\rangle - \gamma\|00\rangle + \beta\|01\rangle$	$\sigma_z \otimes i\sigma_y$
$\alpha\|11\rangle + \mu\|01\rangle + \gamma\|10\rangle + \beta\|00\rangle$	$\sigma_x \otimes \sigma_x$
$\alpha\|11\rangle - \mu\|01\rangle + \gamma\|10\rangle - \beta\|00\rangle$	$\sigma_x \otimes i\sigma_y$
$\alpha\|11\rangle + \mu\|01\rangle - \gamma\|10\rangle - \beta\|00\rangle$	$i\sigma_y \otimes \sigma_x$
$\alpha\|11\rangle - \mu\|01\rangle - \gamma\|10\rangle + \beta\|00\rangle$	$i\sigma_y \otimes i\sigma_y$

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$\alpha\|00\rangle + \mu\|10\rangle + \gamma\|01\rangle + \beta\|11\rangle$	$I \otimes I$
$\alpha\|00\rangle - \mu\|10\rangle + \gamma\|01\rangle - \beta\|11\rangle$	$I \otimes \sigma_z$
$\alpha\|00\rangle + \mu\|10\rangle - \gamma\|01\rangle - \beta\|11\rangle$	$\sigma_z \otimes I$
$\alpha\|00\rangle - \mu\|10\rangle - \gamma\|01\rangle + \beta\|11\rangle$	$\sigma_z \otimes \sigma_z$
$\alpha\|10\rangle + \mu\|11\rangle + \gamma\|00\rangle + \beta\|01\rangle$	$\sigma_x \otimes I$
$\alpha\|10\rangle - \mu\|11\rangle + \gamma\|00\rangle - \beta\|01\rangle$	$\sigma_x \otimes \sigma_z$
$\alpha\|10\rangle + \mu\|11\rangle - \gamma\|00\rangle - \beta\|01\rangle$	$i\sigma_y \otimes I$
$\alpha\|10\rangle - \mu\|11\rangle - \gamma\|00\rangle + \beta\|01\rangle$	$i\sigma_y \otimes \sigma_z$

1. Bidirectional Teleportation of Arbitrary Two-Qubit States

Bidirectional Controlled Quantum Teleportation (BCQT) protocols have been proposed for multi-qubit resource states, such as five-qubit [61], six-qubit [62, 63], seven-qubit [32, 64, 65] and eight-qubit states [66]. Bidirectional Controlled Quantum Teleportation can teleport arbitrary states between two users under the supervision of a third party. Zha et al proposed the first scheme for BCQT of single qubit states using a maximally entangled seve-qubit quantum state [32]. There have been schemes proposed for BCQT that utilise states with the same number of qubits as the quantum channel being used, and thereby realise bidirectional teleportation of arbitrary single- and two-qubit states under the controller Charlie [64, 65].

Let us say Alice and Bob would like to teleport two-qubit states to each other by utilizing the seven-qubit genuinely entangled resource state. We assume the form of the two-qubit states to be

$$|\phi\rangle_{A_1A_2} = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle \tag{26}$$

$$|\phi\rangle_{B_1B_2} = \beta_0|00\rangle + \beta_1|01\rangle + \beta_2|10\rangle + \beta_3|11\rangle \tag{27}$$

For the resource-state, let Alice have the qubits 1,4 and 7, while Bob has the qubits 2, 3 and 6 and Charlie has the qubit 5.

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If we take the composite state of the form

$$
\begin{aligned}
|\Gamma_7\rangle_{1352467}|\phi\rangle_A|\phi\rangle_B = \frac{1}{2\sqrt{2}}(&|000\rangle|\psi_+\rangle\frac{\alpha_0(|00\rangle|00\rangle_A+|11\rangle|00\rangle_A)}{\sqrt{2}}+|000\rangle|\psi_+\rangle\frac{\alpha_1(|00\rangle|01\rangle_A+|11\rangle|01\rangle_A)}{\sqrt{2}}\\
&+|000\rangle|\psi_+\rangle\frac{\alpha_2(|00\rangle|10\rangle_A+|11\rangle|10\rangle_A)}{\sqrt{2}}+|000\rangle|\psi_+\rangle\frac{\alpha_3(|00\rangle|11\rangle_A+|11\rangle|11\rangle_A)}{\sqrt{2}}\\
&+|001\rangle|\phi_-\rangle\frac{\alpha_0(|01\rangle|00\rangle_A+|10\rangle|00\rangle_A)}{\sqrt{2}}+|001\rangle|\phi_-\rangle\frac{\alpha_1(|01\rangle|01\rangle_A+|10\rangle|01\rangle_A)}{\sqrt{2}}\\
&+|001\rangle|\phi_-\rangle\frac{\alpha_2(|01\rangle|10\rangle_A+|10\rangle|10\rangle_A)}{\sqrt{2}}+|001\rangle|\phi_-\rangle\frac{\alpha_3(|01\rangle|11\rangle_A+|10\rangle|11\rangle_A)}{\sqrt{2}}\\
&+|010\rangle|\psi_-\rangle\frac{\alpha_0(|01\rangle|00\rangle_A-|10\rangle|00\rangle_A)}{\sqrt{2}}+|010\rangle|\psi_-\rangle\frac{\alpha_1(|01\rangle|01\rangle_A-|10\rangle|01\rangle_A)}{\sqrt{2}}\\
&+|010\rangle|\psi_-\rangle\frac{\alpha_2(|01\rangle|10\rangle_A-|10\rangle|10\rangle_A)}{\sqrt{2}}+|010\rangle|\psi_-\rangle\frac{\alpha_3(|01\rangle|11\rangle_A-|10\rangle|11\rangle_A)}{\sqrt{2}}\\
&+|011\rangle|\phi_+\rangle\frac{\alpha_0(|00\rangle|00\rangle_A-|11\rangle|00\rangle_A)}{\sqrt{2}}+|011\rangle|\phi_+\rangle\frac{\alpha_1(|00\rangle|01\rangle_A-|11\rangle|01\rangle_A)}{\sqrt{2}}\\
&+|011\rangle|\phi_+\rangle\frac{\alpha_2(|00\rangle|10\rangle_A-|11\rangle|10\rangle_A)}{\sqrt{2}}+|011\rangle|\phi_+\rangle\frac{\alpha_3(|00\rangle|11\rangle_A-|11\rangle|11\rangle_A)}{\sqrt{2}}\\
&+|100\rangle|\phi_+\rangle\frac{\alpha_0(|01\rangle|00\rangle_A+|10\rangle|00\rangle_A)}{\sqrt{2}}+|100\rangle|\phi_+\rangle\frac{\alpha_1(|01\rangle|01\rangle_A+|10\rangle|01\rangle_A)}{\sqrt{2}}\\
&+|100\rangle|\phi_+\rangle\frac{\alpha_2(|01\rangle|10\rangle_A+|10\rangle|10\rangle_A)}{\sqrt{2}}+|100\rangle|\phi_+\rangle\frac{\alpha_3(|01\rangle|11\rangle_A+|10\rangle|11\rangle_A)}{\sqrt{2}}\\
&-|101\rangle|\psi_-\rangle\frac{\alpha_0(|00\rangle|00\rangle_A+|11\rangle|00\rangle_A)}{\sqrt{2}}-|101\rangle|\psi_-\rangle\frac{\alpha_1(|00\rangle|01\rangle_A+|11\rangle|01\rangle_A)}{\sqrt{2}}\\
&-|101\rangle|\psi_-\rangle\frac{\alpha_2(|00\rangle|10\rangle_A+|11\rangle|10\rangle_A)}{\sqrt{2}}-|101\rangle|\psi_-\rangle\frac{\alpha_3(|00\rangle|11\rangle_A+|11\rangle|11\rangle_A)}{\sqrt{2}}\\
&-|110\rangle|\phi_-\rangle\frac{\alpha_0(|00\rangle|00\rangle_A-|11\rangle|00\rangle_A)}{\sqrt{2}}-|110\rangle|\phi_-\rangle\frac{\alpha_1(|00\rangle|01\rangle_A-|11\rangle|01\rangle_A)}{\sqrt{2}}\\
&-|110\rangle|\phi_-\rangle\frac{\alpha_2(|00\rangle|10\rangle_A-|11\rangle|10\rangle_A)}{\sqrt{2}}-|110\rangle|\phi_-\rangle\frac{\alpha_3(|00\rangle|11\rangle_A-|11\rangle|11\rangle_A)}{\sqrt{2}}\\
&+|111\rangle|\psi_+\rangle\frac{\alpha_0(|01\rangle|00\rangle_A-|10\rangle|00\rangle_A)}{\sqrt{2}}+|111\rangle|\psi_+\rangle\frac{\alpha_1(|01\rangle|01\rangle_A-|10\rangle|01\rangle_A)}{\sqrt{2}}\\
&+|111\rangle|\psi_+\rangle\frac{\alpha_2(|01\rangle|10\rangle_A-|10\rangle|10\rangle_A)}{\sqrt{2}}+|111\rangle|\psi_+\rangle\frac{\alpha_3(|01\rangle|11\rangle_A-|10\rangle|11\rangle_A)}{\sqrt{2}})|\phi\rangle_B
\end{aligned}
$$

The steps for the scheme are as follows:

- Alice measures qubit 7 of the resource state and A_1 in the bell basis.
- Bob measures qubit 2 of the resource state and B_1 in the bell basis.
- Charlie, Alice and Bob measure their qubits in the Z-basis.

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- Alice and Bob measure their qubits A_2 and B_2 in the X-basis.

- We apply unitary transformations to the composite state to now get Alice's initial arbitrary state in Bob's terminal and Bob's initial arbitrary state in Alice's terminal.

We will now be looking more closely at these steps with a specific one instance to illustrate each step.

Step 1: Alice measures qubit 7 of the resource state and A_1 in the bell basis. If Alice measures $|\psi_+\rangle$, the remainder state is

$$\begin{aligned}
\frac{1}{4\sqrt{2}}(|000\rangle|\psi_+\rangle &\frac{(\alpha_0|0\rangle_6|0\rangle_{A_2}+\alpha_1|0\rangle_6|1\rangle_{A_2}+\alpha_2|1\rangle_6|0\rangle_{A_2}+\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|001\rangle|\phi_-\rangle\frac{(\alpha_0|1\rangle_6|0\rangle_{A_2}+\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|010\rangle|\psi_-\rangle\frac{(-\alpha_0|1\rangle_6|0\rangle_{A_2}-\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|011\rangle|\phi_+\rangle\frac{(\alpha_0|0\rangle_6|0\rangle_{A_2}+\alpha_1|0\rangle_6|1\rangle_{A_2}-\alpha_2|1\rangle_6|0\rangle_{A_2}-\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|100\rangle|\phi_+\rangle\frac{(\alpha_0|1\rangle_6|0\rangle_{A_2}+\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|101\rangle|\psi_-\rangle\frac{(-\alpha_0|0\rangle_6|0\rangle_{A_2}-\alpha_1|0\rangle_6|1\rangle_{A_2}-\alpha_2|1\rangle_6|0\rangle_{A_2}-\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|110\rangle|\phi_-\rangle\frac{(-\alpha_0|0\rangle_6|0\rangle_{A_2}-\alpha_1|0\rangle_6|1\rangle_{A_2}+\alpha_2|1\rangle_6|0\rangle_{A_2}+\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\\
&+|111\rangle|\psi_+\rangle\frac{(-\alpha_0|1\rangle_6|0\rangle_{A_2}-\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}})|\phi\rangle_B \qquad (28)
\end{aligned}$$

Alice communicates her result to Bob using a classical channel.

Step 2: Bob measures qubit 2 of the resource state and B_1 in the bell basis. If Bob measures $|\psi_+\rangle$, the remainder state is

$$
\begin{aligned}
\frac{1}{2\sqrt{2}}(|000\rangle & \frac{(\alpha_0|0\rangle_6|0\rangle_{A_2}+\alpha_1|0\rangle_6|1\rangle_{A_2}+\alpha_2|1\rangle_6|0\rangle_{A_2}+\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|00\rangle+\beta_1|01\rangle+\beta_2|10\rangle+\beta_3|11\rangle)_{4,B_2} \\
+|001\rangle & \frac{(\alpha_0|1\rangle_6|0\rangle_{A_2}+\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|10\rangle+\beta_1|11\rangle-\beta_2|00\rangle-\beta_3|01\rangle)_{4,B_2} \\
+|010\rangle & \frac{(-\alpha_0|1\rangle_6|0\rangle_{A_2}-\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|00\rangle+\beta_1|01\rangle-\beta_2|10\rangle-\beta_3|11\rangle)_{4,B_2} \\
+|011\rangle & \frac{(\alpha_0|0\rangle_6|0\rangle_{A_2}+\alpha_1|0\rangle_6|1\rangle_{A_2}-\alpha_2|1\rangle_6|0\rangle_{A_2}-\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|10\rangle+\beta_1|11\rangle+\beta_2|00\rangle+\beta_3|01\rangle)_{4,B_2} \\
+|100\rangle & \frac{(\alpha_0|1\rangle_6|0\rangle_{A_2}+\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|10\rangle+\beta_1|11\rangle+\beta_2|00\rangle+\beta_3|01\rangle)_{4,B_2} \\
+|101\rangle & \frac{(-\alpha_0|0\rangle_6|0\rangle_{A_2}-\alpha_1|0\rangle_6|1\rangle_{A_2}-\alpha_2|1\rangle_6|0\rangle_{A_2}-\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|00\rangle+\beta_1|01\rangle-\beta_2|10\rangle-\beta_3|11\rangle)_{4,B_2} \\
+|110\rangle & \frac{(-\alpha_0|0\rangle_6|0\rangle_{A_2}-\alpha_1|0\rangle_6|1\rangle_{A_2}+\alpha_2|1\rangle_6|0\rangle_{A_2}+\alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}}\times \\
& \frac{1}{\sqrt{2}}(\beta_0|10\rangle+\beta_1|11\rangle-\beta_2|10\rangle-\beta_3|01\rangle)_{4,B_2} \\
+|111\rangle & \frac{(-\alpha_0|1\rangle_6|0\rangle_{A_2}-\alpha_1|1\rangle_6|1\rangle_{A_2}+\alpha_2|0\rangle_6|0\rangle_{A_2}+\alpha_3|0\rangle_6|1\rangle_{A_2})}{\sqrt{2}} \\
& \frac{1}{\sqrt{2}}(\beta_0|00\rangle+\beta_1|01\rangle+\beta_2|10\rangle+\beta_3|11\rangle)_{4,B_2}) \quad (29)
\end{aligned}
$$

Bob communicates his result via a classical channel to Alice.

Step 3: Charlie, Alice and Bob measure their qubits in the Z-basis. Let us say they

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all measure 0, we have the state:

$$\frac{1}{2}(|000\rangle \frac{(\alpha_0|0\rangle_6|0\rangle_{A_2} + \alpha_1|0\rangle_6|1\rangle_{A_2} + \alpha_2|1\rangle_6|0\rangle_{A_2} + \alpha_3|1\rangle_6|1\rangle_{A_2})}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(\beta_0|00\rangle + \beta_1|01\rangle + \beta_2|10\rangle + \beta_3|11\rangle)_{4,B_2} \quad (30)$$

Step 4: Let Alice apply a CNOT with A_2 as control and qubit 1 as target, and let Bob apply a CNOT with B_2 as control and qubit 3 as target, to get

$$\frac{1}{2\sqrt{2}}(|0\rangle \frac{(\alpha_0|000\rangle + \alpha_1|101\rangle_{A_2} + \alpha_2|010\rangle + \alpha_3|111\rangle)_{1,6,A_2}}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(\beta_0|000\rangle + \beta_1|101\rangle + \beta_2|010\rangle + \beta_3|111\rangle)_{3,4,B_2} \quad (31)$$

Step 4: Alice and Bob measure their qubits A_2 and B_2 in the X-basis. Let us say they obtain the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, then the composite state is given by

$$\frac{1}{2}(|0\rangle \frac{(\alpha_0|00\rangle + \alpha_1|10\rangle_{A_2} + \alpha_2|01\rangle + \alpha_3|11\rangle)_{1,6}}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(\beta_0|00\rangle + \beta_1|10\rangle + \beta_2|01\rangle + \beta_3|11\rangle)_{3,4} \quad (32)$$

Step 5: We apply unitary transformations to the composite state to now get Alice's initial arbitrary state in Bob's terminal and Bob's initial arbitrary state in Alice's terminal. In this instance, the unitary transformation is simply $I \otimes I \otimes I \otimes I$ with I being the identity matrix.

C. Teleportation of an Arbitrary Three Qubit State

The seven-qubit resource state can be used for the teleportation of an arbitrary three qubit state. In this case, Alice possesses qubits 1, 2, 3, 4 and 5, and the 6th and 7th particles belong to Bob. Alice wants to transport an arbitrary state $|\psi^{(3)}\rangle = a|000\rangle + b|001\rangle + c|010\rangle +$

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$d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ to Bob.

$$\begin{aligned}
|\Gamma_7^{(3)}\rangle = |\psi^{(3)}\rangle \otimes |\Gamma_7\rangle = aA_{000}|000\rangle + aA_{001}|001\rangle + aA_{010}|010\rangle + aA_{011}|011\rangle \\
+ aA_{100}|100\rangle + aA_{101}|101\rangle + aA_{110}|110\rangle + aA_{111}|111\rangle \\
+ bB_{000}|000\rangle + bB_{001}|001\rangle + bB_{010}|010\rangle + bB_{011}|011\rangle \\
+ bB_{100}|100\rangle + bB_{101}|101\rangle + bB_{110}|110\rangle + bB_{111}|111\rangle \\
+ cC_{000}|000\rangle + cC_{001}|001\rangle + cC_{010}|010\rangle + cC_{011}|011\rangle \\
+ cC_{100}|100\rangle + cC_{101}|101\rangle + cC_{110}|110\rangle + cC_{111}|111\rangle \\
+ dD_{000}|000\rangle + dD_{001}|001\rangle + dD_{010}|010\rangle + dD_{011}|011\rangle \\
+ dD_{100}|100\rangle + dD_{101}|101\rangle + dD_{110}|110\rangle + dD_{111}|111\rangle \\
+ eE_{000}|000\rangle + eE_{001}|001\rangle + eE_{010}|010\rangle + eE_{011}|011\rangle \\
+ eE_{100}|100\rangle + eE_{101}|101\rangle + eE_{110}|110\rangle + eE_{111}|111\rangle \\
+ fF_{000}|000\rangle + fF_{001}|001\rangle + fF_{010}|010\rangle + fF_{011}|011\rangle \\
+ fF_{100}|100\rangle + fF_{101}|101\rangle + fF_{110}|110\rangle + fB_{111}|111\rangle \\
+ gG_{000}|000\rangle + gG_{001}|001\rangle + gG_{010}|010\rangle + gG_{011}|011\rangle \\
+ gG_{100}|100\rangle + gG_{101}|101\rangle + gG_{110}|110\rangle + gG_{111}|111\rangle \\
+ hH_{000}|000\rangle + hH_{001}|001\rangle + hH_{010}|010\rangle + hH_{011}|011\rangle \\
+ hH_{100}|100\rangle + hH_{101}|101\rangle + hH_{110}|110\rangle + hH_{111}|111\rangle
\end{aligned} \tag{33}$$

Using the decomposition given by Appendix C, the states possessed by, and the unitary transforms to be performed by, Bob have been recorded, to accomplish the teleportation of an arbitrary three-qubit state.

D. Probabilistic Circular Controlled Quantum Teleportation

In this section, a scheme for the probabilistic teleportation of an arbitrary single-qubit states in a circular manner between three network-nodes (users) using the seven-qubit resource state will be proposed and explored. Let us say I have Alice, Bob and Charlie in the system, with the first qubit used as a control qubit, qubits 1 and 4 given to Alice, qubits 2 and 6 given to Bob and qubits 3 and 7 given to Charlie. Let us say the arbitrary states are

$$|\psi_A\rangle = \alpha_A|0_A\rangle + \beta_A|1_A\rangle \tag{34}$$

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$$|\psi_B\rangle = \alpha_B|0_B\rangle + \beta_B|1_B\rangle \tag{35}$$

$$|\psi_C\rangle = \alpha_C|0_C\rangle + \beta_C|1_C\rangle \tag{36}$$

Then, the composite state is given by

$$|\psi_A\rangle \otimes |\psi_B\rangle \otimes |\psi_C\rangle \otimes |\Gamma_7\rangle_{TA_1B_1C_1A_1B_1C_1} \tag{37}$$

where $|\Gamma_7\rangle_T$ is the control qubit.

Let us now apply a CNOT gate using the qubits A, B and C of the arbitrary states as the control-qubits and the first qubits of each user as the target-qubit. Let us for simplicity only consider the case where $|\Gamma_7\rangle_T = |0\rangle$. Then, this first operation gives the state

$$\begin{aligned}
|\psi'\rangle = \frac{1}{4\sqrt{2}}((&|000000\rangle + |000011\rangle + |001101\rangle + |001110\rangle + |010001\rangle \\
&- |010010\rangle + |011100\rangle - |011111\rangle - |100101\rangle - |100110\rangle \\
&+ |101000\rangle + |101011\rangle + |110100\rangle - |110111\rangle - |111001\rangle \\
&+ |111010\rangle)\alpha_A\alpha_B\alpha_C|0_A0_B0_C\rangle \\
&+ (|001000\rangle + |001011\rangle + |000101\rangle + |000110\rangle + |011001\rangle \\
&- |011010\rangle + |010100\rangle - |010111\rangle - |101101\rangle - |101110\rangle \\
&+ |100000\rangle + |100011\rangle + |111100\rangle - |111111\rangle - |110001\rangle \\
&+ |110010\rangle)\alpha_A\alpha_B\beta_C|0_A0_B1_C\rangle \\
&+ (|010000\rangle + |010011\rangle + |011101\rangle + |011110\rangle + |000001\rangle \\
&- |000010\rangle + |001100\rangle - |001111\rangle - |110101\rangle - |110110\rangle \\
&+ |111000\rangle + |111011\rangle + |100100\rangle - |100111\rangle - |101001\rangle \\
&+ |101010\rangle)\alpha_A\beta_B\alpha_C|0_A1_B0_C\rangle \\
&+ (|011000\rangle + |011011\rangle + |010101\rangle + |010110\rangle + |001001\rangle \\
&- |001010\rangle + |000100\rangle - |000111\rangle - |111101\rangle - |111110\rangle \\
&+ |110000\rangle + |110011\rangle + |101100\rangle - |101111\rangle - |100001\rangle \\
&+ |100010\rangle)\alpha_A\beta_B\beta_C|0_A1_B1_C\rangle
\end{aligned}$$

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$$+ (|100000\rangle + |100011\rangle + |101101\rangle + |101110\rangle + |110001\rangle$$
$$- |110010\rangle + |111100\rangle - |111111\rangle - |000101\rangle - |000110\rangle$$
$$+ |001000\rangle + |001011\rangle + |010100\rangle - |010111\rangle - |011001\rangle$$
$$+ |011010\rangle)\beta_A\alpha_B\alpha_C|1_A0_B0_C\rangle$$
$$+ (|101000\rangle + |101011\rangle + |100101\rangle + |100110\rangle + |111001\rangle$$
$$- |111010\rangle + |110100\rangle - |110111\rangle - |001101\rangle - |001110\rangle$$
$$+ |000000\rangle + |000011\rangle + |011100\rangle - |011111\rangle - |010001\rangle$$
$$+ |010010\rangle)\beta_A\alpha_B\beta_C|1_A0_B1_C\rangle$$
$$+ (|110000\rangle + |110011\rangle + |111101\rangle + |111110\rangle + |100001\rangle$$
$$- |100010\rangle + |101100\rangle - |101111\rangle - |010101\rangle - |010110\rangle$$
$$+ |011000\rangle + |011011\rangle + |000100\rangle - |000111\rangle - |001001\rangle$$
$$+ |001010\rangle)\beta_A\beta_B\alpha_C|1_A1_B0_C\rangle$$
$$+ (|111000\rangle + |111011\rangle + |110101\rangle + |110110\rangle + |101001\rangle$$
$$- |101010\rangle + |100100\rangle - |100111\rangle - |011101\rangle - |011110\rangle$$
$$+ |010000\rangle + |010011\rangle + |001100\rangle - |001111\rangle - |000001\rangle$$
$$+ |000010\rangle)\beta_A\beta_B\beta_C|1_A1_B1_C\rangle) \quad (38)$$

Let us now measure the first qubits of Alice, Bob and Charlie in the Z-basis. Let us say $|\Gamma_7\rangle_{A_1B_1C_1} = |010\rangle$, then we have the state

$$|\psi''\rangle = \frac{1}{4}((|001\rangle - |010\rangle)\alpha_A\alpha_B\alpha_C|0_A0_B0_C\rangle + (|100\rangle - |111\rangle)\alpha_A\alpha_B\beta_C|0_A0_B1_C\rangle$$
$$+ (|000\rangle + |011\rangle)\alpha_A\beta_B\alpha_C|0_A1_B0_C\rangle + (|101\rangle + |110\rangle)\alpha_A\beta_B\beta_C|0_A1_B1_C\rangle$$
$$+ (|100\rangle - |111\rangle)\beta_A\alpha_B\alpha_C|1_A0_B0_C\rangle + (-|001\rangle + |010\rangle)\beta_A\alpha_B\beta_C|1_A0_B1_C\rangle$$
$$+ (-|101\rangle - |110\rangle)\beta_A\beta_B\alpha_C|1_A1_B0_C\rangle + (|000\rangle + |011\rangle)\beta_A\beta_B\beta_C|1_A1_B1_C\rangle) \quad (39)$$

I now measure the control qubits in the X-basis. So, let us say, I have $|Q_AQ_BQ_C\rangle = |+_A -_B +_C\rangle$, then I get the state

$$\begin{aligned}
&|C_1\rangle(|A_1\rangle(-|B_1\rangle + \chi|B_3\rangle - \chi^{-1}|B_4\rangle - |B_2\rangle) + |A_2\rangle(-|B_1\rangle + \chi|B_3\rangle + \chi^{-1}|B_4\rangle + |B_2\rangle))\\
&+ |C_2\rangle(|A_1\rangle(-|B_1\rangle - \chi|B_3\rangle - \chi^{-1}|B_4\rangle + |B_2\rangle) + |A_2\rangle(-|B_1\rangle - \chi|B_3\rangle + \chi^{-1}|B_4\rangle - |B_2\rangle))\\
&+ |C_3\rangle(|A_1\rangle(|B_4\rangle - \chi|B_2\rangle - \chi^{-1}|B_1\rangle - |B_3\rangle) + |A_2\rangle(|B_4\rangle - \chi|B_2\rangle + \chi^{-1}|B_1\rangle - |B_3\rangle))\\
&+ |C_4\rangle(|A_1\rangle(|B_4\rangle + \chi|B_2\rangle - \chi^{-1}|B_1\rangle - |B_3\rangle) + |A_2\rangle(|B_4\rangle + \chi|B_2\rangle + \chi^{-1}|B_1\rangle + |B_3\rangle)) \quad (40)
\end{aligned}$$

where $|C_1\rangle = \beta_C|0\rangle + \alpha_C|1\rangle$, $|C_2\rangle = \beta_C|0\rangle - \alpha_C|1\rangle$, $|C_3\rangle = \beta_C|1\rangle + \alpha_C|0\rangle$, $|C_4\rangle = \beta_C|1\rangle - \alpha_C|0\rangle$, $|B_1\rangle = \alpha_B|1\rangle + \beta_B|0\rangle$, $|B_2\rangle = \alpha_B|1\rangle - \beta_B|0\rangle$, $|B_3\rangle = \alpha_B|0\rangle + \beta_B|1\rangle$, $|B_4\rangle = \alpha_B|0\rangle - \beta_B|1\rangle$, $\chi = \frac{a_2}{a_1}$ with $a_1 = \beta_A + \alpha_A, a_2 = \alpha_A - \beta_A$, $|A_1\rangle = a_1|0\rangle + a_2|1\rangle$, $|A_1\rangle = a_1|0\rangle - a_2|1\rangle$. Therefore I see that the users can obtain states derived from the original state of the users next to them (Alice → Bob → Charlie → Alice). However, as you can see, this can be done in a probabilistic manner with one of the users not quite obtaining the original state but rather a derivative-state based on the original. In this way, I can implement probabilistic circular teleportation using the XZW resource states.

E. Teleportation of arbitrary n-qubit states

We see that the resource state has an interesting 3-2-2 structure that can be generalised to

$$|\psi\rangle = \sum_i \alpha_i|\xi\rangle|\psi^A/\phi^A\rangle|\psi^B/\phi^B\rangle \quad (41)$$

where $|\xi\rangle$ is a three qubit product state in the z-basis, while $|\psi_i^A/\phi_i^A\rangle$ and $|\psi_i^B/\phi_i^B\rangle$ are two-qubit bell state, with a coefficient for each superposition-term α_i. Each superposition-term for each subsystem is orthogonal to all other terms for that specific subsystem. While such a state is seen to be a genuinely entangled, we can think of maximising the entanglement in each group/substructure by employing a maximally entangled three-qubit state for the three-qubit subsystem. This can be a W-state, a GHZ-state or a GHZ-like state. In this paper, this idea is extended further to even look at a superposition of such states. Let us begin by looking at the superposition of GHZ-Bell States, and extending it to (2N+3) arbitrary number of qubits where we have one GHZ state and N Bell states, in a natural extension of the state in equation (34). Saha et al proposed a scheme for teleportation of a

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multiqubit state using the following resource state [67]:

$$|\psi\rangle = [\frac{1}{\sqrt{2}}(|0_A0_A0_B\rangle + |1_A1_A1_B\rangle)][\frac{1}{\sqrt{2}}(|0_A0_B\rangle + |1_A1_B\rangle)]...[\frac{1}{\sqrt{2}}(|0_A0_B\rangle + |1_A1_B\rangle)] \quad (42)$$

To achieve the teleportation of an n-qubit state, we start with a $2n+1$ qubit state of the form given in equation (35), with Alice having $n+1$ qubits and Bob having n qubits. Let us say the arbitrary n-qubit state Alice wants to teleport to Bob is: $\sum_{i=0}^{2^n-1}\alpha_i|a_i\rangle$, where a_i denotes the binary representation of i. The combined state $|\psi_c\rangle$ can then be written in terms of subsystems possessed by Alice and Bob,

$$|\psi_c\rangle = \sum_{i=0}^{2^{2n}-1}|\omega_i\rangle_A|\eta_i\rangle_B \quad (43)$$

where $|\omega_i\rangle_A\forall i \in [0, 2^{2n}-1]$ constitute a mutually orthogonal basis. It is seen that for even n, $|\omega_0\rangle_{1,4,3,2,7,...,2n,2n-3,2n-2,2n+1} = |GHZ_+\rangle|\psi_+\rangle...|\psi_+\rangle$ and for odd n, $|\omega_0\rangle_{1,2,5,4,3,...,2n,2n-3,2n-2,2n+1} = |GHZ_+\rangle|\psi_+\rangle...|\psi_+\rangle$, where $|GHZ_+\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and $|\psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ [67]. A point to note here are the indices and ordering of the particles in these states. We can obtain the other $|\omega_i\rangle_A$ from this

$$|\omega_i\rangle_{1,2,3,...,2n,2n+1} = \otimes_{k=1}^{n}(Z_k)^{b_k}(X_k)^{b_{k+n}}|\omega_0\rangle_{1,2,3,...,2n,2n+1} \quad (44)$$

where i is the decimal representation of the string $b_{2n}......b_2b_1$ with a general b_k being 0 or 1. After measurement is done using these orthogonal states, Alice's state evolves into one of the states $|\omega_i\rangle'$. If her state is $|\omega_i\rangle$ then Bob must apply $\otimes_{k=1}^{n}(Z_k)^{b_k}(X_k)^{b_{k+n}}$ on his n-qubit system to obtain the unknown state, where i is the decimal representation of the string $b_{2n}......b_2b_1$ with a general b_k being 0 or 1.

The form of the resource state can be further generalised to define a class of (2N+3) qubit multi-level entangled states of the form

$$|\psi\rangle = \sum_i |G_i\rangle|B_i^{(1)}\rangle|B_i^{(2)}\rangle...|B_i^{(N)}\rangle \quad (45)$$

where $|G\rangle$ is the GHZ or GHZ-like state, $|B\rangle$ are any Bell states, with the condition that

$$\langle G_j|G_k\rangle = 0 \quad \forall \quad j,k \quad \text{for} \quad j \neq k \quad (46)$$

$$\langle B_i^{(j)}|B_i^{(k)}\rangle = 0 \quad \forall \quad j,k \quad \text{for} \quad j \neq k \quad (47)$$

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$$\langle B_i^{(j)}|B_k^{(j)}\rangle = 0 \;\; \forall \;\; i,k \;\; \text{for} \;\; i \neq k \tag{48}$$

A point to note here is that the three-qubit subsystems can also be formed with the GHZ-like and W states. GHZ and GHZ-like states are defined as where

$$|\Psi_{GHZ}^{0,1}\rangle = \frac{1}{\sqrt{2}}[|000\rangle \pm |111\rangle] \tag{49}$$

$$|\Psi_{GHZ}^{2,3}\rangle = \frac{1}{\sqrt{2}}[|001\rangle \pm |110\rangle] \tag{50}$$

$$|\Psi_{GHZ}^{4,5}\rangle = \frac{1}{\sqrt{2}}[|010\rangle \pm |101\rangle] \tag{51}$$

$$|\Psi_{GHZ}^{6,7}\rangle = \frac{1}{\sqrt{2}}[|100\rangle \pm |011\rangle] \tag{52}$$

Similar to the case for the resource state (35), one can present generalised conditions and transformations required for the teleportation of an n-qubit state. However, the interesting point about this class of states is the nested entanglement. As can be seen, these states have two levels of entanglement, one across the subsystems in the 3-2-2-...-2 partition and the second within each subsystem. This helps in providing a second layer of resilience for the entanglement of the system and allows us to maintain one level of entanglement (intra-subsystem level) even if we end up using the higher state-level entanglement.

IV. QUANTUM SECRET SHARING

Quantum Secret Sharing (QSS) is a procedure for splitting a message into several parts so that no single subset of parts is sufficient to read the message, but the entire set is. In this section, I provide three proposals for doing so. This can also naturally be extended to Quantum Operation Sharing (QOS).

A. Proposal 1

Let us consider the situation in which Alice possesses the 1st qubit, Bob possesses qubits 2, 3, 4, 5, 6 and Charlie possesses the 7th qubit. Alice has an unknown qubit $\alpha|0\rangle + \beta|1\rangle$ which she wants to share with Bob and Charlie.

Now, Alice combines the unknown qubit with $|\Psi_7\;\rangle$ and performs a Bell measurement,

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and conveys her outcome to Charlie by two classical bits. For instance if Alice measures in the $|\Phi_+\rangle$ basis, then the Bob-Charlie system evolves into the entangled state.

$$\alpha|100001\rangle - \alpha|000100\rangle - \alpha|000111\rangle - \alpha|001001\rangle + \alpha|001010\rangle + \alpha|010101\rangle - \quad (53)$$
$$\alpha|010110\rangle - \alpha|011000\rangle + \alpha|011011\rangle + \alpha|100010\rangle + \alpha|101100\rangle + \alpha|101111\rangle - \quad (54)$$
$$\alpha|110011\rangle + \alpha|111101\rangle - \alpha|111110\rangle + \beta|000000\rangle + \beta|000011\rangle + \beta|001101\rangle + \quad (55)$$
$$\beta|001110\rangle + \beta|010001\rangle - \beta|010010\rangle + \beta|011100\rangle - \beta|011111\rangle - \beta|100101\rangle - \quad (56)$$
$$\beta|000000\rangle - \beta|100110\rangle + \beta|101000\rangle + \beta|101011\rangle + \beta|110100\rangle - \beta|110111\rangle - \quad (57)$$
$$\beta|111001\rangle + \beta|111010\rangle \quad (58)$$

Now, Bob can perform a five-qubit measurement and convey his outcome to Charlie through a classical channel. Having known the outcome of both their measurement, Charlie will obtain a certain single qubit quantum state. The outcome of the measurement performed by Bob is correlated with the state obtained by Charlie. If Bob measures $|A_\pm\rangle$ then Charlie obtains the state $\alpha|0\rangle \pm \beta|1\rangle$, while if Bob measures the state $|B_\pm\rangle$ then Charlie obtains the state $\beta|0\rangle \pm \alpha|1\rangle$, where

$$|A_\pm\rangle \quad (59)$$
$$= -|00010\rangle + |00101\rangle - |01011\rangle - |01100\rangle \quad (60)$$
$$+ |10001\rangle + |10110\rangle - |11111\rangle \pm (|00001\rangle \quad (61)$$
$$+ |00110\rangle + |01000\rangle - |01111\rangle - |10010\rangle \quad (62)$$
$$+ |10101\rangle - |11011\rangle - |11100\rangle) \quad (63)$$

$$|B_\pm\rangle \tag{64}$$
$$= \pm(|10000\rangle - |00011\rangle - |00100\rangle + |01010\rangle \tag{65}$$
$$+ |01101\rangle + |10111\rangle - |11001\rangle + |11110\rangle) \tag{66}$$
$$+ |00000\rangle + |00111\rangle - |01001\rangle + |01110\rangle \tag{67}$$
$$- |00000\rangle - |10011\rangle + |10100\rangle + |11010\rangle \tag{68}$$
$$+ |11101\rangle \tag{69}$$
$$\tag{70}$$

B. Proposal 2

Let us consider the situation in which Alice possesses the qubits 1 and 2, Bob possesses qubits 3, 4, 5 and 6 and Charlie possesses the 7th qubit. Alice has an unknown qubit $\alpha|0\rangle + \beta|1\rangle$ which she wants to share with Bob and Charlie. Now Alice can measure in a particular basis. Suppose she measures in the GHZ Basis. Now, Bob can perform a four-qubit measurement and convey his outcome to Charlie through a classical channel. Having known the outcome of both their measurement, Charlie will obtain a certain single qubit quantum state. The outcome of the measurement performed by Bob and the state obtained by Charlie is given as follows: if Bob measures states $|X_\pm\rangle$, Charlie obtains states $\alpha|0\rangle\pm\beta|1\rangle$, while if Bob measures states $|Y_\pm\rangle$ then Charlie obtains the states $\beta|0\rangle\pm\alpha|1\rangle$, where $|X_\pm\rangle = \frac{1}{4}$ $\alpha|0000\rangle+\alpha|0111\rangle+\alpha|1001\rangle+\alpha|1110\rangle \pm (\beta|1001\rangle+\beta|0000\rangle+\beta|1110\rangle$-$\beta|0111\rangle)$ and $|Y_\pm\rangle = \frac{1}{4}$ $\alpha|0001\rangle+\alpha|0110\rangle+\alpha|1000\rangle$-$\alpha|1111\rangle \pm (\beta|1000\rangle+\beta|0001\rangle$-$\beta|1111\rangle$-$\beta|0110\rangle)$

C. Proposal 3

Let us consider the situation in which Alice possesses the qubits 1, 2, 3 and 4, Bob possesses qubits 5 and 6 and Charlie possesses the 7th qubit. Alice has an unknown qubit $\alpha|0\rangle + \beta|1\rangle$ which she wants to share with Bob and Charlie. Based on the state Alice measures $(|A_i\rangle\ \forall i \in \{1,2,3,4,5,6,7,8\})$, Bob and Charlie obtain a corresponding state $|BC_i\rangle$, where

$|A_1\rangle = \frac{1}{4}(|01111\rangle$- $|01011\rangle$+ $|10010\rangle$+ $|11001\rangle$+ $|11100\rangle$+ $|11101\rangle$- $|11000\rangle)$

$|A_2\rangle = \frac{1}{4}(|01111\rangle + |01011\rangle - |10010\rangle - |11001\rangle - |11100\rangle + |11101\rangle - |11000\rangle)$

$|A_3\rangle = \frac{1}{4}(|01111\rangle + |01011\rangle + |10010\rangle + |11001\rangle + |11100\rangle - |11101\rangle + |11000\rangle)$

$|A_4\rangle = \frac{1}{4}(|01111\rangle - |01011\rangle - |10010\rangle - |11001\rangle - |11100\rangle - |11101\rangle + |11000\rangle)$

$|A_5\rangle = \frac{1}{4}(|11111\rangle - |11011\rangle + |00010\rangle + |01001\rangle + |01100\rangle + |01101\rangle - |01000\rangle)$

$|A_6\rangle = \frac{1}{4}(|11111\rangle + |11011\rangle - |00010\rangle - |01001\rangle - |01100\rangle + |01101\rangle - |01000\rangle)$

$|A_7\rangle = \frac{1}{4}(|11111\rangle + |11011\rangle + |00010\rangle + |01001\rangle + |01100\rangle - |01101\rangle + |01000\rangle)$

$|A_8\rangle = \frac{1}{4}(|11111\rangle - |11011\rangle - |00010\rangle - |01001\rangle - |01100\rangle - |01101\rangle + |01000\rangle)$

and

$|BC_1\rangle = \alpha|1\rangle|\Phi_-\rangle + \alpha|0\rangle|\Psi_-\rangle + \beta|0\rangle|\Phi_+\rangle + \beta|1\rangle|\Psi_+\rangle$

$|BC_2\rangle = \alpha|1\rangle|\Phi_-\rangle - \alpha|0\rangle|\Psi_-\rangle - \beta|0\rangle|\Phi_+\rangle + \beta|1\rangle|\Psi_+\rangle$

$|BC_3\rangle = \alpha|1\rangle|\Phi_-\rangle - \alpha|0\rangle|\Psi_-\rangle + \beta|0\rangle|\Phi_+\rangle - \beta|1\rangle|\Psi_+\rangle$

$|BC_4\rangle = \alpha|1\rangle|\Phi_-\rangle + \alpha|0\rangle|\Psi_-\rangle - \beta|0\rangle|\Phi_+\rangle - \beta|1\rangle|\Psi_+\rangle$

$|BC_5\rangle = \beta|1\rangle|\Phi_-\rangle + \beta|0\rangle|\Psi_-\rangle + \alpha|0\rangle|\Phi_+\rangle + \alpha|1\rangle|\Psi_+\rangle$

$|BC_6\rangle = \beta|1\rangle|\Phi_-\rangle - \beta|0\rangle|\Psi_-\rangle - \alpha|0\rangle|\Phi_+\rangle + \alpha|1\rangle|\Psi_+\rangle$

$|BC_7\rangle = \beta|1\rangle|\Phi_-\rangle - \beta|0\rangle|\Psi_-\rangle + \alpha|0\rangle|\Phi_+\rangle - \alpha|1\rangle|\Psi_+\rangle$

$|BC_8\rangle = \beta|1\rangle|\Phi_-\rangle + \beta|0\rangle|\Psi_-\rangle - \alpha|0\rangle|\Phi_+\rangle - \alpha|1\rangle|\Psi_+\rangle$

Bob can now perform a Bell measurement on his particles, and Charlie can obtain a particular resultant state by applying the appropriate unitary operation.

For example, if the joint-state obtained by Bob and Charlie is $\beta|1\rangle|\Phi_-\rangle+\beta\,|0\rangle|\Psi_-\rangle-\alpha\,|0\rangle|\Phi_+\rangle-\alpha|1\rangle|\Psi_+\rangle$, one can see that Charlie will obtain the state $|C_i\rangle$, $i = 1, 2, 3, 4$ corresponding to the state measured by Bob $|B_i\rangle$, where $|B_1\rangle = \frac{1}{\sqrt{2}}|01\rangle$, $|B_2\rangle = \frac{1}{\sqrt{2}}|10\rangle$, $|B_3\rangle = \frac{1}{\sqrt{2}}|11\rangle$, $|B_4\rangle = \frac{1}{\sqrt{2}}|00\rangle$ and $|C_1\rangle = \alpha|0\rangle + \beta\,|1\rangle$, $|C_2\rangle = \alpha|0\rangle - \beta\,|1\rangle$, $|C_3\rangle = \alpha|1\rangle + \beta\,|0\rangle$, $|C_4\rangle = \alpha|1\rangle - \beta\,|0\rangle$

V. CONCLUSION

In this paper, the generation, characterisation and application of the seven-qubit genuine genuinely entangled state found by *Xin-Wei Zha et al* [58] has been studied. The resource state is used for perfect linear teleportation of arbitrary one, two and three qubit states, bidirectional teleportation of arbitrary two qubit quantum states as well as probabilistic circular teleportation of a single qubit state. Three proposed schemes for quantum state sharing of arbitrary one qubit states using the resource state have also been presented.

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Appendix A: Three Qubit States

$|A_{000}\rangle = |0000000\rangle + |0000101\rangle - |0001011\rangle + |0001110\rangle$

$|A_{001}\rangle = |0000010\rangle + |0001001\rangle + |0001100\rangle - |0000111\rangle$

$|A_{010}\rangle = |0000111\rangle - |0000010\rangle + |0001001\rangle + |0001100\rangle$

$|A_{011}\rangle = |0000000\rangle + |0000101\rangle + |0001011\rangle - |0001110\rangle$

$|A_{100}\rangle = |0000011\rangle + |0000110\rangle - |0001000\rangle + |0001101\rangle$

$|A_{101}\rangle = |0000001\rangle - |0000100\rangle + |0001010\rangle + |0001111\rangle$

$|A_{110}\rangle = |0000001\rangle - |0000100\rangle - |0001010\rangle - |0001111\rangle$

$|A_{111}\rangle = |0001101\rangle - |0000011\rangle - |0000110\rangle - |0001000\rangle$

$|B_{000}\rangle = |0010000\rangle + |0010101\rangle - |0011011\rangle + |0011110\rangle$

$|B_{001}\rangle = |0010010\rangle + |0011001\rangle + |0011100\rangle - |0010111\rangle$

$|B_{010}\rangle = |0010111\rangle - |0010010\rangle + |0011001\rangle + |0011100\rangle$

$|B_{011}\rangle = |0010000\rangle + |0010101\rangle + |0011011\rangle - |0011110\rangle$

$|B_{100}\rangle = |0010011\rangle + |0010110\rangle - |0011000\rangle + |0011101\rangle$

$|B_{101}\rangle = |0010001\rangle - |0010100\rangle + |0011010\rangle + |0011111\rangle$

$|B_{110}\rangle = |0010001\rangle - |0010100\rangle - |0011010\rangle - |0011111\rangle$

$|B_{111}\rangle = |0011101\rangle - |0010011\rangle - |0010110\rangle - |0011000\rangle$

$|C_{000}\rangle = |0100000\rangle + |0100101\rangle - |0101011\rangle + |0101110\rangle$

$|C_{001}\rangle = |0100010\rangle + |0101001\rangle + |0101100\rangle - |0100111\rangle$

$|C_{010}\rangle = |0100111\rangle - |0100010\rangle + |0101001\rangle + |0101100\rangle$

$|C_{011}\rangle = |0100000\rangle + |0100101\rangle + |0101011\rangle - |0101110\rangle$

$|C_{100}\rangle = |0100011\rangle + |0100110\rangle - |0101000\rangle + |0101101\rangle$

$|C_{101}\rangle = |0100001\rangle - |0100100\rangle + |0101010\rangle + |0101111\rangle$

$|C_{110}\rangle = |0100001\rangle - |0100100\rangle - |0101010\rangle - |0101111\rangle$

$|C_{111}\rangle = |0101101\rangle - |0100011\rangle - |0100110\rangle - |0101000\rangle$

$|D_{000}\rangle = |0110000\rangle + |0110101\rangle - |0111011\rangle + |0111110\rangle$

$|D_{001}\rangle = |0110010\rangle + |0111001\rangle + |0111100\rangle - |0110111\rangle$

$|D_{010}\rangle = |0110111\rangle - |0110010\rangle + |0111001\rangle + |0111100\rangle$

$|D_{011}\rangle = |0110000\rangle + |0110101\rangle + |0111011\rangle - |0111110\rangle$

$|D_{100}\rangle = |0110011\rangle + |0110110\rangle - |0111000\rangle + |0111101\rangle$

$|D_{101}\rangle = |0110001\rangle - |0110100\rangle + |0111010\rangle + |0111111\rangle$

$|D_{110}\rangle = |0110001\rangle - |0110100\rangle - |0111010\rangle - |0111111\rangle$

$|D_{111}\rangle = |0111101\rangle - |0110011\rangle - |0110110\rangle - |0111000\rangle$

$|E_{000}\rangle = |1000000\rangle + |1000101\rangle - |1001011\rangle + |1001110\rangle$

$|E_{001}\rangle = |1000010\rangle + |1001001\rangle + |1001100\rangle - |1000111\rangle$

$|E_{010}\rangle = |1000111\rangle - |1000010\rangle + |0001001\rangle + |0001100\rangle$

$|E_{011}\rangle = |1000000\rangle + |1000101\rangle + |1001011\rangle - |1001110\rangle$

$|E_{100}\rangle = |1000011\rangle + |1000110\rangle - |1001000\rangle + |1001101\rangle$

$|E_{101}\rangle = |1000001\rangle - |1000100\rangle + |1001010\rangle + |1001111\rangle$

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$|E_{110}\rangle = |1000001\rangle$ - $|1000100\rangle$ - $|1001010\rangle$ - $|1001111\rangle$

$|E_{111}\rangle = |1001101\rangle$ - $|1000011\rangle$ - $|1000110\rangle$ - $|1001000\rangle$

$|F_{000}\rangle = |1010000\rangle + |1010101\rangle$ - $|1011011\rangle + |1011110\rangle$

$|F_{001}\rangle = |1010010\rangle + |1011001\rangle + |1011100\rangle$ - $|1010111\rangle$

$|F_{010}\rangle = |1010111\rangle$ - $|1010010\rangle + |1011001\rangle + |1011100\rangle$

$|F_{011}\rangle = |1010000\rangle + |1010101\rangle + |1011011\rangle$ - $|1011110\rangle$

$|F_{100}\rangle = |1010011\rangle + |1010110\rangle$ - $|1011000\rangle + |1011101\rangle$

$|F_{101}\rangle = |1010001\rangle$ - $|1010100\rangle + |1011010\rangle + |1011111\rangle$

$|F_{110}\rangle = |1010001\rangle$ - $|1010100\rangle$ - $|1011010\rangle$ - $|1011111\rangle$

$|F_{111}\rangle = |1011101\rangle$ - $|1010011\rangle$ - $|1010110\rangle$ - $|1011000\rangle$

$|G_{000}\rangle = |1100000\rangle + |1100101\rangle$ - $|1101011\rangle + |1101110\rangle$

$|G_{001}\rangle = |1100010\rangle + |1101001\rangle + |1101100\rangle$ - $|1100111\rangle$

$|G_{010}\rangle = |1100111\rangle$ - $|1100010\rangle + |1101001\rangle + |1101100\rangle$

$|G_{011}\rangle = |1100000\rangle + |1100101\rangle + |1101011\rangle$ - $|1101110\rangle$

$|G_{100}\rangle = |1100011\rangle + |1100110\rangle$ - $|1101000\rangle + |1101101\rangle$

$|G_{101}\rangle = |1100001\rangle$ - $|1100100\rangle + |1101010\rangle + |1101111\rangle$

$|G_{110}\rangle = |1100001\rangle$ - $|1100100\rangle$ - $|1101010\rangle$ - $|1101111\rangle$

$|G_{111}\rangle = |1101101\rangle$ - $|1100011\rangle$ - $|0000110\rangle$ - $|0001000\rangle$

$|H_{000}\rangle = |1110000\rangle + |1110101\rangle$ - $|1111011\rangle + |1111110\rangle$

$|H_{001}\rangle = |1110010\rangle + |1111001\rangle + |1111100\rangle$ - $|1110111\rangle$

$|H_{010}\rangle = |1110111\rangle$ - $|1110010\rangle + |1111001\rangle + |1111100\rangle$

$|H_{011}\rangle = |1110000\rangle + |1110101\rangle + |1111011\rangle$ - $|1111110\rangle$

$|H_{100}\rangle = |1110011\rangle + |1110110\rangle$ - $|1111000\rangle + |1111101\rangle$

$|H_{101}\rangle = |1110001\rangle$ - $|1110100\rangle + |1111010\rangle + |1111111\rangle$

$|H_{110}\rangle = |1110001\rangle$ - $|1110100\rangle$ - $|1111010\rangle$ - $|1111111\rangle$

$|H_{111}\rangle = |1111101\rangle$ - $|1110011\rangle$ - $|1110110\rangle$ - $|1111000\rangle$

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Appendix B: Decomposition of Arbitrary Three Qubit State with $|\Psi_7\rangle$

$$(a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle)|\Psi_7\rangle \tag{B1}$$

$$= \sum_{permutations} ((-1)^{I_1} A_{a_1a_2a_3} + (-1)^{I_2} B_{b_1b_2b_3} + (-1)^{I_3} C_{c_1c_2c_3} + (-1)^{I_4} D_{d_1d_2d_3} \tag{B2}$$

$$+ (-1)^{I_5} E_{e_1e_2e_3} + (-1)^{I_6} F_{f_1f_2f_3} + (-1)^{I_7} G_{g_1g_2g_3} + (-1)^{I_8} H_{h_1h_2h_3}) \tag{B3}$$

$$\tag{B4}$$

$$((-1)^{I_1} a|a_1a_2a_3\rangle + (-1)^{I_2} b|b_1b_2b_3\rangle \tag{B5}$$

$$+ (-1)^{I_3} c|c_1c_2c_3\rangle + (-1)^{I_4} d|d_1d_2d_3\rangle \tag{B6}$$

$$+ (-1)^{I_5} e|e_1e_2e_3\rangle + (-1)^{I_6} f|f_1f_2f_3\rangle \tag{B7}$$

$$+ (-1)^{I_7} g|g_1g_2g_3\rangle + (-1)^{I_8} h|h_1h_2h_3\rangle) \tag{B8}$$

$$\tag{B9}$$

where I_i (i=1, 2, 3, 4, 5, 6, 7, 8) can take values 0 or 1 independently, and L_j (L=a, b, c, d, e, f, g, h; j = 1, 2, 3) can take values 0 or 1 independently. The summation is over all possible permutation states obtained.

Appendix C: Transformations for Three-Qubit Teleportation

Projection of i^{th} component P_i:

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{C1}$$

Flip and Projection of i^{th} component F_i:

$$F_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, F_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{C2}$$

State Obtained by Bob	Short-Hand Form of Transformation
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes I_2 \otimes I_2$
$a\|000\rangle - b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes I_2 \otimes P_2 + \sigma_z \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1$
$a\|000\rangle + b\|001\rangle + c\|010\rangle - d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes I_2 \otimes P_2 + I_2 \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes I_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle - h\|111\rangle$	$I_2 \otimes I_2 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle - b\|001\rangle + c\|010\rangle - d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes I_2 \otimes P_2 + \sigma_z \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1$
$a\|000\rangle - b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle + h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle - b\|001\rangle + c\|010\rangle - d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle + h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle - d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle - b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle - h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + P_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle - d\|011\rangle + e\|100\rangle + f\|101\rangle + g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle - b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle - h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle + g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$

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$a\|000\rangle$ - $b\|001\rangle$ + $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ + $g\|110\rangle$ - $h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ - $b\|001\rangle$ + $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ - $f\|101\rangle$ + $g\|110\rangle$ - $h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ + $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ + $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $\sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ + $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $\sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ + $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $I_2 \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ + $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $\sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ + $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ + $g\|110\rangle$ - $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $\sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ + $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ + $c\|010\rangle$ + $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ - $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $I_2 \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ + $g\|110\rangle$ - $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $I_2 \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ + $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ - $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2$ - $I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ + $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ - $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $\sigma_z \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2$ - $I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ - $c\|010\rangle$ - $d\|011\rangle$ + $e\|100\rangle$ + $f\|101\rangle$ - $g\|110\rangle$ - $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1$ - $I_2 \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2$ - $I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle$ + $b\|001\rangle$ + $c\|010\rangle$ + $d\|011\rangle$ - $e\|100\rangle$ + $f\|101\rangle$ + $g\|110\rangle$ + $h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1$ - $\sigma_z \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$

$a\|000\rangle + b\|001\rangle - c\|010\rangle + d\|011\rangle - e\|100\rangle - f\|101\rangle + g\|110\rangle + h\|111\rangle$	$\sigma_z \otimes P_1 \otimes P_1 - \sigma_z \otimes P_2 \otimes P_1 - I_2 \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle - e\|100\rangle - f\|101\rangle + g\|110\rangle + h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 - I_2 \otimes P_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle - e\|100\rangle + f\|101\rangle + g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 - \sigma_z \otimes P_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle - g\|110\rangle + h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 - \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle + f\|101\rangle - g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + I_2 \otimes P_1 \otimes P_2 - I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle - g\|110\rangle + h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 - \sigma_z \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle - e\|100\rangle + f\|101\rangle - g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 - \sigma_z \otimes P_1 \otimes P_2 - I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle + e\|100\rangle - f\|101\rangle - g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 + \sigma_z \otimes P_1 \otimes P_2 - I_2 \otimes P_2 \otimes P_2$
$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle - e\|100\rangle - f\|101\rangle - g\|110\rangle - h\|111\rangle$	$I_2 \otimes P_1 \otimes P_1 + I_2 \otimes P_2 \otimes P_1 - I_2 \otimes P_1 \otimes P_2 - I_2 \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle + g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes I_2 \otimes I_2$
$- a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle + g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2 + I\sigma_y \otimes P_1 \otimes P_1$
$a\|001\rangle + b\|000\rangle - c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle + g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes I_2 + I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes I_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle - e\|101\rangle + f\|100\rangle + g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes I_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes I_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$- a\|001\rangle + b\|000\rangle - c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle + g\|111\rangle + h\|110\rangle$	$I\sigma_y \otimes I_2 \otimes P_1 + \sigma_x \otimes I_2 \otimes P_2$

- $a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ + $h\|110\rangle$	$I\sigma_y \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
- $a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$I\sigma_y \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
- $a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
- $a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$I\sigma_y \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
- $a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$I\sigma_y \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
- $a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ + $h\|110\rangle$	$I\sigma_y \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ - $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1$ - $I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ - $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2$ - $I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ + $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ - $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ - $c\|011\rangle$ - $d\|010\rangle$ + $e\|101\rangle$ + $f\|100\rangle$ + $g\|111\rangle$ + $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1$ - $\sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$

$a\|001\rangle + b\|000\rangle + c\|011\rangle - d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle - d\|010\rangle + e\|101\rangle + f\|100\rangle + g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 - I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 - \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle - c\|011\rangle - d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - \sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle - c\|011\rangle - d\|010\rangle + e\|101\rangle + f\|100\rangle + g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - \sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 - I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle - c\|011\rangle + d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - \sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 - I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle - d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - I\sigma_y \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 - \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle - c\|011\rangle - d\|010\rangle + e\|101\rangle + f\|100\rangle - g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 - \sigma_x \otimes P_2 \otimes P_1 + \sigma_x \otimes P_1 \otimes P_2 - \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle - f\|100\rangle + g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 - I\sigma_y \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle - e\|101\rangle - f\|100\rangle + g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + \sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle - e\|101\rangle + f\|100\rangle - g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle - e\|101\rangle + f\|100\rangle + g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2 - I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle - f\|100\rangle - g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 - I\sigma_y \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle + e\|101\rangle - f\|100\rangle + g\|111\rangle - h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 - I\sigma_y \otimes P_1 \otimes P_2 - I\sigma_y \otimes P_2 \otimes P_2$
$a\|001\rangle + b\|000\rangle + c\|011\rangle + d\|010\rangle - e\|101\rangle - f\|100\rangle - g\|111\rangle + h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 - \sigma_x \otimes P_1 \otimes P_2 + I\sigma_y \otimes P_2 \otimes P_2$

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$a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ + $f\|100\rangle$ - $g\|111\rangle$ - $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1 + I\sigma_y \otimes P_1 \otimes P_2$ - $\sigma_x \otimes P_2 \otimes P_2$
$a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ + $e\|101\rangle$ - $f\|100\rangle$ - $g\|111\rangle$ - $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1$ - $I\sigma_y \otimes P_1 \otimes P_2$ - $\sigma_x \otimes P_2 \otimes P_2$
+ $a\|001\rangle$ + $b\|000\rangle$ + $c\|011\rangle$ + $d\|010\rangle$ - $e\|101\rangle$ - $f\|100\rangle$ - $g\|111\rangle$ - $h\|110\rangle$	$\sigma_x \otimes P_1 \otimes P_1 + \sigma_x \otimes P_2 \otimes P_1$ - $\sigma_x \otimes P_1 \otimes P_2$ - $\sigma_x \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$

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$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ - $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + \sigma_z \otimes F_2 \otimes P_1 + \sigma_z \otimes P_2 \otimes P_2 + \sigma_z \otimes P_2 \otimes P_2$
- $a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
- $a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
- $a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
- $a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
- $a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
- $a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	$\sigma_z \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$
- $a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $I_2 \otimes F_1 \otimes P_1 + I_2 \otimes F_2 \otimes P_1 + \sigma_z \otimes F_1 \otimes P_2 + I_2 \otimes P_2 \otimes P_2$

- $a\|010\rangle$ - $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ + $h\|101\rangle$	- $I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2$ + $I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ + $f\|111\rangle$ - $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ + $I_2 \otimes F_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ - $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2$ + $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ - $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ + $\sigma_z \otimes F_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ + $\sigma_z \otimes F_1 \otimes P_2$ + $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ - $g\|100\rangle$ + $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2$ - $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ + $f\|111\rangle$ - $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $\sigma_z \otimes F_1 \otimes P_2$ - $I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ + $e\|110\rangle$ - $f\|111\rangle$ - $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ + $\sigma_z \otimes F_1 \otimes P_2$ - $I_2 \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ + $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2$ + $\sigma_z \otimes P_2 \otimes P_2$
$a\|010\rangle$ + $b\|011\rangle$ + $c\|000\rangle$ + $d\|001\rangle$ - $e\|110\rangle$ - $f\|111\rangle$ - $g\|100\rangle$ - $h\|101\rangle$	$I_2 \otimes F_1 \otimes P_1$ + $I_2 \otimes F_2 \otimes P_1$ - $I_2 \otimes F_1 \otimes P_2$ - $I_2 \otimes P_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ - $f\|110\rangle$ - $g\|101\rangle$ - $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1$ - $\sigma_x \otimes \sigma_x \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes I_2$
- $a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes F_1 \otimes P_1$ + $\sigma_x \otimes F_2 \otimes P_1$ + $\sigma_x \otimes \sigma_x \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes F_1 \otimes P_1$ + $I\sigma_y \otimes F_2 \otimes P_1$ - $I\sigma_y \otimes F_1 \otimes P_2$ + $\sigma_x \otimes F_2 \otimes P_2$

$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes \sigma_x \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes F_1 \otimes P_1 + I\sigma_y \otimes F_2 \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes F_1 \otimes P_1 + I\sigma_y \otimes F_2 \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes \sigma_x \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes \sigma_x \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + I\sigma_y \otimes \sigma_x \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes F_1 \otimes P_1 + I\sigma_y \otimes F_2 \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ - $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes \sigma_x \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
$a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	- $I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + \sigma_x \otimes \sigma_x \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1$ - $I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$

- $a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes I\sigma_y \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1$ - $I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
$a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	- $I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
$a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	- $I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1$ - $I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + \sigma_x \otimes I\sigma_y \otimes P_2$
- $a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes I\sigma_y \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
- $a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes I\sigma_y \otimes P_1$ - $I\sigma_y \otimes F_1 \otimes P_2 + \sigma_x \otimes F_2 \otimes P_2$
- $a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + \sigma_x \otimes I\sigma_y \otimes P_2$
$a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	- $I\sigma_y \otimes F_1 \otimes P_1 + \sigma_x \otimes F_2 \otimes P_1 + \sigma_x \otimes I\sigma_y \otimes P_2$
- $a\|011\rangle$ - $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes I\sigma_y \otimes P_1 + \sigma_x \otimes I\sigma_y \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ - $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + \sigma_x \otimes F_1 \otimes P_2$ - $I\sigma_y \otimes F_2 \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ - $e\|111\rangle$ + $f\|110\rangle$ + $g\|101\rangle$ - $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + \sigma_x \otimes I\sigma_y \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ - $f\|110\rangle$ + $g\|101\rangle$ - $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1$ - $I\sigma_y \otimes \sigma_x \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ - $f\|110\rangle$ - $g\|101\rangle$ + $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + I\sigma_y \otimes I\sigma_y \otimes P_2$
$a\|011\rangle$ + $b\|010\rangle$ + $c\|001\rangle$ + $d\|000\rangle$ + $e\|111\rangle$ + $f\|110\rangle$ - $g\|101\rangle$ - $h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1$ - $\sigma_x \otimes I\sigma_y \otimes P_2$

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$a\|011\rangle + b\|010\rangle + c\|001\rangle + d\|000\rangle - e\|111\rangle - f\|110\rangle - g\|101\rangle + h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 - \sigma_x \otimes F_1 \otimes P_2 + I\sigma_y \otimes F_2 \otimes P_2$
$a\|011\rangle + b\|010\rangle + c\|001\rangle + d\|000\rangle - e\|111\rangle - f\|110\rangle + g\|101\rangle - h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 - \sigma_x \otimes F_1 \otimes P_2 - I\sigma_y \otimes F_2 \otimes P_2$
$a\|011\rangle + b\|010\rangle + c\|001\rangle + d\|000\rangle - e\|111\rangle + f\|110\rangle - g\|101\rangle - h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 + I\sigma_y \otimes F_1 \otimes P_2 - \sigma_x \otimes F_2 \otimes P_2$
$a\|011\rangle + b\|010\rangle + c\|001\rangle + d\|000\rangle + e\|111\rangle - f\|110\rangle - g\|101\rangle - h\|100\rangle$	$\sigma_x \otimes \sigma_x \otimes P_1 - I\sigma_y \otimes F_1 \otimes P_2 - \sigma_x \otimes F_2 \otimes P_2$
$- a\|100\rangle - b\|101\rangle - c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$- I_2 \otimes I_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$I_2 \otimes I_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$\sigma_z \otimes P_1 \otimes F_1 + I_2 \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle + h\|011\rangle$	$I_2 \otimes I_2 \otimes F_1 + \sigma_z \otimes P_1 \otimes F_2 + I_2 \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle - h\|011\rangle$	$I_2 \otimes I_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + \sigma_z \otimes P_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$\sigma_z \otimes I_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle + h\|011\rangle$	$\sigma_z \otimes P_1 \otimes F_1 + I_2 \otimes P_2 \otimes F_1 + \sigma_z \otimes P_1 \otimes F_2 + I_2 \otimes P_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle - h\|011\rangle$	$\sigma_z \otimes P_1 \otimes F_1 + I_2 \otimes P_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + \sigma_z \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle + h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + \sigma_z \otimes P_1 \otimes F_2 + I_2 \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle - h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + \sigma_z \otimes P_2 \otimes F_2$

$a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle - h\|011\rangle$	$I_2 \otimes I_2 \otimes F_1 + \sigma_z \otimes I_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle + h\|011\rangle$	$\sigma_z \otimes I_2 \otimes F_1 + \sigma_z \otimes P_1 \otimes F_2 + I_2 \otimes P_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle - h\|011\rangle$	$\sigma_z \otimes P_1 \otimes F_1 + I_2 \otimes P_2 \otimes F_1 + \sigma_z \otimes I_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle - h\|011\rangle$	$\sigma_z \otimes I_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + \sigma_z \otimes P_2 \otimes \sigma_z$
$a\|100\rangle + b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle - h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + \sigma_z \otimes I_2 \otimes F_2$
$a\|100\rangle - b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle - f\|001\rangle + g\|010\rangle - h\|011\rangle$	$\sigma_z \otimes I_2 \otimes F_1 + \sigma_z \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle - c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 - \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle - g\|010\rangle + h\|011\rangle$	$I_2 \otimes I_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 - \sigma_z \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle - c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$I_2 \otimes \sigma_z \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle - c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle - g\|010\rangle + h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 - \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 - \sigma_z \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle - c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle - h\|011\rangle$	$I_2 \otimes P_1 \otimes - \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + \sigma_z \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle - g\|010\rangle + h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 - \sigma_z \otimes P_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle - g\|010\rangle - h\|011\rangle$	$I_2 \otimes I_2 \otimes F_1 + I_2 \otimes \sigma_z \otimes F_2$
$a\|100\rangle + b\|101\rangle - c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle - g\|010\rangle + h\|011\rangle$	$I_2 \otimes \sigma_z \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle + b\|101\rangle - c\|110\rangle - d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle - h\|011\rangle$	$I_2 \otimes \sigma_z \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + \sigma_z \otimes P_2 \otimes F_2$

$a\|100\rangle$ + $b\|101\rangle$ - $c\|110\rangle$ + $d\|111\rangle$ + $e\|000\rangle$ + $f\|001\rangle$ - $g\|010\rangle$ - $h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1$ - $\sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes \sigma_z \otimes F_2$
$a\|100\rangle$ + $b\|101\rangle$ + $c\|110\rangle$ - $d\|111\rangle$ + $e\|000\rangle$ + $f\|001\rangle$ - $g\|010\rangle$ - $h\|011\rangle$	$I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes \sigma_z \otimes F_2$
$a\|100\rangle$ + $b\|101\rangle$ - $c\|110\rangle$ - $d\|111\rangle$ + $e\|000\rangle$ + $f\|001\rangle$ - $g\|010\rangle$ - $h\|011\rangle$	$I_2 \otimes \sigma_z \otimes F_1 + I_2 \otimes \sigma_z \otimes F_2$
- $a\|100\rangle + b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $\sigma_z \otimes P_1 \otimes F_1 + I_2 \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
- $a\|100\rangle$ - $b\|101\rangle + c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $I_2 \otimes \sigma_z \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
- $a\|100\rangle + b\|101\rangle$ - $c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $\sigma_z \otimes I_2 \otimes F_1 + I_1 \otimes I_2 \otimes F_2$
- $a\|100\rangle + b\|101\rangle + c\|110\rangle$ - $d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $\sigma_z \otimes \sigma_z \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle$ - $b\|101\rangle$ - $c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$\sigma_z \otimes \sigma_z \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
- $a\|100\rangle$ - $b\|101\rangle$ - $c\|110\rangle + d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $I_2 \otimes P_1 \otimes F_1$ - $\sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
- $a\|100\rangle$ - $b\|101\rangle + c\|110\rangle$ - $d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $I_2 \otimes P_1 \otimes F_1 + \sigma_z \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
- $a\|100\rangle + b\|101\rangle$ - $c\|110\rangle$ - $d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	- $\sigma_z \otimes P_1 \otimes F_1$ - $I_2 \otimes P_2 \otimes F_1 + I_2 \otimes I_2 \otimes F_2$
$a\|100\rangle$ - $b\|101\rangle$ - $c\|110\rangle$ - $d\|111\rangle + e\|000\rangle + f\|001\rangle + g\|010\rangle + h\|011\rangle$	$\sigma_z \otimes P_1 \otimes F_1$ - $I_2 \otimes P_2 \otimes F_1 + I_2 \otimes P_1 \otimes F_2 + I_2 \otimes P_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle + c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$\sigma_x \otimes I_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
- $a\|101\rangle + b\|100\rangle + c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$I\sigma_y \otimes P_1 \otimes F_1 + \sigma_x \otimes P_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle$ - $c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 + I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$

$a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes I_2 \otimes F_1 + I\sigma_y \otimes P_1 \otimes F_2 + \sigma_x \otimes P_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ + $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes I_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 + I\sigma_y \otimes P_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes I_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes P_1 \otimes F_1 + \sigma_x \otimes P_2 \otimes F_1 + I\sigma_y \otimes P_1 \otimes F_2 + \sigma_x \otimes P_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ + $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes P_1 \otimes F_1 + \sigma_x \otimes P_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 + I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 + I\sigma_y \otimes P_2 \otimes F_1 + I\sigma_y \otimes P_1 \otimes F_2 + \sigma_x \otimes P_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 + I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 + I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes I_2 \otimes F_1 + I\sigma_y \otimes I_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes I_2 \otimes F_1 + I\sigma_y \otimes P_1 \otimes F_2 + \sigma_x \otimes P_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes P_1 \otimes F_1 + \sigma_x \otimes P_2 \otimes F_1 + I\sigma_y \otimes I_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes I_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 + I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 + I\sigma_y \otimes P_2 \otimes F_1 + I\sigma_y \otimes P_1 \otimes F_2 + I\sigma_y \otimes P_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ - $e\|001\rangle$ + $f\|000\rangle$ - $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes I_2 \otimes F_1 + I\sigma_y \otimes I_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ - $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ - $h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1$ - $I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2$ - $I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle$ + $b\|100\rangle$ + $c\|111\rangle$ - $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1$ - $I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$

$a\|101\rangle + b\|100\rangle + c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes I_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 - I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle - c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$\sigma_x \otimes \sigma_z \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle - c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 + I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 - I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle + c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle - g\|011\rangle + h\|010\rangle$	$I\sigma_y \otimes P_1 \otimes F_1 + \sigma_x \otimes P_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle + c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle - g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes I_2 \otimes F_1 + \sigma_x \otimes \sigma_z \otimes F_2$
$a\|101\rangle + b\|100\rangle - c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle - g\|011\rangle + h\|010\rangle$	$\sigma_x \otimes \sigma_z \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 + I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle - c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes \sigma_z \otimes F_1 + \sigma_x \otimes P_1 \otimes F_2 - I\sigma_y \otimes P_2 \otimes F_2$
$a\|101\rangle + b\|100\rangle - c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle - g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 + I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes \sigma_z \otimes F_2$
$a\|101\rangle + b\|100\rangle + c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle - g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes P_1 \otimes F_1 - I\sigma_y \otimes P_2 \otimes F_1 + \sigma_x \otimes \sigma_z \otimes F_2$
$a\|101\rangle + b\|100\rangle - c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle - g\|011\rangle - h\|010\rangle$	$\sigma_x \otimes \sigma_z \otimes F_1 + \sigma_x \otimes \sigma_z \otimes F_2$
$a\|101\rangle - b\|100\rangle + c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$- I\sigma_y \otimes P_1 \otimes F_1 + \sigma_x \otimes P_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
$- a\|101\rangle - b\|100\rangle + c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$- \sigma_x \otimes \sigma_z \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
$- a\|101\rangle + b\|100\rangle + c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$I\sigma_y \otimes P_1 \otimes F_1 - I\sigma_y \otimes P_2 \otimes F_1 + I\sigma_y \otimes I_2 \otimes F_2$
$a\|101\rangle - b\|100\rangle - c\|111\rangle + d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$- I\sigma_y \otimes \sigma_z \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$
$a\|101\rangle - b\|100\rangle + c\|111\rangle - d\|110\rangle + e\|001\rangle + f\|000\rangle + g\|011\rangle + h\|010\rangle$	$- I\sigma_y \otimes I_2 \otimes F_1 + \sigma_x \otimes I_2 \otimes F_2$

- $a\|101\rangle$ - $b\|100\rangle$ - $c\|111\rangle$ + $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	- $\sigma_x \otimes P_1 \otimes F_1$ + $I\sigma_y \otimes P_2 \otimes F_1$ + $I\sigma_y \otimes I_2 \otimes F_2$
- $a\|101\rangle$ - $b\|100\rangle$ + $c\|111\rangle$ - $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	- $F_1 \otimes P_1 \otimes F_1$ - $I\sigma_y \otimes P_2 \otimes F_1$ + $I\sigma_y \otimes I_2 \otimes F_2$
- $a\|101\rangle$ + $b\|100\rangle$ - $c\|111\rangle$ - $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	$I\sigma_y \otimes I_2 \otimes F_1$ + $\sigma_x \otimes I_2 \otimes F_2$
$a\|101\rangle$ - $b\|100\rangle$ - $c\|111\rangle$ - $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	- $I\sigma_y \otimes P_1 \otimes F_1$ - $\sigma_x \otimes P_2 \otimes F_1$ + $\sigma_x \otimes I_2 \otimes F_2$
- $a\|101\rangle$ - $b\|100\rangle$ - $c\|111\rangle$ - $d\|110\rangle$ + $e\|001\rangle$ + $f\|000\rangle$ + $g\|011\rangle$ + $h\|010\rangle$	- $\sigma_x \otimes I_2 \otimes F_1$ + $\sigma_x \otimes I_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1$ + $I_2 \otimes F_2 \otimes F_1$ + $I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes \sigma_x \otimes F_1$ + $I_2 \otimes \sigma_x \otimes F_2$
+ $a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes F_1 \otimes F_1$ + $\sigma_z \otimes F_2 \otimes F_1$ + $I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ - $h\|001\rangle$	$I_2 \otimes \sigma_x \otimes F_1$ + $I_2 \otimes \sigma_x \otimes F_2$ + $\sigma_z \otimes F_2 \otimes F_1$
$a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ - $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes \sigma_x \otimes F_1$ + $\sigma_z \otimes F_1 \otimes F_2$ + $I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$\sigma_z \otimes \sigma_x \otimes F_1$ + $I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ - $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_1 \otimes F_2 + \sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ - $h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1 + I_2 \otimes F_1 \otimes F_2 + \sigma_z \otimes F_2 \otimes F_2$
$a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ - $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes F_1 \otimes F_1 + \sigma_z \otimes F_2 \otimes F_1 + \sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ - $h\|001\rangle$	$I_2 \otimes F_1 \otimes F_1 + \sigma_z \otimes F_2 \otimes F_1 + I_2 \otimes F_1 \otimes F_2 + \sigma_z \otimes F_2 \otimes F_2$

$a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle + e\|010\rangle$ - $f\|000\rangle + g\|000\rangle$ - $h\|001\rangle$	$I_2 \otimes \sigma_x \otimes F_1 + \sigma_z \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle + c\|100\rangle$ - $d\|101\rangle + e\|010\rangle$ - $f\|000\rangle + g\|000\rangle + h\|001\rangle$	$\sigma_z \otimes \sigma_x \otimes F_1 + \sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle + c\|100\rangle$ - $d\|101\rangle + e\|010\rangle + f\|000\rangle + g\|000\rangle$ - $h\|001\rangle$	$\sigma_z \otimes \sigma_x \otimes F_1 + I_2 \otimes F_1 \otimes F_2 + \sigma_z \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle + c\|100\rangle + d\|101\rangle + e\|010\rangle$ - $f\|000\rangle + g\|000\rangle$ - $h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1 + \sigma_z \otimes F_1 \otimes F_2 + \sigma_z \otimes F_2 \otimes F_2$
$a\|110\rangle + b\|111\rangle + c\|100\rangle$ - $d\|101\rangle + e\|010\rangle$ - $f\|000\rangle + g\|000\rangle$ - $h\|001\rangle$	$I_2 \otimes F_1 \otimes F_1 + \sigma_z \otimes F_2 \otimes F_1 + \sigma_z \otimes F_1 \otimes F_2 + \sigma_z \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle + c\|100\rangle$ - $d\|101\rangle + e\|010\rangle$ - $f\|000\rangle + g\|000\rangle$ - $h\|001\rangle$	$\sigma_z \otimes \sigma_x \otimes F_1 + \sigma_z \otimes \sigma_x \otimes F_2$
- $a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle + e\|010\rangle + f\|000\rangle + g\|000\rangle + h\|001\rangle$	- $\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle$ - $e\|010\rangle + f\|000\rangle + g\|000\rangle + h\|001\rangle$	$I_2 \otimes \sigma_x \otimes F_1$ - $\sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
- $a\|110\rangle$ - $b\|111\rangle + c\|100\rangle + d\|101\rangle + e\|010\rangle + f\|000\rangle + g\|000\rangle + h\|001\rangle$	$I_2 \otimes I\sigma_y \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle$ - $e\|010\rangle + f\|000\rangle + g\|000\rangle + h\|001\rangle$	- $\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1$ - $\sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
- $a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle + e\|010\rangle$ - $f\|000\rangle + g\|000\rangle + h\|001\rangle$	- $\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1 + \sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle + c\|100\rangle + d\|101\rangle$ - $e\|010\rangle + f\|000\rangle + g\|000\rangle + h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1$ - $\sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle$ - $e\|010\rangle$ - $f\|000\rangle + g\|000\rangle + h\|001\rangle$	$I_2 \otimes \sigma_x \otimes F_1 + I_2 \otimes I\sigma_y \otimes F_2$
- $a\|110\rangle$ - $b\|111\rangle + c\|100\rangle + d\|101\rangle$ - $e\|010\rangle + f\|000\rangle + g\|000\rangle + h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1$ - $\sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
- $a\|110\rangle + b\|111\rangle + c\|100\rangle + d\|101\rangle$ - $e\|010\rangle$ - $f\|000\rangle + g\|000\rangle + h\|001\rangle$	- $\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1 + I_2 \otimes I\sigma_y \otimes F_2$

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- $a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ - $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes I\sigma_y \otimes F_1 + \sigma_z \otimes F_1 \otimes F_2 + I_2 \otimes F_2 \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ - $e\|010\rangle$ - $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1 + I_2 \otimes F_2 \otimes F_1 + I_2 \otimes I\sigma_y \otimes F_2$
- $a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ + $d\|101\rangle$ - $e\|010\rangle$ - $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes I\sigma_y \otimes F_1 + I_2 \otimes I\sigma_y \otimes F_2$
$a\|110\rangle$ + $b\|111\rangle$ - $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes F_1 \otimes F_1$ - $\sigma_z \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle$ + $b\|111\rangle$ - $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	- $\sigma_z \otimes F_1 \otimes F_1$ - $\sigma_z \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle$ + $b\|111\rangle$ + $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$\sigma_z \otimes I\sigma_y \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ - $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	- $\sigma_z \otimes I\sigma_y \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ + $b\|111\rangle$ - $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes I\sigma_y \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle$ - $b\|111\rangle$ - $c\|100\rangle$ + $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$I_2 \otimes F_1 \otimes F_1$ - $\sigma_z \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle$ - $b\|111\rangle$ + $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	- $I_2 \otimes F_1 \otimes F_1 + \sigma_z \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle$ + $b\|111\rangle$ - $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	- $\sigma_z \otimes F_1 \otimes F_1$ - $I_2 \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
$a\|110\rangle$ - $b\|111\rangle$ - $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	$\sigma_z \otimes F_1 \otimes F_1$ - $I_2 \otimes F_2 \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|110\rangle$ - $b\|111\rangle$ - $c\|100\rangle$ - $d\|101\rangle$ + $e\|010\rangle$ + $f\|000\rangle$ + $g\|000\rangle$ + $h\|001\rangle$	- $I_2 \otimes \sigma_x \otimes F_1 + I_2 \otimes \sigma_x \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes F_1 \otimes F_1 + I\sigma_y \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$

$a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes \sigma_x \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes \sigma_x \otimes F_1 + \sigma_x \otimes F_1 \otimes F_2 + I\sigma_y \otimes F_2 \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes \sigma_x \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes F_1 \otimes F_1 + I\sigma_y \otimes F_2 \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes F_1 \otimes F_1 + I\sigma_y \otimes F_2 \otimes F_1 + \sigma_x \otimes F_1 \otimes F_2 + I\sigma_y \otimes F_2 \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes \sigma_x \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + I\sigma_y \otimes \sigma_x \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes \sigma_x \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + I\sigma_y \otimes \sigma_x \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes \sigma_x \otimes F_1 + \sigma_x \otimes F_1 \otimes F_2 + I\sigma_y \otimes F_2 \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes F_1 \otimes F_1 + I\sigma_y \otimes F_2 \otimes F_1 + I\sigma_y \otimes \sigma_x \otimes F_2$
- $a\|111\rangle$ + $b\|110\rangle$ - $c\|101\rangle$ + $d\|100\rangle$ - $e\|011\rangle$ + $f\|010\rangle$ - $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes \sigma_x \otimes F_1 + I\sigma_y \otimes \sigma_x \otimes F_2$
$a\|111\rangle$ + $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$\sigma_x \otimes \sigma_x \otimes \sigma_x$
$a\|111\rangle$ - $b\|110\rangle$ + $c\|101\rangle$ + $d\|100\rangle$ + $e\|011\rangle$ + $f\|010\rangle$ + $g\|001\rangle$ + $h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$

$a\|111\rangle + b\|110\rangle + c\|101\rangle + d\|100\rangle + e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes \sigma_x \otimes F_1 - I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$- a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes I\sigma_y \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle + b\|110\rangle + c\|101\rangle + d\|100\rangle + e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 - I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle + e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 - I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle + b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes \sigma_x \otimes F_1 + \sigma_x \otimes I\sigma_y \otimes F_2$
$- a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes I\sigma_y \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$- a\|111\rangle + b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes I\sigma_y \otimes F_1 + I\sigma_y \otimes F_1 \otimes F_2 + \sigma_x \otimes F_2 \otimes F_2$
$a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- I\sigma_y \otimes F_1 \otimes F_1 + \sigma_x \otimes F_2 \otimes F_1 + \sigma_x \otimes I\sigma_y \otimes F_2$
$- a\|111\rangle - b\|110\rangle + c\|101\rangle + d\|100\rangle - e\|011\rangle - f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes I\sigma_y \otimes \sigma_x$
$a\|111\rangle + b\|110\rangle + c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$\sigma_x \otimes F_1 \otimes F_1 - I\sigma_y \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle + b\|110\rangle + c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- I\sigma_y \otimes I\sigma_y \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$a\|111\rangle - b\|110\rangle - c\|101\rangle + d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$I\sigma_y \otimes I\sigma_y \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$a\|111\rangle - b\|110\rangle + c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- I\sigma_y \otimes \sigma_x \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$

$a\|111\rangle + b\|110\rangle - c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- \sigma_x \otimes I\sigma_y \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle - b\|110\rangle - c\|101\rangle + d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- \sigma_x \otimes F_1 \otimes F_1 + I\sigma_y \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle - b\|110\rangle + c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- \sigma_x \otimes F_1 \otimes F_1 - I\sigma_y \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle + b\|110\rangle - c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$I\sigma_y \otimes F_1 \otimes F_1 - \sigma_x \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$a\|111\rangle - b\|110\rangle - c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- I\sigma_y \otimes F_1 \otimes F_1 - \sigma_x \otimes F_2 \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$
$- a\|111\rangle - b\|110\rangle - c\|101\rangle - d\|100\rangle + e\|011\rangle + f\|010\rangle + g\|001\rangle + h\|000\rangle$	$- \sigma_x \otimes \sigma_x \otimes F_1 + \sigma_x \otimes \sigma_x \otimes F_2$