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Multi-Objective Optimization Concept Based on Periodical and Permanent Objective within a Process

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ABSTRACT: Multi-objective optimization (MOO) is the subset of optimization which deals with minimization of several objective functions more than the conventional one objective optimization. These have useful application in decision making. Many of the current methodologies addresses challenges and solutions to multi-objective optimization problem which attempt to simultaneously solve several objectives with multiple constraints subjoined to each objective. Such as evolutionary algorithm, genetic algorithm, flower pollination algorithm, and many more. However, most challenges in MOO are generally subjected to linear inequality constraints that prevent all objectives from being optimized at once. This paper discusses some approaches presented by scholars in MOO and then presents some new concepts by introducing methods in solving problem in MOO which comes due to periodical objectives that do not stay for the entire duration of process life time unlike permanent objectives which are optimized once for the entire process life time. A methodology based on partial optimization which optimizes each objective iteratively and weight convergence method which optimizes sub-group of objectives is given. Furthermore, another methods is introduced which involve objective classification, ranking, estimation and prediction where objectives are classified base on their properties, and rank using a given criteria and in addition estimated for its optimal weight point if it certifies a coveted optimal weight point. Then finally predicted to find how much it deviates from the estimated point.

KEYWORDS: optimization; multi-objective optimization; decision making; algorithm

1. INTRODUCTION

Optimization of process is of paramount eminence in science, engineering, finance and social issue. Its application ranges from biological process control, chemical industrial control to physical process control and many more. Optimal control succor in abridging cost and wastage of resources and minimizes time for process execution within given constraints to meet desire objectives as illustrated in many method in Optimal Control[1] and adaptive control [2]. Take for example a case in a flight or marine vessel which is required to maximize the amount of load (goods) it carries meanwhile minimizing it weight and size. Some common approach is to minimize the amount of fuel it uses as reduction in fuel use is directly proportional to decrease in overall weight of a vessel and ultimately the size of the fuel tank. But how does one reduce fuel without affecting the distance to be covered by the vessel? Some engineers would use the shortest path/rout possible for navigation, others might use the approach of redesigning an energy efficient engine to ensure minimum consumption and minimum wastage of fuel[3][4] during navigation. All these are targeted at minimizing cost and wastage of resources such as fuel.

Another example, in chemical industrial setting, where the objective is to maximize the end product of reaction. Some engineers would device a methodology by maximizing rate of reaction but some reaction are endothermic which requires too much input energy to run the reaction process fully and efficiently. So how does one minimize energy inputs required to run such endothermic reaction meanwhile maximizing end product of the reaction? All these are some of the notable situation that requires good optimization strategy.

A simple System often consists of a single objective and perhaps a single constraint and they are linear normally. However a system start getting complicated when it’s non-linear and in addition, when there are more than one or many constraints and many or multiple objectives to be solve simultaneously. Such systems are quite not unexacting to optimize due to their Complexity.

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Most challenges in multi-objective optimization often referred to as multi-criteria programming are generally subjected to linear inequality constraints that prevent all objectives from being simultaneously solve for instance a case where number of objectives are more than that of controllable variable or, perturbation that generates uncertainties. Several Approaches and model for finding optimal weight point of multi-objective optimization has been put in place by many scholars. The methods addresses several challenges in multi-objective optimization and their application in many field such as science, finance, engineering and many more for instance in field of electricity in electrical power balance where trade-off between voltage and electrical grid requirement[5] where demand of electricity and electricity generation needs to be balance with considering constraints bonded to it. The challenges involve in multi-objective optimization is finding an optimal weight point solution thought there may exist multiple solution for a given multi-objective optimization problem, however the problem is finding such solution which isn't straight forward such as that of a single objective optimization.

2. RELATED WORK.

Below are some of the notable multi-objective optimization methods and their application in many problem domains in addition to currently and often use standard approach by setting a fix optimal weight point among multiple objectives. Their approaches are categories into four major categories [6][7] as below: No preferences, Priori method (lexicographical programming, Goal programming, Utility programming), Posteriori Method and Interactive Method.

A case studies by Bialaszewski, Tomasz et el[8] showed their method base on genetic gender approach for solving multi-objective optimization challenges of detection observers. In their method the previous knowledge about a single gender of all included solutions is use for the purpose of making difference among groups of objective. The knowledge is got from fitness of a single person and use during a current parental crossover in evolutionary multi-objective optimization process.

An approach by Fazlollah, Samira et el[9] presented a work on multi-objective optimization model for sizing and operation optimization district heating system with heat storage tanks. The model includes process design and energy integration method for optimizing the temperature interval, the volume and the operation strategy of thermal storage tanks.

The application of multi-objective optimization in water distribution system by Shokoohi, Meisam[10], they use Ant-colony-Optimization for the optimization algorithm which concern with water quality base objectives in Water Distribution System design alongside other common objectives.

A multi-objectives decision support system developed for rehabilitation planning of public infrastructure by farranMazan[11]. Their method provides decision makers a collection of optimal rehabilitation tradeoff over a preferred analysis period. They handled two main objectives function cost and performance at once, together with the collection of attached constraints. The mechanism is based on a fitness-oriented method where challenges information is taken into account. To further analyze cost and performance at once, a normalization methods of all objectives is attained through time-value concept for both cost and condition states. Their proposed methodology is based on life-cycle costing approach using a dynamic markov chain to constitute the degeneration methodology and optimal rehabilitation profile is found using algorithm.

In conclusion, several methods are proposed by many scholars however some method might be feasible for one or more situations while in another situation it might not be feasible.

3. PROPOSED CONCEPT IN MULTI-OBJECTIVE OPTIMIZATION.

This portion discusses and presents some of the challenges and solution involve in multi-objective optimization which do not appear in single objectives optimization and then it gives ways forward in solving those problem.

First we classify objectives base on how long they take or are actively needed in a process life time during process execution. They are classified into two categories as describe here below.

3.1 PERIODICAL OBJECTIVES

In this context, a temporal objective is the one that is needed only for a particular period of time or for a short while less than the time for process execution and it's not throughout the entire process. Periodical objectives can pop in or out at any moment during process execution time, these could be due to perturbation or any other factor which may cause it, for example consider the objectives function given below (1) in MOO

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$$\min(f_1(x), f_2(x) \dots f_k(x)) \tag{1}$$

Where integers, $k \geq 2$ are the number of objectives. Out of total objective $f_k(x)$, one or more might be very crucial from the start of the process or in the middle or even towards the end of process execution but not throughout the entire process execution and somewhere somehow, the periodical objective will be no longer be relevant once it is not needed.

This periodical objectives which switch on and off, results in the entire multi-objectives being solve again and again, with and without the periodical objective in and out of the process execution as the optimal weight point of multi-objectives varies and are not the same when one include or remove a given objective from many objectives.

For example, the optimal weight $W_1 \neq W_2$ of the equation in multi-objective (2) and (3)are not the same because of the absent of the objective in other word, W_1 is not an optimal weight point of equation (3) whose optimal weight point is W_2

$$W_1 \coloneqq \min(f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)) \tag{2}$$

The MOO (2) contains five objectives function which need to be minimized.

$$W_2 \coloneqq \min(f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_6(x)) \tag{3}$$

And the MOO (3) contains six objectives functions not the same as equation (2) however all the five objectives are the same with additional objective.

These gives challenge of redesigning another new optimal weight point which is optimal for the remaining objectives which are only relevant during execution excluding the objective which is no longer needed or including the additional objectives as in equation(3).

3.2 PERMANENT OBJECTIVES

Unlike in periodical or temporary objective in multi-objective optimization, permanent or long term objective is defined as the one that stay or is needed from the starts of a process execution up to the end of process execution which does not result in redesigning the optimal weight point since the optimal weight point remains the same.

3.3 PARTIAL OPTIMIZATION CONCEPT IN MULTI_OBJECTIVE OPTIMISATION

In response to the challenges introduced by the presents of periodical objective in multi-objective optimization, I present this theorem which deals with partial optimization of many objectives in multi-objective optimization, two initial idea related to multi-objective optimization is presented.

i. ITERATIVE MULTI-LEVEL APPROACH.

The approach is iterative in that, it involves taking two or more solvable set from multiple objectives set and their optimal weight $W_\mu$for the sub-set solve. This solution becomes or is set as constraints of the next solvable set or objectives from the multiple set. This is done iteratively until all objectives are finished and the final optimal weight point is assumed to be the most optimal weight among multi-objectives.

Consider objective function below:

$$\min(f_1(x), f_2(x), f_3(x), \dots, f_k(x)) \tag{4}$$
$$s.t \;\; x \in X$$

Where integers $k \geq 2$ are the number of objectives and the set $X$are number of feasible set of decision vectors.

However, element $x^* \in X$ is further defined as a feasible solution or feasible decision of an objectives vector.

$$z^* \coloneqq f(x^*) \in \mathbb{R}^k \tag{5}$$

Initially, we first take the second objective function $f_2(x)$ and set the previous (first) objective function $f_1(x)$ as a constraint and find the optimal weight W_0 as their optimal weight between the two objectives just like the one presented in a paper [12] which attempt to solve two objective by setting one objective as a constraint and another objective to be minimize.

$$W_0 \coloneqq \min(f_2(x)) \tag{6}$$
$$s.t \;\; f_1(x) \leq \varepsilon_0$$

Next iteration is to take objective function $f_3(x)$ and set the previous optimal weigh W_0 for objectives function $f_1(x)$, $f_2(x)$ as a constraint to find optimal weight W_1.

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$$W_1 \coloneqq \min\big(f_3(x)\big) \qquad (7)$$
$$s.t \quad W_0 \leq \varepsilon_1$$

The process continues until all the functions in multi-objectives $f_k(x)$are finished and the final weight W_μ is assumed to be the optimal weight for all objectives.

An overall formulation is as below:

$$W_\mu \coloneqq \min\big(f_{\mu+2}(x)\big) \qquad (8)$$
$$s.t \quad W_{\mu-1} \leq \varepsilon_\mu$$

ii. WEIGHT CONVERGENCE OPTIMIZATION

The second approach is by dividing a given set of objectives into several small sub-sets of objectives which is easily solvable without much burden, and each sub-set solve separately. Let $W_{\mu,i}$ where μ, and i are integers of weight level and set of weight respectively.

Weight Level zero $W_{0,i}$

$$\min\begin{pmatrix} W_{0,0}\big(f_1(x), f_2(x), f_3(x), \dots, f_{k1}(x)\big), W_{0,1}\big(f_{k1+1}(x), f_{k1+2}(x), f_{k1+3}(x), \dots, f_{k2}(x)\big), \dots \\ \dots, W_{0,i}\big(f_{kn+1}(x), f_{kn+2}(x), f_{kn+3}(x), \dots\dots, f_k(x)\big) \end{pmatrix} \qquad (9)$$
$$s.t \quad W_{0,i} \leq \varepsilon_0$$

The process continues up to a set of weight with a single element (weight point)$W_{\mu,0}$ and that is assumed to be an optimal weight point for multi-objective optimization.

Weight Level μ for $W_{\mu,0}$

$$\min\Big(W_{\mu,0}(\dots\dots), W_{\mu,1}(\dots\dots.), W_{\mu,2}(\dots\dots.), \dots., W_{\mu,i}(\dots\dots)\Big) \qquad (10)$$
$$s.t\, W_{\mu-1,i} \leq \varepsilon_\mu$$

3.4 OBJECTIVES CLASSIFICATION, RANKING, ESTIMATION, PREDICTION.

The second approach which is base on the following: objective classification, Ranking, Estimation, and Predictive measurement to find how far a system will deviate from a preferred optimal weight. This basically in the present of a decision maker (DM), where there could be preference.

The following steps are how the process can be executed in order to find an optimal weight point.

Step 1: *Objectives Classification*

Several criteria can be use to classify objectives. For instance each objective can be categories as independence and dependence objective. Independence is objectives which does not rely other objectives as oppose to dependence objective which depend on other objectives in other word solving one objective affect the other which is dependant. Another way of classifying objectives is by categorizing them into either temporary or permanent objective as discussed above. This is mainly to priorities and takes care of objectives such that when dynamically choosing weight, such objectives are taken care of.

Step 2; *Objective Ranking*

In this stage 2 of objective ranking, in a set of multi-objectives, objectives is first ranked in order of preference or merit base. Second each individual participating objective its minimum and maximum optimal weight point is determined such that when finding weight such points are taken care of. Furthermore, objective can be rank in order with which they exit process execution, with the one that exit earliest being the last to be optimize and the one that exit last being the first to be optimize in partial optimization.

For instance, b^- and b^+ are defined as lower and upper bound limits for both minimum and maximum optimal weight point for a given single objective see condition (11)

$$b^- \leq x^* \leq b^+ \;: s.t \;\; \{b^-, \dots\dots. b^+\} \leq \varepsilon_i \qquad (11)$$

Step 3: *Objective Estimation*

In estimation stage, this requires prior knowledge of objectives ranking and classification then an optimal weight point is first estimated that certifies the ranking and classification in step 1 and 2.

Step 4: *Objective Prediction*.

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This step is base on the principle of predictive measurement in that control variable can be predicted base on prior and current state, and hence a control strategy is laid such that a controller parameter is adjusted accordingly to meet the estimated weight point of multi-objective optimization base on step 1 and 2.

3.5 OBJECTIVE ALIGNMENT IN PARTIAL OPTIMIZATION.

In the above method presented under partial optimization, Objectives has to first be align in order to make it easily possible to remove an objective or many objective without needs to redesign the entire equation. This purposely is done to give temporary objective less priority and permanent objective very high priority.

a. Classification.

First before beginning to optimize the entire multi-objective using method of either iterative or weight convergence, it is first classified into either periodical or permanent objectives. This is basically to give temporary objective less priority and permanent objective very high priority

b. Ranking

After classifying the objective, it further rank based on the time the objectives takes to exit the process before the process execution ends, then optimization is done in a way that objective which are classified as permanent are optimized first without any preference. And the one that are classified as temporary or periodical objective are then optimized later in the order with which they exit the process and the one that exit earliest will be optimized last such that removing will not affect or need to redesign the entire equation. See equation below (12) and (12) where $f_5(x)$ is removed and the previous weight W_2 automatical become the weight of the current equation without the present of the objective function $f_5(x)$.

$$W_3 \coloneqq \min\big(f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)\big) \tag{12}$$

The MOO (12) contains five objectives function which need to be minimized.

$$W_2 \coloneqq \min\big(f_1(x), f_2(x), f_3(x), f_4(x)\big) \tag{13}$$

The same applies to weight convergence method where permanent objectives are classified and optimized separately and the one which are temporary or periodical are optimized separately.

Then later they are finally optimized such that removing portion of the periodical objectives does not affect the entire solution weight (optimal weight point).

4. ANALYSIS OF THE CONCEPT.

The following analysis tries to shows the drawback and some advantages of this propose methodology compared to the currently existing methods

4.1 DRAWBACK

In most of the current methodologies presented by many scholars tries to solve all multi-objective problem simultaneously which when periodical objectives is added or remove will result in destabilizing the optimal weight point hence need to redesign the entire solution again. This isn't must problem, however during runtime it could be tiresome and the hardship to resolving the same thing time and again for example in equation given in (2) and (3) where objective is removed and the entire weight are not the same.

4.2 ADVANTAGE

The advantage of this solution presented here is that unlike simultaneous solution, partial objective optimization an objective can be remove safely without need to redesign the entire equation as the previous solution become the current solution without need to solve again.

Also addition of an objective does affect much, all is needed is to optimize that objective together with existing solution as explained above. However objective classification and ranking should be carefully done to ensure alignment before optimization.

5. CONCLUSION

Due to ultimatum in seeking the best optimal weight point among multi-objective optimization, this paper then discusses some notable published paper byscholars which presents the novel technique in tackling problems in multi-criteria optimization such as evolutionary algorithm methods, flower pollination algorithm and many more.

However, the author further notices some dare in the area of multi-objective optimization where there are situation when one or more objectives are needed either in the beginning or towards the ends or even in the middle of the process execution. The author called this as periodical or temporary objectives unlike usual objectives which are needed from the start up to the end of process execution. This is called permanent or long term objectives.

The problem with this is that every time an objective is added or removed from the running process with multiple objectives will results in destabilization of existing optimal weight point as shown in the above example above in (2) and (3).

In respond to the challenges, the author presented a theoretical concept which uses partial optimization iteratively and weight convergence including some methods which use objective classification, ranking, estimation and predictive measurement. These concepts are very convincing theoretically however their feasibility is far beyond the scope of this paper. Further in-depth studies will still be conducted to check the practical application and its feasibility in real world scenario including a numerical simulation or any analytical solution when conducted.

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