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Huge Filaments as Regions of Space-Time Deformation

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ABSTRACT

Huge filaments with scales from several hundred megaparsecs to gigaparsecs are detected in the distribution of galaxies and clusters, quasars, gamma-bursters. The hypothesis on the nature of the huge filaments as regions of space-time deformation is proposed. An anisotropic deformation of the local region is described by the strain tensor, it depends on the velocities of matter. Galaxies get an extra velocity in the region, which leads to the formation of filamentary structures. The class of exact solution of the GR equations is constructed by introducing the special definition of the Christoffel symbols as function of the velocity of matter. With a definition of these symbols, the motion matter equation turns into identity. For the sake of simplicity, an ideal fluid is considered.

Key words: cosmology, large scale structure, huge filaments, space-time deformation.

1. Introduction

It is popularly believed that the dominant structural elements of the large-scale structure are filaments – chains consisting of galaxies, groups of

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galaxies, galaxy clusters, intergalactic gas, and dust. Huge filaments with scales from several hundred megaparsecs to gigaparsecs are detected in the distribution of galaxies and clusters, quasars, gamma-bursters (Great Wall, Great GRB Wall, Hyperion, LQG) [1 – 5]. The sizes of these structures are several times greater than the maximum scale length for correlation function of galaxies in standard cosmological ΛCDM model (usually not more than 300 Mpc). The characteristic time of formation of the huge filaments from the initial density contrasts of matter is comparable to or even exceeds the characteristic lifetime of the Universe in the ΛCDM model.

The previous article [6] considered a filament model of primary scalar and vector perturbations of the metric tensor in homogeneous and isotropic cosmology in the framework of the General Theory of Relativity (GR).

This *Letter* proposes a hypothesis on the relationship of huge filaments with local space-time anisotropy. This anisotropy is described with the strain tensor of space-time. It is assumed that additional velocities of particles of matter correspond to the local anisotropy of space-time, and the strain tensor depends on the velocities of matter. Due to additional velocities, the convergence of world trajectories of matter particles can occur, i.e. the formation of matter flows occurs. Filaments of various scales may be the result of these flows. To describe this picture, a special definition of the Christoffel symbols of the 1st kind as function of the velocity of matter is introduced. With the chosen definition of these symbols, the motion matter equation turns into identity. The metric tensor is determined from the GR equations. For the sake of simplicity, an ideal fluid is considered. Note, that examples of using the strain tensor for the analysis of cosmological models with rotation are considered in the monograph [7].

The astronomer Eddington wrote that the Christoffel symbols are observable physical characteristics of the gravitational field (gravitational field strength) a hundred years ago [8]. In the physical sense, they are primary with respect to the metric tensor of space-time. Here, this Eddington's idea is implemented in the following logical chain: giant filaments are associated with the velocities of the matter flows – the matter flows are associated with the deformation of space-time - the Christoffel symbols are determined through the speeds of matter. We note that in the modern theory of gauge fields, the Christoffel symbols are determined from equations of the Yang - Mills type for these fields, and the metric tensor is determined from the GR equations.

2. The strain tensor of space-time

Consider the perfect fluid with the energy-momentum tensor $T_{ik} = \varepsilon u_i u_k$, where an energy density of fluid is ε, four-dimensional velocity is u_i, and $u_i u^i = 1$. The metric tensor is g_{ik}, the space-time interval is $ds^2 = g_{ik} dx^i dx^k$, the lateen indices run through the values 0, 1, 2, 3, the metric signature (+ - - -). Here, the physical system of units is maintained.

To describe the deformation of space-time, we will use the technique developed in the deformation theory of continuous matter. Suppose, due to deformation in the vicinity of the world point x^i a coordinate differential dx^i increases by a shift dy^i: $dx^i \to dx^i + dy^i = dx^i + \frac{\partial y^i}{\partial x^k} dx^k$. Then the space-time interval in the coordinate system x^i is $ds^2 = g_{ik}\left(dx^i + dy^i\right)\left(dx^k + dy^k\right) = \left(g_{ik} + 2Y_{ik}\right) dx^i dx^k$. Here, the strain tensor is introduced:

$$Y_{ik} = \frac{1}{2}\left(g_{mk}\frac{\partial y^m}{\partial x^i} + g_{mi}\frac{\partial y^m}{\partial x^k} + g_{mn}\frac{\partial y^m}{\partial x^i}\frac{\partial y^n}{\partial x^k} \right). \tag{1}$$

The local shift of world lines characterizes the vector $Y_{ik} dx^i$, and the velocity of this shift is $Y_{ik}\frac{dx^i}{ds} = Y_{ik} u^i$.

With isotropic deformation of space-time, we have $\frac{\partial y^i}{\partial x^k} = y\delta^i_k$, and $y = const$. In this case, the tensor (1) is the volumetric strain tensor $I_{ik} = \frac{1}{4} I g_{ik}$, here $I^i_i = I = 2y\left(2 + y\right)$ is the trace of tensor I_{ik}. The velocity of displacement of world lines due to an isotropic deformation is $V_k = I_{ik} u^i = \frac{1}{4} I u_k$. This velocity is parallel to the velocity u_k, therefore, the grid of coordinate lines does not change, only a compression or an expansion of the space volume occurs. In the case of the Friedmann-Lemaître-Robertson-Walker (FLRW) model, we have $ds^2 = \left(g_{ik} + \frac{1}{2} I g_{ik} \right) dx^i dx^k = \left(1 + \frac{1}{2} I \right) a^2 \eta_{ik} dx^i dx^k$, here Minkowski's tensor is

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η_{ik}, the scale factor is a. For the values of the scale factor at the moments of conformal time x^0 and x^0+dx^0 we have $2I_{ik}dx^i dx^k = ds^2\left(x^0+dx^0\right)-ds^2\left(x^0\right)$. Then

$$\frac{1}{2}Ig_{ik}=\left(a^2\left(x^0+dx^0\right)-a^2\left(x^0\right)\right)\eta_{ik}=2a\left(x^0\right)\frac{da}{dx^0}dx^0\left(\eta_{ik}\right)=2\left(\frac{da}{a}\right)g_{ik}. \quad (2)$$

It can be seen from formula (2) that the local change in the scale factor depends on the strain tensor trace, $\frac{da}{a}=\frac{1}{4}I$. During an isotropic expansion, the local Hubble parameter $H=\frac{1}{2}g^{ik}\frac{dg_{ik}}{dx^0}=\frac{1}{a}\frac{da}{dx^0}$ can be expressed in terms of the isotropic strain tensor $H=\frac{1}{2I_{ik}}\frac{dI_{ik}}{dx^0}$. Thus, the isotropic deformation of space-time of the FLRW model describes the isotropic expansion of space-time in the vicinity of any world point. In this case, there are no matter flows.

In the general case, the strain tensor Y_{ik} determines the local change of the metric tensor:

$$2Y_{ik}=g_{ik}\left(x^m+dy^m\right)-g_{ik}\left(x^m\right)=dg_{ik}, \quad (3)$$

and $Y=Y_i^i=\frac{1}{2}g^{ik}dg_{ik}$.

The anisotropic strain tensor is determined by the following formula:

$$D_{ik}=Y_{ik}-\frac{1}{4}Yg_{ik}. \quad (4)$$

The velocity of the relative displacement of the world lines of matter particles is calculated using the formula:

$$W_k=D_{ik}u^i. \quad (5)$$

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In the general case, the direction of velocity w_k does not coincide with the direction of velocity u_k, therefore, the world lines of particles (geodesic lines of space-time) are curved relative to world lines in space-time without anisotropic deformation. This leads, in particular, to the convergence of the world lines of particles in the direction of speed (6) and the formation of elongated structures.

3. Determination of the Christoffel symbols

Using direct calculations, we can see that the equation of continuity of matter

$$\frac{\partial\left(\varepsilon u^k\right)}{\partial x^k}+\Gamma^k_{nk}\varepsilon u^n=0, \qquad (6)$$

and the motion equation

$$\frac{\partial T^k_i}{\partial x^k}-\Gamma^n_{ik}T^k_n+\Gamma^k_{nk}T^k_i=0, \qquad (7)$$

become identities if the velocity u_i is a solution of the equation for a geodesic line in space-time, i.e. $u_{i,k}=\frac{\partial u_i}{\partial x^k}-\Gamma^n_{ik}u_n=0$. We determine the Christoffel symbols so that the geodesic line equation is identity:

$$\Gamma^n_{ik}=\gamma\frac{\partial u_i}{\partial x^k}\left(u^n+b^n\right), \qquad (8)$$

with $u_n b^n=\frac{1-\gamma}{\gamma}$ and b^n is a velocity, which is proportional to speed (5). According to definition (8), the coefficients Γ^n_{ik} are proportional to the acceleration of matter particles $\frac{du_i}{ds}=\frac{\partial u_i}{\partial x^k}u^k$.

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Note, that when we consider the homogeneous and isotropic cosmological models, the form of the metric tensor $g_{ik} = a^2 \eta_{ik}$ is specified at the first step. Then, the Christoffel symbols, the Ricci tensor, the velocity and the energy density are calculated as a function of the scale factor. The dependence of the scale factor on cosmological time is determined from the GR equations for the metric tensor. Thus, the scale factor plays the role of the geometric phase coordinate for space-time, and it is not directly measured.

In the proposed method, the Christoffel symbols (8) are specified as functions of velocity u_i, the Ricci tensor and the metric tensor are also calculated as functions of the velocity u_i. Thus, velocity u_i is the physical phase coordinate, which is a measurable quantity.

Consider the case of the potential motion of matter without any rotation and when the velocity u_i is a derivative of some scalar function $\varphi\left(x^i\right)$: $u_i = \dfrac{\partial \varphi}{\partial x^i}$.

Then we have an equation

$$\frac{\partial u_i}{\partial x^k} = \frac{\partial^2 \varphi}{\partial x^k \partial x^i} = \frac{\partial^2 \varphi}{\partial x^i \partial x^k} = \frac{\partial u_k}{\partial x^i} . \tag{9}$$

The equation (9) means that the vorticity is equal to zero, $r_{ik} = u_{i,k} - u_{k,i} = 0$, and the coefficients (8) are symmetric in the lower indices. For covariant conservation of definition (8), the covariant derivatives must be equal to zero: $u_{i,k} = 0$, $b_{i,k} = 0$, $\left(\dfrac{\partial u_i}{\partial x^k}\right)_{,m} = 0$. Then we find that

$$\frac{\partial^2 u_i}{\partial x^m \partial x^k} = \Gamma^n_{mk} \frac{\partial u_i}{\partial x^n} + \Gamma^n_{mi} \frac{\partial u_k}{\partial x^n} = \gamma \left(\frac{\partial u_m}{\partial x^k} \frac{\partial u_i}{\partial x^n} + \frac{\partial u_m}{\partial x^i} \frac{\partial u_k}{\partial x^n} \right) \left(u^n + b^n \right). \quad (10)$$

The derivatives of contravariant vectors u^i and b^i are calculated using the formulae:

$$\frac{\partial u^i}{\partial x^k} = \frac{\partial g^{im} u_m}{\partial x^k} = g^{im} \frac{\partial u_m}{\partial x^k} + u_m \left(-\Gamma^i_{nk} g^{nm} - \Gamma^m_{nk} g^{ni} \right) = -u^n \Gamma^i_{nk} = -\gamma \frac{\partial u_n}{\partial x^k} u^n \left(u^i + b^i \right), \quad (11)$$

$$\frac{\partial b^i}{\partial x^k} = -\gamma \frac{\partial u_n}{\partial x^k} u^n \left(u^i + b^i \right). \quad (12)$$

The Ricci tensor for the coefficients (8) is equal to:

$$R_{ik} = \gamma^2 \left(\frac{\partial u_i}{\partial x^m} \frac{\partial u_k}{\partial x^n} \left(u^m + b^m \right) \left(u^n + b^n \right) - \frac{\partial u_i}{\partial x^k} \frac{\partial u_m}{\partial x^n} \left(u^m b^n + u^n b^m + b^m b^n \right) \right). \quad (13)$$

It takes into account the equality $\frac{\partial u_n}{\partial x^m} u^m u^n = 0$ that holds for a velocity u^n and an acceleration $\frac{du_n}{ds} = \frac{\partial u_n}{\partial x^m} u^m$. The metric tensor is determined from the GR equations:

$$g_{ik} = 2 u_i u_k - \frac{2}{\kappa \varepsilon} R_{ik}, \quad (14)$$

where $\kappa = \frac{8\pi G}{c^4}$ is Einstein's constant. It follows from equation (14): $\kappa \varepsilon = 2 R_{ik} u^i u^k$.

All these formulae describe the anisotropy of space-time in terms of velocities u_i, b_i and shifts y^i. Therefore, the constructed solution allows you to simulate filaments with various geometric properties using the functions u_i, b_i and y^i. The choice of these functions must be made so that formula (8) does not contradict the dependence of the Christoffel symbols on the metric tensor:

$$\Gamma^m_{ik} = \frac{1}{2} g^{mn} \left(\frac{\partial g_{ni}}{\partial x^k} + \frac{\partial g_{nk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^n} \right).$$

Here is an example with fairly simple formulas. The total differential of shift is $dy^i = \delta^i_k dx^k + \Gamma^i_{km} y^m dx^k$, and $\frac{\partial y^i}{\partial x^k} = \delta^i_k + \Gamma^i_{km} y^m$. Let the vector y^i be chosen so that

$$\frac{\partial y^i}{\partial x^k} = \delta^i_k + \beta \left(u_k b^i + u^i b_k \right), \qquad (15)$$

where $\beta = const$. In this case, one can find for the strain tensor:

$$Y_{ik} = \frac{3}{2} g_{ik} + \frac{1}{2} \beta \left(4 + \beta u_m b^m \right) \left(u_i b_k + u_k b_i \right) + \frac{1}{2} \beta^2 \left(u_i u_k + b_i b_k \right), \qquad (16)$$

$$Y = 6 + \beta \left(4 + \beta u_m b^m \right) u_m b^m + \beta^2, \qquad (17)$$

$$D_{ik} = \frac{1}{2} \beta \left(4 + \beta u_m b^m \right) \left(u_i b_k + u_k b_i \right) + \frac{1}{2} \beta^2 \left(u_i u_k + b_i b_k \right), \qquad (18)$$

$$W_i = D_{ik} u^k = \beta \left(2 + \beta u_m b^m \right) b_i, \qquad (19)$$

with $\beta = -\frac{4 u_m b^m}{1 + \left(u_m b^m \right)^2}$. The velocity W_i is parallel to the velocity b_i. The geometry of the deformed region in space-time depends on the type of functions $u_i\left(x^k\right)$ and $b_i\left(x^k\right)$.

4. Conclusions

In this Letter, a hypothesis is proposed on the nature of huge filaments as regions of space-time deformation. An anisotropic deformation of the local region of space-time is described by the strain tensor, it depends on the velocities of matter. Galaxies get an extra velocity (5) in the region of deformation, which leads to the formation of filamentary structures. We constructed the exact solution of GR equations by introducing the special definition of the Christoffel symbols as function of the velocity of matter. With the chosen definition of these symbols, the motion matter equation turns into identity.

The local Hubble parameter in the deformation region $\tilde{H} = \frac{1}{2}\tilde{g}^{ik}\frac{d\tilde{g}_{ik}}{dx^0}$ for $\tilde{g}_{ik} = \left(1+\frac{1}{2}Y\right)g_{ik} + 2D_{ik}$ and the solution (15) is

$$\tilde{H} \approx \frac{1}{2g_{ik}}\frac{\partial g_{ik}}{\partial x^0}\left(1-2\frac{D_{ik}}{g_{ik}}\right) + \frac{1}{1+\frac{1}{2}Y}\frac{1}{g_{ik}}\frac{\partial D_{ik}}{\partial x^0} = H - \left(2H\frac{D_{ik}}{g_{ik}} - \frac{1}{1+\frac{1}{2}Y}\frac{1}{g_{ik}}\frac{\partial D_{ik}}{\partial x^0}\right) \tag{20}$$

with $\frac{D_{ik}}{\left(1+\frac{1}{2}Y\right)g_{ik}} \ll 1,\ Y = const$. From formula (20) it can be seen that if the metric tensor g_{ik} corresponds to a homogeneous and isotropic space-time of a distant observer, then such an observer will detect the difference between the Hubble parameter $\tilde{H}$ and the value $H = \frac{1}{2g_{ik}}\frac{\partial g_{ik}}{\partial x^0}$. Thus, the proposed model can be used to model the spatial variations of the Hubble constant [9].

References

1. S. Bharadwaj, et al., Astrophys. J. 606 (2004) 25.

2. I. K. Rozgacheva, et al., arXiv:1201.5554.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 May 2020 doi:10.20944/preprints202005.0320.v1

3. R. G. Clowes, et al., Mon. Not. R. Astron. Soc. 429 (2013) 2910.

4. S. I. Shirokov, et al., Vestnik SPbGU Ser. 1, (59), No. 4 (2014) 659.

5. I. Horváth et al., Astron. Astrophys. 561 (2014) L12.

6. I. K. Rozgacheva, et al., Astrophysics 63, No. 1 (2020) 56.

7. A. L. Zelmanov, V. G. Agakov, Elements of the General Theory of Relativity, Moscow, Nauka, Main Editorial Board for Physical and Mathematical Literature, 1988.

8. A. S. Eddington, Space, Time and Gravitation: An Outline of the General Relativity Theory, Cambridge University Press, 1920.

9. A. G. Riess, et al., Astrophys. J. 826 (2016) 56.