

Analysis of the COVID-19 pandemic spreading in India by an epidemiological model and fractional differential operator.

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Abstract

Fractional differential mathematical model unfolding the dynamics of the COVID-19 pandemic in India is presented and explored in this paper. The purpose of this study is to estimate the future outbreak of disease and potential control strategies using mathematical models in India as a whole country as well as in some of the states of the country. This model is calibrated based on reported cases of infections over the month of April 2020 in India. We have used iterative fractional complex transform method to find approximate solutions of the model having modified Riemann Liouville fractional differential operator. We have also carried out a comparative analysis between actual and estimated cumulative cases graphically, moreover, most sensitive parameters for basic reproduction number (R_0) are computed and their effect on transmission dynamics of COVID-19 pandemic is investigated in detail.

Keywords : compartmental model; COVID-19; modified Riemann Liouville fractional differential operator; basic reproduction number; numerical simulations.

2010 Mathematics Subject Classifications : Primary: 35A20, 35A22, 34A08, 35R11

1 Introduction

The COVID-19 pandemic has taken the globe by storm and caused a global awareness of the public health emergency [1]. It has already taken a toll in advanced countries

known for their health care infrastructure and accessibility. On December 31, 2019 in Wuhan city, Hubei Province of China, the coronavirus disease initially identified. This exceptionally infectious and lethal disease spreads fundamentally when individuals contact surfaces where the infection has been deposited by tainted individuals and contact their eyes, nose or mouth. Most of countries are counting actual reported cases rather than number of people have the virus because the symptoms of COVID-19 are mild and self-treated which misleads the count of disease evolution and forecasting the outbreak by general practitioner and hospital reports at early stage. India reported the first case of coronavirus late January, when a three students having a travel history from Wuhan, China landed in Kerala and tested positive. The disease transmission gradually increased over the month of March 2020, after a significant number of cases were accounted for in the country, a large portion of which were connected to individuals landed from influenced countries. Since then, India is focused on detecting Infections, referring identified patients for treatment to dedicated COVID-19 centers. As of 10 May 2020, India had seen 62,939 confirmed cases out of which 41472 are active cases, 19,358 recovered peoples and 2,109 deaths due to COVID-19 infections [2].

1.1 Mathematical Model

The Covid-19 pandemic has just begun demonstrating colossal negative effect on politics, education, socio-economics and other important worldwide aspects. Besides this, the condition of health related crisis is turning out to be increasingly more grim by each spending day. Hence, to analyze, forecast and examine the behaviour of viruses, threads, infections and others it is necessary to develop a mathematical model that effectively depicts transmission dynamics of the disease to assist policymakers with making significant choices dependent on the compelling presumptions given by the model. The benefit of simulations with the help of computational approaches of epidemic mathematical models is that they adjust dynamically and sensitive to various parameters, like behaviour of people, government strategies of disease control and many more which affect the model output and helps to analyze infections within populations.

In recent times, researchers around the globe have done remarkable studies about the transmission dynamics, forecasting future growth, control strategies and estimation for the infection of COVID-19. For example, In [3] Li et al. estimated the epidemic doubling time and the basic reproductive number. Khan et al. [4] formulated Mathematical model of coronavirus versus people. Asymptomatic carrier transmission of COVID-19 is analysed in [5]. In [6] chen et. al examined a mathematical model for simulating the phasebased transmissibility of a novel coronavirus. In [7], Lin et. al. suggested conceptual model for the coronavirus disease 2019, which successfully captures the course of the COVID-19 outbreak. More research pertaining to COVID-19 is found in ([8, 9, 10]).

Mathematical models, utilizing system of ordinary differential equations with integer-order, have been demonstrated significantly for understanding the dynamics

of biological systems. The study of epidemiological dynamical processes involving memory effects is appropriate on the grounds that such frameworks rely on strength of memory which is constrained by the order of fractional derivative operator. The purpose of formulating mathematical models using fractional differential equations is to improve and generalize several ordinary differential systems. Hence, demonstrating some real world phenomena using fractional derivative operator has attracted the consideration of many researchers in the field of applied sciences [11, 12, 13, 14, 15, 16]. In this study, we consider the compartmental model to analyse and simulate the transmission dynamics of the COVID-19 pandemic [18]. We use the total number of confirmed cumulative cases reported over the month of April 2020 as per the data collected through public health authorities announcements and Ministry of health and family welfare [2]. The total population is compartmentalized into eight components of the epidemic flow at time t given as $N = S + E + I + R + S_p + E_p + I_p + Q$, where,

- S is the number of susceptible individuals which are not traced.
- E denotes the latent but not traced population.
- I is the number of infectituous population which are not traced.
- R is the number of recovered (through self-isolation or treatment) population.
- S_p represent the uninfected but traced population which stay at home and take self isolation.
- E_p is the number of infectituous and traced individuals.
- I_p is the number of infected and quarantined population which are traced.
- D is the number of dead population.

The complete transformation process is depicted in Fig. 1.

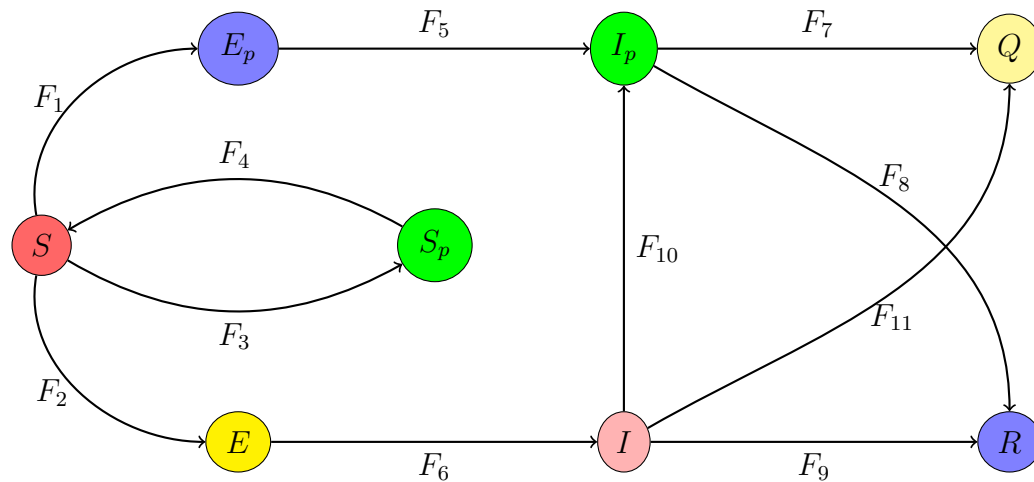


Figure 1: Transformation process of Model

Where $F_1 = \eta\alpha\mu S(I + \gamma E)$, $F_2 = (1 - \eta)\alpha\mu S(I + \gamma E)$, $F_3 = \eta(1 - \alpha)\mu S(I + \gamma E)$, Infected and traced $= F_4 = \eta\alpha\mu S(I + \gamma E)$, $F_5 = \tau E_p$, $F_6 = \tau E$, $F_7 = \rho\kappa I_p$, $F_8 = (1 - \rho)\kappa I_p$, $F_9 = (1 - \sigma)(1 - \phi)\delta I$, $F_{10} = \phi\delta I$, $F_{11} = \delta(1 - \phi)\delta I$

The contact rate, trace rate and probability of infection of are denoted by μ , η α respectively. Also the the proportion of infectious ability in latent individuals is denoted by γ . Then the variation of the susceptible S consists of three parts: the uninfected but traced F_3 , the infected and traced F_1 and the infected but not traced F_2 .

The infected but not traced move to the latent compartment E , while the infected and traced move to E_p , and the uninfected but traced move to S_p . The latent individuals no matter whether they were quarantined are becoming the infectious with the rate τ . The infectious but not quarantined individuals are sent to some hospital settings (i.e., they move to I_p) with the rate $\phi\delta$. The infectious but not quarantined individuals are dead with the rate $(1 - \phi)\sigma\delta$. The infectious and quarantined individuals are dead with the rate $\rho\kappa$. The infectious but not quarantined individuals recover with the rate $(1 - \phi)(1 - \sigma)\delta$. The infectious and quarantined individuals recover with the rate $(1 - \phi)\delta$.

Further, the infection process of coronavirus virus is displayed as a system of differential equations which describes the change in the population size of each compartment with respect to time t . Inspired by the above significant applications of fractional calculus in modeling of infectious diseases, we are simulating dynamics of COVID-19 model suggested by wang et.al.[18] in the form of modified Riemann-Liouville fractional differential system of equations of order β such that $\beta \in (0, 1]$. given by

$$\begin{cases} D_t^\beta S(t) = -\eta(1 - \alpha)\mu S(I + \gamma E) - \eta\alpha\mu S(I + \gamma E) - (1 - \eta)\alpha\mu S(I + \gamma E) + \psi S_p, \\ D_t^\beta E(t) = (1 - \eta)\alpha\mu S(I + \gamma E) - \tau E, \\ D_t^\beta I(t) = \tau E - (1 - \phi)(1 - \sigma)\delta I - (1 - \phi)\sigma\delta I - \phi\delta I, \\ D_t^\beta R(t) = (1 - \phi)(1 - \sigma)\delta I + (1 - \rho)\delta I_p, \\ D_t^\beta S_p(t) = \eta(1 - \alpha)\mu S(I + \gamma E) - \psi S_p, \\ D_t^\beta E_p(t) = \eta\alpha\mu S(I + \gamma E) - \tau E_p, \\ D_t^\beta I_p(t) = \tau E_p + \phi\delta I - (1 - \rho)\kappa I_p - \rho\kappa I_p, \\ D_t^\beta Q(t) = (1 - \phi)\sigma\delta I, \end{cases} \quad (1.1)$$

with initial conditions

$$\begin{aligned} S(0) = S_0 > 0, \quad E(0) = E_0 > 0, \quad I(0) = I_0 > 0, \quad R(0) = R_0 > 0, \\ S_p(0) = S_{p_0} > 0, \quad E_p(0) = E_{p_0} > 0, \quad I_p(0) = I_{p_0} > 0, \quad D(0) = D_0 > 0. \end{aligned} \quad (1.2)$$

The rest of this paper is sorted out as follows. In Section 2, we have discussed the iterative fractional complex method by presenting preliminary definitions. In Section 3, Simulations result and forecast of the COVID-19 pandemic in India as a whole and some of the states along with plots and tables is presented. Further, various control strategies and prevetive measures predicted by using plots is discussed in section 4. Section 5 is about conclusions.

2 Analysis of iterative fractional complex transform method

2.1 Preliminaries

In this section we present some required definitions of fractional derivative operators. Further, we describe one of the reliable and efficient techniques to find approximate solutions of fractional differential equations known as iterative fractional complex transform method. This method is a combination of fractional complex transform and new iterative method.

Definition 2.1. The Riemann-Liouville fractional integral of order β is defined as

$$I_t^\beta u(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t - \zeta)^{\beta-1} u(\zeta) d\zeta,$$

where, $\beta \in (-\infty, \infty)$.

Definition 2.1. The Caputo fractional derivative operator of order $\beta (\beta \geq 0)$ and $n \in \mathbb{N} \cup \{0\}$ is defined as

$$D_t^\beta(u(t)) = \frac{1}{\Gamma(n - \beta)} \int_0^t (t - \zeta)^{n-\beta-1} \frac{d^n}{dt^n} u(\zeta) d\zeta, \quad (2.1)$$

where $n - 1 \leq \beta < n$.

While Riemann–Liouville definition of derivative is given as:

$$D_t^\beta(u(t)) = \frac{1}{\Gamma(n - \beta)} \frac{d^n}{dt^n} \int_0^t (t - \zeta)^{n-\beta-1} u(\zeta) d\zeta, \quad (2.2)$$

where $n - 1 \leq \beta < n$, $n \in \mathbb{N} \cup \{0\}$.

Recently, G. Jumarie [19] suggested an alternate definition to the Riemann-Liouville derivative which is known as modified Riemann-Liouville derivative. It has advantages over previously defined operators and is denoted by the expression.

$$D_t^\beta(u(t)) = \frac{1}{\Gamma(n - \beta)} \frac{d^n}{d\zeta^n} \int_0^t (t - \zeta)^{n-\beta-1} [u(\zeta) - u(0)] d\zeta, \quad (2.3)$$

it is defined over continuous but not necessarily differentiable functions.

2.2 Fractional complex transform

Fractional complex transform which is suggested by Li and He [20] is highly reliable method to transform the differential equations in non integer form into ordinary differential equations. Hence, analytical techniques dedicated to the advanced calculus

can be effectively applied to the fractional calculus.

Here we begin with the fractional differential equation given by

$$f(u, u_t^{(\alpha)}, u_t^{(2\alpha)}, \dots) = 0, \quad (2.4)$$

where $u_t^{(\alpha)} = \frac{d^\alpha u(t)}{dt^\alpha}$, $0 < \alpha \leq 1$ denotes modified Riemann–Liouville derivatives.

Introducing the fractional complex transform given as

$$T = \frac{qt^\alpha}{\Gamma(1 + \alpha)},$$

where q is unknown constant. Using the basic properties of the fractional derivative and the above transforms, we can convert fractional derivatives into ordinary derivatives as:

$$\frac{d^\alpha u}{dt^\alpha} = q \frac{du}{dT}. \quad (2.5)$$

Therefore, the system of fractional differential equations are effectively transforms into system of ordinary differential equations. This system further solved by new iterative method.

2.3 New Iterative Method for a System of Equations

In recent years, Daftardar-Gejji and Jafari [13] introduced very simple and efficient method to solve linear and nonlinear differential equation known as New iterative method (NIM). This technique is a improvements of terms of Adomian Decomposition Method so that it is easily implemented on computer with the help of symbolic computational packages like Mathematica.

Consider a system of equation

$$u_i(t) = f_i + L_i(u_1(t), u_2(t), \dots, u_n(t)) + N_i(u_1(t), u_2(t), \dots, u_n(t)), \quad i = 1, 2, \dots, n. \quad (2.6)$$

where f_i is a known function, L_i is linear and N_i non-linear operator.

Let $u = (u_1(t), u_2(t), \dots, u_n(t))$ be a solution of system Eq.2.6 where $u_i(t)$ is of the series form given as:

$$u_i(t) = \sum_{j=0}^{\infty} u_{i,j}(t), \quad i = 1, 2, \dots, n. \quad (2.7)$$

Since L is linear,

$$L_i \left(\sum_{j=0}^{\infty} u_{i,j}(t) \right) = \sum_{j=0}^{\infty} L_i(u_{i,j}(t)). \quad (2.8)$$

The operator N_i is decomposed as :

$$N_i(u) = N_i \left(\sum_{j=0}^{\infty} u_{i,j}(t) \right) = N_i \left(\sum_{j=0}^{\infty} u_{1,j}(t), \dots, \sum_{j=0}^{\infty} u_{n,j}(t) \right)$$

$$= N_i(u_{1,0}(t), \dots, u_{n,0}(t)) + \sum_{k=1}^{\infty} \left\{ N_i \left(\sum_{j=0}^k u_{1,j}(x, t), \dots, \sum_{j=0}^k u_{n,j}(t) \right) - N_i \left(\sum_{j=0}^{k-1} u_{1,j}(t), \dots, \sum_{j=0}^{k-1} u_{n,j}(t) \right) \right\} \quad (2.9)$$

In view of Eq.2.7, 2.8 and 2.9, the Eq.2.6 is equivalent to

$$\sum_{j=0}^{\infty} u_{i,j}(t) = f_i + \sum_{j=0}^{\infty} L_i(u_{i,j}(t)) + N_i(u_{1,0}(t), \dots, u_{n,0}(t)) + \sum_{k=1}^{\infty} \left\{ N_i \left(\sum_{j=0}^k u_{1,j}(t), \dots, \sum_{j=0}^k u_{n,j}(t) \right) - N_i \left(\sum_{j=0}^{k-1} u_{1,j}(t), \dots, \sum_{j=0}^{k-1} u_{n,j}(t) \right) \right\} \quad (2.10)$$

where $i = 1, 2, \dots, n$.

Further consider the recurrence relation as given below for $i = 1, 2, \dots, n$

$$\begin{aligned} u_{i,0} &= f_i \\ u_{i,1} &= L_i(u_{1,0}(t), \dots, u_{n,0}(t)) + N_i(u_{1,0}(t), \dots, u_{n,0}(t)) \\ u_{i,m+1} &= \sum_{j=1}^m L_i(u_{i,j}(t)) + N_i \left(\sum_{j=0}^m u_{1,j}(t), \dots, \sum_{j=0}^m u_{n,j}(t) \right) - \\ &\quad - N_i \left(\sum_{j=0}^{m-1} u_{1,j}(t), \dots, \sum_{j=0}^{m-1} u_{n,j}(t) \right) \quad m = 1, 2, \dots \end{aligned} \quad (2.11)$$

The k-term series solution is given as

$$u_i = u_{i,0} + u_{i,1} + u_{i,2} + \dots + u_{i,k-1} \quad (2.12)$$

In [21] the detail criteria of convergence of the series $\sum u_{i,j}$ is given.

3 Simulations and Forecast of the COVID-19 pandemic in INDIA

In this section, we have presented data fitting, numerical simulations and graphical demonstration of the Caputo COVID-19 model (1.1) for the population of India and some states in it which contributes highest infected cases to total infected cases of India. These states includes Maharashtra Gujrat and Capital Delhi with several important parameters which will be best fitted or estimated based on a variety of factors such as timing of the first contamination (earlier states were caught unprepared), to population density and social dynamics, to timing and efficiency of state-wide mandated directives for travel bans and social distancing, to timing and availability of testing. The parameters were estimated based on the some assumptions and facts such as the Mean incubation period (τ) is set to 5.2 days, quarantined rate of the

infectious people is ($\phi = 0.0952$), we set death rate for the quarantined and infectious population as $\rho = 0.00161$ and death rate for the not quarantined and infectious population as $\sigma = 0.00392$ induced because of COVID-19. Further, as per the from 30th March to 24th April 2020 available on websites of public health authorities of respective states [2, 22, 23, 24] the removal or recovery rate of the infected as well as quarantined (κ) and infected but not quarantined (δ) is estimated as 0.08 and 0.066 respectively. The period of quarantined (home or institutional or hospital) is 14 days, hence we set release rate of traced and uninfected population as $\psi = 0.0714$. The other parameters plays significant role in estimating R_0 (basic reproductive number) for different states or cities depend upon a various factors is given in tables 1 and 2. Using the concepts of next generation matrix and calculation about reproduction number presented in [25] the basic reproduction number (R_0) for model 1.1 is defined by $R_0 = (1-\eta)\alpha\mu\left(\frac{\gamma}{\tau} + \frac{1}{\delta}\right)$. The epidemiology, R_0 is expected number of cases directly generated by one individual in a population. When $R_0 > 1$ the infection will be able to start spreading in a population, but if $R_0 < 1$ the disease will die out.

Parameters	Value			
	India	Maharashtra	Gujrat	Delhi
Contact rate μ	0.000000022	0.000000030	0.000000017	0.000000031
Traced rate η	0.381	0.0181	0.0181	0.0581
Transmission rate α	0.0123	0.0523	0.021	0.023
Infectious rate of latent γ	0.0914	0.914	0.0914	0.0914
R_0	3.59854	3.73882	3.77176	3.37776

Table 1: Estimated value of parameters for COVID-19 model (1.1) with high R_0

Parameters	Value			
	India	Maharashtra	Gujrat	Delhi
Contact rate μ	0.000000012	0.000000040	0.000000011	0.000000029
Traced rate η	0.681	0.0281	0.020	0.181
Transmission rate α	0.0123	0.023	0.012	0.013
Infectious rate of latent γ	0.0914	0.914	0.0914	0.0914
R_0	1.01155	2.16997	1.30996	1.46153

Table 2: Estimated value of parameters for COVID-19 model (1.1) with low R_0

Next, we calculate and demonstrate the number of infectious cases using various plots for different parts of India. We have analyse and simulated only those regions which contributes maximum number of infected cases in total number of cases in India as on 12th April 2020 [2, 26].

We start our model from initial time ($t = t_0$) as per data reported on 30th March 2020. Hence, the required initial values are

$$S(0) = 1376420550, \quad E(0) = 171520, \quad I(0) = 200, \quad R(0) = 50, \quad S_p(0) = 25000, \quad E_p(0) = 1200, \quad I_p(0) = 645, \quad Q(0) = 10.$$

We have not considered the birth and death rate for the total population. By applying iterative fractional complex transform using (2.5) and (2.11) successively upto four

terms we get series form approximate solution of the fractional COVID-19 model (1.1) as given below

$$\begin{aligned}
S(t) &= 1376420550 - \frac{171814.t^{2\beta}}{\Gamma(\beta+1)^2} + \frac{17043.2t^{3\beta}}{\Gamma(\beta+1)^3} + \frac{9.61362t^{4\beta}}{\Gamma(\beta+1)^4} - \frac{0.979314t^{5\beta}}{\Gamma(\beta+1)^5} \\
&\quad + \frac{0.0000688057t^{6\beta}}{\Gamma(\beta+1)^6} + \frac{5.580619712441739*10t^{7\beta}}{\Gamma(\beta+1)^7} - \frac{185050.t^\beta}{\Gamma(\beta+1)} \\
E(t) &= 171520 + \frac{6310.86t^{2\beta}}{\Gamma(\beta+1)^2} - \frac{660.95t^{3\beta}}{\Gamma(\beta+1)^3} - \frac{0.178714t^{4\beta}}{\Gamma(\beta+1)^4} + \frac{0.0191867t^{5\beta}}{\Gamma(\beta+1)^5} \\
&\quad - \frac{1.34804 \times 10^{-6}t^{6\beta}}{\Gamma(\beta+1)^6} - \frac{1.09335 \times 10^{-11}t^{7\beta}}{\Gamma(\beta+1)^7} - \frac{29322.8t^\beta}{\Gamma(\beta+1)} \\
I(t) &= 200 - \frac{3907.4t^{2\beta}}{\Gamma(\beta+1)^2} + \frac{490.489t^{3\beta}}{\Gamma(\beta+1)^3} - \frac{0.0150454t^{4\beta}}{\Gamma(\beta+1)^4} + \frac{32970.1t^\beta}{\Gamma(\beta+1)} \\
R(t) &= 50 + \frac{970.065t^{2\beta}}{\Gamma(\beta+1)^2} - \frac{67.9354t^{3\beta}}{\Gamma(\beta+1)^3} + \frac{54.398t^\beta}{\Gamma(\beta+1)} \\
S_p(t) &= 25000 + \frac{166174.t^{2\beta}}{\Gamma(\beta+1)^2} - \frac{16628.9t^{3\beta}}{\Gamma(\beta+1)^3} - \frac{9.3006t^{4\beta}}{\Gamma(\beta+1)^4} + \frac{0.948318t^{5\beta}}{\Gamma(\beta+1)^5} \\
&\quad - \frac{0.0000666279t^{6\beta}}{\Gamma(\beta+1)^6} - \frac{5.40398769 \times 10^{-10}t^{7\beta}}{\Gamma(\beta+1)^7} + \frac{179137.t^\beta}{\Gamma(\beta+1)} \\
E_p(t) &= 1200 + \frac{1954.59t^{2\beta}}{\Gamma(\beta+1)^2} - \frac{283.12t^{3\beta}}{\Gamma(\beta+1)^3} - \frac{0.11t^{4\beta}}{\Gamma(\beta+1)^4} + \frac{0.0118096t^{5\beta}}{\Gamma(\beta+1)^5} \\
&\quad - \frac{8.2972939 \times 10^{-7}t^{6\beta}}{\Gamma(\beta+1)^6} - \frac{6.729679 \times 10^{-12}t^{7\beta}}{\Gamma(\beta+1)^7} + \frac{2022.29t^\beta}{\Gamma(\beta+1)} \\
I_p(t) &= 645 - \frac{319.008t^\beta}{\Gamma(\beta+1)} + \frac{434.286t^{2\beta}}{\Gamma(\beta+1)^2} - \frac{6.56525t^{3\beta}}{\Gamma(\beta+1)^3} - \frac{0.0092606t^{4\beta}}{\Gamma(\beta+1)^4} \\
Q(t) &= 10 + \frac{3.6396t^{2\beta}}{\Gamma(\beta+1)^2} - \frac{0.105785t^{3\beta}}{\Gamma(\beta+1)^3} + \frac{0.933968t^\beta}{\Gamma(\beta+1)}
\end{aligned}$$

Figures 2a and 2b shows dynamical behavior of infected and quarantined population $I_p(t)$ with $R_0 = 3.60$ and $R_0 = 1.02$ respectively for various values of $\beta = 1, 0.9, 0.8, 0.7$ with respect to time(days). We clearly see the major difference in both plots when value of fractional parameter β decreases the number of infected cases also decreases after reaching to its peak in less time. Also lowering the value of R_0 near to one reduces the number of infected cases significantly. Also in figures 3a and 3b we examine the possible impact of enhanced interventions on COVID-19 infection at varying disease transmission rate per contact ($\alpha = 0.0123, 0.0129, 0.0133$) and traced rate ($\eta = 0.0351, 0.0361, 0.0381, 0.0391$) respectively for $\beta = 1$. It is observed that reducing contact rate persistently decreases the peak value of the infected population and further enforce delay in the peak value. We also observe that this flattens the curve of infection.

Meanwhile, increasing the trace rate of susceptible people leads to increase in the peak value and also further delay in the peak, as shown in Figure 3b. Next, Increasing

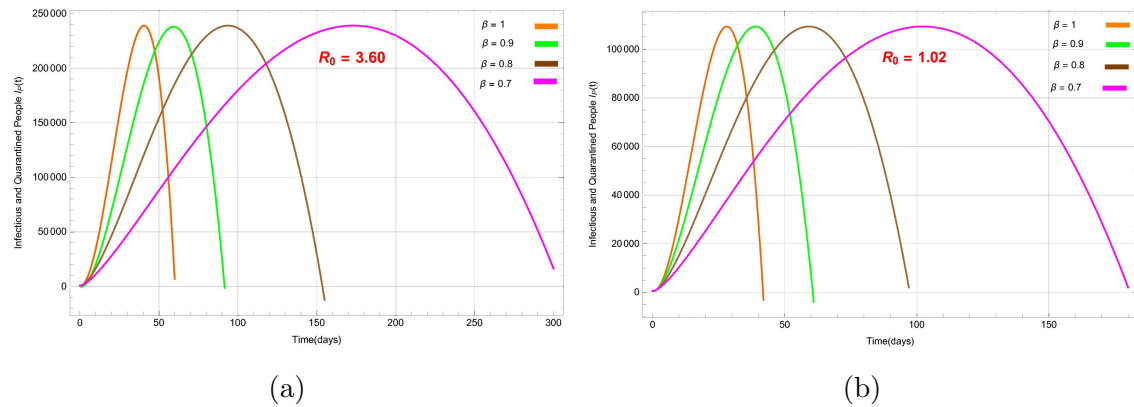


Figure 2: Dynamical behavior of infected and quarantined population $I_p(t)$ with different R_0 for various values of β with respect to time(days)

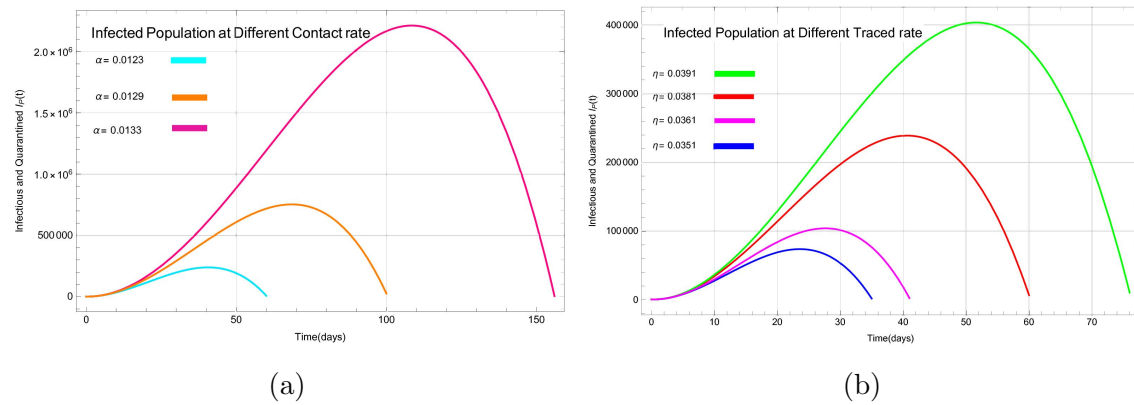


Figure 3: Dynamical behavior of infected and quarantined population $I_p(t)$ at various values of a) contact rate (α) and b) traced rate (η) with respect to time(days)

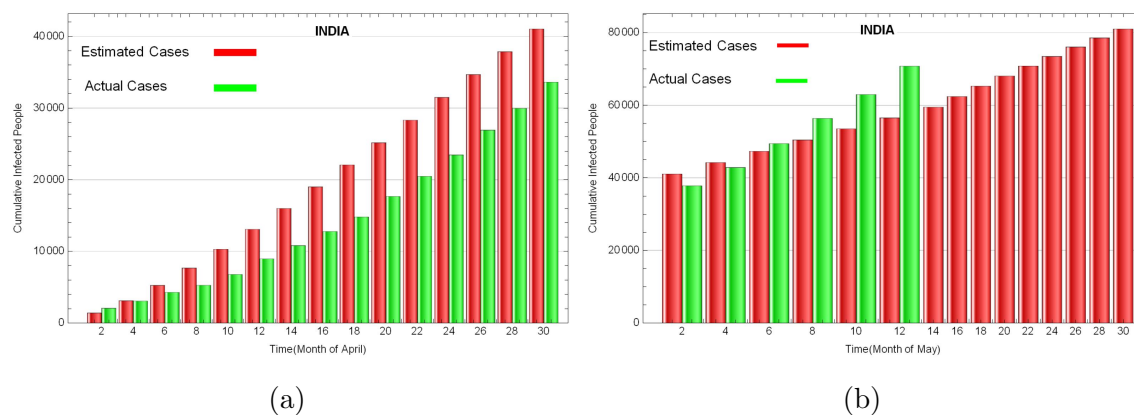


Figure 4: Cumulative number of cases of infected and quarantined population $I_p(t)$ with respect to time(days)

trace rate the infected cases also increases continuously. Figures 3a and 3b shows estimated and actual cumulative cases starting from 30th March 2020 till 12th May

2020. Moreover we have estimated cases for the month of May 2020.

3.1 Forecast of the COVID-19 pandemic in MAHARASHTRA

We consider the data reported on 30th March 2020 as initial time ($t = t_0$). Hence, the required initial values are

$S(0) = 121924973$, $E(0) = 11520$, $I(0) = 400$, $R(0) = 39$, $S_p(0) = 10000$, $E_p(0) = 800$, $I_p(0) = 200$, $Q(0) = 10$. By applying iterative fractional complex transform using (2.5) and (2.11) successively upto four terms we get series form approximate solution of the fractional COVID-19 model (1.1) and we get the following graphical results

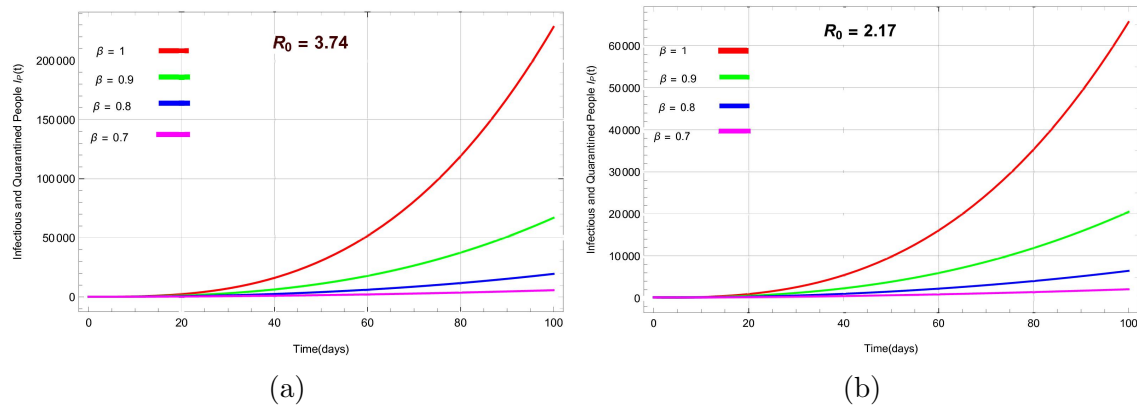


Figure 5: Dynamical behavior of infected and quarantined population $I_p(t)$ with different R_0 for various values of β with respect to time(days)

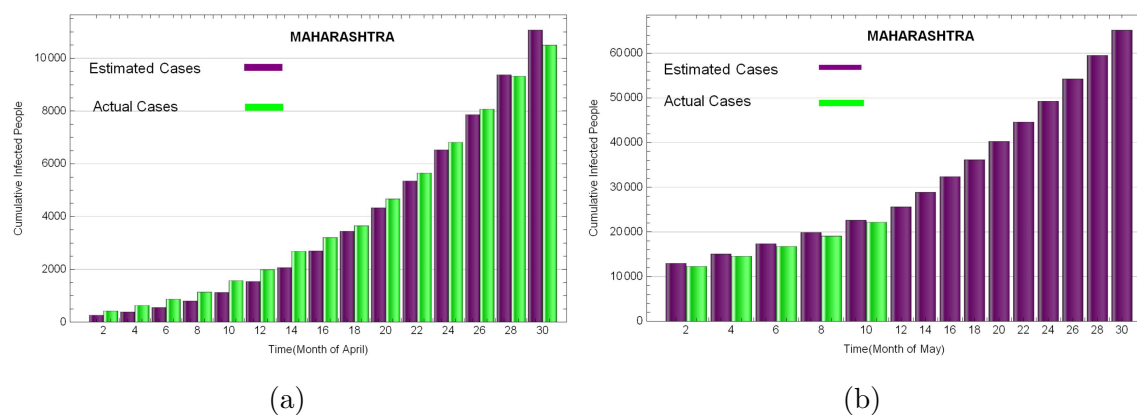


Figure 6: Cumulative number of cases of infected and quarantined population $I_p(t)$ with respect to time(days)

3.2 Forecast of the COVID-19 pandemic in GUJRAT

As per the above discussion and the data reported on 30th March 2020 at initial time ($t = t_0$) the required initial values are

$S(0) = 64801901$, $E(0) = 9100$, $I(0) = 100$, $R(0) = 6$, $S_p(0) = 10000$, $E_p(0) = 306$, $I_p(0) = 70$, $Q(0) = 2$. Also, we get the following graphical results by applying iterative fractional complex transform successively

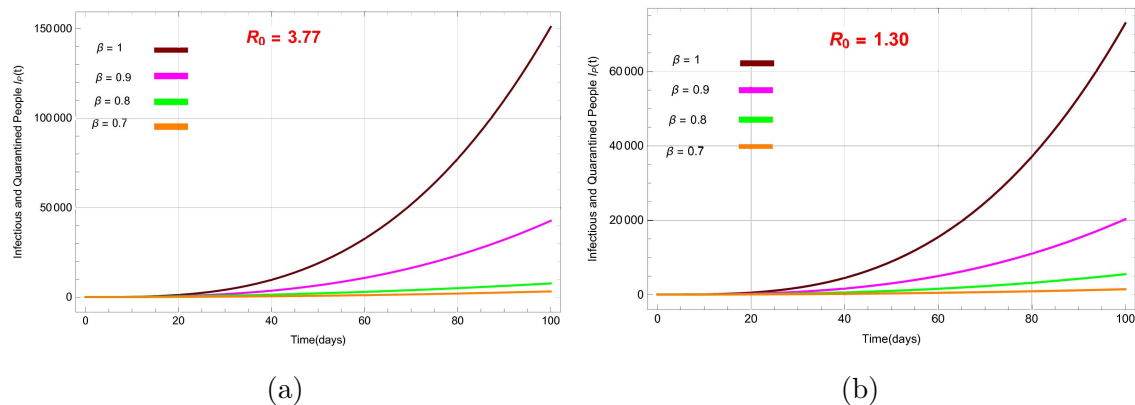


Figure 7: Dynamical behavior of infected and quarantined population $I_p(t)$ with different R_0 for various values of β with respect to time(days)

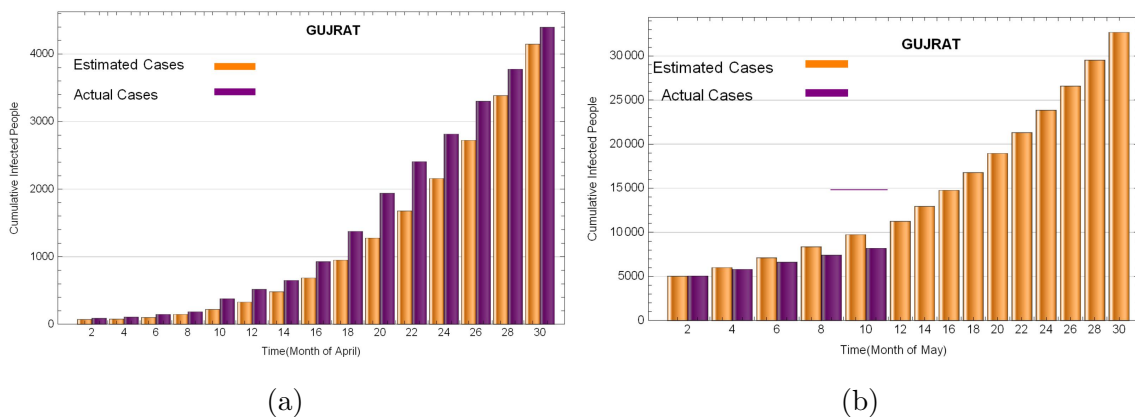


Figure 8: Cumulative number of cases of infected and quarantined population $I_p(t)$ with respect to time(days)

3.3 Forecast of the COVID-19 pandemic in DELHI

As per the above discussion and the data reported on 30th March 2020 at initial time ($t = t_0$) the required initial values are

$S(0) = 30290936$, $E(0) = 12100$, $I(0) = 100$, $R(0) = 6$, $S_p(0) = 10000$, $E_p(0) = 600$, $I_p(0) = 152$, $Q(0) = 2$. Also, we get the following graphical results by applying iterative fractional complex transform successively

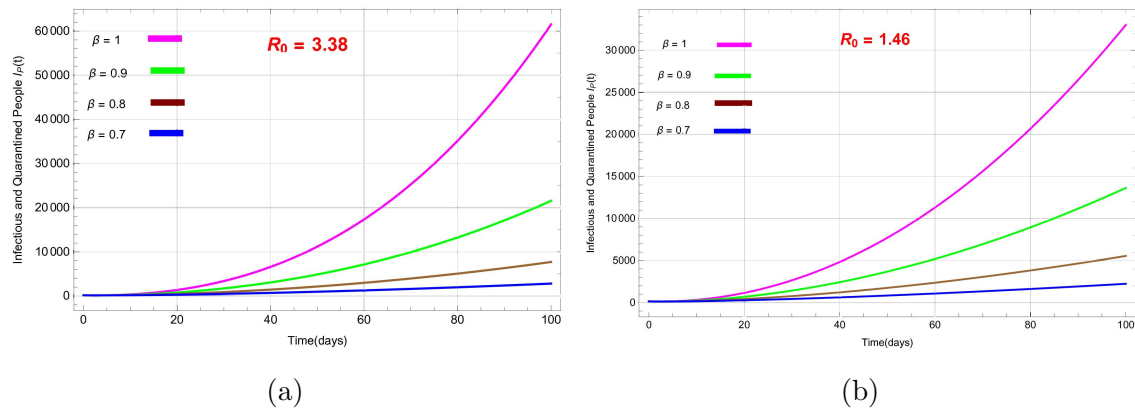


Figure 9: Dynamical behavior of infected and quarantined population $I_p(t)$ with different R_0 for various values of β with respect to time(days)

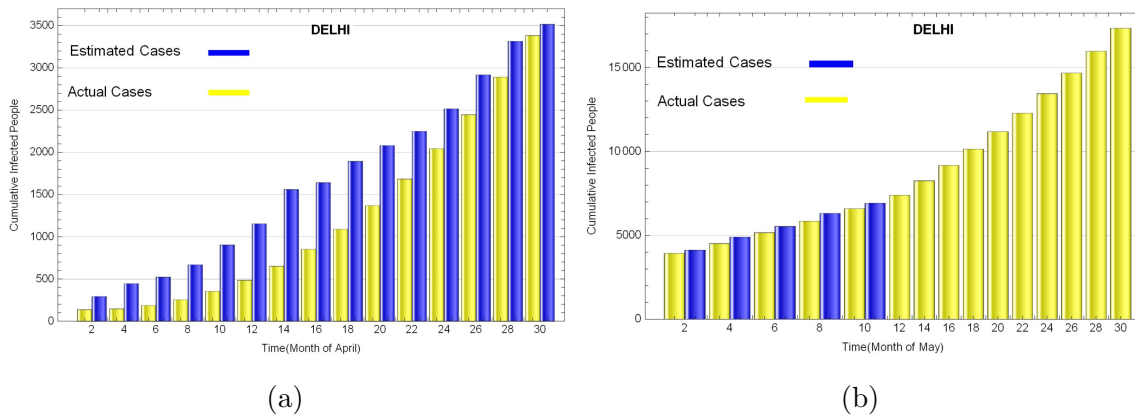


Figure 10: Cumulative number of cases of infected and quarantined population $I_p(t)$ with respect to time(days)

4 Discussion

This model of the COVID-19 pandemic for India incorporates some key features of this pandemic which includes 1) the planning of execution of the significant government policies about restrictions intended to mitigate the seriousness of the pandemic. 2) the importance of quarantined, infectituous but not traced and latent but not traced population in estimating the number of cumulative reported cases. In this model the various parameters are estimated by considering effects of several control measures like complete nationwide lockdown from 25th March 2020 and further extension of it, social distancing and restricting population movement implemented by government of India. Since the daily reported cases could contain more commotion during the beginning time, we utilize the total cumulative as compared to the daily frequency to make the simulations which is the limitation of our estimation in this work. In tables 1 and 2 we have estimated some important parameter values like contact rate μ , traced rate η , transmission rate α and infectious rate of latent γ are estimated with different R_0 for various regions. From figures 5a, 5b, 7a, 7b, 9a and 9b it

is observed that if $R_0 > 3$ the number infected population increases very rapidly whereas for $1 < R_0 < 2$ rate of increase is quite low. The basic reproduction number R_0 is depends upon most sensitive parameters like contact rate μ , transmission rate α and traced rate η . Estimating or fitting these parameters to appropriate values reduces rate of infection significantly.

Figures 6a, 6b, 8a,8b, 10a and 10b shows the comparison between actual and estimated number of cumulative cases of infected and quarntined population for Maharashtra state, Gujrat state and Delhi respectively. These figures also includes The bars in plots shows that there estimated cases are very nera to actual cases on a particular day. As per our estimation from this model total number of cumulative cases of infected population might go upto 100000 in India, 60000 in Maharshttra, 30000 in Gujrat and 15000 in Delhi till the end of May 2020. It is noted that there is the non-integer order time derivative affects the dynamics of the pandemic, moreover there is a striking difference at various values of time fractional parameter β on which the present COVID-19 model depend persistently. Moreover, these figures indicates that interventions to reduce transmission rate come first in control of the outbreak followed by traced rate.

5 Conclusions

In this manuscript, we have investigated the appropriate fractional-order COVID-19 model which demonstrate the current scenario and future estimate of infected cases by using iterative fractional complex transform method. The effectiveness of this technique can be drastically enhanced by reducing steps and computing more components. The control measures such as quarantine startergies at home or institutional level or at hospitals, tracing infected individuals at early stage and treatment along with the policy of reducing the transmission rate can help to minimize the growth of daily infected cases of the virus. In the future to forecast conceivable behaviors of the present dynamical system, simulations with several possible fitting of parameter values can be implemented. We have analysed and estimated COVID-19 outbreak in India using this model, but it is applicable to predict and analyse transmission dynamics of any part of the world affected by COVID-19 pandemic.

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