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A Complementary Micro Coronavirus Model Under Uncertainty and Utility

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ABSTRACT

A micro decision-making utility model under uncertainty is presented as a complementary foundation for macro coronavirus models. The micro model consists of two functions, a risk averse utility function depending on wellness and a wellness random output which is a function of the input variable called "treatment" consisting of such elements as social distance, washing hands, wearing a face mask, and others. The decision maker selects a level of treatment that maximizes her/his expected utility, given the probabilities of the respective outputs. The focus is on how changes in a person's psychological attitude towards the macro determined (announced) probabilities affects the optimum results of the model. Such changes create a micro-macro dynamic interaction which is briefly outlined. A short discussion of the model's behavioral implications for health policy is also given.

Key Words: Utility, Uncertainty, Risk Averse, Wellness Output, Treatment Inputs, Coronavirus, Psychological Risk Attitude, and Dynamic Interactions.

JEL Classification: C02, C61, D11, D20, D81, D84, D91, I1, and I18.

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1. Introduction

As near as we can determine, most of the Coronavirus models are designed by epidemiologists and fall under the category of statistical forecasting models (for example, the Murray COVID-19 Model, 2020). The Murray Model uses a normal (Gaussian) probability function for the rate of spread of the virus with integration limits fixed from zero to x(t, α, β, p), a collection of parameters. From this the forecasted cumulative death rate is obtained, subject to the three parameters in x(.). The assumption behind the empirical value of these parameters is based on the Wuhan, China data.

Our model uses the microeconomic theory of utility under uncertainty to build a complementary micro foundation to the macro models. The individual is the decision maker in the model and is assumed to be risk averse. The decision involves selecting a level of treatment (physical inputs like social distance, washing hands, and others) that maximizes his expected utility. The expected utility is a function of the expected random output or wellness (in terms of measurable elements yet to be determined) that is dependent on the level of treatment. The focus of the paper is on the micro behavior of the model and how the results from the behavior are affected by changes in a person's psychological attitude about the probabilities used in the model by the individual. The changes set up a dynamic interaction between the micro and macro behaviors. The dynamics will be discussed shortly.

Before proceeding with the development of our complementary model, a brief description for comparison is given about another related but complementary utility model called

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prospect theory developed by Kahneman and Tversky (1979) with advances in terms of several prospects and non-monetary outcomes by Tversky and Kahneman (1992). Using a simple example, prospect theory involves the choice a person may make between (say) prospect A with random outcomes x and y and corresponding utilities, U(x) and U(y), with probabilities p and (1-p) and prospect B with a fixed and certain outcome, z, where ($x > z > y$). Our complementary model outlined above has only one prospect with random outcomes from a given level of treatment and corresponding utilities and probabilities.

Other relevant literature to our paper is the Platt and Huettel (2008) paper on different aspects of uncertainty (e.g., investment choices, allocation of effort, and attitudes towards uncertainty) and how they affect decision making. Another useful paper is a review of the literature on the psychology of decision making under uncertainty prepared for the United States Agency for International Development (USAID) by Pasquini, Steynor, and Waagsaether (2019). While the review is directly related to the science of climate change and uncertainty, its general topics dealing with psychology, individual decision making, factors affecting decision making, and strategies to deal with uncertainty are relevant to our paper's topic. More to our topic is the paper by Tversky and Kahneman (1971) dealing with decision making under risk. It will be discussed later in the relevant places in the paper.

In what follows, the next section gives a verbal intuitive description of the model. The next section contains a mathematical description of the model, its behavior, and an analysis of the risk premium the decision maker is willing to cope with. The last section has a summary and conclusions.

2. A Verbal Intuitive Description of the Micro Model

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The typical individual in a geographic environment infested with the coronavirus is assumed to select an optimum level of treatment (T*) that maximizes his expected utility EU(W) of wellness (W(T)) which is assumed to be an increasing function of T, at an increasing rate (or marginal). For the sake of simplification two random wellness functions are used, one with a high (W_h) and one with a low (W_l) with probabilities ṕ = βp and (1 - ṕ), respectively. The β < > 1 is some index of the persons psychological degree of belief in the officially announced p. Using two random functions makes it easier to speculate about the value of β and its effect on the level of the optimum treatment, T*. For example, for a β > 1, Tversky and Kahneman (1971) argue that (financial) decision makers tend to overestimate the probability of success. When it comes to health issues, this may also be the case, so β > 1. The macro consequence of this outcome will be discussed shortly. The use of β allows for a comparative-static analysis, when expectations differ from reality.

As indicated earlier, the utility function U(W) is increasing in W with diminishing marginal utility, dU(W)/dW > 0 and falling with T for a risk averse individual. Given the perceived probability, ṕ, the person maximizes his expected utility, EU(W(T*)) = EU(T*) when dEU(W(T*))/dT = 0. The optimization process results, ex ante, in a total risk premium and a marginal risk premium. These risk concepts can be interpreted as a "total cost" and a "marginal cost" that are covered by the person's expected wellness, E(W(T*)) and expected marginal wellness, EMW(T*), at the optimum level of treatment, T*. More on these risks later.

One interesting outcome of the model occurs when the actual results, ex post, are not what the person expected. If the results are better than expected (the high W_h occurs), then there is no regret. If the results are worse than expected, the person may play the "gambler's

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game" and try again. Or, the person may rethink his β and decide that β and thus ṕ were too high. As a consequence of this judgement, with a lower ṕ for the next trial he would end up selecting a lower optimum T*. As we will show in the mathematical section, the person's total and marginal risk premiums will be lower and the lower T* if selected by everyone will have adverse macro consequences due to the more rapid spread of the virus, when the aggregate treatment is less.

One policy solution to this adverse macro outcome is to change peoples' psychological attitude about risk taking and its probability. This could be a very long-run endeavor for health officials and a topic the discussion of which is beyond the scope of this paper.

3. The Mathematical Structure of the Micro Utility Virus Model

The utility optimization approach we model follows in the spirit of that given by Baron (1970). A similar application of utility and risk taking is given by Gander (1985 and 2007).

To recap, the utility function, U(W(T)), is strictly concave under risk averse and the two production functions, W_l (T) and W_h (T), are increasing at an increasing rate with the rate higher for the high function than for the low function. The use of two separate functions greatly simplifies the discussion about the psychology behind a person's acceptance of the announced probability, as discussed earlier. Also, the use of two functions implies that there exist causes or technologies that result in two different outputs of W for the same input treatment, T (one could assume one function with a probability density function, f(u), like in $W = aT^b e^u$, with a > 0 and b > 1, where the probability for u is given by f(u)). The increasing marginal rate assumption is necessary for the covariance of the marginal utility and marginal wellness to be negative, as shown shortly.

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The two expectation functions are given by

(1) $EU(W(T)) = ṕU(W_h(T)) + (1 - ṕ)U(W_l(T))$,

for a given T and β and by

(2) $EW(T) = ṕ W_h(T) + (1 - ṕ) W_l(T)$,

where (1) is the expected utility and (2) is the expected wellness, given T and β.

The individual selects T* that maximizes his expected utility. The first-order condition for a maximum is given by the marginal expected utility

(3) $dEU(W(T))/dT = ṕ[(dU(W_h)/dW)(dW_h(T)/dT)] = (1 - ṕ)[dU(W_l)/dW)(dW_l(T)/dT)] = 0$,

solved implicitly giving the optimum, T = T*. The second-order condition is assumed to hold, $d^2 EU(W)/dT^2 < 0$. Since both terms in the brackets are random variables, we can use the property that the expected function of the product of two random variables (say, XY) is equal to the product of the two expectations, EXEY plus the cov(X', Y'), where X' and Y' are marginals.. This property allows us to rewrite (3) and obtain

(4) EMW(T*)EMU(W(T*)) + cov(U', W') = 0,

which when rearranged gives the expected marginal wellness evaluated at T*

(5) EMW(T*) = [-cov(U',W')/EMU(W(T*))] = σ(T*),

where σ is the marginal risk premium (MRP). It is positive under risk aversion for the cov(.) is negative, as shown above. We note for future use that the EMW(t*) is equivalent to MEW(t*), since the expectation of marginals (the dW/dT's) is equivalent to the margin of the expectation (or average) of the W(t)'s.

As indicated earlier, the MRP can be interpreted as the "marginal cost" of the T* decision. In effect, the MRP is covered or absorbed by the expected marginal wellness, EMW(T*). An increase in ṕ due either to an increase in β and/or an increase in the announced probability, p, will result in an increase in the optimum T*, an increase in the EU(W), and an increase in the EMW(T*) due to the decrease in the EMU(W(T*)) under risk aversion. In other words, since the person's perception that the probability of success, ṕ, is higher, the person is willing to endure a higher T*, for he expects the gain in wellness to offset the increase in the MRP. The reverse outcome occurs if the person believes that β should be lower and thus ṕ will be lower for a given, p.

This analysis can also be done in terms of the total risk premium (TRP) = E(W(T*)) – U^{-1}[EU(W(T*))], where the inverse function gives the EU(W(T*)). It can be shown that the TRP is an increasing function of the treatment T*. Briefly. the derivative of the TRP with respect to T* is dTRP(T*)/dT* = dE(W(T*))/dT* - dEU(W(T*))/dT*. The last derivative is equation (3) and equal to zero at the optimum T*. From (5) the left hand derivative (in its equivalent form given earlier) is [-cov(U',W')/EMU(W(T*))], which is positive. Hence, the TRP is an increasing function of T*.

If for some reason the psychological attitude of the person towards risk taking changes resulting in an increase in β, so ṕ increases, then the optimum T* will increase. In other words, if for some reason a person becomes more optimistic about his chances of winning (greater wellness output and higher expected utility), then T* and the TRP will increase (as suggested earlier based on the Tversky and Kahneman (1971) work). At the

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macro level, if all individuals increase their perception of ṕ, the macro consequence, all other things given, will be a lower death rate due to the lower spread rate of the virus.

However, if for some reason the actual empirical macro results are less than anticipated or expected, people may revise downward their probability of wellness, ṕ, so in the aggregate the consequence worsens with a higher death rate due to lower treatment inputs. What is possible then at the macro level is the occurrence of a dynamic counter-cyclical behavior in the relevant variables of the model (as suggested earlier). This dynamic outcome needs more discussion which is beyond the scope of this paper.

4. **Summary and Conclusions**

A complementary micro decision-making utility model under uncertainty is presented as a foundation for macro models of the behavior of the coronavirus. In the micro model, the individual decision maker is risk averse and selects an optimum level of treatment (e.g., social distancing) that maximizes his expected utility. The utility function depends on the level of wellness which is a random output of a two-level wellness function which depends on the level of treatment as an input. The probabilities of the random wellness outputs are parameters of the model and are determined by the person's psychological perception and acceptance to a degree of the publicly announced probabilities by the health officials.

The degree of acceptance was indexed by $\beta > < 1$, so the probabilities used by the decision maker were given by ṕ = βp and (1- ṕ). The use of this index allowed for a comparative-static analysis of the behavior of the micro model. Changes in a person's degree of acceptance will result in changes in the optimum level of treatment and the total and marginal risk premiums tolerated by the decision maker. A dynamic counter-

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cyclical process due to deviations between a person's expected results and the actual macro results was described and briefly discussed.

A basic micro conclusion from the model is that a person's, ex ante, expectations are increasing functions of the person's attitude towards and degree of acceptance of the publicly announced probabilities. These announced probabilities are essentially data driven. pertaining to the successfulness of various treatments to improve the output of wellness and thus to control the spread of the virus. Just how perceptions and degrees of acceptance relevant to the model can be changed for the better is a topic left for psychology experts. Hopefully, our micro model and its behavior will provide a useful complementary foundation for the macro model behavior of the virus.

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