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Getting rid of contextuality from quantum physics

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Abstract

This paper is aimed to dissociate from quantum physics contextuality, where the latter is standardly defined on the basis of families of compatible observables. In quantum physics, contextuality is just an exhibition of incompatibility of observables and, hence, Bohr's principle of complementarity. At the same time we recognize that contextuality can play the important role outside of physics.

1 Introduction

In a series of previous papers [1]-[3], I tried to dissociate nonlocality from quantum physics by pointing that so called "quantum nonlocality" is just a special exhibition of Bohr's complementarity principle [4, 5]. Now, it is time to do the same with contextuality. A system of quantum observables $X_0, ..., X_{n-1}$ can be contextual if at least two of them are incompatible, i.e., represented by noncommuting operators. Thus contextuality is also just an exhibition of Bohr's complementarity principle. We emphasize that that specialty of quantum observables plays the crucial role. Contextuality of nonquantum observables can have a nontrivial component, i.e., not reducible to incompatibility. We can mention, e.g., application of the Bell type inequalities in cognitive science and psychology, e.g., [6, 7]. However, since the majority of papers on contextuality are published in physical journals, disillusion of coupling of this notion from physics is important for quantum foundations.

Of course, one may say that operating with the words "nonlocality" and "contextuality" is merely the language problem. This is correct and if one likes, then he can play with these words as the special expressions for incompatibility of quantum observables. However, the wide use of this terminology mystifies quantum theory and overshadows its straightforward foundational structure based on the complementarity principle. We stress that the latter is in turn based on the quantum postulate about the existence of the indivisible quantum of action given by the Planck constant [8] (see also my recent paper [9]).

Finally, we remark that the standardly used definition of contextuality is determined by a family of compatible observables [10] does not match completely to Bohr's definition as dependence of measurement output on the complex of experimental conditions, "experimental context" [4]. Bohr's contextuality is the nutshell of quantum foundations and in particular of the complementarity principle [11], [1]-[3], [9].

2 Quantum contextuality vs. incompatibility

We follow paper [12] (one of the best and clearest representations of contextuality). The n-cycle contextuality scenario is given by n observables $X_0, ..., X_{n-1}$ and the set of maximal contexts

$$\mathcal{C}_n = \{\{X_0, X_1\}, ..., \{X_{n-2}, X_{n-1}\}, \{X_{n-1}, X_0\}\}. \tag{1}$$

Theorem 1 from paper [12] describes noncontextuality inequalities and Theorem 2 demonstrates that they are violated by quantum correlations. We do not question these statements, they are of course correct. But, what is the root of quantum violations? We claim that this is simply incompatibility expressed in the very special form.

Now, we turn to the quantum physics, i.e., $X_0, ..., X_{n-1}$ are not arbitrary observables, but quantum physical ones. In the quantum formalism, they are represented by Hermitian operators $\hat{X}_0, ..., \hat{X}_{n-1}$. We consider observables with purely discrete spectra, $X_j \in \{\alpha_{j1}, ..., \alpha_{jm}, ...\}$. Denote the orthogonal projectors onto the corresponding eigenspace by $\hat{E}_{j\alpha}$.

Suppose now that these observables are compatible with each other, i.e., any two observables X_i, X_j can be jointly measurable, so in the operator formalism, $[\hat{X}_i, \hat{X}_j] = 0$.

The quantum theory has one amazing feature that is not so widely emphasized. In fact, *pairwise joint measurability implies k-wise joint measurability for any $k \leq n$.* If all pairs can be jointly measured, then even any family of observables $\{X_{i_1}, ..., X_{i_k}\}$ can be jointly measured as well. In principle, there is no reason for this. This is the specialty of quantum theory.

The joint probability distribution (JPD) is defined by the following formula [13]:

$$P(X_{i_0} = \alpha_0, ..., X_{i_{k-1}} = \alpha_{k-1}) = \mathrm{Tr}\rho \hat{\mathrm{E}}_{\mathrm{i}_0\alpha_0} \cdot \hat{\mathrm{E}}_{\mathrm{i}_{\mathrm{k}-1}\alpha_{\mathrm{k}-1}}. \tag{2}$$

In particular, by setting $k = n$ we obtain JPD of all observables,

$$P_n(\alpha_0, ..., \alpha_{n-1}) = P(X_0 = \alpha_0, ..., X_{n-1} = \alpha_{n-1}).$$

We remark that the probability distributions given by (2) can be obtained from the latter JPD as the marginal probability distributions. So, we have the classical probability framework; the Kolmogorov probability model with the probability measure P_n. In this classical probabilistic framework we can prove any noncontextual inequality. So, it is impossible to violate them for compatible quantum observables.

3 Concluding remarks

Thus incompatibility is the source of so-called contextuality. The natural questions arise:

1. Do we need contextuality in quantum physics?
2. What is its supplementary impact for quantum foundations?[1]

This paper may be seen as provocative. I would be happy to get some input regarding these questions. All my previous attempts to clarify the physical meaning of contextuality in conversations with well qualified people did not lead to clarification. For the moment, I think that:

For quantum observables, violation of noncontextual inequalities is the very special expression of the Bohr complementarity principle; just its expression and nothing more.

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