EXPLORING STUDENTS' UNDERSTANDING OF STATISTICAL MEAN

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This study explores students' understanding of one measure of central tendency, the mean. A teaching experiment was conducted to understand how sixth-grade students made sense of this concept. Findings suggest that the students know how to solve mathematical problems related to mean using procedural understanding and lack of conceptual understanding.

Keywords: statistics, mean, weighted mean, average, mathematical thinking

Introduction

Statistical mean is a critical topic covered during the middle school years. The National Council of Teachers of Mathematics (NCTM) has recommended the inclusion of statistical mean in middle grade mathematics curriculum as far back as 1989 (CCSSM, 2010; NCTM 1998; NCTM 2000). The widespread adoption of the Common Core State Standards for Mathematics (CCSSM) also recognize the importance of statistical mean as a topic in middle grades, emphasizing a relationship between variance and measures of central tendency (CCSSM, 2010). Underscoring the push for developing an understanding statistical mean, studies advocating numeracy have shown that statistical mean is utilized in the activities of daily life (Pollatsek, Lima, & Well, 1981).

The idea of whether students are truly learning statistical concepts have been explored from several different perspectives (Amiruzzaman, 2016; Pollatsek et al, 1981; Strauss & Bichler, 1988). Students can often solve known mathematics problems by remembering rules or procedures. However, students may not be able to transfer such procedural knowledge to solve and explain new or unknown problems. This issue is outlined in detail by Skemp (2006), as he describes two types of teaching methods: (a) teaching for understanding and (b) teaching for fluency. Another perspective manifests itself in the exploration of whether students can utilize their understanding of statistical mean in their daily lives. A large body of research indicates that students have not been able to successfully apply such understanding (Amiruzzaman, 2016; Pollatsek et al, 1981).

The purpose of this study was to explore sixth grade students' understanding of mean, and how they identify similar concepts in real world scenarios that added dimensions of cognitive demand. To fulfill this purpose, this report presents a report on results of a semester long teaching experiment (Steffe & Thompson, 2000).

Literature Review

Several research studies have been conducted in order to learn more about students' understanding of basic statistical concepts (Pollatsek et al, 1981; Strauss & Bichler, 1988). Such investigations typically focus on whether students understand the concept of mean and know how to apply it to real-world scenarios (Pollatsek et al, 1981). In weighted mean, we multiply each observation by associated weights, add those values together and then divide by the sum of the weights. Pollatsek et al (1981) found that most students understand the concept of simple mean (i.e., $\frac{\sum a_1 + a_2 + \cdots + a_{n-1} + a_n}{n}$); but fail to understand the concept of weighted mean (Pollatsek et al, 1981). Strauss et al (1988) drew a distinction between computational and conceptual

understanding of mean. Their research revealed that as students grow older, their conceptual understanding of mean increases accordingly (Strauss & Bichler, 1988).

It is apparent that age plays a vital role in mathematics learning, and this explains why NCTM recommends introducing different statistical concepts during different grade levels (NCTM, 1998; NCTM, 2000). Mokros and Russell (1995) explored the characteristics of fourth through eighth graders' constructions of mean as a representative number summarizing a data set. They interviewed 21 students of various ages (i.e., grade 4, grade 6 and grade 8), using a series of open-ended problems. In this paper, the word mean and average are interchangeable and used alternatively. Mokros and Russell (1995) observed that not all students understood the five basic characteristics of mean: (a) mean as mode (i.e., when mean and mode is same), (b) mean as algorithm (i.e., step-by-step process of find mean), (c) mean as reasonable (i.e., mean as a tool to make sense of data), (d) mean as midpoint (i.e., mean of three consecutive data elements), and (e) mean as mathematical point of balance (i.e., differentiate total and mean). Examining students' conceptions of mean with a different approach, Capraro, Kulm and Capraro (2005) observed students' misconceptions for mean when zero values were included in the data. These students often did not consider zeros when calculating mean (Capraro et al., 2005), possibly avoiding zeroes because of a belief it has no value.

The purpose of this study was to explore the nature of student understanding of the mean; thus a teaching experiment was conducted. The interview included verbal and written answers to one close-ended and six open-ended questions. The experiment was conducted on an in depth one-to-one basis.

Methods

Data collection

Six sixth-grade students were chosen for the teaching experiment. A teacher known to the researchers was contacted in advance in order to obtain permission for the research and to secure a location for conducting the teaching experiment. The study was conducted for a semester long (i.e., 15 weeks). However, because of the relevance and length of this paper, we report one episode's analyzed data. We chose a series of tasks and followed the students' completion with probing questions to solicit their mathematical thinking. The idea of procedural vs. conceptual understanding was explored through questions such as question (a) Do you know the meaning of mean, and what does the mean tell you? (Pollatsek et al, 1981) As question (b) In its last 4 games, our basketball team scored 55, 87, 46, and 62 points. What must we score in the next game so that the average score of all 5 games is 60? (Baker & Beisel, 2001).

Figure 1. Procedural understanding: How the student was trying to calculate the mean is shown in above figure.

The Vet Club problem (Capraro et al., 2005) was modified slightly in question (c) to explore zero as a misconception in the mean concept. As question (c) Jenny is writing a

newsletter article about the members of the Future Veterinarians Club. On average, how many pets are there per club member? Her notes were shown with fourteen club members' initials and their associated number of pets ranging from 'NO PETS' to 6 pets. The student answered this question by both calculation and plotting data on a number line. The question was answered incorrectly, as the student did not include members with no pets in the calculation or number line. He was asked to explain the answer and he explained that those with no pets should not be considered when calculating the mean (see Figure 2).

Jenny is writing a newsletter article about the members of the Future Veterinarians Club. Her notes are listed below. She asked each of the members: "What pets live in your house?" On average, how many pets are there per club member?

- B.J. none (NO PET)
- A.P. 1 dog and 1 cat (2 PETS)
- K.K. 2 dogs (2 PETS)
- R.B. 1 bird, 1 dog, and 4 goldfish (6 PETS)
- G.L. 1 dog (1 PET)
- W.F. none (NO PET)
- J.A. 1 rabbit (1 PET)
- J.M. 1 dog (1 PET)
- J.Z. none (NO PET)
- S.F. 3 cats (3 PETS)
- E.P. none (NO PET)
- K.B. 1 bird (1 PET)
- S.L. 2 dogs, 1 bird, and 2 fish (5 PETS)
- D.L. 6 fish (6 PETS)

3pds

Figure 2. Misconceptions: student skipped persons who owned no pets, basically, student skipped zeros during the calculation.

To ensure the trustworthiness all names and identification remakes were removed from the transcript. The transcript was read and reread to make sure the accuracy of the transcript.

Results

The interview data was analyzed twice for the integrity. Students worksheets were scanned and reviewed to further understand the student's thinking. They successfully calculated average for straightforward problems. Observed data indicates that they knew if a number was added to the set it would change the mean value. One student said, "Numbers control the mean, it's not the mean controlling the numbers." He also recognized that two different data sets may have same mean. However, for the new problem type (b), he incorrectly applied the straightforward procedure when an addend was missing and average was given.

Calculation and number line results revealed a misconception in mean calculation, as the student did not plot data the zero values. This is visualized in the number line where ten values were plotted and the four zero data values were not included. (see Figure 3). When the student was asked to explain why he did not plot the zero values, then he mentioned that there is no need to consider zero values, because the zero values are worthless.
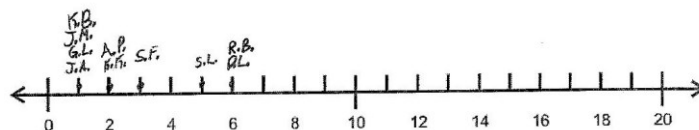
Figure 3. Number line: student was asked to show position of each numbers using the number line. Students original marking are shown above.

Some of Steffe and Thompson's (2000) writings suggest that multiplication schemes are partly dependent upon arithmetic schemes, and this suggests that this student's understanding of mean may be contingent upon those schemes as well. However, the student does not appear to see the entire set of data because of this misconception. Coincidentally, this relates directly to his

completion of the task (a) in which he missed a part of the data in his calculation. Thus, the student, while not seeing 0 as part of the data, also does not see other values (unless given) as part of the data set. Thus, the student's conception of mean lacks an understanding of variability in that he does not understand the sample space (in both tasks, he has missed the sample space for the data).

It appears that the student has a procedural knowledge of the mean. He knows that it requires adding all numbers and dividing them by the frequency. However, because the student's knowledge is not conceptual, he experienced issues. However, since adding numbers with the zero values does not make any difference, rather, the student considers zeroes as value-less, they student demonstrates with his actions that including them as part of the sample space (or the data total to be divided by in the algorithm) that zeroes can be avoided during the calculating of mean. If the student had a more robust conceptual understanding of mean (i.e., understanding the sample space), then he would have included the zero values as part of the sample space.

Conclusion

The study explored the nature of students' understanding of mean, with an eye directed toward the type of understanding and possible misconceptions. The student answered the initial question about what the mean was with an explanation of the procedure used to calculate the mean which suggests only a procedural understanding. This may have contributed to the misconception regarding how to treat exercises with zero data points. These findings about misconceptions of data with zero in mean calculation support the findings in previous research that pointed to difficulty with graphing zero. As students struggle with the concept of 0, teachers who are aware of the misconception can plan activities to address it as they introduce the concept of mean.

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