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Technical Note

STRUCTURAL DISCONTINUITIES IN A MULTI BOND GRAPH

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Abstract: The hybrid bond graph has been studied in depth for scalar bond graphs, but how does this translate to the multi-bond graph? Here, the controlled junction – used to model structural switching such as contact – is extended to the multi-bond case. This is a simple process, assuming that all bonds switch simultaneously (which makes physical sense). A controlled 0-junction is applied to multi-bond graph of a car, which can lose contact with the ground in cornering. Dynamic causality features, but this can be accommodated using an equational submodel in 20-Sim (in a manner similar to that used with scalar bond graphs). The junction is proposed for subsequent work to develop a validated multi-body dynamics car model in cornering.

Keywords: Bond graph; multi bond; vector bond; hybrid; switching; multi-body; dynamics; system; model

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1. Introduction

There has been a body of work in recent decades on hybrid dynamical models, including hybrid bond graphs. This has been well explored in simple subsystems using scalar bond graphs, but has not yet been formalised for multi-bond graphs (also known as vector bond graphs). This variant of a bond graph concatenates scalar bonds for each degree of freedom into single vector bonds, which facilitates multi-body dynamics (MBD) modelling. Hybrid models frequently give rise to dynamic causality, which can create conflicts in a multi-bond graph if applied inconsistently. The author was therefore approached with the task of formalising how to capture switching behaviour like contact with a hybrid multi-bond graph.

The controlled junction [2] [3] [4] is used, as it describes the variable structure system which arises with contact problems well. This is an example of structural switching, where the structure of the model varies (yielding a VSS). There is dynamic causality present, reflecting the physics of the problem i.e. two independent bodies have (dis)joined. A mathematical model can be generated as a state equation with Boolean terms in the coefficients, and simulated in widely-available off-the-shelf software like Matlab and 20-Sim.

As an aside, a switching element has been defined for scalar bond graphs to capture 'parametric switching' [5], where the constitutive equation(s) of an element are so highly nonlinear that they are represented by some form of Heaviside function or mode-switching. It is possible to define a switching element for use in multi-bond graphs, but in reality this kind of behaviour may be better captured by a nonlinear RC field. In either case, dynamic causality is not normally associated with these elements.

2. The controlled junction for multi-bond graphs

In broad terms, vector bonds and elements correspond to scalar ones, with each signal now being a vector quantity. It is therefore straightforward to define a controlled junction for multi-bond graphs. The most important consideration here is that switching occurs in all directions or parts of the vector.

Proposition 1. *In a multi-bond graph, any structural switching occurs simultaneously in all degrees of freedom.*

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Consider the case study of contact between a vehicle and the ground: when the wheel ceases to contact the ground, it does so in all directions. There is no vertical reaction from the ground, nor is there any lateral or longitudinal force. It would be erroneous to apply the switching to only the vertical. This is the case in any multi-body dynamics system where multi-bonds are used to describe the three dimensions: the structure is either connected or it isn't. It is physically impossible for a single dimension to be connected while another isn't.

Controlled junctions for multi-bond graphs can then be easily defined by analogy to the scalar bond graph, as shown in figure 1.

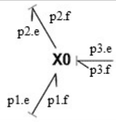
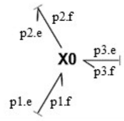
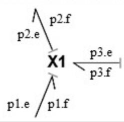
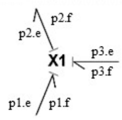
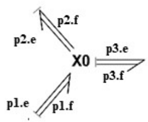
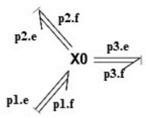
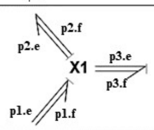
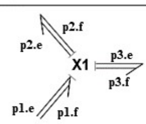
Element Type	Controlled Junction 'ON'	Constitutive Equation when 'ON'	Controlled Junction 'OFF'	Constitutive Equation when 'OFF'
Scalar X0-junction		$p1.e = p3.e$ $p2.e = p3.e$ $p3.f = p1.f + p2.f$		$p1.e = 0$ $p2.e = 0$ $p3.e = 0$
Scalar X1-junction		$p1.f = p3.f$ $p2.f = p3.f$ $p3.e = p1.e + p2.e$		$p1.f = 0$ $p2.f = 0$ $p3.f = 0$
Vector X0-junction		$\boldsymbol{p1.e = p3.e}$ $\boldsymbol{p2.e = p3.e}$ $\boldsymbol{p3.f = p1.f + p2.f}$		$\boldsymbol{p1.e = 0}$ $\boldsymbol{p2.e = 0}$ $\boldsymbol{p3.e = 0}$
Vector X1-junction		$\boldsymbol{p1.f = p3.f}$ $\boldsymbol{p2.f = p3.f}$ $\boldsymbol{p3.e = p1.e + p2.e}$		$\boldsymbol{p1.f = 0}$ $\boldsymbol{p2.f = 0}$ $\boldsymbol{p3.f = 0}$

Figure 1. Descriptions of scalar and vector controlled junctions, with their causal assignments and constitutive equations in 'ON' and 'OFF' states.

3. Application: Vehicle contact with ground

This case study relates to a passenger car which rolls during cornering. The wheels will lose contact with the ground, and [hopefully] make contact again. The model is similar to that of an aircraft landing gear as it descends and makes contact with the ground. However, in this case the whole car is modelled and this is facilitated by using multi-bond graphs.

The equations for a tyre in contact with ground are well established, and the bond graphs developed by Pacejka [1] are used as a starting point here. In the vertical sense, the tyre is usually represented by a spring (often as part of the well-known quarter-car model). Lateral slip is typically modelled using the well-described linear, Dugoff or Pacejka tyre models, but in all of these cases is essentially a resistive force i.e. encapsulated in an R-element in the bond graph framework. Likewise, longitudinal slip is considered as a resistive force and expressed as an R-element. The scalar bond graphs for the wheel on the ground are shown in figure 2.

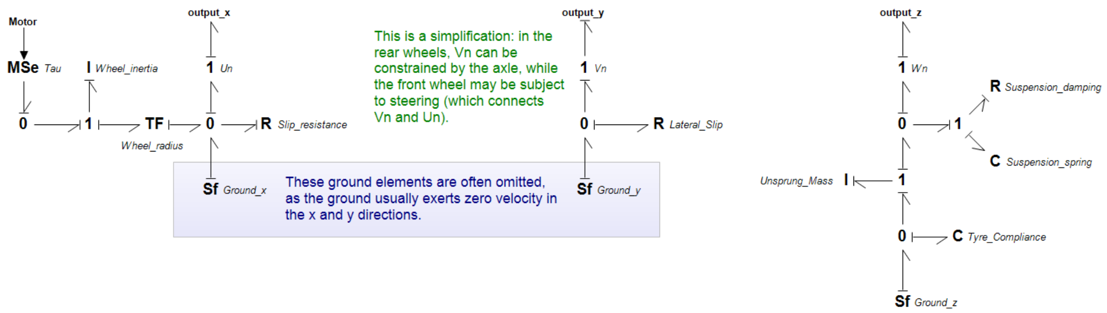


Figure 2. Scalar bond graph of a quarter car in contact with the ground

When the car is in contact with the ground, modelled by a flow source in the bond graph framework, the ground constrains the velocity of the tyre. When the tyre is no longer in contact with the ground, the constraint is no longer there. There is no compliant force from the tyre, which is no longer under compression. There is no slip resistance between the tyre and the ground. The tyre ceases to be deformed, and ground no longer has any effect on the velocity or position of the car. The scalar hybrid bond graphs for the wheel while not on the ground (i.e. controlled junctions set to 'OFF') are shown in figure 3.

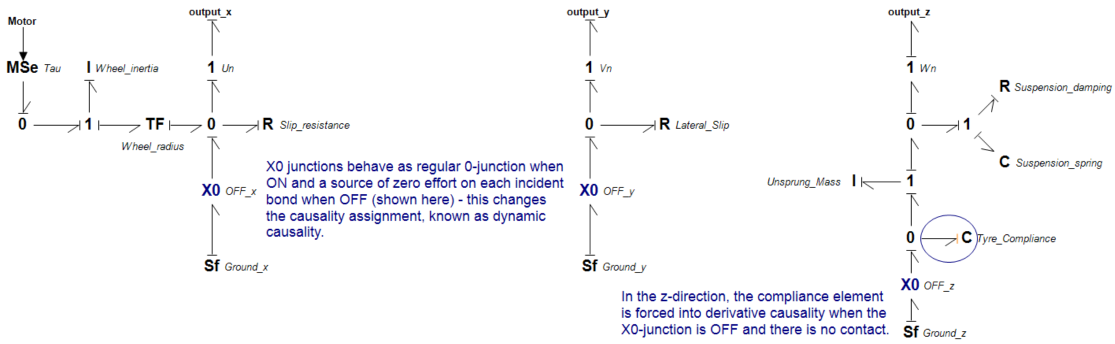


Figure 3. Scalar hybrid bond graph of a quarter car not in contact with the ground

Pacejka's model [1] does not display a causality assignment, and reproducing this model in 20-Sim (as in figure immediately shows that the causality assignment must be different in the multi-bond graph to the scalar bond graph. In the scalar bond graphs (figures 2 and 3) the bonds outputting to the rest of the vehicle model take differing causal assignments: the x- and y-ports are effort-out, and the z-port is velocity-out. Bonds can only be combined into vector bonds if they share the same causality assignment. In the multi-bond graph, the output to the rest of the car model takes a velocity-out casual assignment. This is not necessarily a problem – causality in a bond graph is computational causality. Besides, the multi-bond graph will incorporate inertias and weights on all senses (which can be neglected in the scalar bond graphs in some directions).

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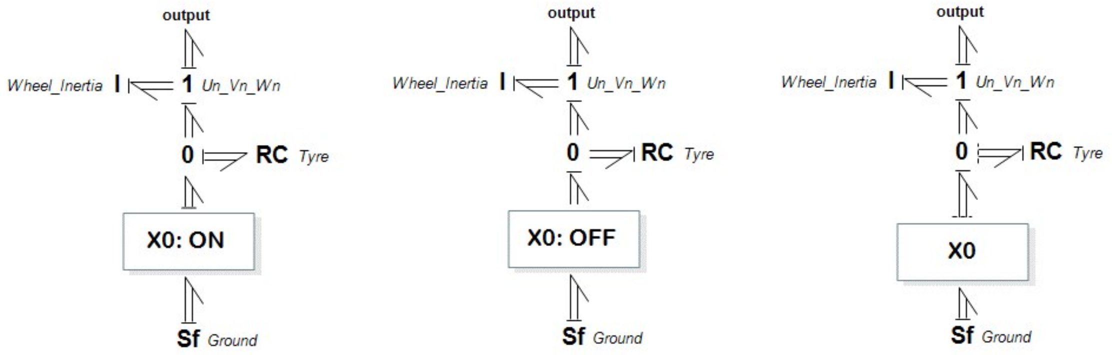

Figure 4. Hybrid multi-bond graph of a quarter car in contact with the ground (**a**) In contact, (**b**) Not in contact, (**c**) HMBG with Dynamic Causality.

Note that the RC element 'absorbs' the dynamic causality, acting like a causality resistance [6] [7]. While applying causality resistance without due consideration can be problematic, in this case it is entirely physically relevant: the tyre does indeed deform and exert/dissipate energy. Although dynamic causality is not supported in most software packages, the controlled junction and RC-element can be converted into an equational submodel with logical statements. This has been well-described for scalar bond graphs [examples], and can be extended to the multi-bond graph as proposed in figure 5.

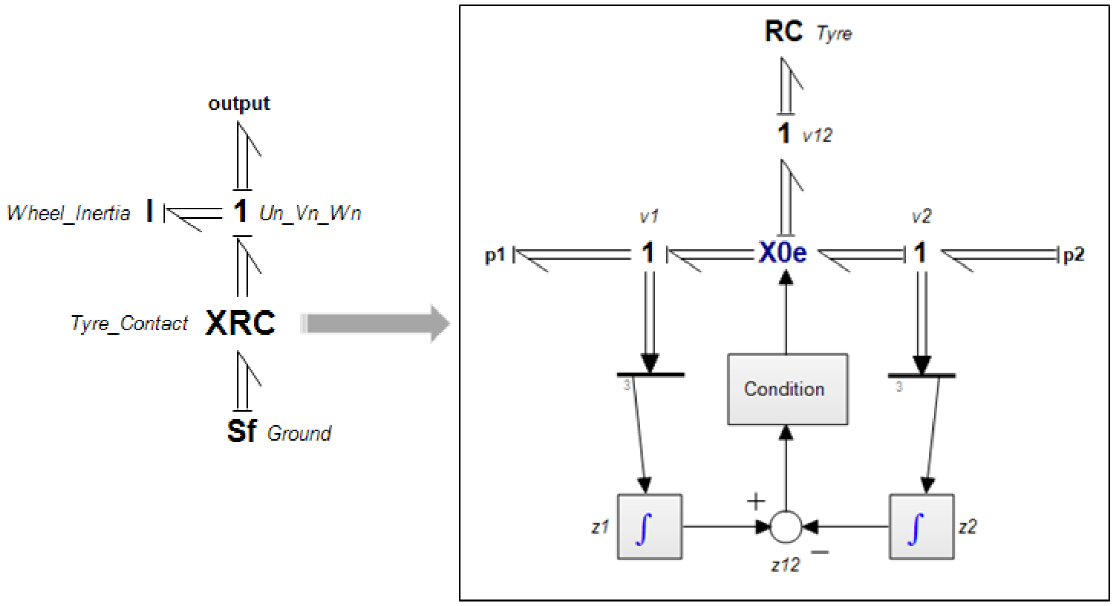

Figure 5. Hybrid multi-bond graph of a quarter car utilising the equational submodel in 20Sim

The 'Condition' equational submodel defines whether contact has occurred based on what the z-displacement between the wheel and ground is i.e.:

```
contact = if (dz) < 0 then true else false end;
```

The X0e element is another equational submodel, which defines the effort and flow on each bond according to the condition:

```
p1.e = if condition then p12.e else 0 end;
p2.e = p1.e;
p12.f = if condition then p2.f-p1.f else 0 end;
```

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Alternatively, a state space model(s) can be derived from the bond graph manually, and simulated in Matlab in the same way that scalar bond graphs have been [8].

4. Conclusions

This short technical note proposes an approach to hybrid multi-bond graphs, which is a logical extension to previous work on scalar hybrid bond graphs. Further work will use this in validated whole system models.

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Abbreviations

The following abbreviations are used in this manuscript:

HBG	Hybrid Bond Graph
MBD	Multi-Body Dynamics
VSS	Variable Structure System

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