

The shadow of M87* black hole within rational nonlinear electrodynamics

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Abstract

We consider rational nonlinear electrodynamics coupled to General Relativity. The effective geometry induced by nonlinear electrodynamics corrections are found. We determine shadows of regular non-rotating magnetic black holes and compare them with the shadow of the super-massive M87* black hole imaged by the Event Horizon Telescope collaboration. This allows us to obtain the black hole magnetic charge. The size of the shadow is very close to the shadow of non-regular neutral Schwarzschild black holes. As a result, we can interpret the super-massive M87* black hole as a regular (without singularities) magnetized black hole.

1 Introduction

The black hole (BH) shadow is a dark region surrounded by bright optically thin (a geometrically thick) emission. It separates capture orbits and scattering orbits of photons around BHs [1]-[3]. The size and shape of the BH shadow strongly depends on the BH mass and very weakly on its spin. For a non-rotating black hole the shape of a shadow is a circle, whereas for a BH with a spin the shadow will be deformed due dragging effects. Shadows are formed due to gravitational lensing near a BH event horizon because the gravitational field is very strong and circular photon orbits are unstable forming light rings. Very long baseline interferometry can detect the shadows of super-massive black holes (SMBHs) [4], [5]. Event Horizon Telescope (EHT) collaboration detected the shadow of M87* at the center of the elliptical galaxy Messier 87, [5]-[10]. The image of the M87* shadow is consistent

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with the Kerr BH in General Relativity (GR). At the same time BH shadows can test deviations from GR [11]-[14]. It is possible to extract an information from the M87* shadow, regarding properties of the BH such as constraints on fundamental physics, see e.g. [15]-[23]. Gravitational collapse in GR leads to the undesirable existence of singularities [24], [25]. The cosmic censorship conjecture states that all singularities of gravitational collapse are hidden by event horizons of BHs, and should not be naked [26]. But it is desirable to have solutions which avoid singularities. For a Schwarzschild (non-rotating) BH, the shadow diameter is $3\sqrt{3}r_{Sch}$, where r_{Sch} is the Schwarzschild radius [27].

Nonlinear electrodynamics (NLED) represent models having regular BH solutions, the first example is Born–Infeld electrodynamics [28], in which the charge has finite self-energy. Another example is Euler–Heisenberg NLED with action of QED taking into account loop corrections [29]. NLED models and their BH solutions were studied in (an incomplete list) [30]-[52]. It was shown in [34] that NLED coupled to GR does not possess a static, spherically symmetric solution with a regular center and non-zero electrical charge if NLED has the Maxwell asymptotic at weak-field limit. Therefore, only magnetically charged BH can have regular solution when NLED becomes Maxwell electrodynamics at weak fields.

In this letter we consider non-spinning BHs and investigate regular solutions carrying a magnetic charge within rational NLED [53] coupled to GR. By computing the shadow size within our model and compare it with the size of a shadow of the M87* BH, we evaluate the magnetic charge of the BH. It worth noting that the shadow size is independent of the spin of the BH with an uncertainty $\leq 10\%$ [5].

2 The model

Let us consider the motions of photons in a regular spacetime which being a solution to the Einstein and rational NLED theories. This will allow us to determine the shadows of magnetically charged BH in the framework of rational NLED. Light propagates in NLED along null geodesics and induces effective geometry modifying original background spacetime [54], [55].

We consider the rational NLED, proposed in [53], which is converted to linear Maxwell’s electrodynamics at weak field limit ($\beta\mathcal{F} \geq 0$) and described

by the Lagrangian

$$\mathcal{L} = -\frac{\mathcal{F}}{2\beta\mathcal{F} + 1}. \quad (1)$$

The parameter β possesses the dimension of $(\text{length})^4$, $\mathcal{F} = (1/4)F_{\mu\nu}F^{\mu\nu} = (B^2 - E^2)/2$ is the field tensor. The action of GR coupling to NLED is given by

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} R + \mathcal{L} \right), \quad (2)$$

where G is the Newton constant, $16\pi G \equiv M_{Pl}^{-2}$, and M_{Pl} is the reduced Planck mass. The variation of the action (2) with respect to the metric tensor gives the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi GT_{\mu\nu} = 8\pi G \left(\frac{F_{\mu}^{\alpha} F_{\nu\alpha}}{(1 + 2\beta\mathcal{F})^2} + g_{\mu\nu}\mathcal{L} \right). \quad (3)$$

In order to solve the field equations, we use the ansatz of the static and spherically symmetric metric with the squared of the line element

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2). \quad (4)$$

The metric function of the magnetic BH within rational NLED is given by [56]

$$f(x) = 1 - \frac{q_m^3 G}{4\sqrt{2}\beta x} \left(\ln \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} + 2 \arctan(\sqrt{2}x + 1) - 2 \arctan(1 - \sqrt{2}x) \right), \quad (5)$$

where we introduced the unitless variable $x = r/\sqrt[4]{\beta q_m^2}$ (q_m is the magnetic charge which is unitless in our Gaussian units with $c = 1$). To have the regular BH we ignore the Schwarzschild mass so that the total mass of the BH is the magnetic mass [56]

$$m_M = \frac{\pi q_m^3}{4\sqrt{2}\beta^{1/4}} \approx 0.56 \frac{q_m^3}{\beta^{1/4}}. \quad (6)$$

Equation (5) can be rewritten as

$$f(x) = 1 - \frac{Bg(x)}{x}, \quad B = \frac{q_m G}{\sqrt{\beta}},$$

$$g(x) = \frac{1}{4\sqrt{2}} \left(\ln \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} + 2 \arctan(\sqrt{2}x + 1) - 2 \arctan(1 - \sqrt{2}x) \right), \quad (7)$$

where the B is the unitless constant (do not confuse the constant B with the magnetic field). We will explore the method of [54], [55] to describe the effective geometry induced by our NLED. Photons propagate along the null geodesics of this effective geometry. The null geodesics of the photon paths are described by the effective spacetime with the static and spherically symmetric metric [54], [55], [57]

$$g_{eff}^{\mu\nu} = \mathcal{L}_{\mathcal{F}}\eta^{\mu\nu} - \mathcal{L}_{\mathcal{F}\mathcal{F}}F_{\alpha}^{\mu}F^{\alpha\nu}, \quad (8)$$

where $\mathcal{L}_{\mathcal{F}} = \partial\mathcal{L}/\partial\mathcal{F}$. The effective metric for photons found from Eqs. (4) and (8) is given by

$$ds_{eff}^2 = A(r) \left(-f(r)dt^2 + \frac{1}{f(r)}dr^2 \right) + h(r)r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2), \quad (9)$$

where

$$A(x) = -\mathcal{L}_{\mathcal{F}} = \frac{x^8}{(x^4 + 1)^2}, \quad h(x) = -\mathcal{L}_{\mathcal{F}\mathcal{F}}\frac{q_m^2}{r^4} = \frac{x^8(x^4 - 3)}{(x^4 + 1)^3}. \quad (10)$$

The metric functions $A(r)$ and $h(r)$ ($r = x\sqrt[4]{\beta q_m^2}$) must be positive in order that the effective geometry does not change its signature during the photon's motion. This leads to the requirement $x > \sqrt[4]{3} \approx 1.32$ or $r > r_{eff} \equiv \sqrt[4]{3}\beta^{1/4}\sqrt{q_m}$. Another restriction is $r > r_h$, where r_h is the radius of the event horizon ($f(r_h) = 0$). When $r_{eff} < r_h$ the exterior region of the BH is realized by $r > r_h$ but if $r_{eff} > r_h$ the range of the photon motion outside of the BH is given by $r > r_{eff}$. The equations of motion along the equatorial plane, for which $\vartheta = \pi/2$ for a null geodesic in terms of an effective potential $V(r)$, reads [57]

$$\left(\frac{dr}{d\phi} \right)^2 = V(r) = r^4 \left(\frac{E^2 h(r)^2}{L^2 A(r)^2} - \frac{f(r)h(r)}{A(r)r^2} \right), \quad (11)$$

where $E = f(r)A(r)\dot{t}$ and $L = r^2 h(r)\dot{\phi}$ are the photon's constants of motion, the photon total energy and angular momentum, respectively. Unstable circular orbits take place when $dV/dr = V = 0$. Then Eq. (11) leads to

$$b^{-2} \equiv \frac{E^2}{L^2} = \frac{f(r)A(r)}{r^2 h(r)},$$

$$2f(r)A(r)h(r) + rf(r)h'(r)A(r) - rf'(r)A(r)h(r) - rf(r)h(r)A'(r) = 0, \quad (12)$$

where b is the impact parameter and the prime means the derivative with respect to r . Making use of Eqs. (5), (10) and (12) we obtain relations as follows:

$$b^{-2} = \frac{1}{q_m \sqrt{\beta}} \left(1 - \frac{Bg(x)}{x} \right) \frac{x^4 + 1}{x^2(x^4 - 3)},$$

$$2x(x^8 + 6x^4 - 3) - Bg(x)(3x^8 + 10x^4 - 9) + Bx^3(x^4 - 3) = 0. \quad (13)$$

The size of the shadow depends on the impact parameter b . The circle with the radius b in the center of the BH mass is the “photon ring”. The solution of Eq. (13) depends on B which includes two independent parameters q_m and β .

3 The BH shadow

Because Quantum Electrodynamics (QED) is well proven theory, we imply that rational NLED (1) at the weak-field limit becomes QED with loop corrections. Therefore, the model parameter β can be evaluated by comparing (1), at weak electromagnetic fields, with the Euler–Heisenberg Lagrangian [29]. Making use of series of Lagrangian (1) in the small parameter $\beta\mathcal{F}$ ($\beta\mathcal{F} \ll 1$) we obtain

$$\mathcal{L} = -\mathcal{F} + 2\beta\mathcal{F}^2 - 6\beta^2\mathcal{F}^3 + \mathcal{O}((\beta\mathcal{F})^4). \quad (14)$$

The the Euler–Heisenberg Lagrangian (the QED Lagrangian with one loop correction) can be approximated as [58]

$$\mathcal{L}_{EH} = -\mathcal{F} + c_1\mathcal{F}^2, \quad c_1 = \frac{8\alpha^2}{45m_e^4}, \quad (15)$$

where the electron mass is $m_e = 0.51$ MeV and the coupling constant $\alpha = e^2/(4\pi) \approx 1/137$. Making the identification of Eqs. (14) and (15) up to $\mathcal{O}((\beta\mathcal{F})^2)$, we obtain

$$\beta = \frac{4\alpha^2}{45m_e^4} = 69 \times 10^{-5} \text{ MeV}^{-4}. \quad (16)$$

Now, we in the position to compare the shadow for the BH within our model with the shadow of M87* detected by the EHT [5]. We use the M87* mass

$M = (6.5 \pm 0.9) \times 10^9 M_\odot \approx 72.4 \times 10^{68}$ MeV and the magnetic mass in our model (6) with the value for the parameter β (16). Making use of the identity $m_M = M$ and the value of the parameter $B = q_m G / \sqrt{\beta}$ in Eq. (7) and the Newton constant $G = 67.9 \times 10^{-58}$ eV², one obtains the magnetic charge (in Gaussian units with $c = 1$) and the unitless parameter B

$$q_m \approx 1.6 \times 10^{46}, \quad B \approx 4205. \quad (17)$$

With the help of values (17) we find the numerical solutions to Eq. (13)

$$x_{ph} \approx 7005, \quad b \approx 2.5 \times 10^{20} \text{ eV}^{-1}. \quad (18)$$

The horizon radius is the solution for the equation $f(x_h) = 1 - Bg(x_h)/x_h = 0$ which, for $B = 4205$, is $x_h \approx 4670$. Because $x_{ph} > x_h$ the BH shadow is defined by the photon capture radius (18). By virtue of the value of the impact parameter (18) (the photon capture radius), one obtains the diameter of the M87* shadow within our model

$$\frac{2b}{m_M G} \approx 10.2. \quad (19)$$

The diameter of the M87* shadow measured by EHT is $d_{M87*} = 11.0 \pm 1.5$. Thus, within 1σ uncertainties our result (19) is in very good agreement with the experimental data for the angular size of the shadow $\delta = (42 \pm 3) \mu\text{as}$, the distance for M87* $D = (16.8 \pm 0.8)$ Mpc and the diameter of the M87* shadow $d_{M87*} = D\delta/(MG)$.

4 Conclusion

Our result (19) for the diameter of the M87* shadow is very close to the estimation within the Schwarzschild BH, where a photon capture radius is $b = 3\sqrt{3}G \approx 2.6 \times 10^{20}$ eV⁻¹, $2b/(MG) \approx 10.4$. The model of regular magnetic BH based on rational NLED has an advantage compared to the singular Schwarzschild BH and with Euler–Heisenberg BH [57] because of the absences of singularities [53]. We considered the spherically symmetric spacetime because of its simplicity and ignored therefore the rotation of the BH. It worth noting that the rotational uncharged Kerr BH possesses singularities on a ring. The next step in this direction is to find the corrections to the shape of the shadow due to the spin of magnetically charged BHs based on rational NLED.

References

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