Peer-reviewed version available at *Appl. Sci.* **2020**, *10*, 3197; doi:10.3390/app10093197

Article

Traffic Flow Catastrophe Border Identification for Urban High-density Area Based on Cusp Catastrophe Theory: A Case Study under Sudden Fire Disaster

Ciyun Lin [1], Yongli Yu[1], Dayong Wu[2 *] and Bowen Gong [1,3]

[1] Department of Traffic Information and Control Engineering, Jilin University, Changchun 130022, China; linciyun@jlu.edu.cn (C.L.); yuyl18@mails.jlu.edu.cn (Y.Y.); gongbowen@jlu.edu.cn (B.G.);
[2] Texas A&M Transportation Institute, Texas A&M University, College Station, Texas, TX 77843, USA; J-Wu@tti.tamu.edu (D.W.)
[3] Jilin Engineering Research Center for ITS, Changchun 130022, China;
* Correspondence: Dayong Wu Email address: J-Wu@tti.tamu.edu

Received: date; Accepted: date; Published: date

Abstract: For traffic management under sudden disaster in high-density areas, the first and foremost step is to prevent traffic congestion in the disaster-affected area by traffic flow control, as to provide enough and flexible traffic capacity for emergency evacuation and emergency rescue. Catastrophe border identification is the foundation and the key to traffic congestion prediction under sudden disaster. This paper uses a mathematical model to study the regional traffic flow in the high-density area under sudden fire disaster based on the Cusp Catastrophe Theory (CCT). The catastrophe border is identified by fitting the CCT-based regional traffic flow model to explore the stable traffic flow changing to the instable state, as to provide a theoretical basis for traffic flow manage and control in disaster-affected areas, and to prevent the traffic flow being caught into disorder and congestion. Based on VISSIM simulator data by building simulation scenarios with and without sudden fire disaster in a Sudoku traffic network, the catastrophe border is identified as 439pcu/lane/h, 529pcu/lane/h, 377pcu/lane/h at 5s, 10s, 15s data collection interval respectively. The corresponding relative precision, which compares to the method of Capacity Assessment Approach (CAA), is 89.1%, 92.7% and 76.5% respectively. It means that 10s data collection interval would be the suitable data collection interval in catastrophe border identification and regional traffic flow control in high-density area under sudden fire disaster..

Keywords: catastrophe border identification; sudden fire disaster; emergency management; cusp catastrophe theory; high-density urban area

---

1. Introduction

With the rapid development of China's social economy and the accelerating urbanization process, population, buildings, wealth and other infrastructures are more highly concentrated in urban areas. Cities, especially urban centers, have gradually formed in a high-density state. The high-density characteristics of spatial structure, population, buildings and wealth make the urban safety system more fragile, with more potential disaster, higher disaster risk, harder in identification of disaster sources, more complicated in disaster risk management, more serious in damage and loss, and so on.

With the characteristics of high density, high intensity and high complexity, these urban center areas present a situation of multiple disasters and secondary disasters coexisting and concurrent. These disasters include natural disasters and man-made disasters. They are may be traditional disasters or new types of disasters (such as virus infection, terrorist). This situation makes the disaster management more difficult to control and withstand. In addition, the disaster is easier to propagate and diffuse in high-density environment. In view of the high-density urban central area's disaster-formative environment and the disaster-prone characteristics, the high-density urban

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

central area has become the core area of urban disaster prevention and mitigation[1]. The research on the high-density urban central area disaster prevention and mitigation system has become the basic premise and difficult problem in ensuring urban security and keeping sustainable urban development.

The emergency traffic organization guarantee under the sudden disaster in the high-density urban center area is an important part in the disaster prevention and mitigation system of the whole city. Fast and effective emergency traffic organization guarantees can significantly reduce the loss of life and property, and minimize the loss of disasters. According to statistics, effective emergency transportation organization guarantees can reduce disaster losses to 6% of without emergency transportation organization guarantees[2].

However, the concentration of facilities in high-density urban centers, strained traffic land, and frequent activities of residents make the contradiction between traffic demand and supply more prominent. According to the statistics of the competent traffic authorities, 90% of the surface roads in the central urban areas of many large and medium-sized cities in China are saturated or supersaturated[3], and even traffic congestion in urban centers' road segments has become normalized in some cities[4,5]. Under such circumstances, in the event of a sudden disaster, surface road traffic system which is supposed to play an emergency safe-guarder in urban disaster prevention, disaster mitigation and relief, may have adverse effects on disaster relief due to traffic congestion in the rescue traffic route.

In addition, when the urban traffic management department responds to urban sudden disasters, they often rely on administrative orders and historical experiences when formulating emergency traffic organization safeguard measures and resolving control strategies, and lacks of theoretical and technical supports. Due to the characteristics of uncertainties, multi-category, information loss, companionship, derivative, and chain reactions, the sudden disaster is hard to predict and control by traffic agents. Moreover, they are unable to consider the characteristics of traffic flow aggregation, propagation and distribution under the incompleteness information of traffic network condition in sudden disaster. These may lead to make the traffic agent making a simple, inefficient and high-vulnerability emergency traffic organization plan. This plan may pose great risks to disaster prevention, mitigation and relief work under sudden disaster in high-density urban centers.

For traffic management under sudden disaster in high-density area, the first and foremost step is traffic flow control to prevent traffic congestion in the disaster-affected area, as to provide enough and flexible traffic capacity for emergency evacuation and emergency rescue. Therefore, identification catastrophe border which will transfer the traffic flow unstable, disorder and trending toward congestion, is the foundation and the key in regional traffic flow control under sudden disaster. Furthermore, most high-density urban areas in China are in the old town of inner cities, the electrical and gas systems are older, degradation and prone to trigger sudden fire disaster. Many sudden fire disasters reported were caused by the ageing electrical and gas systems in high-density areas. With the aim of understanding the aggregation, propagation and distribution of traffic flow characteristics in high-density area under sudden fire disaster, providing a theoretical basis method for traffic agents to manage and control regional traffic flow in disaster-affected area, and to prevent the regional traffic flow being caught into disorder and congestion. This paper tempts to identify the catastrophe border of traffic flow in high-density area under sudden fire disaster. This study focuses on:

(1) Using catastrophe theory to analysis the traffic flow characteristics in high-density area under sudden fire disaster, modeling a regional traffic flow model based on Cusp Catastrophe Theory (CCT) to describe the changing of traffic flow and to identify the catastrophe border of traffic flow.

(2) For the aim of regional traffic flow control, using VISSIM simulator data with different collection intervals to identify the catastrophe border, and analysis the suitable data collection interval for adaptive and automatic traffic control in sudden fire disaster.

The rest of the paper is structured as follows: Section 2 introduces the state-of-the-art research under fire disaster. Section 3 presents the proposed methods for analyzing traffic flow under sudden fire disaster. Simulation and calibration are presented in Section 4 and Section 5 concludes the paper with contributions and limitations, as well as the perspectives on future work.

2. Literature Review

In the last decade, researchers mainly focused on the evacuation problem in the case of sudden fire disaster. Zheng[6] investigated the dynamics of pedestrian evacuation with the influence of the fire spreading. A numerical model based on cellular automaton was proposed by Yuan [7] to simulate the human behavior termed "flow with the stream" in emergency evacuation from a large smoke-filled compartment. Evacuation features from a terrace classroom were investigated by Xi [8] through simulations using both the models and experiments. A modified particle swarm optimization algorithm was proposed by Li[9] to investigate the dynamic of pedestrian evacuation from a fire occurred in a public building—a supermarket with multiple exits and configurations of counters. Sahin[10] proposed an approach which combines a multi-agent model with fuzzy logic to smoothly and successfully handle multiple features of each individual to simulate common human and group behavior during safety egress. Delcea[11] analyzed the possibility of using agent-based simulations in Net-Logo for a classroom with two exits in order to increase pupils' awareness on how to act during an evacuation process caused by sudden-onset fire disaster. The floor field model consisted of static floor field, dynamic floor field and emergency diffusion field was established by Lin[12], the corresponding updating rules were defined. Based on the stair-unit model, a schedule-line model was proposed by Wen[13] to calculate evacuation paths in stair-units; a modified algorithm was proposed to describe the evacuee movements in stairwells as to calculate the pedestrian forces; and a projection strategy was proposed to model the 3-dimensional evacuation process in multi-floor buildings. Wen[14] found that, with the increase of the occupant density, the evacuation efficiency would decline.

In the past few years, scholars focused on studying the sudden fire disaster in tunnels. Sýkora[15] designed a simple but realistic model to calculate the hazards posed by tunnel sudden fire disaster. Aralt[16] exanimated the smoke and heat detection systems to research what kind of principle is the best suitable for detecting a fire in an early stage. Numerical study was conducted by Hua[17] to investigate the smoke control strategies for a designed sudden-onset fire scenario in the Beijing Center Business District (CBD). Caliendo[18] presented a model to simulate the effects of sudden fire disaster due to different vehicle types in a bi-directional road tunnel. A set of full-scale experiments was conducted by Yu[19] to study thermal and smoke control strategies using transverse ventilation system in a sloping urban traffic link tunnel and the results showed that it was not the case: the slower the smoke spread longitudinally, the better the smoke being controlled. Experimental studies on the smoke spread in a titled tunnel were carried out by Li[20] with a reduced-scale tunnel model.

From the literature mentioned above, researchers mainly focused on fire disaster dynamic diffusion mechanism analysis, emergency evacuation models establishment, traffic behavior in sudden disaster occurring in the building and specially points of traffic network, such as tunnel. The studies on traffic flow characteristics in traffic network under sudden fire disaster are limited in the state-of-the-art research literature. However, Traffic flow characteristics analysis is the foremost step in emergency traffic organization, such evacuation route optimization, emergency vehicle dispatching, and disaster affected area traffic control and management. Therefore, this paper tries to analysis traffic flow characteristics in high-density urban central areas under the sudden fire disaster.

Traditional traffic flow theory is intuitive and easy to understand, which can explain the relationship between traffic flow parameters by the two-dimensional relationship. However, when traffic flow approaches to the capacity of road, the traditional traffic flow theory does not explain traffic characteristics well. Traffic congestion is common in road segments after sudden fire disaster.

It is not suitable to use the traditional traffic flow theory to analyze the road traffic system after a sudden fire disaster.

The catastrophe theory can handle complex linear and nonlinear relationships simultaneously, using a high order probability density function that has the advantage of being able to describe mutation behavioral [21]. It is particularly suitable for systems in which continuous changes of parameters can result in discontinuous changes in outcome variables. It was widely used in social and behavioral sciences such as sudden changes in a person's behavior[22], crowd congestion in public buildings[23]. In the field of traffic system, Dendrinos[24] first applied catastrophe theory to traffic analysis, and used two-dimensional catastrophe model to describe the relationship between traffic volume and VC ratio. However, due to the complexity of traffic flow system, the model could not describe traffic flow system in detail. Navin[25] proposed a three-dimensional cusp catastrophe model in 1986, which can express the relationship of traffic flow's speed, volume and density more clearly. Forbes and Hall[26] argued that the traditional relationship between speed, volume and density was not enough to analyze the characteristics of traffic flow when traffic flow approaching to the traffic capacity of road segment, and used the cusp catastrophe theory to study the traffic flow characteristics of Queen Elizabeth Avenue in Ontario during peak hours.

3. CCT-based Traffic Flow Model

3.1. Cusp Catastrophe Theory

The catastrophe theory is a principle to describe the phenomenon that transitions from a stable configuration to another stable configuration. It points out that the status of motion in nature or in human society has two states: stability and instability. Under the action of tiny accidental disturbance factors, the status still keeping stability is called the stability status. On the contrary, the status quickly changing to the unstable status with slightly disturbed is called the instability status. The stability status and instability status are interlaced. The transformation of a nonlinear system from one steady status or equilibrium status to another stabilized state occurs in a mutant form. As a powerful mathematical tool in studying the evolution of systematic order, catastrophe theory can better explain and predict sudden phenomena in nature and society. Catastrophic change emphasizes the meaning of discontinuities or mutational changes in the process of changing. In most high-density urban centers, the traffic flow always within a stable equilibrium and unstable equilibrium boundary in the peak hours. With sudden fire disaster, the traffic flow status in high-density urban central area will be catastrophic changing and the traffic flow status will be transiting. The continuous steady traffic flow status will be interrupted and lead to qualitative changes.

In the case of sudden fire disaster, the traffic flow in the disaster affected-area can be managed and controlled by intelligent transportation systems (such as adaptive traffic signal control system, advance route guidance system) or human being with insulate fences. Through controlling the inflow and outflow in the boundary of disaster-affected area and managing the speed within the disaster-affected area, the density or occupancy of regional traffic flow can be estimated and manage by the relationship of traffic flow characteristics and intelligent transportation systems. From the perspective of catastrophe theory, the cusp catastrophe is the simplest model to describe the catastrophe characteristics with two control variables and one state variable. Therefore, the relationship of flow-speed-occupancy can correspond to the variables in cusp catastrophe model. The relationship of variables in cusp catastrophe can be described as formula (1), when there is a slight change in control variable, then the catastrophic change will be happened in state variable.

$$\mathrm{U(x,y,z)} = x^4 + yx^2 + zx \quad (1)$$

where: x is the internal state variable of the system; y and z are the external control variables. By taking the derivative of formula (1), the equilibrium surface can be obtained as formula (2) and Figure 1[27].

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 April 2020 doi:10.20944/preprints202004.0157.v1

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

$$4x^3 + 2yx + z = 0 \tag{2}$$

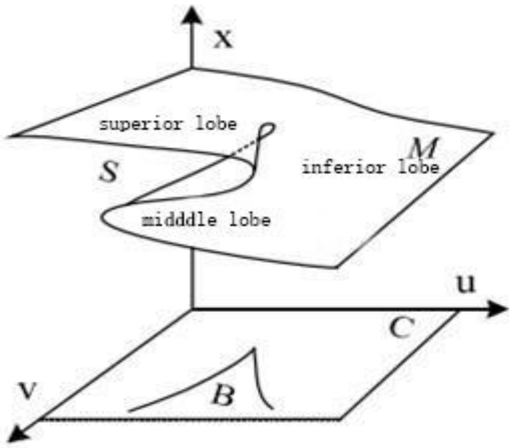


Figure 1. Cusp catastrophe equilibrium surface.

The critical equilibrium surface is divided into upper, middle and lower parts. As shown in Figure 1, the upper and lower parts represent stable regions. In the middle part, the state of the system jumps and becomes unstable. The middle lobe is composed of folding regions, and the folding area can be projected to the bifurcation set. The bifurcation set is shown as B section in Figure 1. When (u,v) is outside the bifurcation set, the changing in the control variables will lead a continual change in the system status. However, when the point enters the bifurcation set, a sudden jump of the state variable occurs. Therefore, the critical value of mutation corresponds to a discriminant equal to zero[28].

Due to sudden fire disaster, the traffic flow will presents as an aggregation characteristic in disaster-affected area. As to evacuate, the traffic flow will concentrate to the road segments and intersections. Because of traffic congestion, the vehicle speed decreases sharply. This phenomenon is consistent with the characteristics of the cusp catastrophe theory. Therefore, this paper focuses on using the cusp catastrophe theory to describe the catastrophe change of traffic flow under sudden fire disaster. Before the sudden fire disaster happen, the traffic network is in a stable state of high speed and low occupancy, which corresponds to the upper leaf of cusp catastrophe theory. Under the sudden fire disaster, the capacity of traffic network is decreasing, and the traffic network is changing to the state of low speed and high occupancy, which corresponds to the lower leaf of cusp catastrophe theory. It is considering that there is a sudden transition between two states with a catastrophe border. Therefore, a regional traffic flow model is proposed to identify the catastrophe border based on the formula (2).

3.2. CCT-based Traffic Flow Model under Sudden Fire Disaster

There are two critical issues need to be considered in constructing the traffic flow model based on cusp catastrophe theory: (a) the selection of traffic flow parameters and (b) the corresponding relationship between traffic flow parameters and variables in catastrophe theory.

(a) Traffic flow parameters selection

When a sudden fire disaster happening, the range of affection to traffic flow is not only the location where fire disaster happened, but also the adjacent road segments and intersections. It is a regional affection to traffic network. Therefore, the parameters of regional traffic flow are suitable to describe the status and degree of affection and changing for disaster-affected traffic network. The cusp catastrophe model has three variables, state variables x and control variables y and z. Considering the possibility of traffic data collection and analysis under sudden fire disaster, the average traffic flow volume, average traffic flow occupancy and average traffic flow speed are selected to describe the characteristics of traffic flow in the sudden fire disaster affected area.

(b) Relationship between traffic flow parameters and variables in catastrophe theory

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

In the corresponding relationship between traffic flow parameters and cusp catastrophe model's variables, the most important thing is to determine the traffic flow parameter corresponding to the state variables. In reality, the most intuitive indicator of traffic state is speed, so the regional average speed is chosen as the state variable. From the perspective of traffic flow theory, the catastrophic change of speed may be caused by a slow change of occupancy or flow. Therefore, it is feasible to select the regional average speed as the state variable. Then, the remaining parameters regional average flow and regional average occupancy, correspond to the control variables y and z in catastrophe theory.

The regional traffic flow model based on cusp catastrophe theory can be constructed as follow:

$$\mathrm{E}(\mathrm{x},\mathrm{y},\mathrm{z}) = \mathrm{a}x^4 + \mathrm{b}yx^2 + zx \tag{3}$$

Where: x is the average traffic flow speed in sudden fire disaster affected area. y and z are the average traffic flow volume and average traffic flow occupancy in the sudden fire disaster affected area, respectively. By calculating the first partial differential of the formula (3), the critical equilibrium surface equation of the state of disaster affected traffic network can be described as follows:

$$4\mathrm{a}x^3 + 2byx + z = 0 \tag{4}$$

For further simplification, formula (4) can be simplified as:

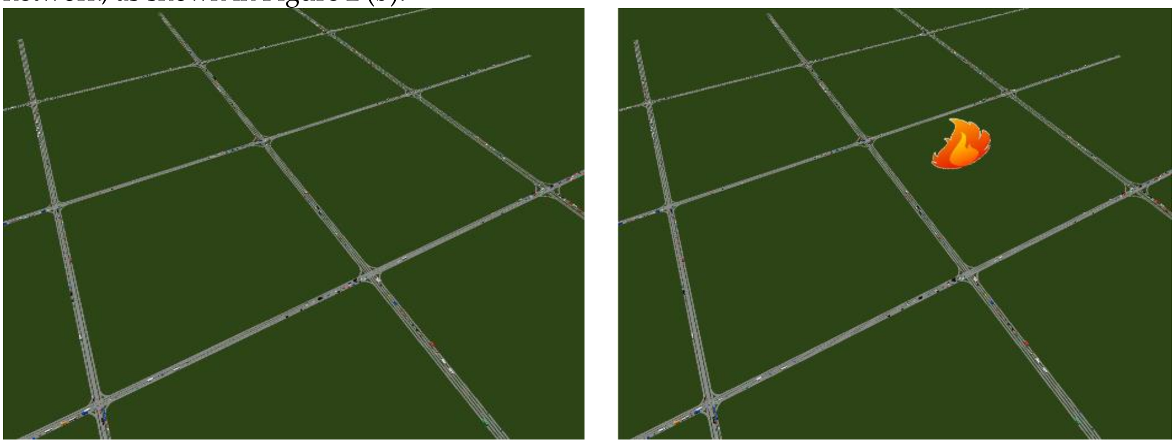
$$\mathrm{c}x^3 + dyx + z = 0 \tag{5}$$

Furthermore, the stable state traffic flow of disaster-affected area can be demonstrated by high speed and low occupancy of traffic flow before the sudden fire disaster. The instable state traffic flow of disaster-affected area can be demonstrated by low speed and high occupancy of traffic flow after the sudden fire disaster. The distribution and transition of the states can be analyzed in the three-dimensional space to identify the catastrophe border.

4. Simulation and Calibration

4.1. Simulation Scenarios and Data Collection

In this paper, we use the microscopic traffic flow simulator VISSIM 8.0 (PTV Group, Karlsruhe, Germany) to simulate and collect data to calibrate the proposed model. As to collect the traffic flow stable state and instable state dataset, we first simulate the saturated traffic flow without sudden fire disaster in a Sudoku traffic network, as shown in Figure 2. Then, with the same traffic flow input, we run the simulator with sudden-onset fire disaster happening in the Sudoku traffic network, as shown in Figure 2 (b).



(**a**) (**b**)

Figure 2. Snapshot of traffic network in simulator

There are nine intersections in Figure 2, and each entrance direction of intersection has two lanes, where the inside lane is a straightforward lane, the outside lane is a straight and right-turn lane. In the Sudoku traffic network, the length of each arterial is 2400 meters, and including 3 intersections. The traffic flow is composed of 80% of cars, 10% of trucks and 10% of buses. The input traffic flow in VISSIM simulator is setting as 800 pcu/lane/h. The expected vehicle speed in simulator is setting as 45km/h. As to simulate traffic flow under sudden fire disaster, we use the VISSIM COM and VC++ to restrict the capacity, speed, vehicle routes based on the temporal and spatial distribution influence model under traffic accident in the urban traffic network[29-34]. Then, the simulator was performed three times with and without sudden fire disaster respectively. The output evaluation data including flow, occupancy and speed of the traffic network are collected with different time intervals each time.

4.2. Data Processing and Analysis

Different time interval of traffic data will show out different accuracy and performance in regional traffic flow managing and controlling. In this paper, traffic data was collected at 5s interval, 10s interval and 15s interval with and without sudden fire disaster in the simulation network respectively as to identify the catastrophe border of traffic flow.

Data from simulator was set to obtain the regional average flow, occupancy and speed of traffic flow. From the running time sequences, we get out of 240 groups of data in each case and each interval from the simulator.

As to highlight the characteristics of traffic data, the regional average flow, occupancy and speed of traffic flow data are given normalized processing. The processing method is as follows:

$$X = \text{speed} - \text{speed at capacity} \tag{6}$$

$$Y = (\text{flow} - \text{capacity})/100 \tag{7}$$

$$Z = \text{occupancy} - \text{maximum occupancy at maximum flow} \tag{8}$$

The capacity in the formula is the statistical capacity, and the maximum flow is collected in each interval. After normalization processing, the traffic flow parameters in different traffic conditions can be converted into corresponding to the variables in cusp catastrophe theory, respectively.

Figure 3 shows the traffic flow data at 5s interval with sudden fire disaster after normalization processing. Figure 4 shows the traffic flow data at 5s interval without sudden fire disaster after normalization processing.

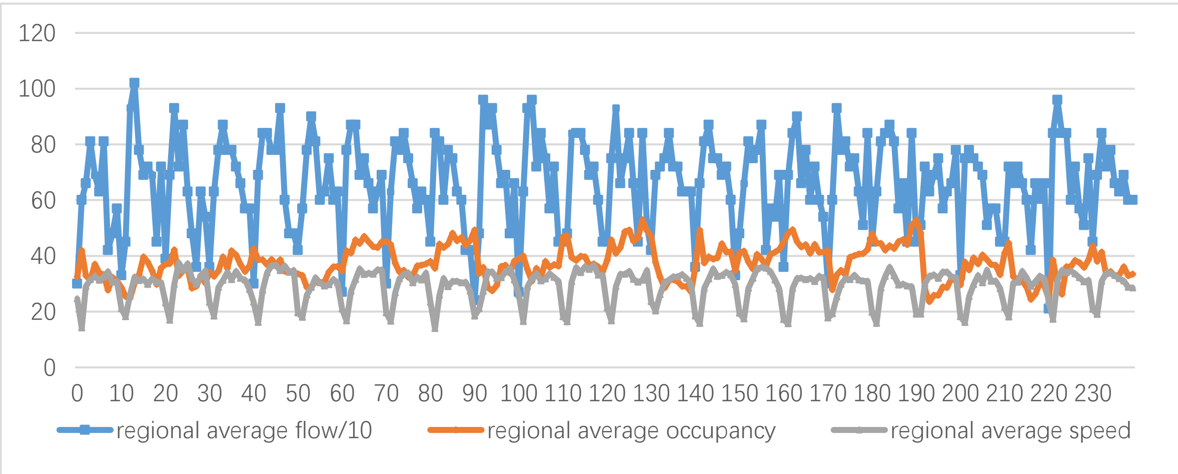


Figure 3. Traffic flow data at 5s interval with sudden fire disaster.

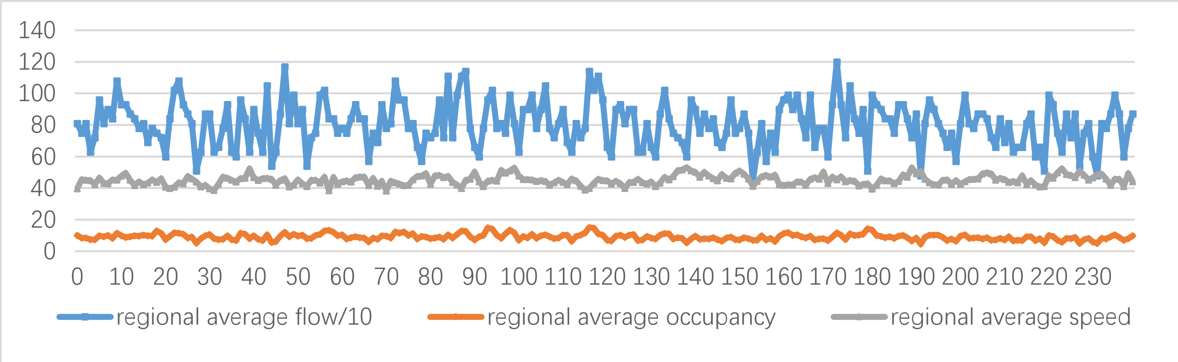


Figure 4. Traffic flow data at 5s interval without sudden fire disaster.

By transformation using the formula (6) to (8), the traffic flow data of Figure 3 and Figure 4 are converted as follows:

$$X = \text{speed} - 45.2375 \quad (9)$$

$$Y = (\text{flow} - 1200)/100 \quad (10)$$

$$Z = \text{occupancy} - 30.175 \quad (11)$$

The corresponding three-dimensional point diagrams obtained from the transformed data in Figure 3 and Figure 4 are shown in Figure 5.

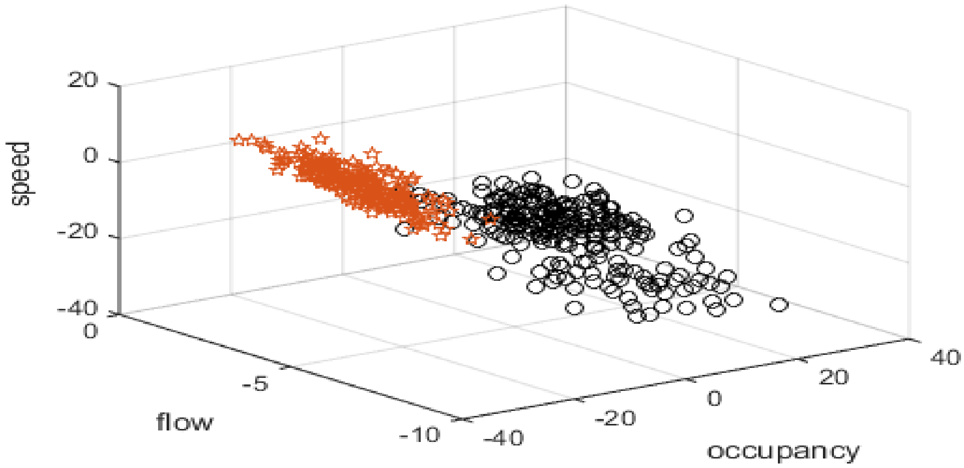


Figure 5. Distribution of traffic flow data at 5s interval with and without sudden fire disaster.

Note: The red pentagon in the Figure 5 represents the traffic flow data without sudden fire disaster situation, and the black circle represents the traffic flow data with sudden fire disaster.

In Figure 5, it can be clearly seen that after conversion, the traffic flow data of regional average flow, occupancy and speed without sudden fire disaster situation are distributed in a small area, relatively concentrated, and located on the positive half axis of speed. In generally, the traffic status without sudden fire disaster is in a high-speed stable state. The regional average flow, occupancy and speed of traffic flow under sudden fire disaster are more discretization comparing to those without sudden fire disaster. Before sudden fire disaster happening, the traffic network was in a state of high speed and low occupancy (as shown by the red pentagon in Figure 5). During the sudden fire disaster happening, the traffic flow is changing to the status of low speed and high occupancy (as shown by the black circle in Figure 5). According to the catastrophe theory, there is a catastrophe border to make the catastrophe change occur. Therefore, the catastrophe border can be identified by catastrophe fitting with the transformed data. The transformed data was fitted as Figure 6.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 April 2020 doi:10.20944/preprints202004.0157.v1

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

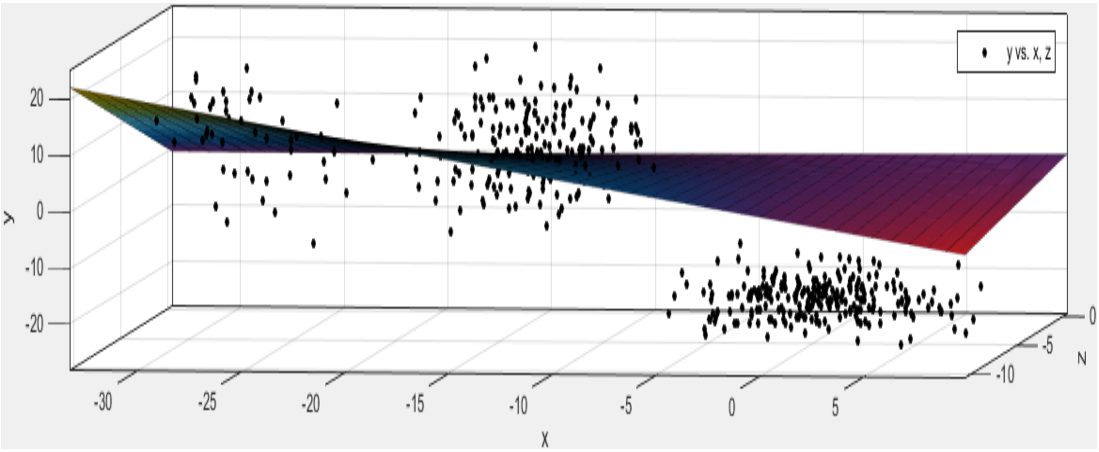


Figure 6. Fitting chart of traffic flow data at 5s interval.

In Figure 6, X represents the regional average speed, Y represents the regional average flow, and Z represents the regional average occupancy. From the angle of x-axis, we can see the obvious catastrophe border.

The parameters obtained after the fitting are as follows:

$$c = -0.0001511 \quad d = -0.0621 \tag{12}$$

Therefore, the equation of critical equilibrium surface obtained by the fitting is as follows:

$$0.0001511X^3 + 0.0621\mathrm{YX} - \mathrm{Z} = 0 \tag{13}$$

In order to find the catastrophe border under sudden fire disaster and determine the singularity set, the potential function is quadratically differentiated according to the catastrophe dynamics method, and the singularity set formula is obtained as follows:

$$\begin{cases} 0.0001511X^3 + 0.0621YX - Z = 0 \\ 0.0004533X^2 + 0.0621Y = 0 \end{cases} \tag{14}$$

The singularity set of the equilibrium surface is two creases in the folded part of the equilibrium surface. By eliminating the state variable X and satisfying formula (14), the bifurcation set of the stability of the traffic flow state can be obtained as follows:

$$Z^2 + 0.234805Y^3 = 0 \tag{15}$$

Therefore, the discriminant is formed as follows:

$$\Delta = Z^2 + 0.234805Y^3 \tag{16}$$

When $\Delta < 0$, the traffic flow is in an instable state. When $\Delta > 0$, the traffic flow is in a stable state (including the traffic states with high speed and low occupancy, low speed and high occupancy). When $\Delta = 0$, the traffic flow is in catastrophe border.

Small changes in traffic flow and occupancy can cause huge changes in speed when the above discriminant equals zero. According to the traffic data obtained at 5s interval, the critical occupancy value is based on the border between traffic flow with sudden fire disaster and traffic flow without sudden fire disaster. Solution the formula (14) and (15), the occupancy value is 20%, and the catastrophe border is 439 pcu/lane/h. That is to say, for the aim of prevent traffic congestion and disorder under sudden fire disaster, the traffic flow should not exceed 439 pcu/lane/h within the disaster-affected area.

Figure 7 shows the traffic flow data at 10s interval with sudden fire disaster after normalization processing. Figure 8 shows the traffic flow data at 10s interval without sudden fire disaster after normalization processing.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 April 2020 doi:10.20944/preprints202004.0157.v1

Peer-reviewed version available at *Appl. Sci.* **2020**, *10*, 3197; doi:10.3390/app10093197

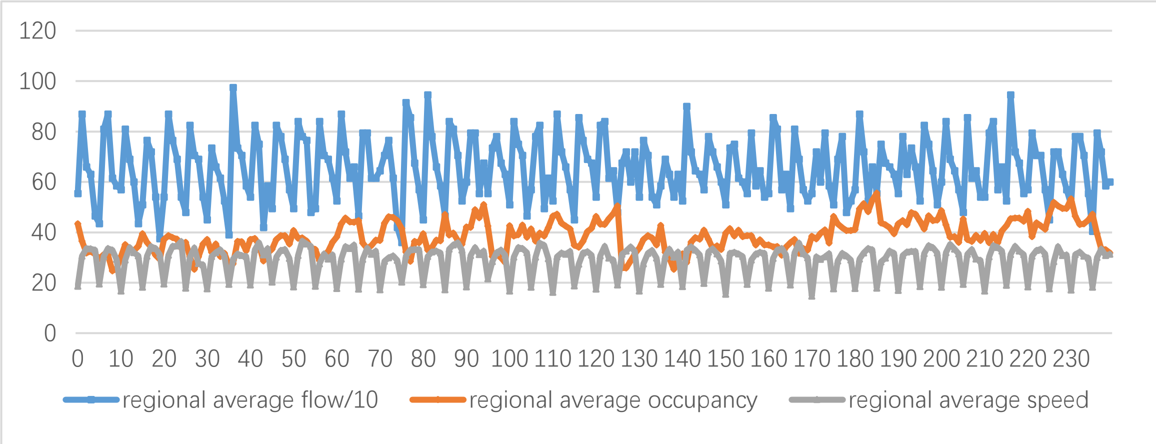


Figure 7. Traffic flow data at 10s interval with sudden fire disaster.

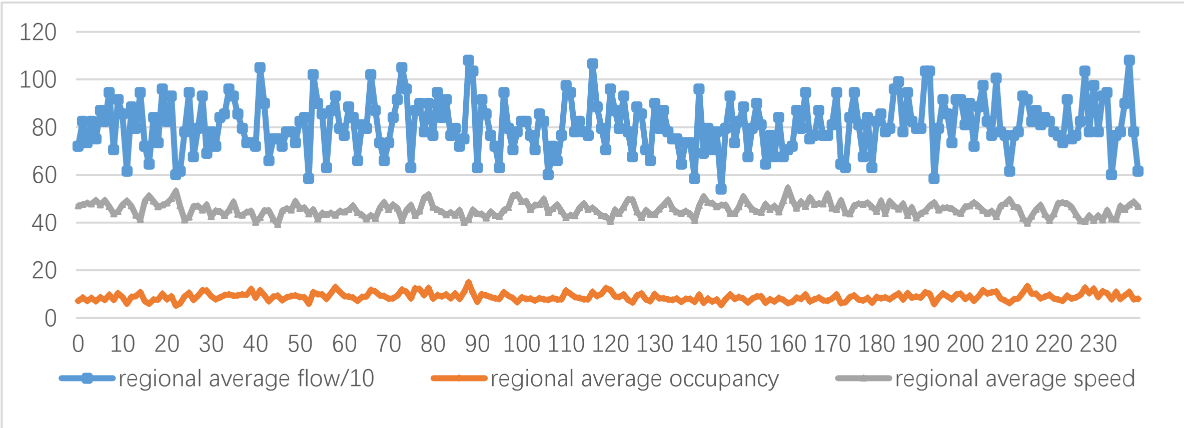


Figure 8. Traffic flow data at 10s interval without sudden fire disaster..

By transformation using the formula (6) to (8), the traffic flow data of Figure 7 and Figure 8 are converted as follows:

$$X = \text{speed} - 47.4333 \quad (17)$$

$$Y = (\text{flow} - 1080)/100 \quad (18)$$

$$Z = \text{occupancy} - 27.49167 \quad (19)$$

The corresponding three-dimensional point diagrams obtained from the transformed data in Figure 7 and Figure 8 are shown in Figure 9.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 April 2020 doi:10.20944/preprints202004.0157.v1

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

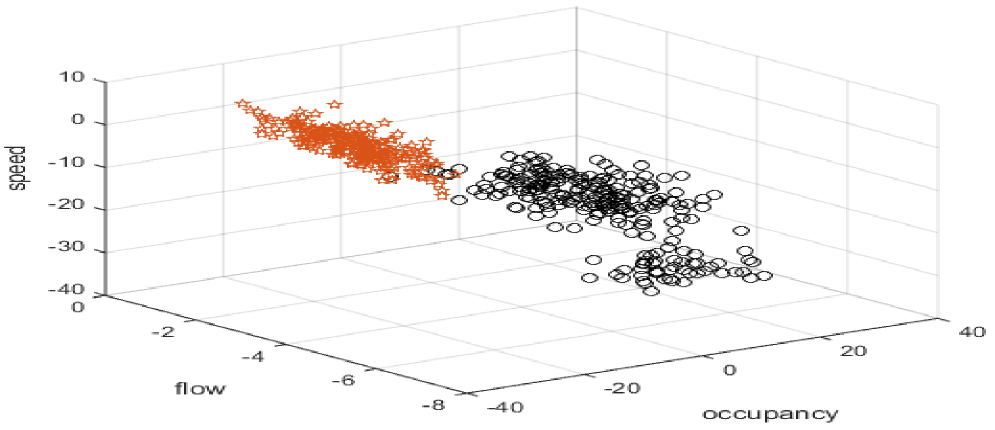


Figure 9. Distribution of traffic flow data at 10s interval with and without sudden fire disaster

The distribution of traffic flow data in Figure 9 is similar to that in Figure 5. The traffic status without sudden fire disaster is in a high-speed stable state while the traffic status under sudden fire disaster is in the instable state. The traffic flow data under sudden fire disaster is more discretization comparing to those without sudden fire disaster. Considering the transformation of two states, he catastrophe border can be identified by catastrophe fitting with the transformed data. The transformed data was fitted as Figure 10.

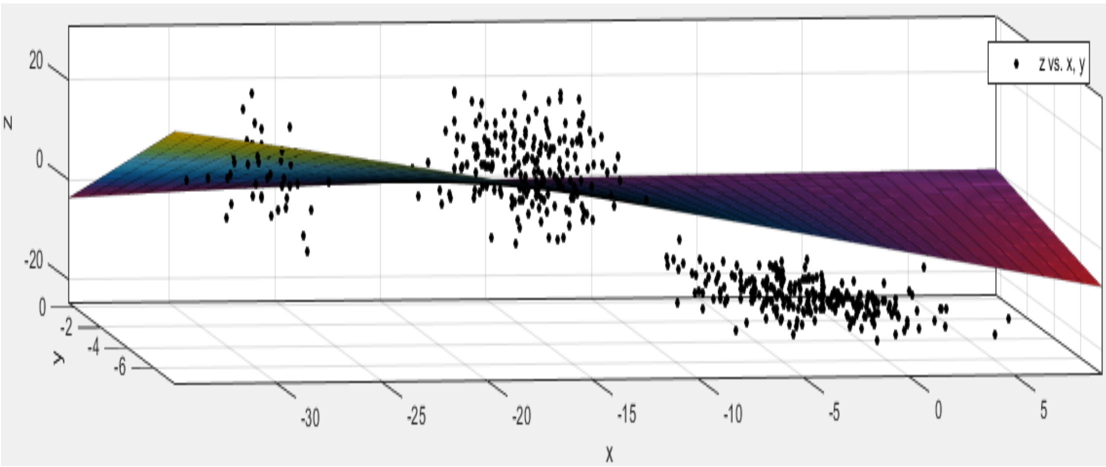


Figure 10. Fitting chart of traffic flow data at 10s interval.

In Figure 10, X represents the regional average speed, Y represents the regional average flow, and Z represents the regional average occupancy. From the angle of x-axis, we can see the obvious catastrophe border.

The parameters obtained after the fitting are as follows:

$$c = -0.0005264 \quad d = -0.1059 \tag{20}$$

Therefore, the equation of critical equilibrium surface obtained by the fitting is as follows:

$$0.0005264X^3 + 0.1059YX - Z = 0 \tag{21}$$

In order to find the catastrophe border under sudden fire disaster and determine the singularity set, the potential function is quadratically differentiated according to the catastrophe dynamics method, and the singularity set equation is obtained as follows:

$$\begin{cases} 0.0005264X^3 + 0.1059YX - Z = 0 \\ 0.0015792X^2 + 0.1059Y = 0 \end{cases} \tag{22}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 April 2020 doi:10.20944/preprints202004.0157.v1

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

The singularity set of the equilibrium surface is two creases in the folded part of the equilibrium surface. By eliminating the state variable X and satisfying formula (22), the bifurcation set of the stability of the traffic flow state structure can be obtained as follows:

$$Z^2 + 0.334248Y^3 = 0 \quad (23)$$

Therefore, the discriminant is formed as follows:

$$\Delta = Z^2 + 0.334248Y^3 \quad (24)$$

When $\Delta < 0$, the traffic flow is in an instable state. When $\Delta > 0$, the traffic flow is in a stable state (including the traffic states with high speed and low occupancy, low speed and high occupancy). When $\Delta = 0$, the traffic flow is in catastrophe border.

According to the traffic data obtained at 10s interval, the critical occupancy value is based on the border between traffic flow with sudden fire disaster and traffic flow without sudden fire disaster. Solution the formula (22) and (23), the occupancy value is 20%, and the catastrophe border is 529 pcu/lane/h. That is to say, for the aim of prevent traffic congestion and disorder under sudden fire disaster, the traffic flow should not exceed 529 pcu/lane/h within the disaster-affected area.

Figure 11 shows the traffic flow data at 15s interval with sudden fire disaster after normalization processing. Figure 12 shows the traffic flow data at 15s interval without sudden fire disaster after normalization processing.

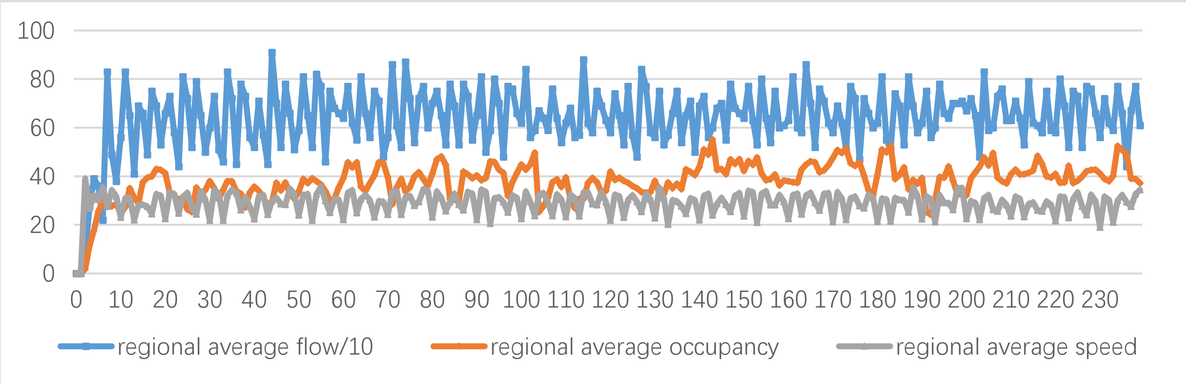


Figure 11. Traffic flow data at 15s interval with sudden fire disaster.

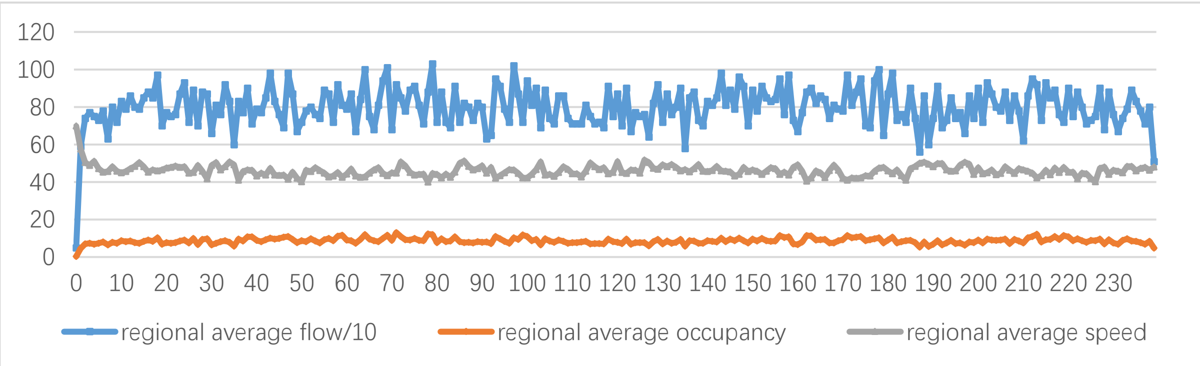


Figure 12. Traffic flow data at 15s interval without sudden fire disaster.

By transformation using the formula (6) to (8), the traffic flow data of Figure 11 and Figure 12 are converted as follows:

$$X = \text{speed} - 44.62913 \quad (25)$$

$$Y = (\text{flow} - 1030)/100 \quad (26)$$

$$Z = \text{occupancy} - 27.34583 \quad (27)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 April 2020 doi:10.20944/preprints202004.0157.v1

Peer-reviewed version available at Appl. Sci. 2020, 10, 3197; doi:10.3390/app10093197

The corresponding three-dimensional point diagrams obtained from the transformed data in Figure 11 and Figure 12 are shown in Figure 13.

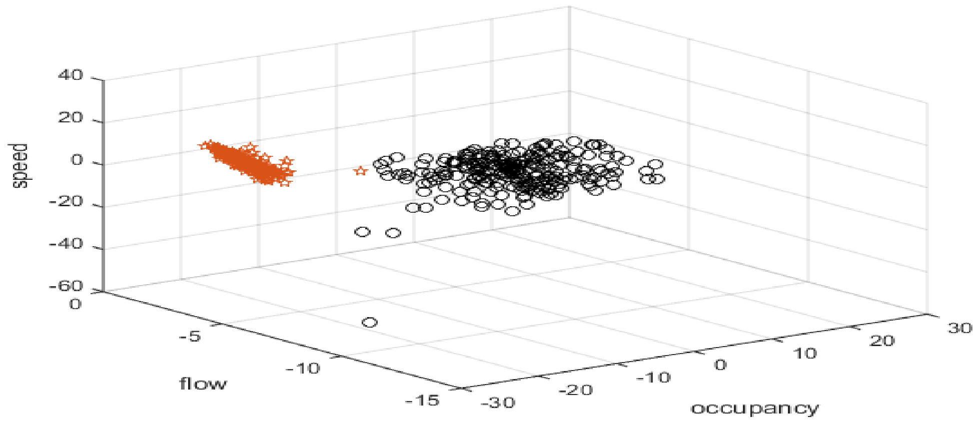


Figure 13. Distribution of traffic flow data at 15s interval with and without sudden fire disaster

The distribution of traffic flow data in Figure 13 is similar to that in Figure 9 and Figure 5. The traffic status without sudden fire disaster is in a high-speed stable state while the traffic status under sudden fire disaster is in the instable state. The traffic flow data under sudden fire disaster is more discretization comparing to those without sudden fire disaster. Considering the transformation of two states, he catastrophe border can be identified by catastrophe fitting with the transformed data. The transformed data was fitted as Figure 14.

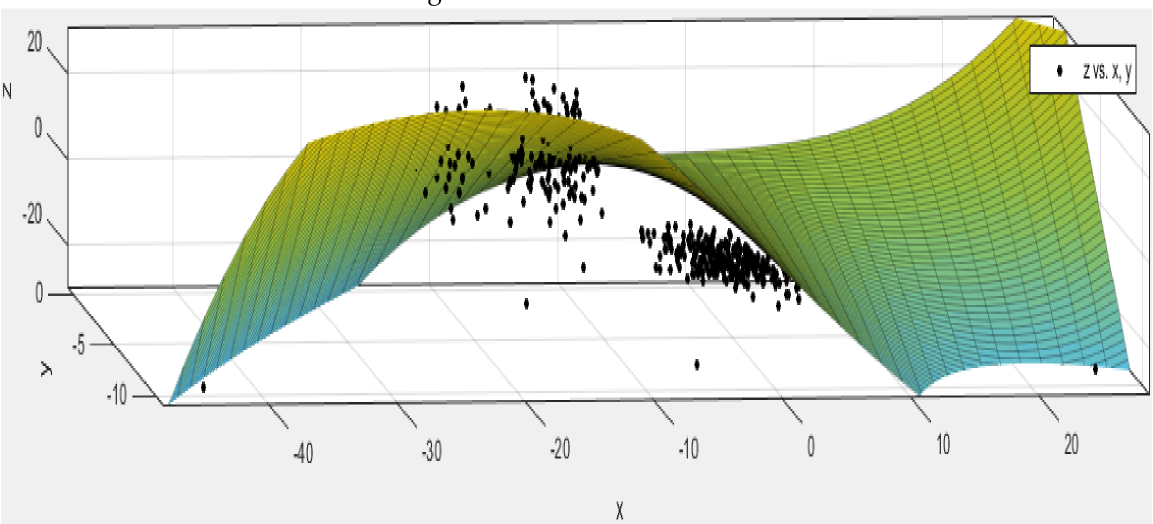


Figure14. Fitting chart of traffic flow data at 15s interval.

In Figure 14, X represents the regional average speed, Y represents the regional average flow, and Z represents the regional average occupancy. From the angle of x-axis, we can see the obvious catastrophe border.

The parameters obtained after the fitting are as follows:

$$c = -0.001583 \quad d = -0.2745 \tag{28}$$

Therefore, the equation of critical equilibrium surface obtained by the fitting is as follows:

$$0.001583X^3 + 0.2745YX - Z = 0 \tag{29}$$

In order to find the catastrophe border under sudden fire disaster and determine the singularity set, the potential function is quadratically differentiated according to the catastrophe dynamics method, and the singularity set equation is obtained as follows:

$$\begin{cases} 0.001583X^3 + 0.2745YX - Z = 0 \\ 0.004746X^2 + 0.2745Y = 0 \end{cases} \tag{30}$$

The singularity set of the equilibrium surface is two creases in the folded part of the equilibrium surface. By eliminating the state variable X and satisfying formula (30), the bifurcation set of the stability of the traffic flow state structure can be obtained.

$$Z^2 + 1.935719Y^3 = 0 \tag{31}$$

Therefore, the discriminant is as follows:

$$\Delta = Z^2 + 1.935719Y^3 \tag{32}$$

When $\Delta < 0$, the traffic flow is in an instable state. When $\Delta > 0$, the traffic flow is in a stable state (including the traffic states with high speed and low occupancy, low speed and high occupancy). When $\Delta = 0$, the traffic flow is in catastrophe border.

According to the traffic data obtained at 15s interval, the critical occupancy value is based on the border between traffic flow with sudden fire disaster and traffic flow without sudden fire disaster. Solution the formula (30) and (31), the occupancy value is 20%, and the catastrophe border is 377 pcu/lane/h. That is to say, for the aim of prevent traffic congestion and disorder under sudden fire disaster, the traffic flow should not exceed 377 pcu/lane/h within the disaster-affected area. The catastrophe border at different data collection interval is shown as Table 1.

Table 1. Model fitting and catastrophe border identification at different data collection interval

Data Collection Interval	**Fitting result**		**Catastrophe Border**	
	c	d	regional average occupancy	regional average flow (pcu/lane/h)
5s	-0.0001511	-0.0621	20%	439
10s	-0.0005264	-0.1059	20%	529
15s	-0.001583	-0.2745	20%	377

Chen and You[35] analyzed the traffic flow characteristics under sudden fire disaster in road segment used a Capacity Assessment Approach (CAA) based on VISSIM simulator data. The study results showed that the remaining capacity of the road after the fire is 61.611%. It can be concluded that if the traffic flow is 800 pcu/lane/h in traffic network in normal traffic condition, the capacity of traffic flow will be remain about 493 pcu/lane/h under sudden fire disaster. If the volume of traffic flow exceed this value, the traffic network will be caught in congestion and disorder. This value can be considered as the catastrophe border of traffic network under sudden disaster. So the model ability at different data collection interval based on cusp catastrophe theory can be seen in Table 2.

Table 2. Model at different data collection interval based on cusp catastrophe theory.

Data Collection Interval	**Catastrophe Border based on CCT (pcu/lane/h)**	**Catastrophe Border based on CAA (pcu/lane/h)**	**Relative Precision (%)**
5s	439		89.1
10s	529	493	92.7
15s	377		76.5

In Table 2, the relative accuracy of catastrophe border identification at 5s interval and 10s interval are 89.1% and 92.7% compare to the method of capacity assessment approach (CAA) respectively, which shows that the study in this article based on cusp catastrophe theory can describe the traffic flow characteristics effectively at fine granularity data collection interval. However, the catastrophe border identification error at 15s interval is larger, which is more than 20%. The cause of the larger error may be: traffic flow parameters in coarse granularity can not catch the traffic flow changing accurately in the simulation; and traffic flow mutation happening under sudden fire disaster always at smaller data collection interval.

5. Conclusion

In this paper, we use the method of cusp catastrophe theory to model the regional traffic flow in high-density area under sudden fire disaster. The mathematical modeling method is proposed to study the changing process of traffic flow from the stable state to instable state based on cusp catastrophe theory, as to improve the understanding of traffic flow aggregation, propagation and distribution characteristics in disaster affected area under sudden fire disaster. This paper explores the catastrophe border of regional traffic flow as to provide a theoretical basis for traffic agents to manage and control regional traffic flow in disaster-affected area, to prevent the regional traffic flow being caught into disorder and congestion. Finally, we use the microcosmic traffic flow simulator VISSIM to simulate and collect the traffic data in high-density area under sudden fire disaster. Throughout the data normalization and analysis, the catastrophe border is identified as 439pcu/lane/h, 529pcu/lane/h, 377pcu/lane/h at 5s, 10s, 15s data collection interval respectively. The relative precision, which compares to the method of capacity assessment approach, is 89.1%, 92.7% and 76.5% respectively. It means that 10s data collection interval may be the most suitable data collection interval in catastrophe border identification and regional traffic flow control in high-density area under sudden fire disaster. The low accuracy at 15s data collection interval may infer that the stable state of traffic flow catastrophe changing to instable state may be happen in short time, coarse granularity data collection interval can not capture the transformation process accurately.

However, there are some topics remain to be studied, further research work includes: (1) the aggregation, propagation mechanism of reginal traffic flow under sudden disaster and their influence factors should be identified and analyzed. The changing of spatial-temporal distribution during the sudden disaster should be considered in the mathematical model. (2) In high-density areas of urban central, the types of land uses, traffic networks, and transportation patterns are different. In this paper, we just consider the vehicle stream in a simple Sudoku traffic network. Pedestrian flow and its disturb to traffic flow should be considered and analyzed in a real traffic network. (3) At present, only the simulator-based tests are performed, the field traffic data in high-density area before, during and after sudden fire disaster should be carried out in the future.

Author Contributions: Conceived and designed the experiments: C.L. D.W. Performed the experiments: B.G. Y.Y. Analyzed the data: C.L. Y.Y. Contributed reagents/materials/analysis tools: C.L. Y.Y. D.W. B.G. Wrote the paper: C.L. Y.Y. D.W.

Funding: The authors acknowledge the National Natural Science Foundation of China (Grant No. 51408257 and 51308248), Youth Scientific Research Fund of Jilin (Grant No. 20180520075JH), and the Science and technology project of Jilin Provincial Education Department (Grant No. JJKH20170810KJ and JJKH20180150KJ) are partly support this work.

Acknowledgments: We are very thankful to the reviewers for their time and efforts, their comments and suggestions have greatly improved the quality of this paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Qiao, W. Disaster Prevention Planning Study for the High-Density Area of City Center. Ph.D, Tianjing Universirt, Tianjing, 2013.
2. Ciyun, L. Research on Key Technologies of Decision Support System for Emergency Traffic Guarantee in Disaster. Ph.D, Jilin University, Changchun, China, 2010.
3. Wang, Y.; Rong, J.; Zhou, C.; Gao, Y. Dynamic Estimation of Saturation Flow Rate at Information-Rich Signalized Intersections. *Information* **2020**, *11*, doi:https://doi.org/10.3390/info11040178.
4. Guangsheng, J. Study on Emergency Traffic Organization and Evacuation Strategy in Metropolises. Beijing University of Technology, Beijing, 2006.

5. Wang, Y.; Zhao, L.; Li, S.; Wen, X.; Xiong, Y. Short Term Traffic Flow Prediction of Urban Road Using Time Varying Filtering Based Empirical Mode Decomposition. *Applied Sciences* **2020**, *10*, doi:https://doi.org/10.3390/app10062038.
6. Zheng, Y.; Jia, B.; Li, X.-G.; Zhu, N. Evacuation dynamics with fire spreading based on cellular automaton. *Physica A: Statistical Mechanics and its Applications* **2011**, *390*, 3147-3156, doi:10.1016/j.physa.2011.04.011.
7. Yuan, W.; Tan, K.H. A model for simulation of crowd behaviour in the evacuation from a smoke-filled compartment. *Physica A: Statistical Mechanics and its Applications* **2011**, *390*, 4210-4218, doi:10.1016/j.physa.2011.07.044.
8. Guo, X.; Chen, J.; You, S.; Wei, J. Modeling of pedestrian evacuation under fire emergency based on an extended heterogeneous lattice gas model. *Physica A: Statistical Mechanics and its Applications* **2013**, *392*, 1994-2006, doi:10.1016/j.physa.2012.12.033.
9. Li, L.; Yu, Z.; Chen, Y. Evacuation dynamic and exit optimization of a supermarket based on particle swarm optimization. *Physica A: Statistical Mechanics and its Applications* **2014**, *416*, 157-172, doi:10.1016/j.physa.2014.08.054.
10. Şahin, C.; Rokne, J.; Alhajj, R. Human behavior modeling for simulating evacuation of buildings during emergencies. *Physica A: Statistical Mechanics and its Applications* **2019**, *528*, doi:10.1016/j.physa.2019.121432.
11. Delcea, C.; Cotfas, L.-A. Increasing awareness in classroom evacuation situations using agent-based modeling. *Physica A: Statistical Mechanics and its Applications* **2019**, *523*, 1400-1418, doi:10.1016/j.physa.2019.04.137.
12. Zheng, L.; Peng, X.; Wang, L.; Sun, D. Simulation of pedestrian evacuation considering emergency spread and pedestrian panic. *Physica A: Statistical Mechanics and its Applications* **2019**, *522*, 167-181, doi:10.1016/j.physa.2019.01.128.
13. Li, W.; Li, Y.; Yu, P.; Gong, J.; Shen, S.; Huang, L.; Liang, J. Modeling, simulation and analysis of the evacuation process on stairs in a multi-floor classroom building of a primary school. *Physica A: Statistical Mechanics and its Applications* **2017**, *469*, 157-172, doi:10.1016/j.physa.2016.11.047.
14. Lei, W.; Li, A.; Gao, R.; Hao, X.; Deng, B. Simulation of pedestrian crowds' evacuation in a huge transit terminal subway station. *Physica A: Statistical Mechanics and its Applications* **2012**, *391*, 5355-5365, doi:10.1016/j.physa.2012.06.033.
15. Sýkora, J.; Jarušková, D.; Šejnoha, M.; Šejnoha, J. Fire risk analysis focused on damage of the tunnel lining. *Fire Safety Journal* **2018**, *95*, 51-65, doi:10.1016/j.firesaf.2017.10.008.
16. Aralt, T.T.; Nilsen, A.R. Automatic fire detection in road traffic tunnels. *Tunnelling and Underground Space Technology* **2009**, *24*, 75-83, doi:10.1016/j.tust.2008.04.001.
17. Hua, G.Y.; Wang, W.; Zhao, Y.H.; Li, L. A study of an optimal smoke control strategy for an Urban Traffic Link Tunnel fire. *Tunnelling and Underground Space Technology* **2011**, *26*, 336-344, doi:10.1016/j.tust.2010.11.004.
18. Caliendo, C.; Ciambelli, P.; De Guglielmo, M.L.; Meo, M.G.; Russo, P. Simulation of fire scenarios due to different vehicle types with and without traffic in a bi-directional road tunnel. *Tunnelling and Underground Space Technology* **2013**, *37*, 22-36, doi:10.1016/j.tust.2013.03.004.
19. Yu, L.-X.; Liu, F.; Liu, Y.-Q.; Weng, M.-C.; Liao, S.-J. Experimental study on thermal and smoke control using transverse ventilation in a sloping urban traffic link tunnel fire. *Tunnelling and Underground Space Technology* **2018**, *71*, 81-93, doi:10.1016/j.tust.2017.08.012.
20. Li, J.; Liu, S.; Li, Y.; Chen, C.; Liu, X.; Yin, C. Experimental Study of Smoke Spread in Titled Urban Traffic Tunnels Fires. *Procedia Engineering* **2012**, *45*, 690-694, doi:10.1016/j.proeng.2012.08.224.
21. Wagenmakers, E.-J.; Molenaar, P.C.M.; Grasman, R.P.P.P.; Hartelman, P.A.I.; van der Maas, H.L.J. Transformation invariant stochastic catastrophe theory. *Physica D: Nonlinear Phenomena* **2005**, *211*, 263-276, doi:https://doi.org/10.1016/j.physd.2005.08.014.
22. Hu, B.; Xia, N. Cusp catastrophe model for sudden changes in a person's behavior. *Information Sciences* **2015**, *294*, 489-512, doi:https://doi.org/10.1016/j.ins.2014.09.055.
23. Xiaoping, Z.; Jiahui, S.; Yuan, C. Analysis of crowd jam in public buildings based on cusp-catastrophe theory. *Building and Environment* **2010**, *45*, 1755-1761, doi:https://doi.org/10.1016/j.buildenv.2010.01.027.
24. Dendrinos, D.S. Operating speeds and volume to capacity ratios: The observed relationship and the fold catastrophe. *Transportation Research* **1978**, *12*, 191-194, doi:https://doi.org/10.1016/0041-1647(78)90124-7.
25. Navin, F.P.D. Traffic congestion catastrophes. *Transportation Planning and Technology* **1986**, *11*, 19-25, doi:10.1080/03081068608717327.

26. Forbes, G.J.; Hall, F.L. The applicability of catastrophe theory in modelling freeway traffic operations. *Transportation Research Part A: General* **1990**, *24*, 335-344, doi:https://doi.org/10.1016/0191-2607(90)90046-9.

27. Yanguo, H. Research on Traffic Congestion Mechanism and Traffic Control Method for Urban Road. South China University of Technology, Guangzhou, China, 2015.

28. Tian, Y.; Zheng, B.; Shen, H.; Zhang, S.; Wang, Y. A novel index based on the cusp catastrophe theory for predicting harmful algae blooms. *Ecological Indicators* **2019**, *102*, 746-751, doi:10.1016/j.ecolind.2019.03.044.

29. Zhu, J. Spatio-temporal Distribution and Environmental Factors of Urban Traffic Accidents: A Case Study of Shanghai, China. Master Dissertation, East China Normal University, Shanghai, China, 2017.

30. Yuan, S. Study on the Mechanism of Urban Traffic Congestion Propagation and Its Control Strategies. PhD Dissertation, Chang'an Univeristy, Xi'an, China, 2012.

31. Shen, Y. Simulation and Analysis of Traffic Flow under Accident. Master Dissertation, Beijing Jiaotong University, Beijing, China, 2009.

32. Ciyun, L.; Kang, W.; Dayong, W.; Bowen, G. Research on Residents' Travel Behavior under Sudden Fire Disaster Based on Prospect Theory. *Sustainability* **2020**, *12*, doi: https://doi.org/10.3390/su12020487.

33. Chen, C.; Zhao, X.H.; Liu, H.; Ren, G.C.; Zhang, Y.L.; Liu, X.M. Assessing the Influence of Adverse Weather on Traffic Flow Characteristics Using a Driving Simulator and VISSIM. *Sustainability* **2019**, *11*, doi:ARTN 830 10.3390/su11030830.

34. Wei, L.Y.; Xu, J.L.; Lei, T.; Li, M.H.; Liu, X.L.; Li, H.R. Simulation and Experimental Analyses of Microscopic Traffic Characteristics under a Contraflow Strategy. *Appl Sci-Basel* **2019**, *9*, doi:ARTN 2651 10.3390/app9132651.

35. Hong, C.; Yue-sheng, Y.; Bin, Z. A Capacity Assessment Method on Urban Expressway after Traffic Incident. *Procedia - Social and Behavioral Sciences 96*, 1921-1928.